Xu Gao

Curriculum Vitae

Education

2016–2023 **Doctor of Philosophy**
Mathematics Department, University of California at Santa Cruz
Santa Cruz, California, USA

2012–2015 **Master of Mathematics**
Chern Institute of Mathematics, Nankai University
Tianjin, China

2008–2012 **Bachelor of Science**
School of Mathematical Sciences, Nankai University
Tianjin, China

Ph.D Thesis

Title Simplicial distance in Bruhat-Tits buildings of split classical type

Advisors Prof. Suh, Junecue and Prof. Dong, Chongying

Experience

2023–present **Postdoctor**
School of Mathematical Sciences, Tongji University
Shanghai, China

Research Interests

My current research focuses on conformal blocks from vertex operator algebras, specifically the algebro-geometric approach initiated by Beilinson-Drinfeld and Frenkel-BenZvi, with an emphasis on the general geometric framework.

I'm also interested in Bruhat-Tits buildings and p-adic representations, particularly in problems related to simplicial distance and concave functions, as well as tensor triangulated geometry, especially the (co)stratification of tt categories arising from important algebraic objects.

Other interests include algebraic analysis, p-adic geometry, higher category theory, homotopical algebras, transcendental number theory, etc.

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Ongoing Research Projects

- *Tensor category of conformal blocks of vertex operator algebras*
  (with with Angela Gibney, Daniel Krashen, and Jianqi Liu)
  We aim to develop a tensor product theory for modules of vertex operator algebras, using an algebro-geometric approach based on recent work of Damiolini-Gibney-Krashen-Tarasca and Gao-Liu-Zhu. Our ultimate goal is to establish a robust framework for the homological study of VOA-modules.

- *Twisted conformal blocks of vertex operator algebras*
  (with with Jianqi Liu and Yiyi Zhu)
  The goal is to extend the algebro-geometric theory of conformal blocks of vertex operator algebras, as developed by Damiolini-Gibney-Krashen-Tarasca, to orbifold theory. Additionally, we seek to provide geometric versions of various constructions in VOA representation theory, with a better fit into the conformal block framework.

- *Stratification for Hopf algebroids*
  (with with Changhan Zou)
  The (co)stratification of representations of finite group schemes plays a crucial role in the development of tensor triangulated geometry and serves as a test for many important theories. This project aims to extend the work of Barthel-Benson-Iyengar-Krause-Pevtsova to finite Hopf algebroids. The ultimate goal is to study the stratification of Adams Hopf algebroids, a class of important objects in homotopy theory.

- *Simplicial study of Bruhat-Tits building*
  The goal is to develop a combinatorial geometry on Bruhat-Tits buildings based on the notion of simplicial distance, and to apply this geometry to study p-adic representations. In this context, the notion of concave functions plays a crucial role in bridging the combinatorial and group-theoretic aspects of a Bruhat-Tits building.

Publications and Preprints

- *Fusing ring, modular invariance, and the Verlinde formula*
  (with Jianqi Liu)
  In preparing.
  In the representation theory of VOAs, the fusion ring, modular invariance, and the Verlinde formula are fundamental results. In this paper, we derive these results from the algebro-geometric theory of conformal blocks of vertex operator algebras, providing short and intuitive proofs.

- *Twisted restricted conformal blocks of vertex operator algebras III: sheaves of twisted restricted conformal blocks and coherence*
  (with Jianqi Liu and Yiyi Zhu)

In preparing.
In this paper, we develop the theory of twisted restricted conformal blocks on general admissible G-covers, leading to sheaves on the moduli space $\overline{\mathcal{M}}_{g,n}^{G}$. Furthermore, we clarify the relation between these sheaves and prove their coherence under certian conditions.

- *Twisted restricted conformal blocks of vertex operator algebras II: twisted restricted conformal blocks on totally ramified orbicurves*
(with Jianqi Liu and Yiyi Zhu)
Available at arXiv:2403.00545.
In this paper, we introduce a notion of twisted restricted conformal blocks on totally ramified orbicurves and establish an isomorphism between the space of twisted restricted conformal blocks and the space of twisted conformal blocks. The relationships among twisted (restricted) conformal blocks, g-twisted (restricted) correlation functions, and twisted intertwining operators are explored. Furthermore, by introducing a geometric generalization of Zhu's algebra and its modules, we obtain a description of the space of coinvariants by modules over associative algebras and show it is finite-dimensional under some conditions. In particular, a more conceptual proof of the g-twisted fusion rules theorem in vertex operator algebra theory is provided.

- *Twisted restricted conformal blocks of vertex operator algebras I: g-twisted correlation functions and fusion rules*
(with Jianqi Liu and Yiyi Zhu)
Available at arXiv:2312.16278.
In this paper, we introduce a notion of g-twisted restricted conformal block on the three-pointed twisted projective line $\mathfrak{x}\colon \overline{C} \to \mathbb{P}^1$ associated with an untwisted module M^1 and the bottom levels of two g-twisted modules M^2 and M^3 over a vertex operator algebra V. We show that the space of twisted restricted conformal blocks is isomorphic to the space of g-twisted (restricted) correlation functions defined by the same datum and to the space of intertwining operators among these twisted modules. As an application, we derive a twisted version of the Fusion Rules Theorem.

- *The stable Picard group of finite Adams Hopf algebroids with an application to the $\mathbb{R}$-motivic Steenrod subalgebra $\mathcal{A}^{\mathbb{R}}(1)$*
(with Ang Li)
Journal of Pure and Applied Algebra, Volume 228, Issue 11, 2024,
`https://doi.org/10.1016/j.jpaa.2024.107732`
In this paper, we investigate the rigidity of the stable comodule category of a specific class of Hopf algebroids known as *finite Adams*, shedding light on its Picard group. Then, we establish a reduction process through base changes, enabling us to effectively compute the Picard group of the $\mathbb{R}$-*motivic mod* 2 *Steenrod subalgebra* $\mathcal{A}^{\mathbb{R}(1)}$. Our computation shows that $\mathrm{Pic}(\mathcal{A}^{\mathbb{R}}(1))$ is isomorphic to $\mathbb{Z}^4$, where two ranks come from the motivic grading, one from the algebraic loop functor, and the last is generated by

the $\mathbb{R}$-*motivic joker* J.

- *Simplicial distance in Bruhat-Tits buildings of split classical type*
  UCSC Ph.D. Dissertation.
  `https://escholarship.org/uc/item/1g02s4hw`
  This study delves into *simplicial distances* on Bruhat-Tits buildings of split classical types (namely, types of A_n, B_n, C_n, D_n, and any combination thereof). Simplicial distance serves for measuring the proximity between vertices within the simplicial structure of a building. The purpose of this research is three-fold: (i) to present a concrete characterization of the simplicial distance; (ii) to better understand simplicial balls; and (iii) to derive a formula for the simplicial volume and explore its asymptotic growth. To accomplish these goals, vertices in Bruhat-Tits buildings are carefully analyzed under three frameworks: root systems, norms, and lattices. By leveraging concave functions, we interpret simplicial balls as fixed-point sets of Moy-Prasad subgroups and deduce a formula for the simplicial volume. Additionally, the theory of q-exponential polynomials is developed to facilitate the asymptotic study.

- *Extensions and Non-abelian Cohomology of Pre-Lie Algebras*
  Master degree thesis, 2015, Nankai University. Available on my website.

- *Rota-Baxter Operators on Witt and Virasoro Algebras*
  (with Ming Liu, Chengming Bai, and Naihuan Jing)
  Journal of Geometry and Physics, vol.108, 2016, pp.1-20.
  `https://doi.org/10.1016/j.geomphys.2016.06.007`

Academic Talks

Oct.31, 2024	*Twisted restricted conformal blocks of vertex operator algebras* Math-Physics Joint Seminar, University of Pennsylvania, USA
Oct.29, 2023	*p-adic representations and simplicial distance in Bruhat-Tis buildings* Conference on Algebraic Combinatorics, Zhuhai, China
July 21, 2023	*p-adic representations and simplicial distance in Bruhat-Tis buildings* 18th National Lie Theory Conference, Shanghai, China
Apr.14, 2023	*p-adic representations and simplicial distance in Bruhat-Tis buildings* UC Santa Cruz - Algebra & Number Theory Seminar
Mar.16, 2023	*p-adic representations and simplicial distance in Bruhat-Tis buildings* UC San Diego - Number Theory Seminar

Jan.31, 2023 *p-adic representations and simplicial balls in Bruhat-Tits buildings*
University of Arizona - Algebra and Number Theory Seminar

May 9, 2022 *How many vertices are there in a simplicial ball of radius r (in a Brihat-Tits Building)?*
UCSC Graduate Colloquium

Nov.22, 2021 *Stable Simplexes of p-adic Representations in Bruhat-Tits Buildings.*
UCSC Graduate Colloquium

May 24, 2019 *Transcendence of Periods.*
PhD qualifying oral Presentation

Teaching Experiences

Graduate Student Instructor at **University of California at Santa Cruz**

Duties include instructing students, holding office hours, responding to questions, preparing the course materials such as sliders, course notes, and website, and preparing and grading quizzes, homework, and exams.

Summer 2023 MATH 110: Introduction to Number Theory
Winter 2023 MATH 110: Introduction to Number Theory
Fall 2022 MATH 110: Introduction to Number Theory

Teaching Assistants at **University of California at Santa Cruz**

Duties include organizing discussion sections, holding office hours, responding to questions, reviewing quizzes, writing solutions, and grading homework and exams.

Summer 2022 MATH 22: Introduction to Calculus of Several Variables
Spring 2022 MATH 19B: Calculus for Science, Engineering, and Mathematics
Winter 2022 MATH 19A: Calculus for Science, Engineering, and Mathematics
Fall 2021 MATH 19B: Calculus for Science, Engineering, and Mathematics
Summer 2021 MATH 110: Introduction to Number Theory
Spring 2021 MATH 19B: Calculus for Science, Engineering, and Mathematics
Winter 2021 MATH 110: Introduction to Number Theory
Fall 2020 MATH 11A: Calculus with Applications
Summer 2020 MATH 110: Introduction to Number Theory
Spring 2020 MATH 111B: Algebra
Winter 2020 MATH 110: Introduction to Number Theory

Fall 2019	MATH 100: Introduction to Proof and Problem Solving
Summer 2019	MATH 117: Advanced Linear Algebra
Spring 2019	MATH 19B: Calculus for Science, Engineering, and Mathematics
Winter 2019	MATH 19A: Calculus for Science, Engineering, and Mathematics
Fall 2018	MATH 19B: Calculus for Science, Engineering, and Mathematics
Summer 2018	MATH 21: Linear Algebra
Spring 2018	MATH 21: Linear Algebra
Winter 2018	MATH 110: Introduction to Number Theory
Winter 2018	MATH 111: Algebra
Fall 2017	MATH 21: Linear Algebra
Fall 2017	MATH 100: Introduction to Proof and Problem Solving
Summer 2017	The Calculus series
Spring 2017	MATH 19B: Calculus for Science, Engineering, and Mathematics
Winter 2017	MATH 19B: Calculus for Science, Engineering, and Mathematics
Fall 2016	MATH 19A: Calculus for Science, Engineering, and Mathematics

Teaching Assistants at **Nankai University**

Duties include responding to questions, grading homework, and exams.

2014-2015	Calculus