COE CST Fif h Ann al Technical Mee ing

Space En ironmen MMOD Modeling and Predic ion

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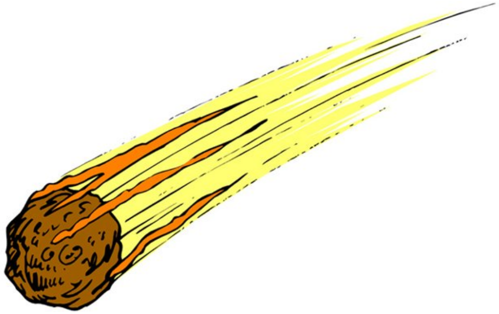


Team Member

- Sigrid Clo e, S anford Uni er i (PI)
- Alan Li, S anford Uni er i (grad a e den)

- Loren o Limon a, S anford Uni er i (grad a e den ppor ed b NSF)

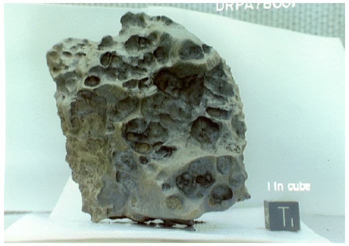
P rpo e of Ta k

- **Spacecraf are ro inel impac ed b microme eoroid and orbi al debri (MMOD)**
  - Mechanical damage: “well-known”, larger (> 120 microns), rare
  - Electrical damage: “unknown”, smaller/fast, more numerous

- **Gro ing need o charac eri e MMOD do n o maller i e and pro ide predic i e hrea a e men**

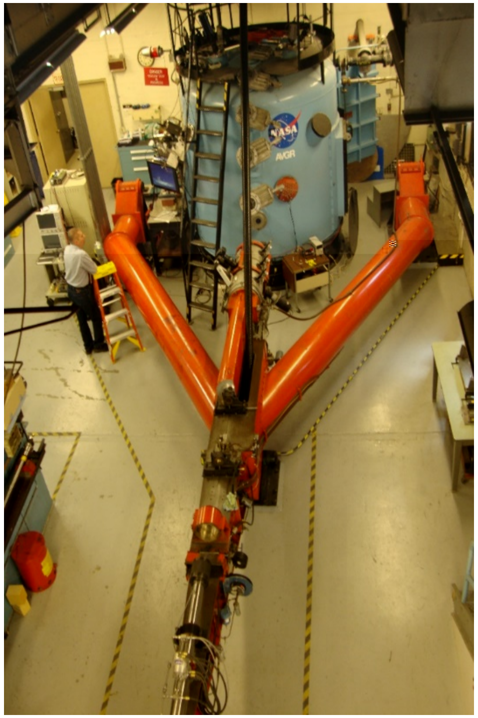
MMOD Cla ifica ion

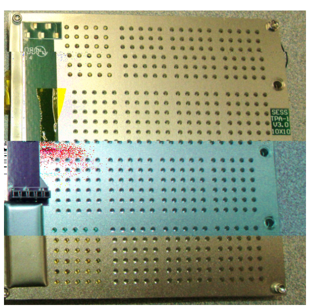
- **Me eoroid**
  - **Speed**
    - 11 to 72.8 km/s (interplanetary)
    - 30-60 km/s (average)
  - **Den i ie**
    - ≤1 g/cm^3 (icy) or > 1 g/cm^3 (rocky/stony)
  - **Si e**
    - < 0.3 m (meteoroid)
    - < 62 μm (dust)

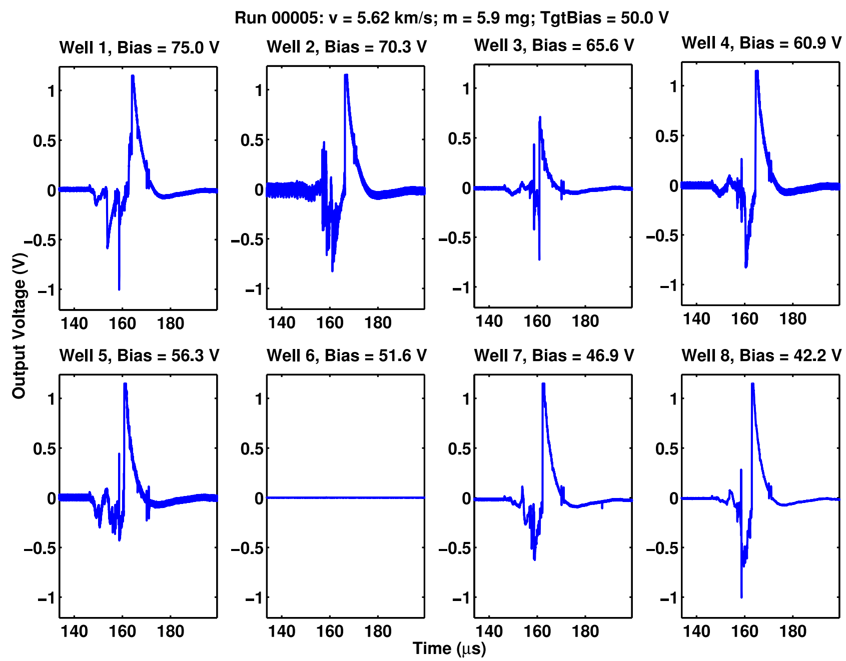


- **Space Debri**
  - **Speed in LEO**
    - < 12 km/s
    - 7-10 km/s (average)
  - **Den i ie**
    - > 2 g/cm^3
  - **Si e**
    - < 10 cm (small)

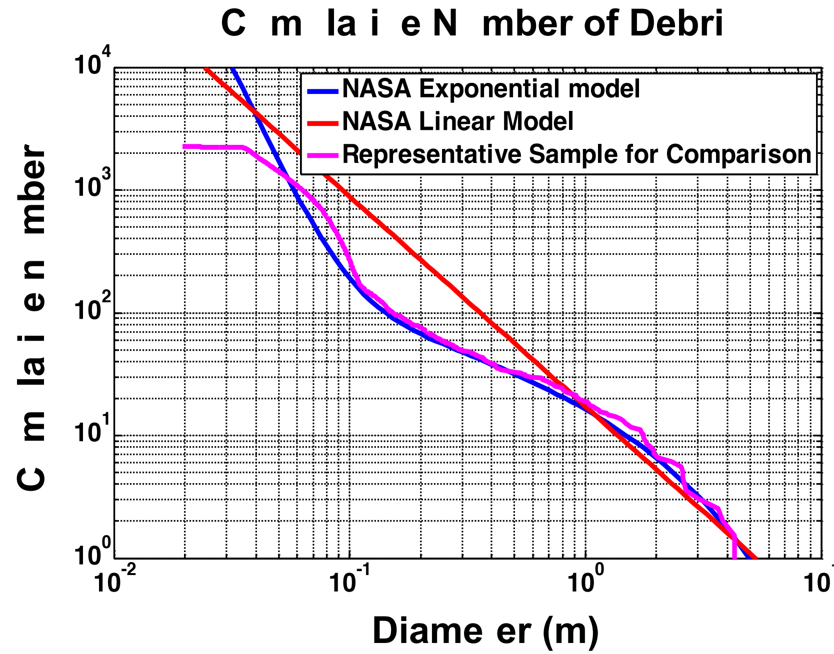
MMOD Pre io Re earch







MMOD Pre io Re earch



- **EISCAT S albard radar**
  - 78.1°N, 16.0°E
  - 500 MHz, 32 m dish, 0.8 MW peak power
  - Data collected March 2007 – March 2009 (following Chinese ASAT test in January 2007)

MMOD and Ne ral Den i ie

- **Space j nk WT1190F**
  - Approximately 1-2 m long
  - Most likely discarded rocket body “lost” by SSN
  - Reentry on November 13 (point of impact over Indian Ocean?)
- **Can e impro e he 15-50% error?**

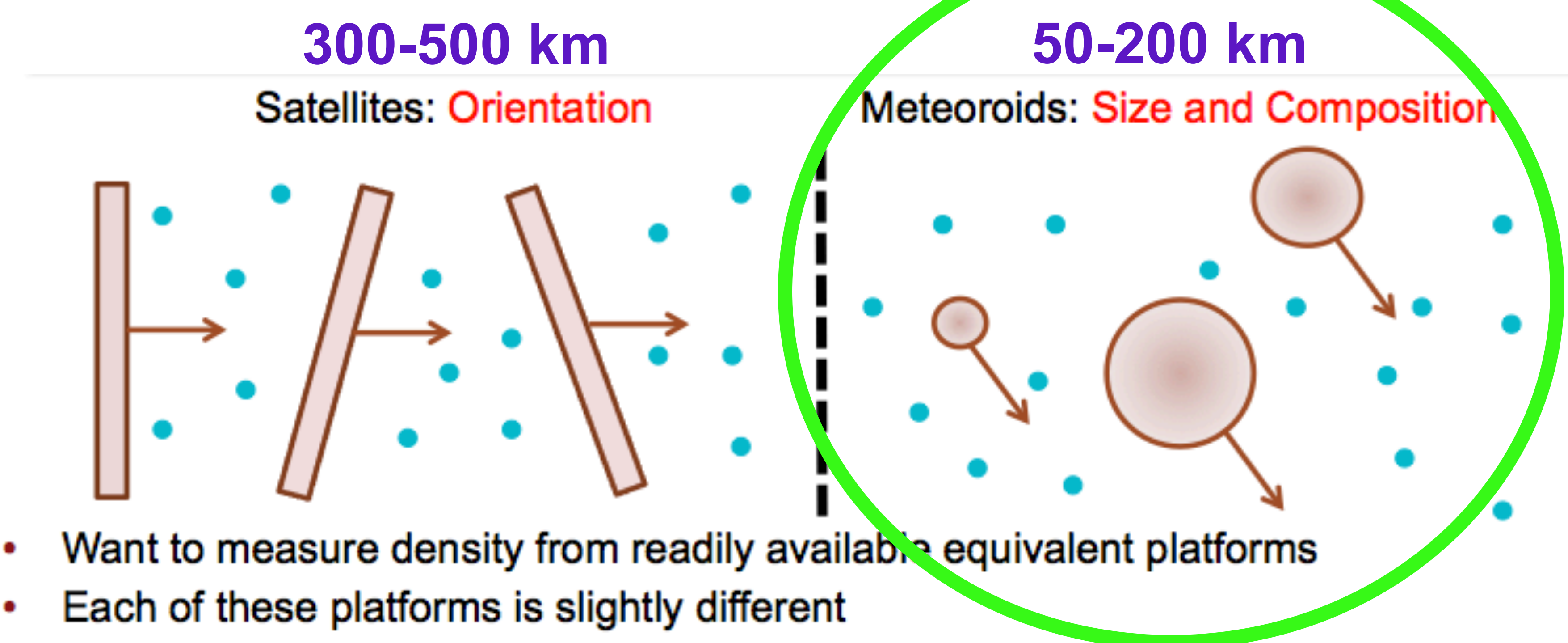
Goal: Ne ral Den i E ima ion

Source: http://eijournal.com/print/articles/entrepreneurial-blueprint-bolsters-satellite-development

Source: http://www.huffingtonpost.com/2014/04/21/lyrid-meteor-shower-2014_n_5186204.html

- Le erage he increa ing n mber of con ella ion of a elli e in orbi
- Le erage he ab ndance of me eoroid abla ing in he a mo phere
- Good emporall and pa iall ar ing profile of ne ral den i
- Differen o rce of den i e ima ion

Me hodolog



- Want to measure density from readily available equivalent platforms
- Each of these platforms is slightly different
- Measurements made at a certain time contains certain biases (across all platforms)

A mp ion and Eq a ion

- **A mp ion**
  - C_D constant (spherical shape)
  - Variation arises from mass/size/bulk density
  - Multiple layers of atmosphere traversed
  - Ablation and mass loss
- **Go erning eq a ion**

Drag: $$\frac{dv}{dt} = -\frac{3}{8}\frac{\rho_a}{\rho_m}\frac{C_D}{r}|v|^2$$

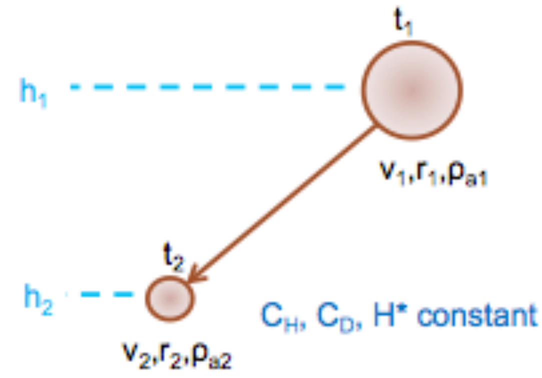
Ablation: $$\frac{dr}{dt} = -\frac{1}{8}\frac{C_H}{H^*}\frac{\rho_a}{\rho_m}|v|^3$$

Velocity:	v	Enthalpy of Destruction:	H^*
Radius:	r	Coefficient of Heat Exchange:	C_H
Atmospheric Density:	ρ_a		
Meteoroid Density:	ρ_m		

Den i Ra io

- **Combine drag and abla ion eq a ion and compare ra io of radii a differen poin in ime**

$$\frac{r_1}{r_2} = \exp\left(\frac{1}{6}\frac{C_H}{C_D}\frac{1}{H^*}(v_1^2 - v_2^2)\right)$$



- **For i h me eoroid a j h al i de**

$$\underbrace{\ln\left(\frac{dv_{i,j+1}}{dt}\frac{1}{v_{i,j+1}^2}\right) - \ln\left(\frac{dv_{i,j}}{dt}\frac{1}{v_{i,j}^2}\right)}_{LHS_i} = \underbrace{\frac{1}{6}D_i\left(v_{i,j}^2 - v_{i,j+1}^2\right) + \ln(\rho_{rj})}_{RHS_i}$$

- **Gi en da a on eloci and decelera ion, e ima e D_i and ρ_{rj} for each me eoroid and al i de**

Minimize: $$\min\left(\sum (LHS_{i,j} - RHS_{i,j})^2\right)$$

Subject to: $$D_i > 0$$

$$D_i = \frac{C_{Hi}}{C_{Di}}\frac{1}{H_i^*}$$

$$\rho_{rj} = \frac{\rho_{a,j+1}}{\rho_{a,j}}$$

Ra io Di rib ion

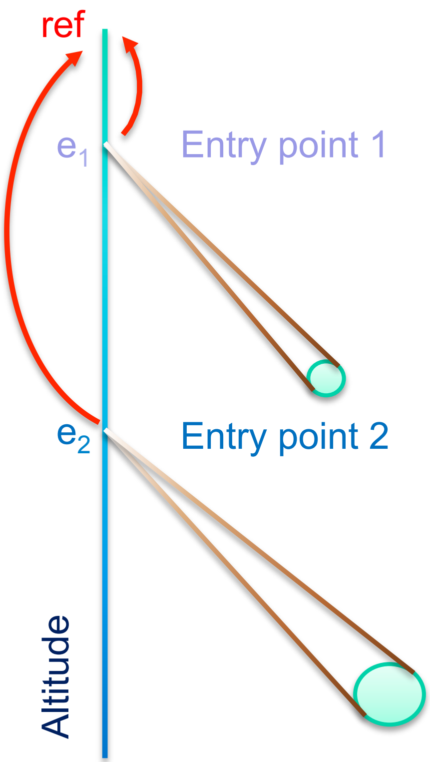
- Tran la e poin of en r mea remen o a reference poin in al i de

$$\frac{dv}{dt}\frac{1}{v^2} \sim \frac{\rho_{a,e}}{r_e \rho_m} \longrightarrow \frac{dv}{dt}\frac{1}{v^2}\frac{\rho_{a,ref}}{\rho_{a,e}} \sim \frac{\rho_{a,ref}}{r_e \rho_m} = K$$

- Calc la e K for each me eoroid and define minim m ra io ing order a i ic

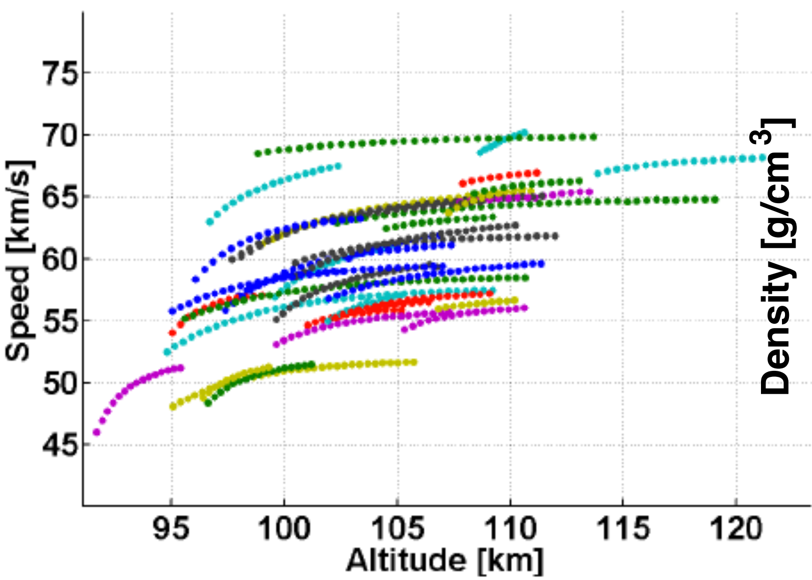
$$\frac{K_j}{K_{mk}} \approx \frac{\left(\frac{1}{r_e \rho_m}\right)_j}{\left(\frac{1}{r_e \rho_m}\right)_{mk}}$$

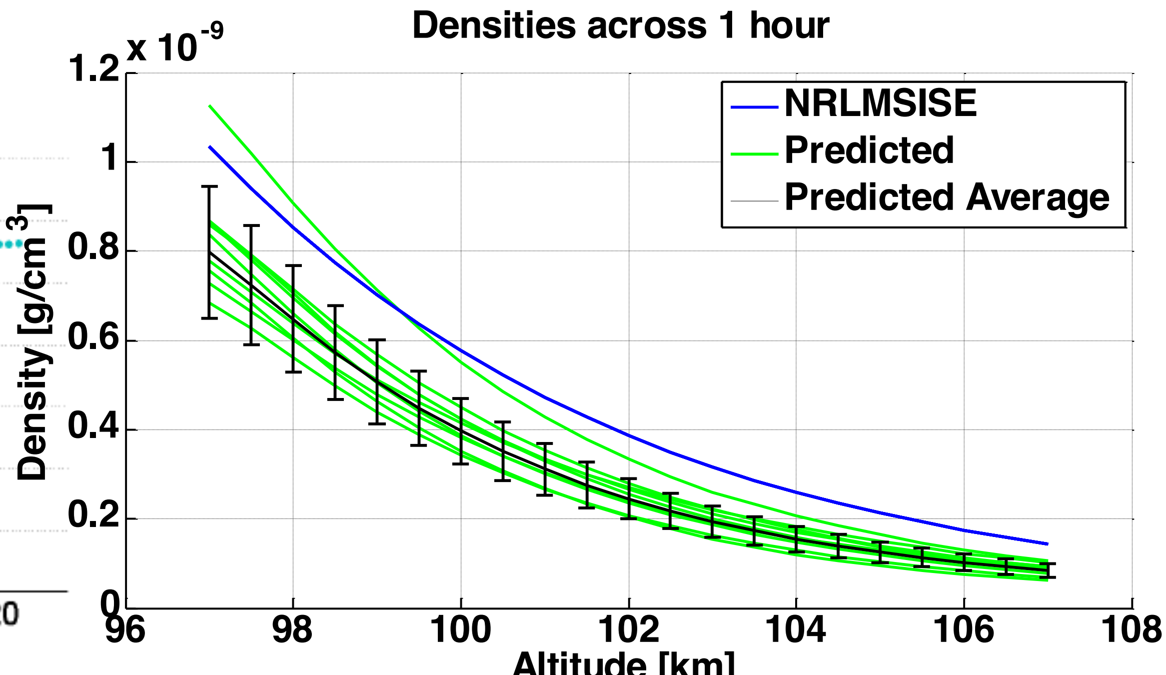
- Calc la e di rib ion



Re l

- **ALTAIR radar**
  - 9°N, 167°E
  - 160 and 422 MHz, 46 m dish, 6 MW peak power
  - Data collected November 8th 2007 (6 AM local time)





Concl ion and F re Work

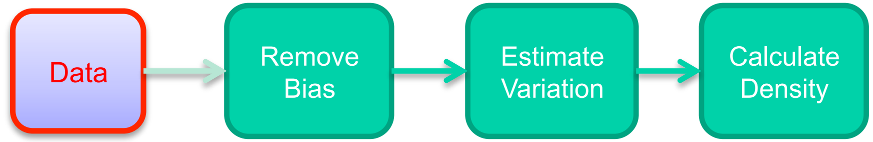
- **Ne me hod for e ima ing ne ral den i from m l iple mea remen acro eq i alen pla form**
  - Errors < 10% using CubeSats (not shown), 12% for meteoroids
  - Additional data to modeling community
- **Ne ep**
  - Satellites: precision orbit determination
  - Meteoroids: ablation physics
  - Space debris: highly variable C_D

Li, A., and Mason, J. *Optimal Utility of Satellite Constellation Separation with Differential Drag*. 2014 AIAA/AAS Astrodynamics Specialist Conference. AIAA 2014-4112.

Li, A., and Close, S. *Mean Thermospheric Density Estimation derived from Satellite Constellations.* Advances in Space Research 56 (2015),pp. 1645-1657. DOI: 10.1016/j.asr. 2015.07.022

Thank o !

Balli ic Fac or



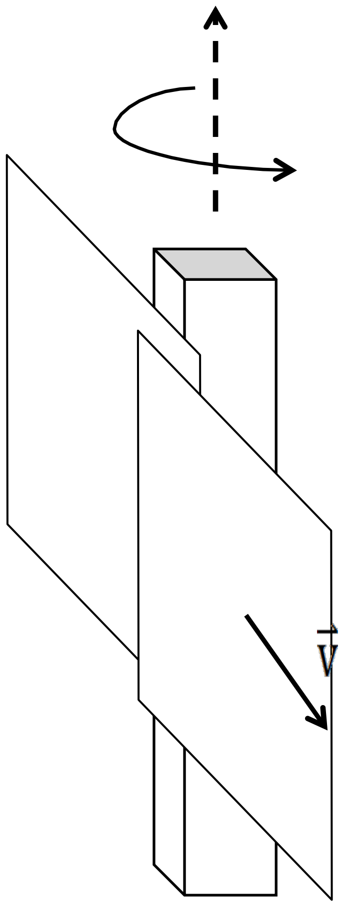
- θ: Rotation about the satellite spin axis (IID)
- Ballistic factor:

$$B(\theta) = \frac{C_D(\theta)A(\theta)}{m}$$

  - B is IID with some unknown distribution
  - B_{min} defined when θ=0 (absolute minimum)

- Ignore rotations about other axes

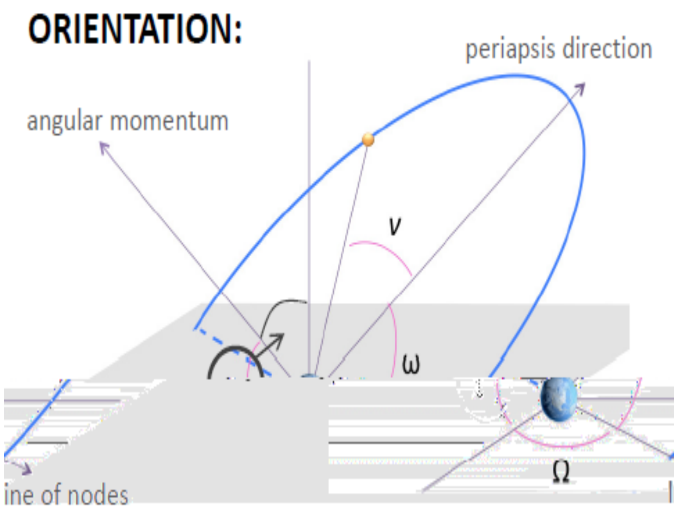
$$B_{min} = \frac{C_{D,min}A_{min}}{m}$$



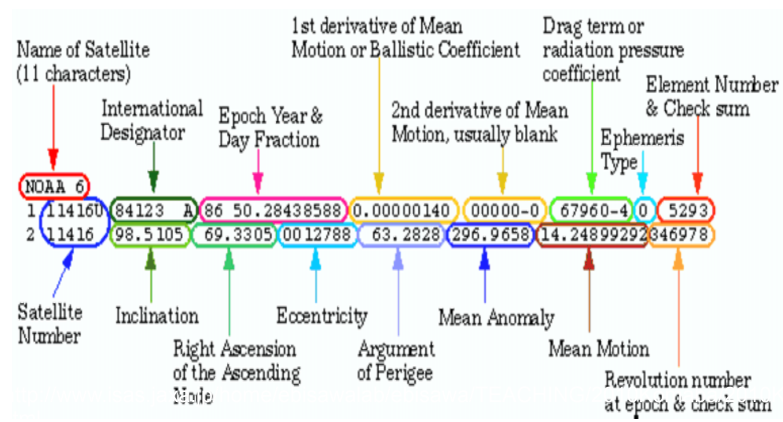
IID = Independent and Identically Distributed

Orbi al Elemen

Data → Remove Bias → Estimate Variation → Calculate Density



- Kept up to date by NORAD (Space-track)
- Uses Simplified General Perturbations (SGP) model
- Within few km of error over 1 day



Da a

Data → Remove Bias → Estimate Variation → Calculate Density

From TLE

- $$\left.\frac{da}{dt}\right|_D = \frac{2a^2 v}{\mu}\frac{d\vec{v}_D}{dt}\cdot\hat{e}_v = \frac{2a^2 B v^3 F}{\mu}$$

- $$\Delta a_{SGP4}(t_i,t_k) = \int_{t_i}^{t_k} \left.\frac{da}{dt}\right|_D + \left.\frac{da}{dt}\right|_G + \left.\frac{da}{dt}\right|_U \, dt$$

  mall

- $$\Delta\bar{n}_M(t_i,t_k) = \frac{3}{2}\mu^{-\frac{2}{3}}\oint_{t_i}^{t_k}\bar{n}^{\frac{1}{3}}\rho B v^3 F\,dt$$

→ $$\bar{\rho}\bar{B} \cong \frac{2\mu^{\frac{2}{3}}\Delta\bar{n}_M(t_i,t_k)}{3\oint_{t_i}^{t_k}\bar{n}^{\frac{1}{3}} v^3 F\,dt} = K$$

From ranging da a

- $$\mathbf{X} = \begin{bmatrix}\mathbf{r}\\ \mathbf{v}\\ K\end{bmatrix},\ \mathbf{X}_0 = \begin{bmatrix}\mathbf{r}_0\\ \mathbf{v}_0\\ K_0\end{bmatrix}$$

- $$\dot{\mathbf{X}} = F_D(\mathbf{X}) + F_g(\mathbf{X}) + F_U(\mathbf{X})$$

- $$\tilde{\mathbf{b}} = R\left(\dot{\mathbf{X}}(t_i, X_0)\right) - R_{meas}(t_i)$$

Loop until RMS < Threshold:

$$\boldsymbol{\delta}\mathbf{x} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\tilde{\mathbf{b}}$$

→ $$\mathbf{X}_0 = \mathbf{X}_0 + \boldsymbol{\delta}\mathbf{x}$$

$$RMS = \sqrt{\frac{\tilde{\mathbf{b}}^T\mathbf{W}\tilde{\mathbf{b}}}{n_{obs}}}$$

Ho o Remo e Bia

- Density estimated as:

$$\bar{\rho} \cong \frac{2\mu^{\frac{2}{3}}\Delta\bar{n}_M(t_i, t_k)}{3\bar{B}\oint_{t_i}^{t_k} \bar{n}^{\frac{1}{3}} v^3 F\, dt} = \frac{K}{\bar{B}} \quad \text{or} \quad \bar{\rho}\bar{B} = K$$

Ballistic factor:	$B = \frac{C_D A}{m}$
Mean motion:	n
Wind Factor:	F
Density:	ρ

- K can be calculated by:
  - SGP4 in the case of TLEs
  - Ranging or GPS measurements; propagator needs to account for higher order gravity terms, SRP, etc…
- Internal bias within K because K is composed from varying densities

The Dilemma: If we have N satellites, we have K_N measurements but need to estimate n+1 values (ρ and B_N), where B_N is randomly distributed

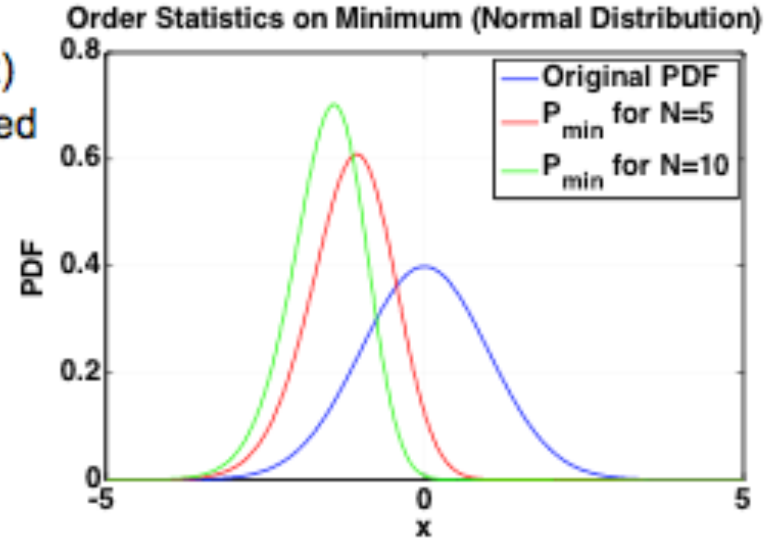
Order S a i ic

- What is order statistics?
  - Let $X_1, X_2, \ldots, X_N$ be IID with some CDF $C(x)$
  - Then the r^{th} order statistic can be expressed as:

$$C_{(r)}(x) = \sum_{i=r}^{N} \binom{N}{i} C^i(x)\,[1 - C(x)]^{N-i}$$

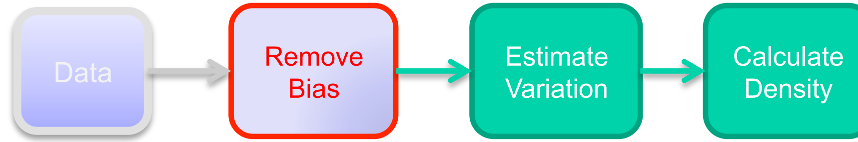
  - And the minimum as:

$$C_{(1)}(x) = 1 - [1 - C(x)]^N$$



- Why do we use it?
  - We know something about the minimum of C_D from physics
  - We have many satellites
  - Estimation of C_D is difficult due to coupling with ρ

Remo e Bia



- Define the minimum of our observations:

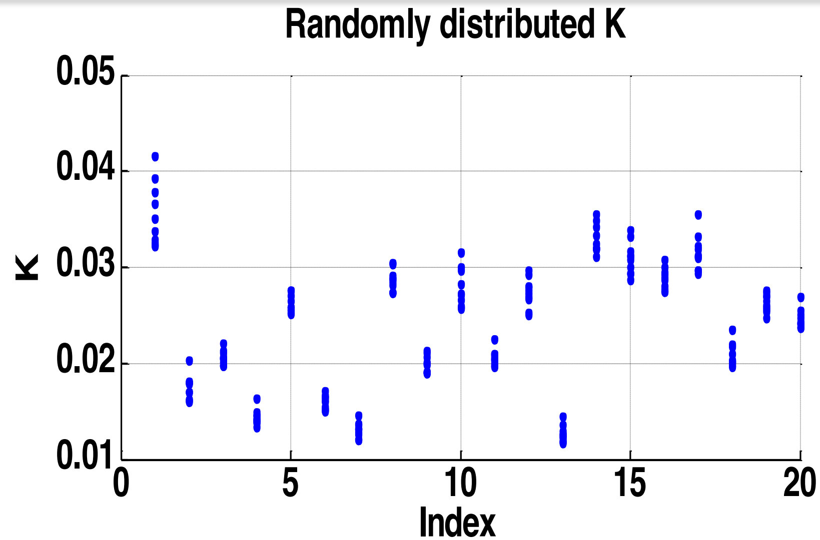
$$K_{mk}(t_k) = \min_j K_j(t_k)$$

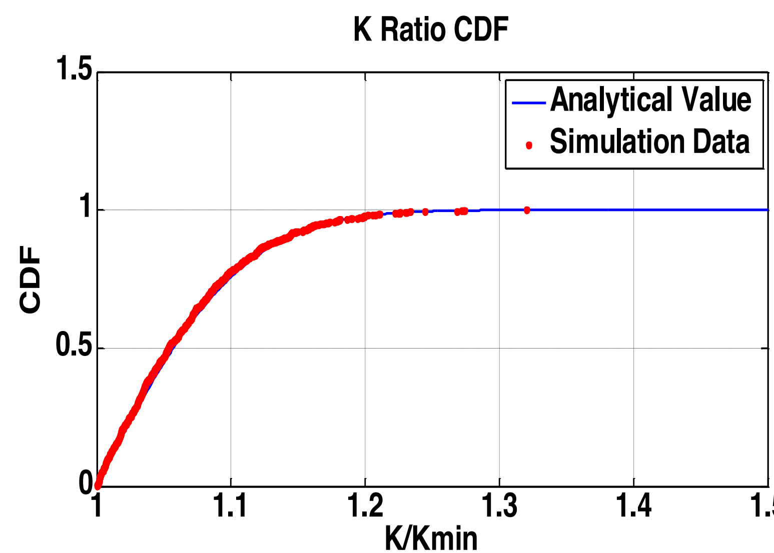
$$\frac{K_j(t_k)}{K_{mk}(t_k)} \approx \frac{B_j(t_k)}{B_{mk}(t_k)}$$

- Amalgamate measurements across all time periods to construct CDF ratio:

- Results in ratio distribution

$$CDF\left(\frac{B}{B_{mk}} \middle| \frac{B}{B_{mk}} > 1\right)$$





Ra io Di rib ion

- Probability of ratios defined as:

$$P(z) = \int_{-\infty}^{+\infty} |y| P_{B,y}(zy, y)\, dy$$

↓ Math

$$C(z) = \frac{N}{N-1} \int_{B_{min}}^{B_{max}} F_B(z \cdot y) \cdot \frac{d\left(F_B^{N-1}(y)\right)}{dy} dy + 1$$

↓ Discretize

$$C(z_j) - 1 = \frac{N}{N-1} \sum_{i=1}^{m} F_B(z_j y_i) \left(F_B^{N-1}(y_{i+1}) - F_B^{N-1}(y_i)\right)$$

- Limits:

$$\lim_{B \to B_{min}} F_B(B) \to 1 \qquad \lim_{B \to B_{max}} F_B(B) \to 0$$

$$y = B_{mk}$$

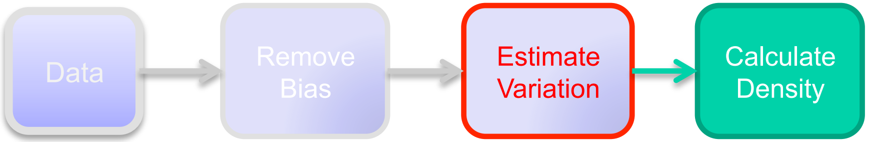
$$z = \frac{B}{B_{mk}}$$

$N = \#$ of platforms

$$C_B = CDF(B)$$

$$F_B(B) = 1 - C_B(B)$$

Di cre i a ion



- Matrix form:

$$\frac{N-1}{N}\left(\begin{bmatrix} C(z_m) \\ C(z_{m-1}) \\ \vdots \\ C(z_2) \end{bmatrix} - 1\right) = \begin{bmatrix} F_{B,m} & 0 & \cdots & 0 \\ F_{B,m-1} & F_{B,m} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ F_{B,2} & F_{B,3} & \cdots & F_{B,m} \end{bmatrix} \begin{bmatrix} F_{B,2}^{N-1} - F_{B,1}^{N-1} \\ F_{B,3}^{N-1} - F_{B,2}^{N-1} \\ \vdots \\ F_{B,m}^{N-1} - F_{B,m-1}^{N-1} \end{bmatrix}$$

- Minimize:

$$\min\left(\sum (LHS - RHS)^2 + \kappa \cdot \max\left(\frac{dC_B}{dz}\right)\right)$$

Subject to:

$$0 = F_{B,1} > F_{B,2} > \cdots > F_{B,m} = 1$$

Data → Remove Bias → Estimate Variation → ? → Calculate Density

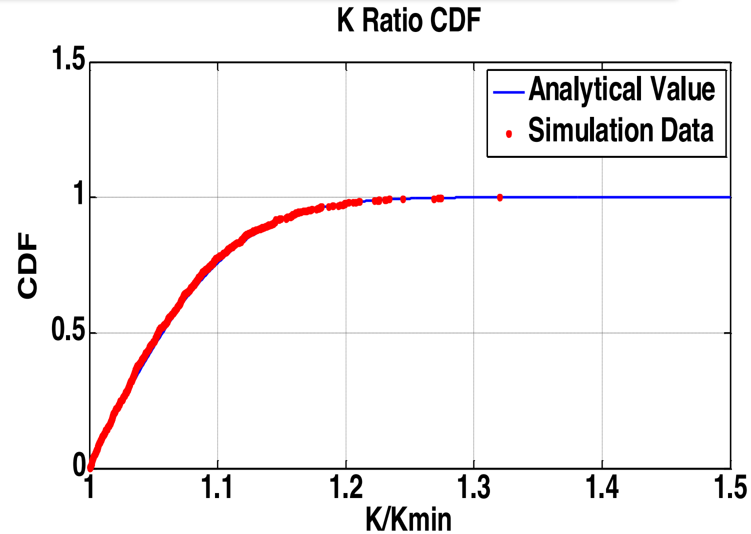
Effec of Error

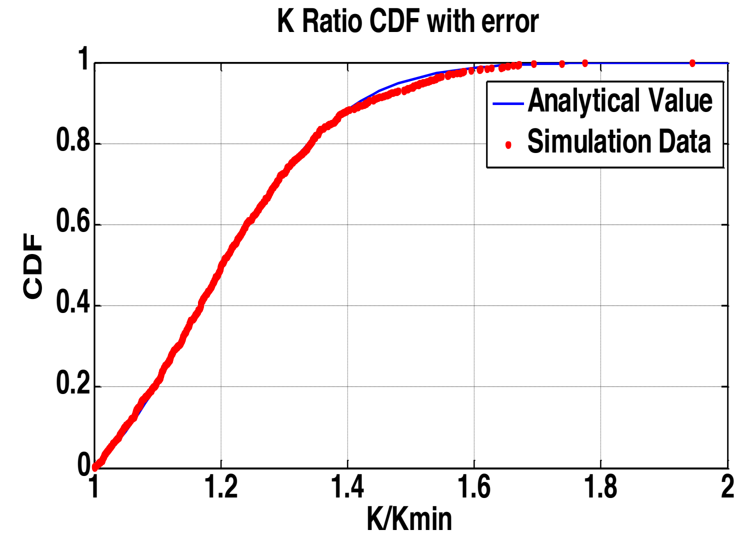
- Any estimation scheme is prone to error
- These errors affect the minimum ratio and hence its CDF

$$CDF\left(\frac{(B+dB)}{(B+dB)_{mk}}\middle|\frac{(B+dB)}{(B+dB)_{mk}}>1\right)$$

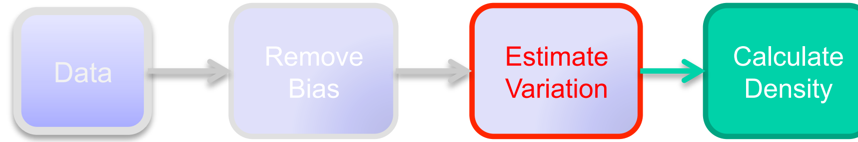
- Estimate (B + dB) using similar method
- Require statistics on the error of dB
  - Estimate from previous filtering methods (non-linear least squares to estimate K)

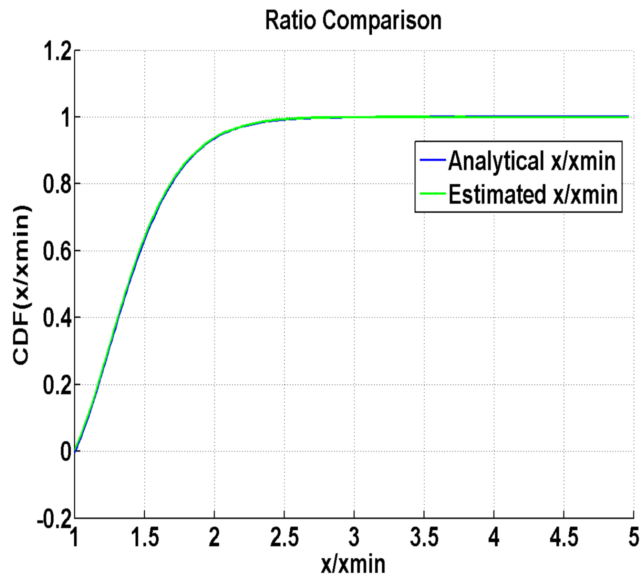
- $dB \sim \mathcal{N}\left(0, \frac{\sigma_K}{\bar{\rho}}\right)$ $C_{B+dB}(x) = [C_B * P_{dB}](x)$

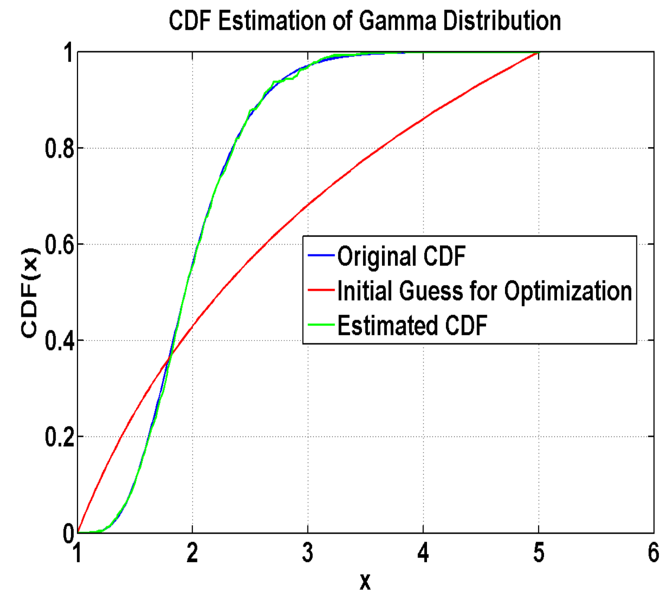




Sol ing for F





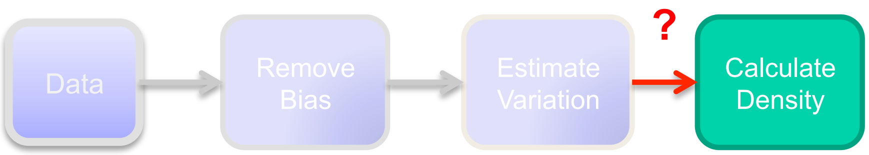


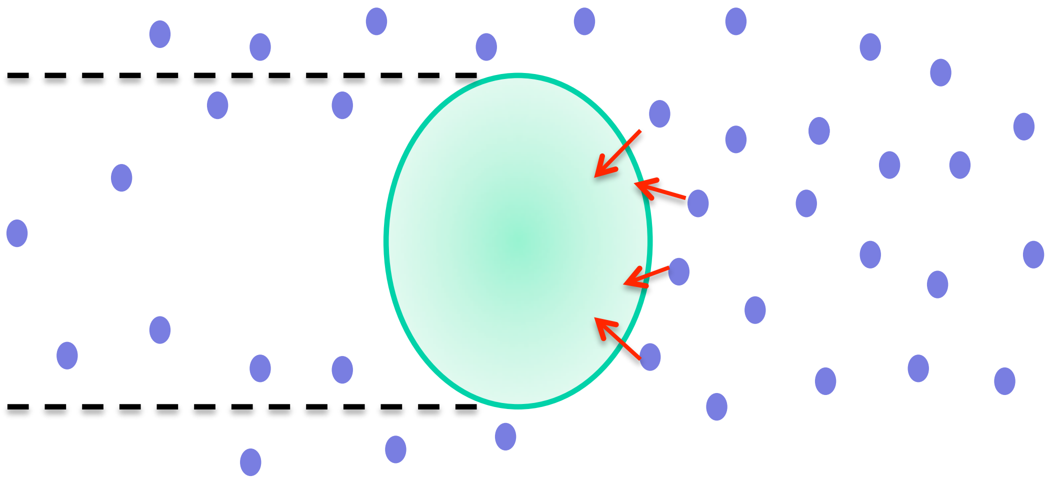
- Test Case: Gamma Distribution
-

Problem: If the distribution shifted left or right and is scaled appropriately, get same observed result (unknown integration constant)!

How to determine the minimum, B_{min}?

Free Molec lar Flo





Mean free path:	λ
Characteristic length:	L
Number density:	N
Collision Area:	σ_A

- Knudsen number: $Kn = \frac{\lambda}{L}$ $\qquad \lambda \sim \frac{1}{N\sigma_A}$
- High Knudsen numbers: Free molecular flow (Kn >> 10)
  - Basically collisionless, not a continuum (no bulk properties)
  - Random thermal motions dominant: Maxwellian distribution

$$B = \frac{C_D A}{m}$$

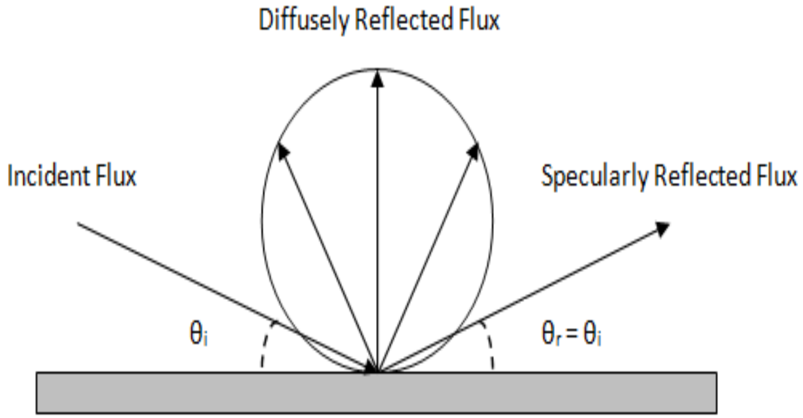
min ↓

$$B_{min} = \frac{C_{D,min} A_{min}}{m}$$

Free Molec lar Flo

Data → Remove Bias → Estimate Variation → ? → Calculate Density

- Accommodation coefficient: $$\alpha = \frac{E_i - E_r}{E_i - E_w}$$



- Reflected particles classified as:
  - Specular – perfect reflection about surface normal
  - Diffuse – random
- Surfaces for satellites in LEO tend to become coated with adsorbed atomic oxygen; most reflections are diffuse (80-99%)

C_D on Fla Pla e

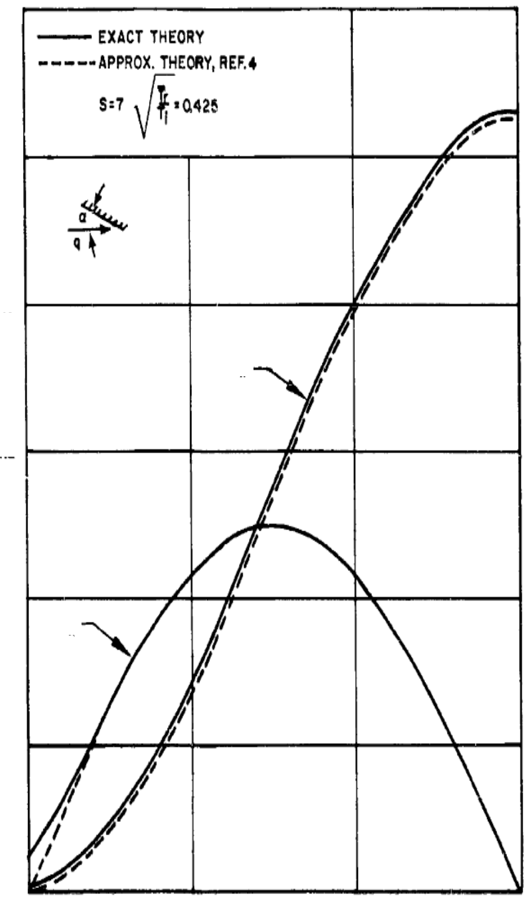
Data → Remove Bias → Estimate Variation → ? → Calculate Density

$$C_D = \frac{A}{A_{ref}}\left[(2-\sigma_N)\cos\theta\left(\cos\theta\left(1+erf(\gamma)\right)+\frac{1}{S\sqrt{\pi}}e^{-\gamma^2}\right)\right.$$

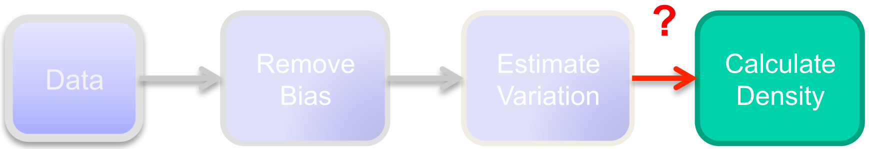
$$\left.+\frac{2-\sigma_N}{2S^2}\left(1+erf(\gamma)\right)+\frac{\sigma_N}{2}\sqrt{\frac{T_r}{T_i}}\left(\frac{\sqrt{\pi}}{S}\left(1+erf(S)\right)+\frac{1}{S^2}e^{-S^2}\right)\right]$$

$$\gamma = S\cos\theta \qquad T_r = T_i(1-\alpha)+\alpha T_w$$

$$S = \frac{U}{V_a} = \frac{U}{\sqrt{2R_{sp}T_a}}$$



Uncer ain in α



- Combine this with earlier results:

$$P(B,\alpha) = P(B|B_{min})P(B_{min}|\alpha)P(\alpha)$$

- B_{min} is a function of α:

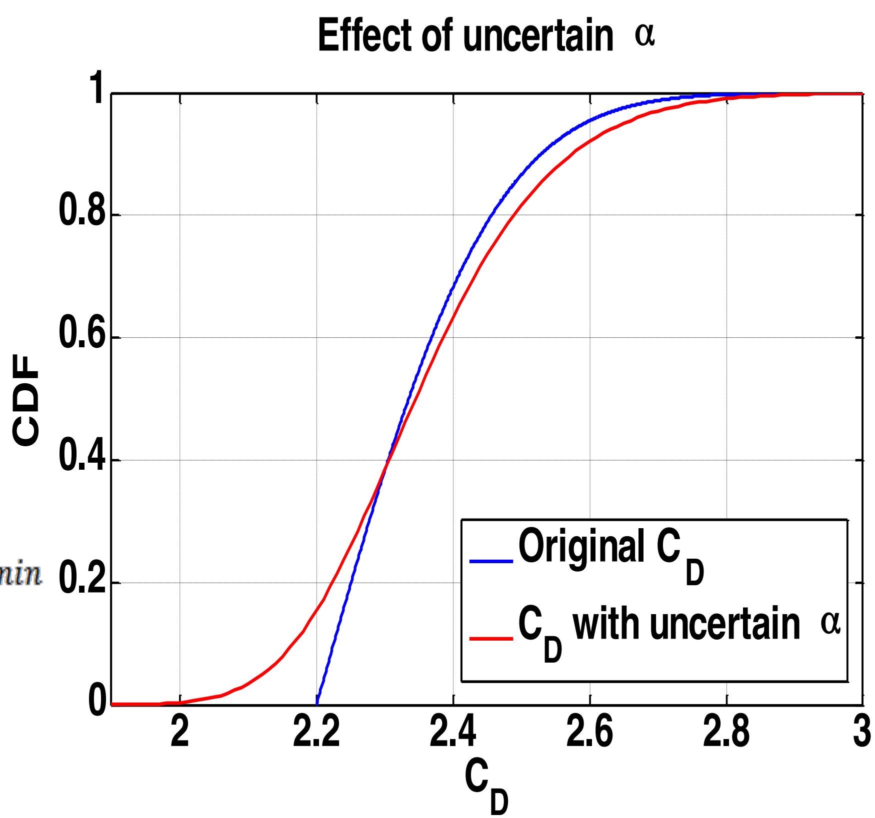
- Effect on minimum:

$$C(B) = \int_{B_{min,\alpha=1}}^{B_{max}} C(B|B_{min})P(B_{min})\, dB_{min}$$

- Limits:

$$C(B_{mk}) = \int_{B_{min,\alpha=1}}^{B_{max}} C(B_{mk}|B_{min})P(B_{min})\, dB_{min}$$

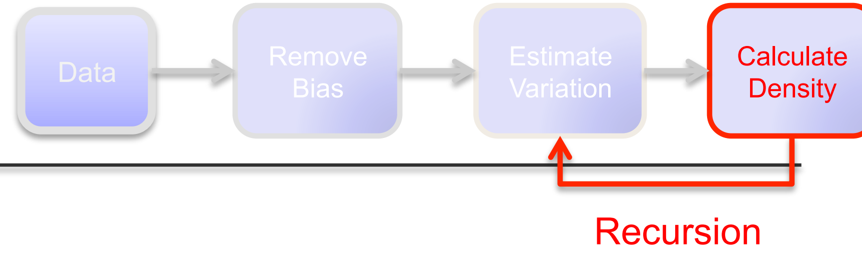
$$\lim_{N\to\infty} P(B_{mk}) \to P_{min}$$



C_D ~ Half Normal

α ~ $\mathcal{N}$(2.2, 0.1)

Calc la ing Den i



- Calculate density ρ:

Mean Estimate: $$\rho_k = \frac{\bar{K}_k}{\bar{B}}$$

Minimum Estimate: $$\rho_k = \frac{K_{mk}}{\bar{B}_{min}}$$

- Minimum K
- Maximum B

- Maximum K_{mk}
- Minimum B_{min}

- K contains estimation error
- B contains error associated with platform
- Nullify large estimation errors in K from affecting estimation of B
-

Ha e o choo e hich one o minimi e!

Separate estimation error from the random elements of the platform in question