Particle Acceleration in Shocks

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USA

Particle Acceleration in Shocks

1. Introduction
2. Parker Transport Equation
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5. Wave Excitation at Shocks
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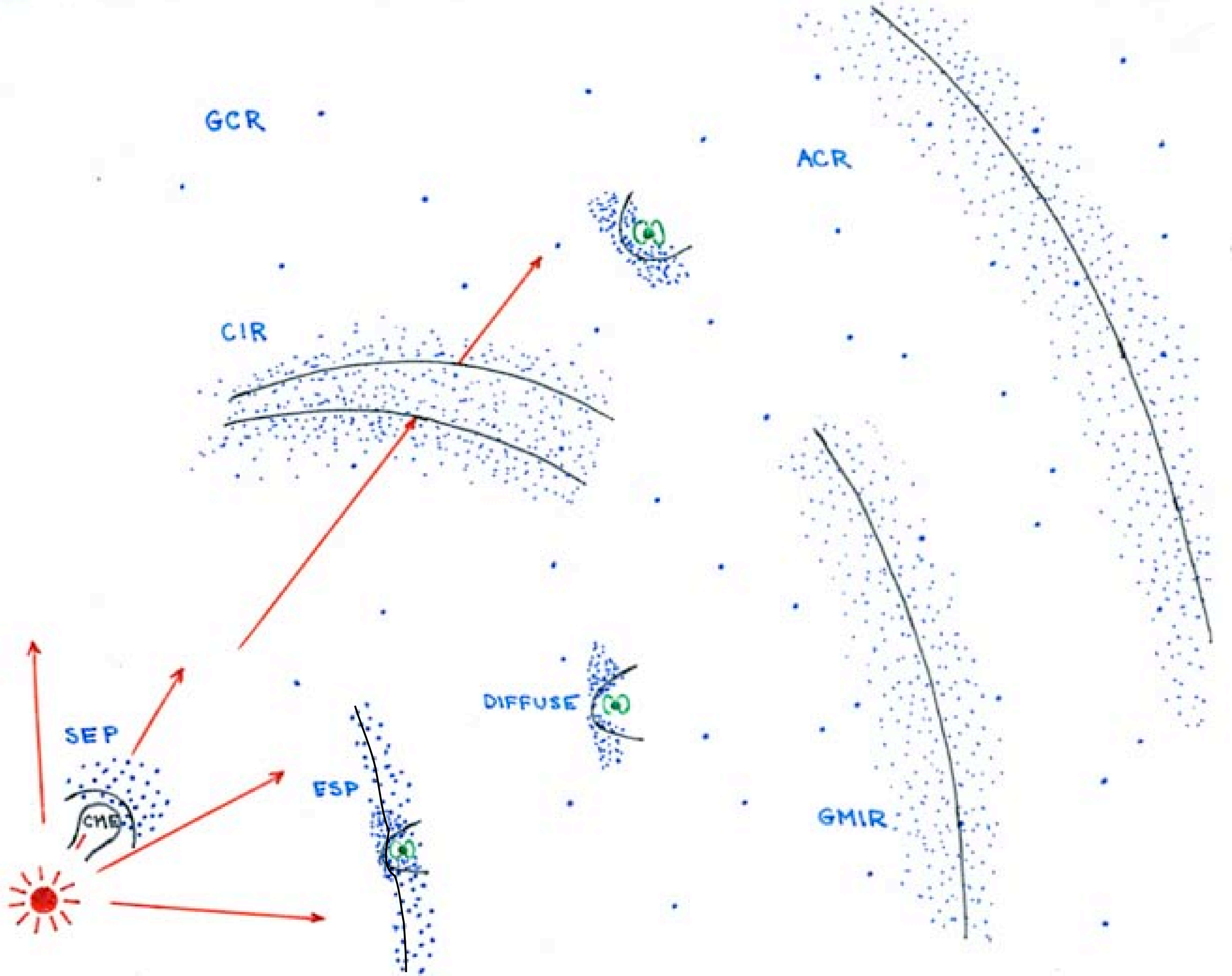
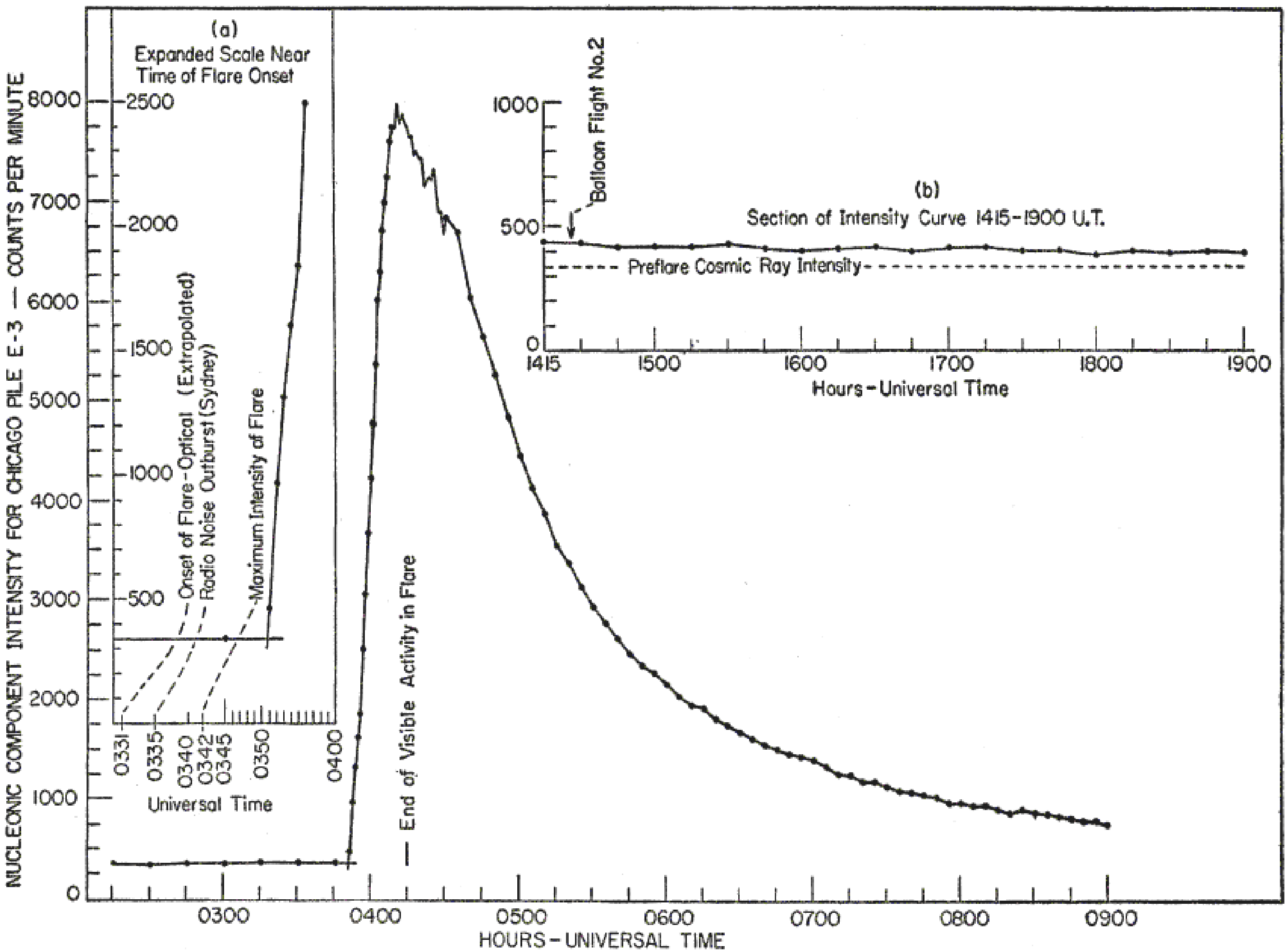
1. Introduction

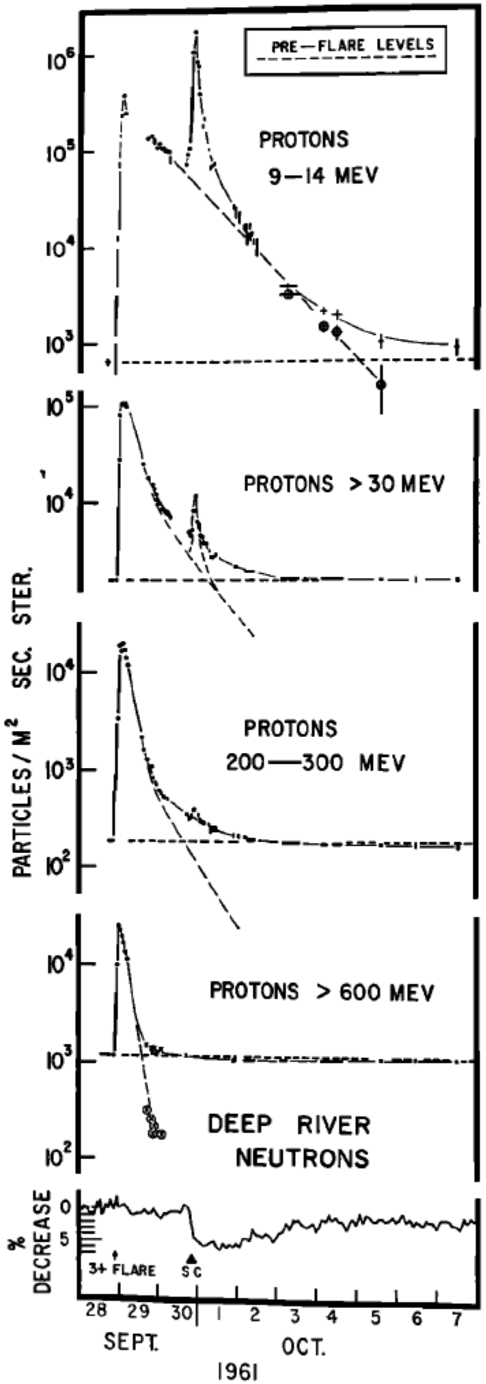

GCR
ACR
CIR
DIFFUSE
SEP
CME
ESP
GMIR

23 February 1956 GLE Event



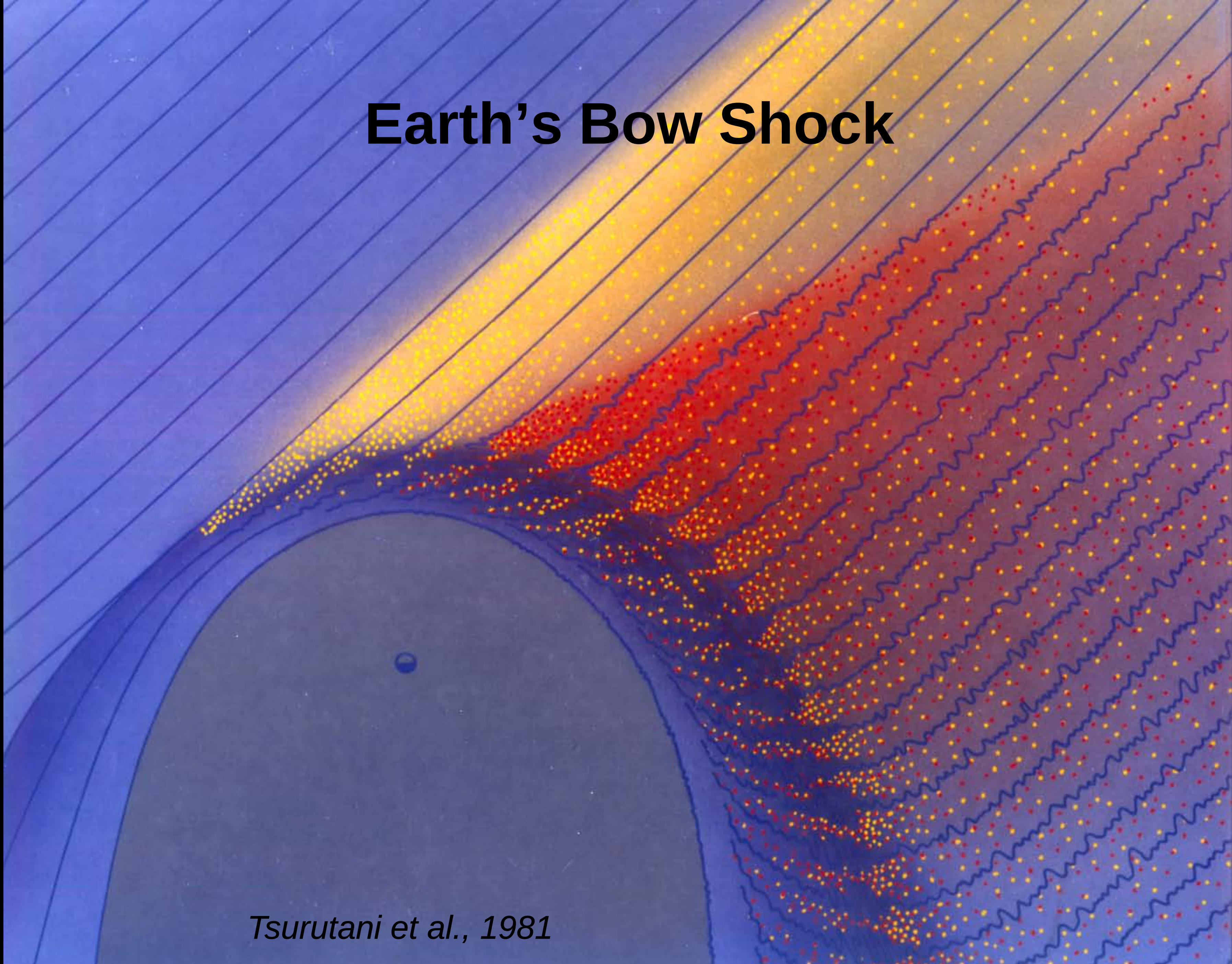
Meyer, Parker and Simpson, 1956

28 September 1961 Event
Explorer 12



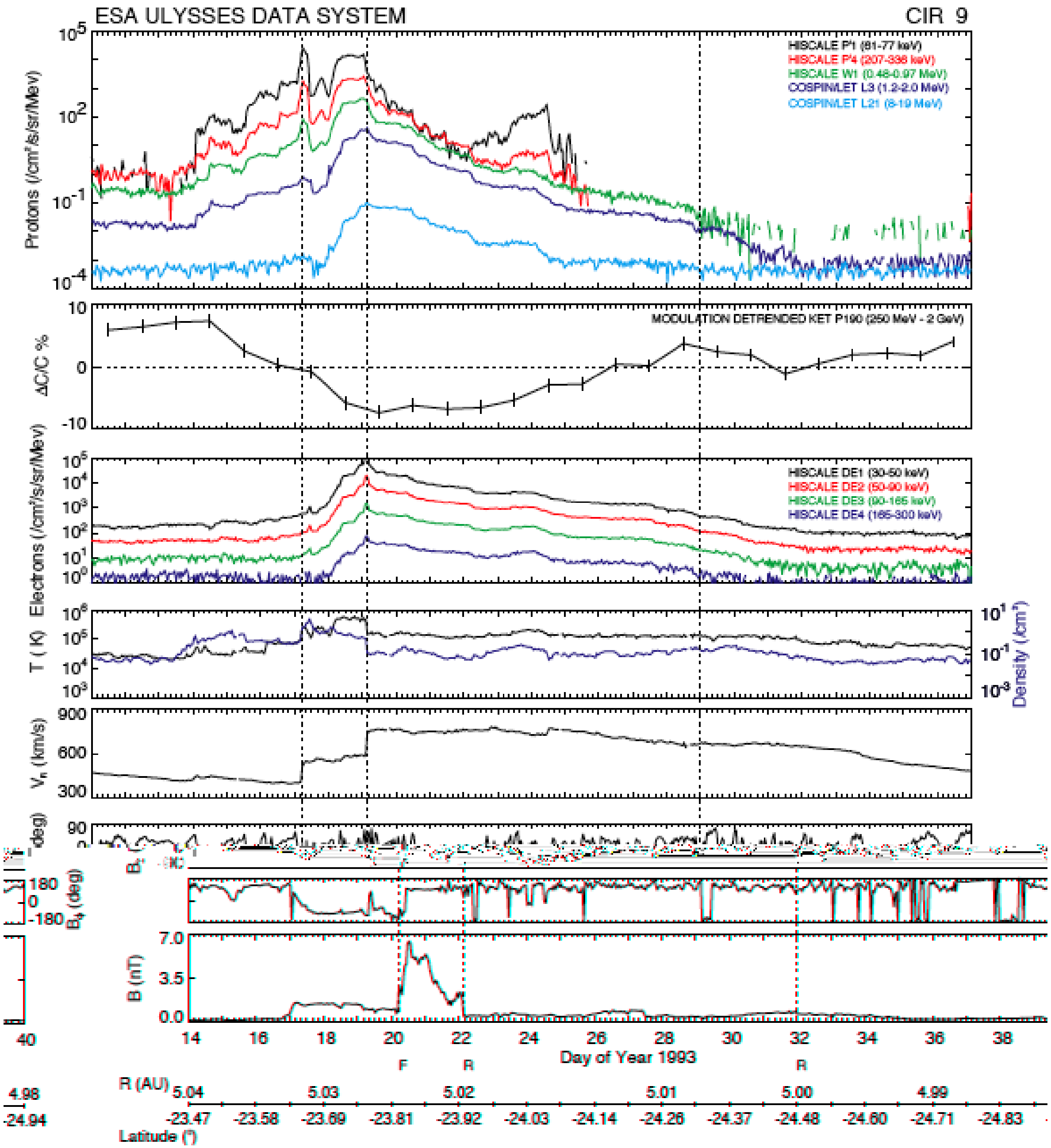
Bryant, Cline, Desai
and McDonald, 1962

Earth’s Bow Shock



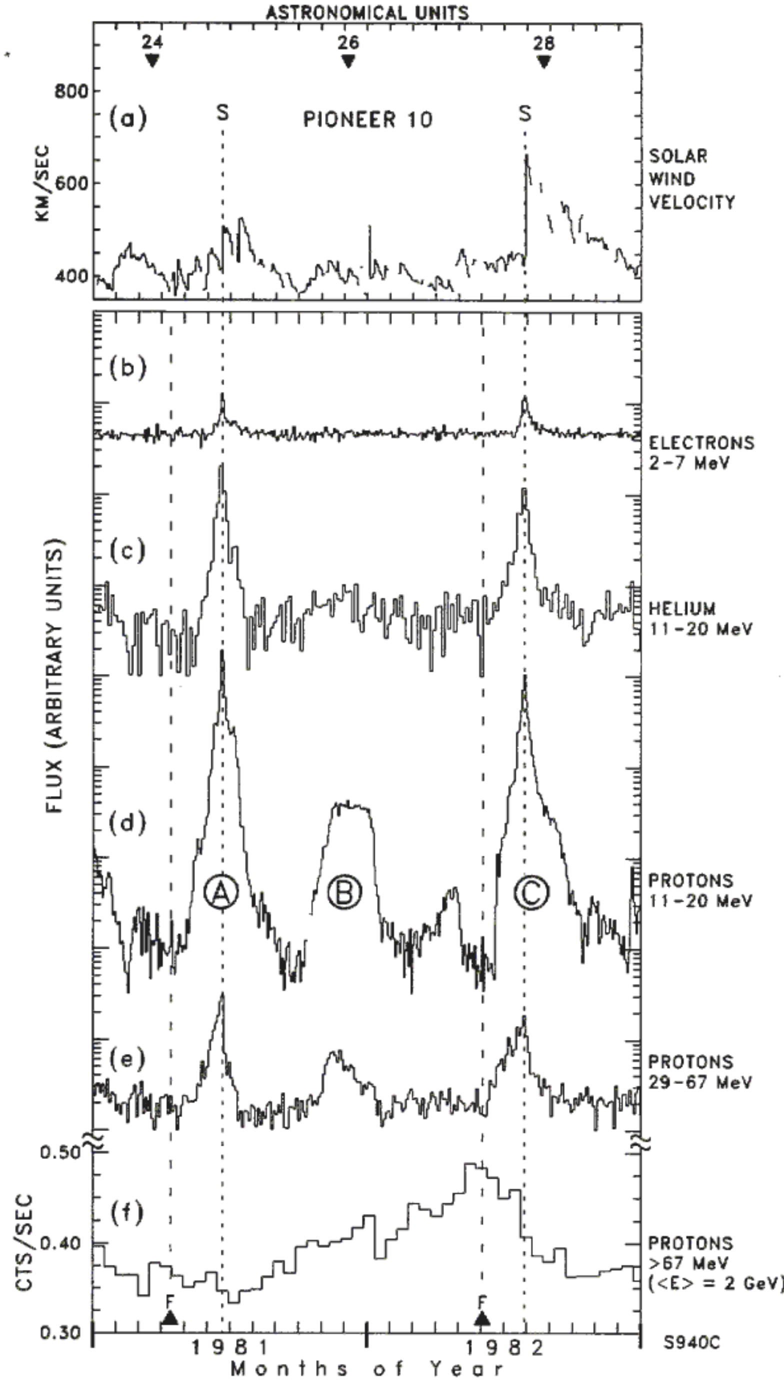
Tsurutani et al., 1981

CIR Event: Ulysses



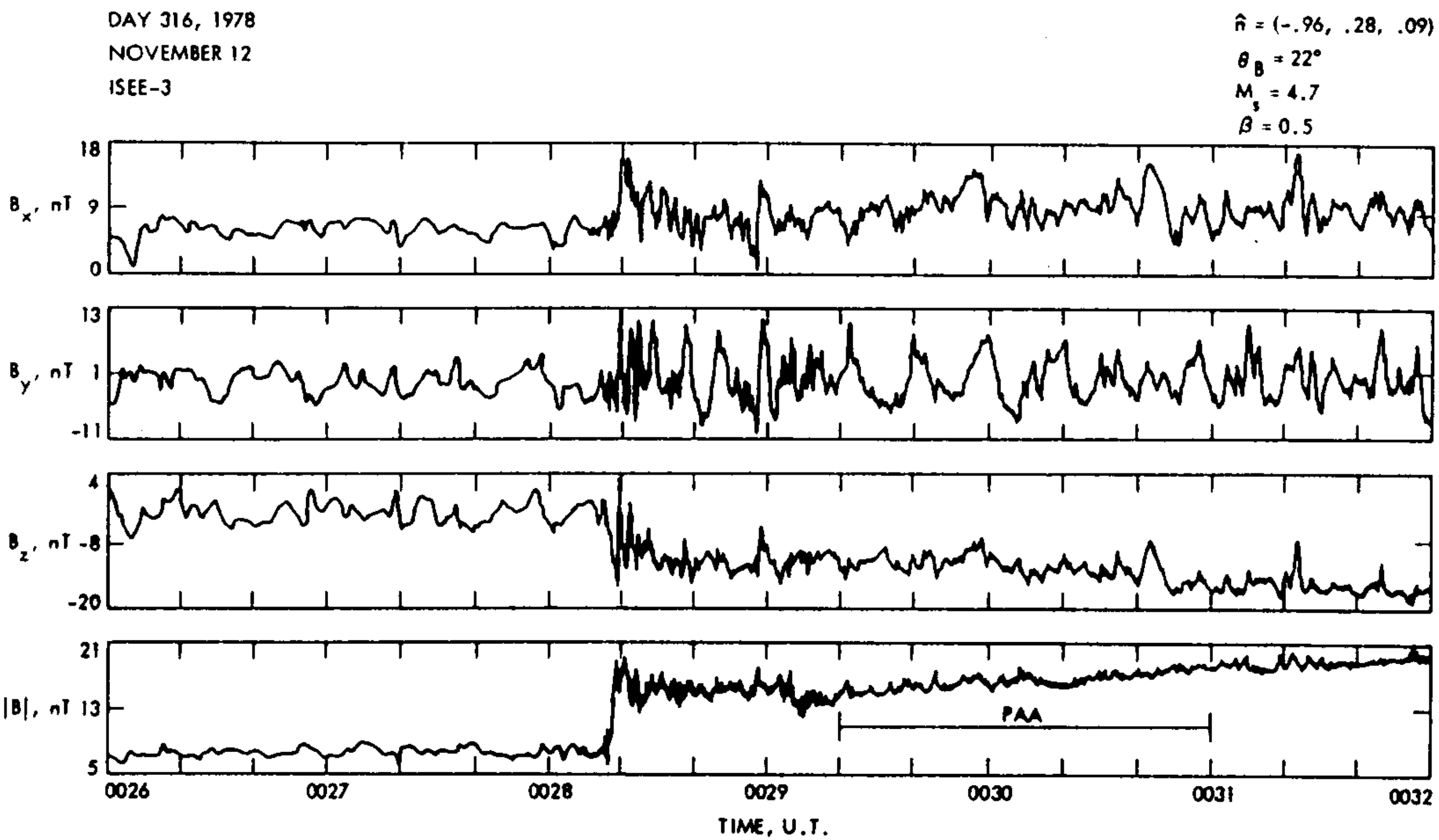
Kunow et al., 1999

Pioneer Super Events

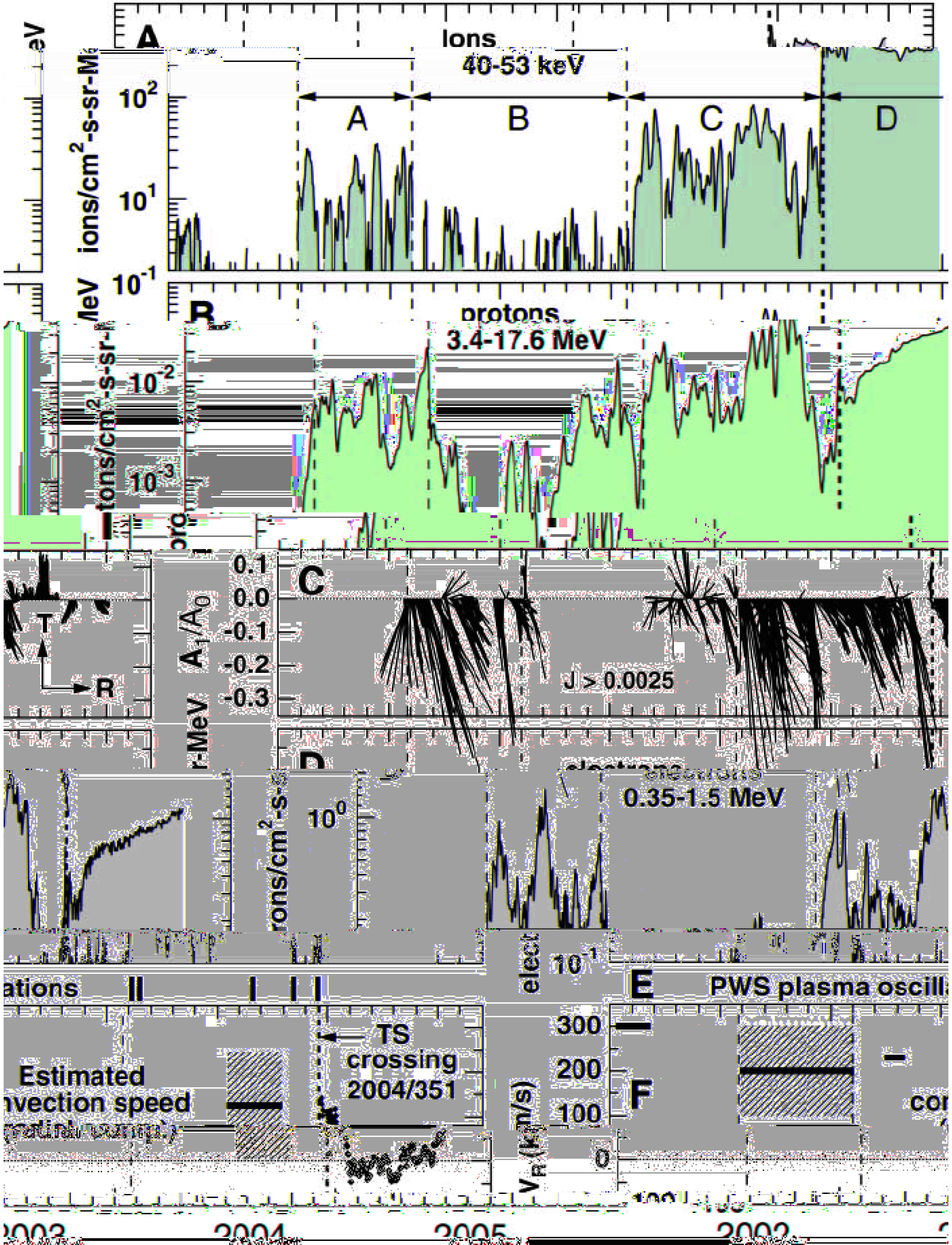


Pyle et al., 1984

Collisionless Shock on 11/12/78: ISEE-3



Tsurutani et al., 1983



Voyager 1
Ions

Decker et al., 2005

“Termination Shock” in Your Sink

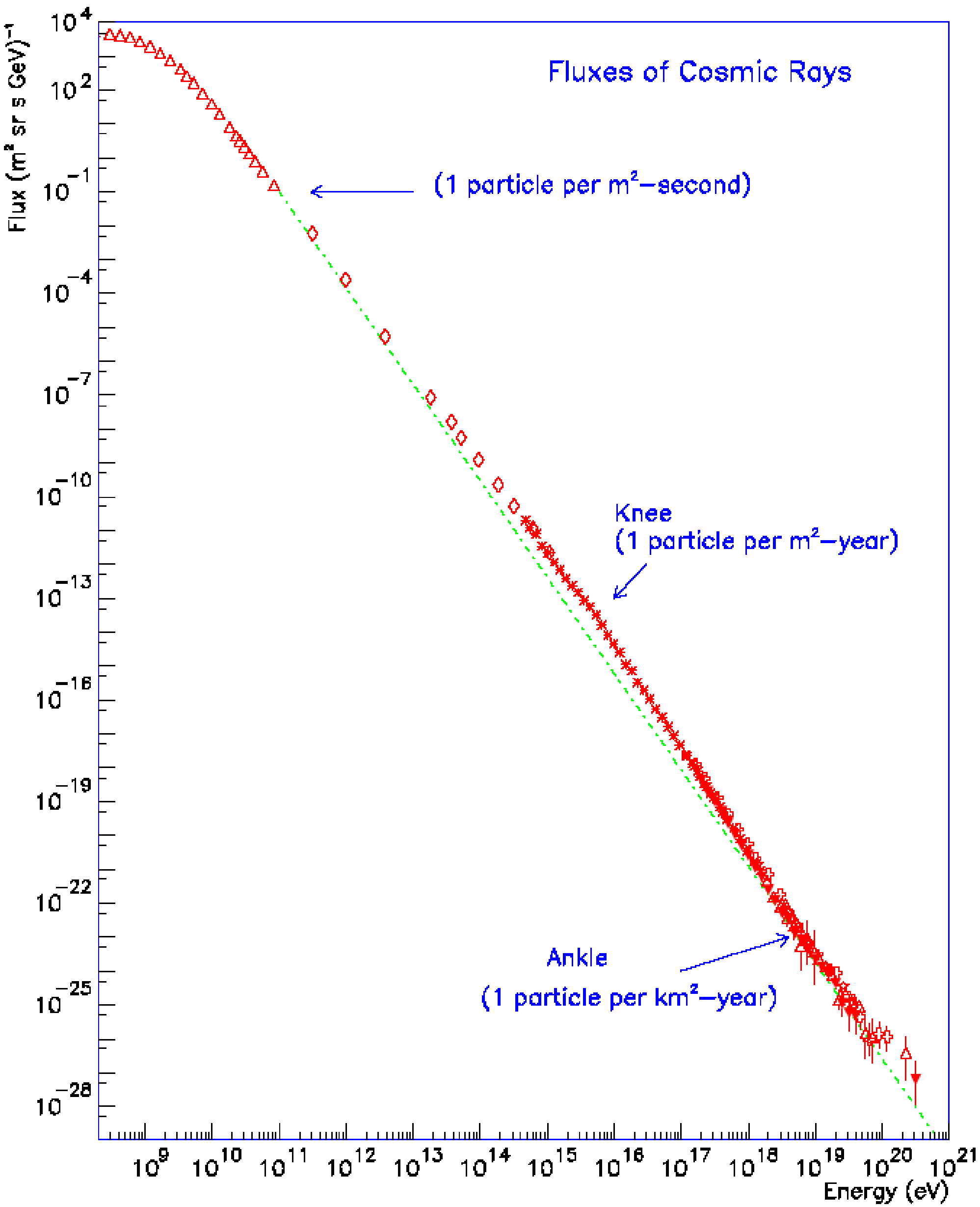
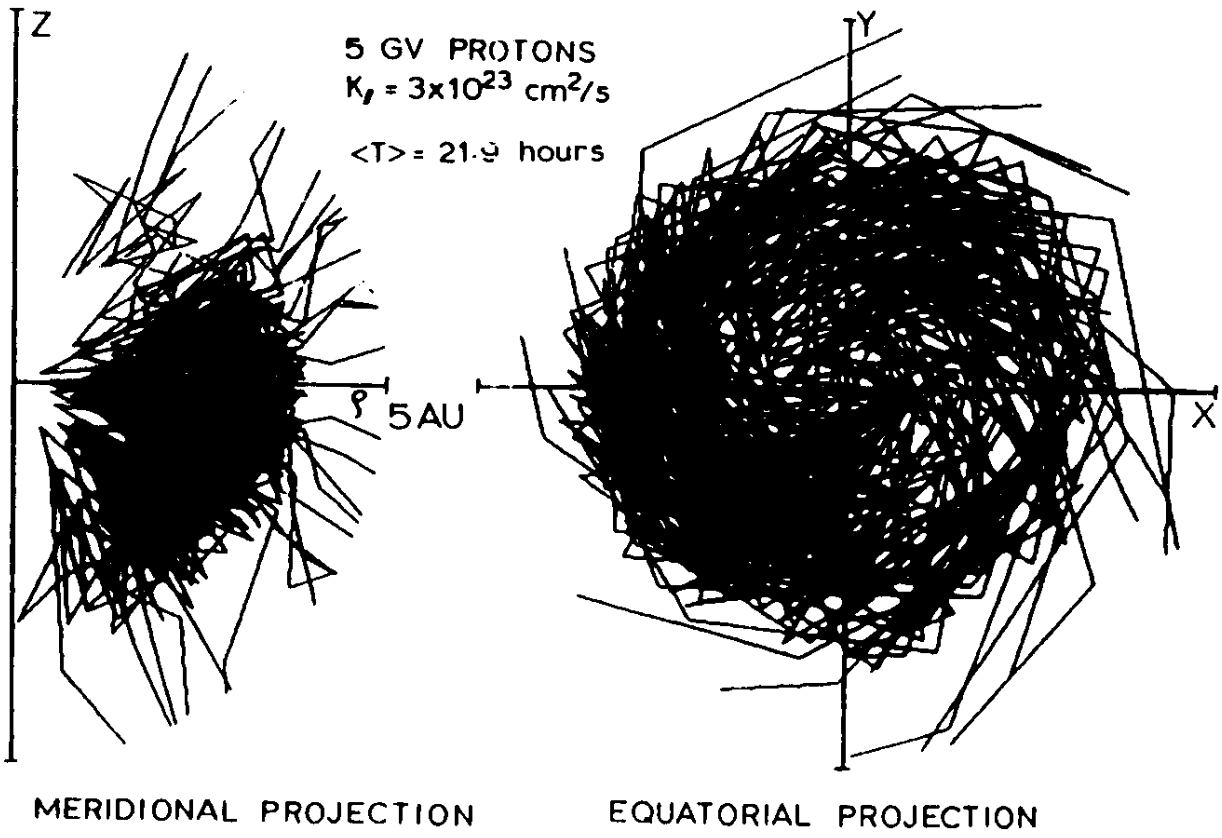


Figure 1. The all particle spectrum of cosmic rays - Cronin, Gaisser, Swordy 1997

The GCR spectrum continues as a power, in energy (index of about -2.7)

Highest energy cosmic rays have the kinetic energy of a major league baseball.

The Hairy Ball?



Thomas and Gall, 1984

Distribution Functions

$$F(\mathbf{p}, \mathbf{x}, t)$$ (phase-space distribution function)

$$n(\mathbf{x}, t) = \int d^3\mathbf{p} F(\mathbf{p}, \mathbf{x}, t)$$ (number density)

$$f(p, \mathbf{x}, t) = (4\pi)^{-1} \int d\Omega F(\mathbf{p}, \mathbf{x}, t)$$

(omnidirectional distribution function)

$$Flux = vFp^2 dp d\Omega$$

$$J = Flux/(d\Omega dE) = p^2 F$$ (differential intensity)

Vlasov Equation

$$\frac{\partial F}{\partial t} + \mathbf{v} \cdot \frac{\partial F}{\partial \mathbf{x}} + q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \cdot \frac{\partial F}{\partial \mathbf{p}} = 0$$

2. Parker Transport Equation

Parker (1964)

$$\underline{B} = B_0 \left[\frac{dF}{dz} \hat{e}_x + \frac{dG}{dz} \hat{e}_y + \hat{e}_z \right]$$

field lines: $x = F(z) + x_0$

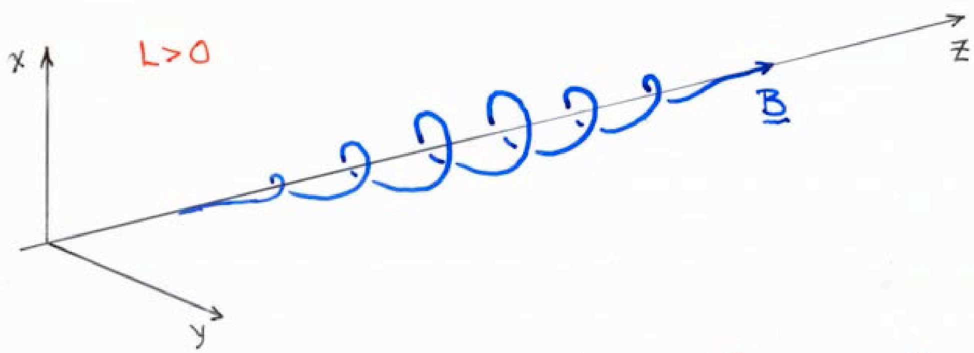
$y = G(z) + y_0$

if $F, G \rightarrow F_\pm, G_\pm$ as $z \rightarrow \pm\infty$

particle remains on field line — Jokipii, Kota

take: $F(z) = \varepsilon \sin\left(2\pi \frac{z}{L}\right) e^{-z^2/\ell^2}$

$G(z) = \varepsilon \cos\left(2\pi \frac{z}{L}\right) e^{-z^2/\ell^2}$ $\quad \varepsilon \ll 1$



$$\Delta v_z = \varepsilon \sin\Phi \, \pi^{1/2} \ell \, v_\perp (\Omega^2/v_z^2) \, e^{-\frac{1}{4}\left(\frac{\Omega \ell}{v_z}\right)^2 \left(\frac{2\pi v_z}{\Omega L} - 1\right)^2}$$

resonance: $\tau_g = \frac{L}{v_z}$

Cyclotron Resonance Condition

$$\omega - kv_z + \Omega = 0$$

$$kv_z \approx \Omega$$

Streaming Anisotropy

Reames et al., 2001

Impulsive Events

Mason et al., 1999

Parker's "Confusion-Defection" Equation

$$\int UdE = n = \int 4\pi p^2 f dp$$

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial E}(\mathbf{V} \cdot \nabla pvU/3)$$
$$+\nabla \cdot \left[-\mathbf{K} \cdot \nabla U + \frac{pvc}{3qB^2}\mathbf{B} \times \nabla U - \frac{1}{3}\mathbf{V}\frac{p^3}{v}\frac{\partial}{\partial p}\left(U\frac{v}{p^2} \right) \right] = 0$$

$$\Rightarrow \frac{\partial f}{\partial t} + (\mathbf{V} + \mathbf{V}_D) \cdot \nabla f - \nabla \cdot \mathbf{K} \cdot \nabla f - \frac{1}{3}\nabla \cdot \mathbf{V} p \frac{\partial f}{\partial p} = 0$$

$$\left(\mathbf{E} \cong -c^{-1}\mathbf{V} \times \mathbf{B} \right)$$

Parker, 1965

Contours of $\nabla \cdot \mathbf{V} > 0$ and < 0

Stochastic Compressions and Rarefactions: Quasi-Linear Theory

$$\frac{\partial f_0}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left\{\frac{v^4}{9}\int_{-\infty}^{\infty} d^3\mathbf{x}'\int_{-\infty}^{t} dt' G(\mathbf{x},t;\mathbf{x}',t')\left\langle(\nabla\cdot\delta\mathbf{V})(\nabla'\cdot\delta\mathbf{V}')\right\rangle\frac{\partial f_0(v,t)}{\partial v}\right\}$$

$$G(\mathbf{x},t;\mathbf{x}',t') = [4\pi K(t-t')]^{-3/2}\exp\{-|\mathbf{x}-\mathbf{x}'|^2[4K(t-t')]^{-1}\}$$

$$\frac{\partial f_0}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2 D\frac{\partial f_0}{\partial v}\right]$$

Jokipii and Lee, 2010

Stochastic Acceleration

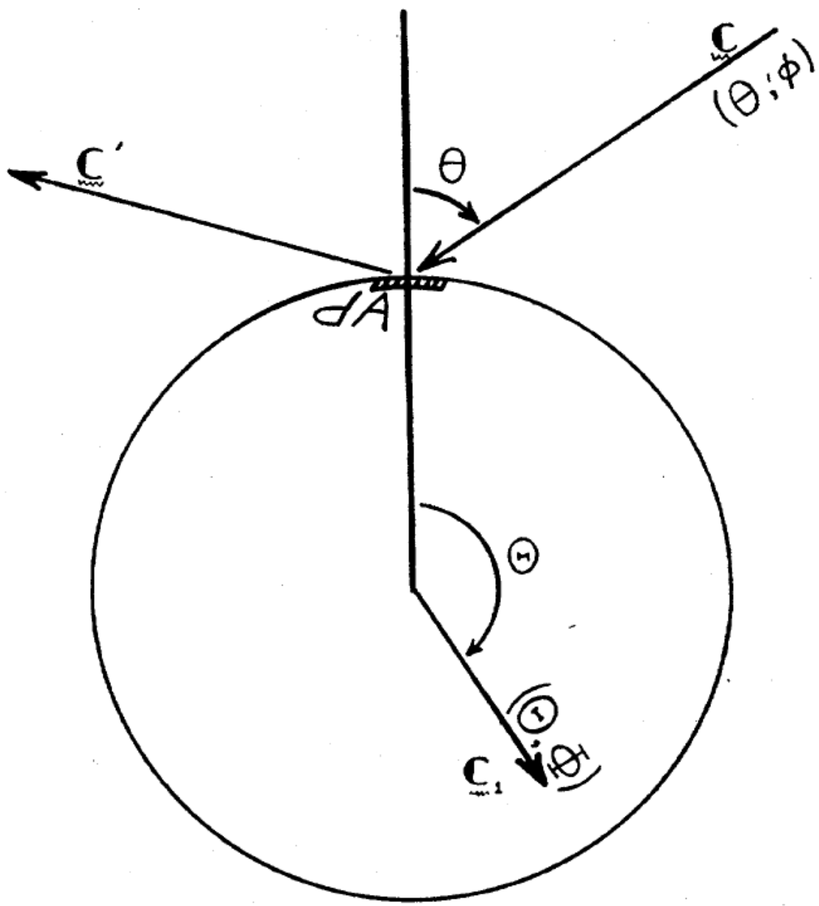


FIG. 2. Coordinate system for calculation of Δc, etc.

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D(p)\frac{\partial f}{\partial p}\right)$$

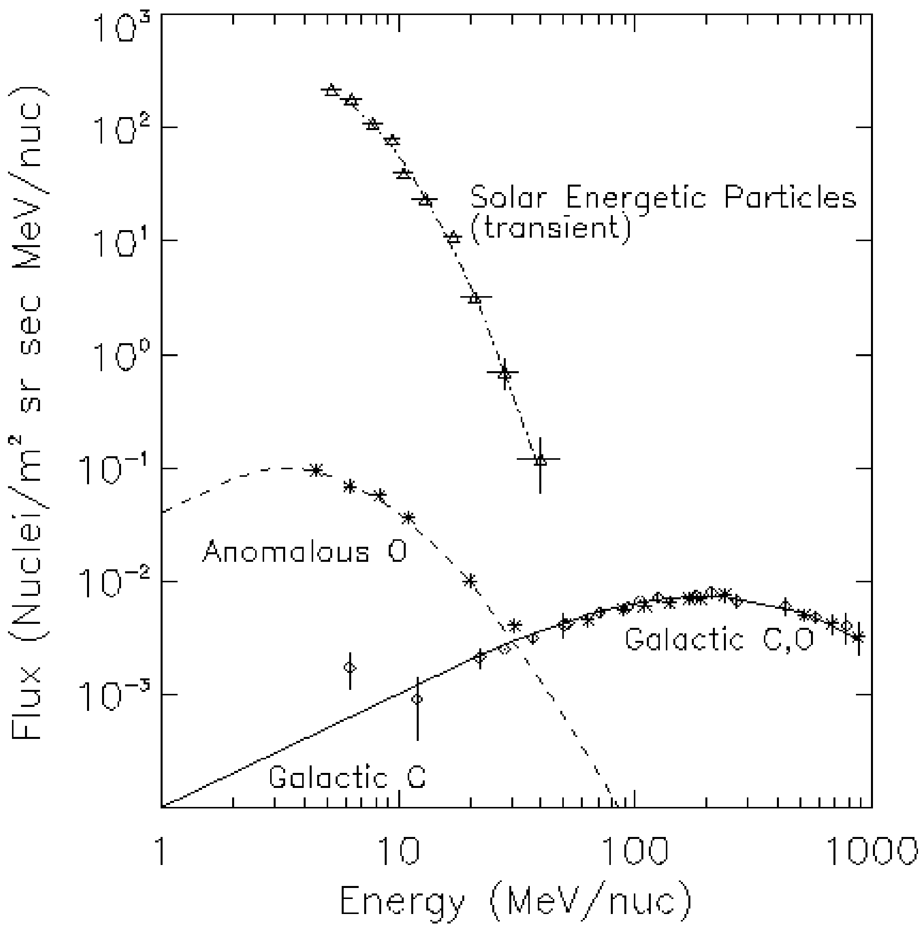
$$D(p) = \frac{1}{3}\left\langle V^2 \right\rangle \frac{1}{\lambda}\frac{p^2}{v}$$

$$\lambda \equiv (\pi R^2 N)^{-1}$$

Parker and Tidman, 1958

3. Applications of the Parker Equation

Charged Particle Spectrum



Ions not marked by source

Energy and timing help separate sources

Charge state also:

- AC singly charged
- GCR full stripped
- SEP partially stripped

Solar Modulation of GCR: A Simple Case

$$n = \int 4\pi p^2 f dp, \;\; \mathbf{V}_D = 0, \;\; \nabla = \mathbf{e}_r d/dr, \;\; \partial/\partial t = 0, \;\; K = K(r), \;\; \mathbf{V} = \mathbf{e}_r V$$

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\left(Vn - K\frac{dn}{dr}\right)\right] = 0$$

$$Vn - K\frac{dn}{dr} = \frac{C}{r^2} \qquad\qquad C = 0$$

$$n(r) = n(r = R)\exp\left(-\int_r^R \frac{Vdr'}{K(r')}\right)$$

Solar Energetic Particle Event

Reames et al., 2001

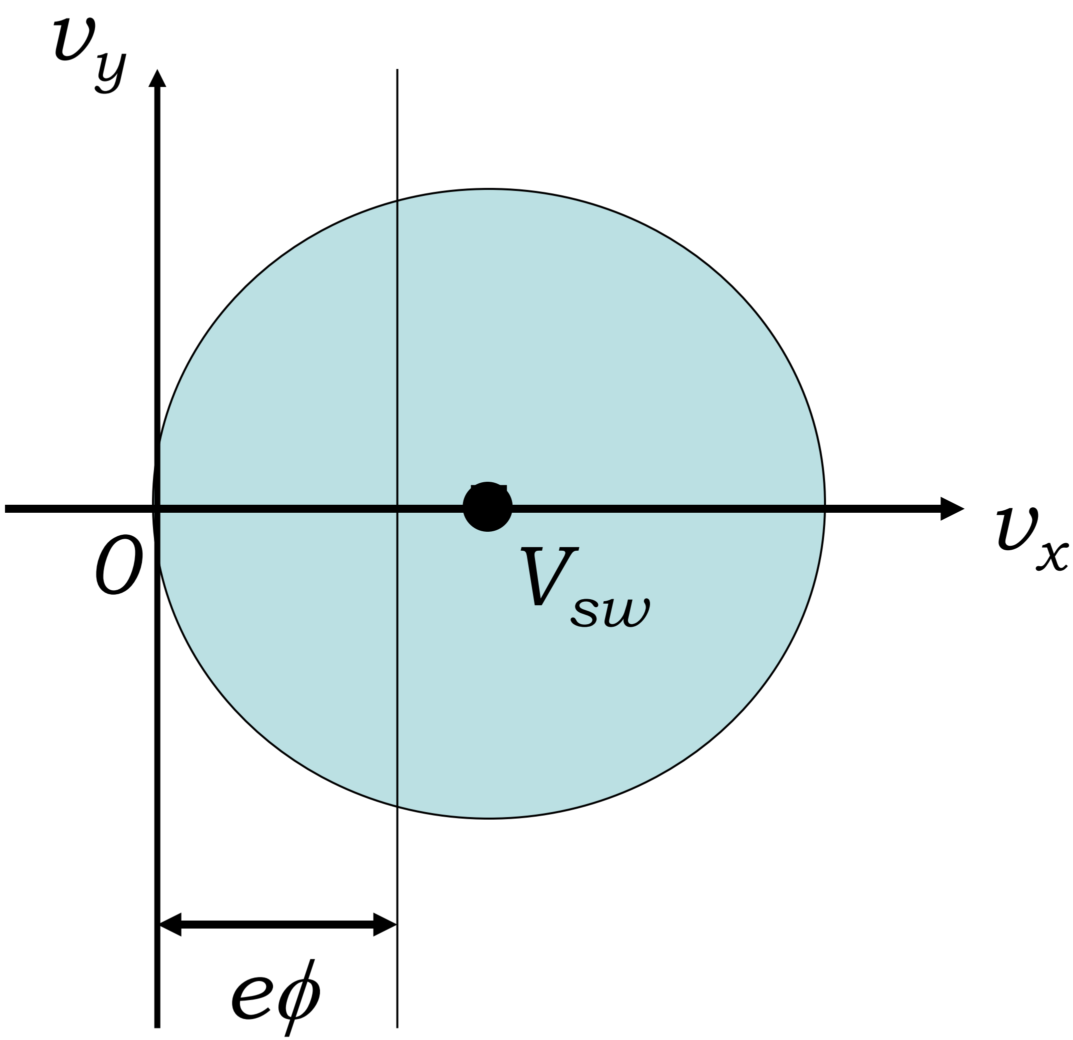
SEP Propagation: A Simple Case

$$\mathbf{V} \cong 0,\ \ \mathbf{V}_D \cong 0,\ \ \mathbf{K} = \mathbf{K}(p),\ \ \nabla = \mathbf{e}_r \partial/\partial r$$

$$\frac{\partial f}{\partial t} = K\nabla^2 f + f_0(p)\delta(\mathbf{x})\delta(t)$$

$$f(p,r,t) = \frac{f_0(p)}{[4\pi K(p)t]^{3/2}} \exp\left(-\frac{r^2}{4K(p)t}\right)$$

Pickup Ion Mediated Termination Shock



Interstellar Pickup Ion Transport

$$\mathbf{V}_D \cong 0, \ \ \mathbf{K} \cong 0, \ \ \mathbf{V} = \mathbf{e}_r V, \ \ \partial/\partial t = 0$$

$$\frac{\partial f}{\partial t} + \mathbf{V} \cdot \nabla f - \frac{1}{3} \nabla \cdot \mathbf{V} v \frac{\partial f}{\partial v} = \beta_0 \left(\frac{r_0}{r} \right)^2 n_g(\mathbf{x}) \frac{\delta(v - V)}{4\pi v^2}$$

$$f(r, v < V) = \frac{3\beta_0 r_0^2}{8\pi V^{5/2}} \frac{1}{r v^{3/2}} n_g \left[r(v/V)^{3/2}, \theta, \phi \right]$$

4. Diffusive Shock Acceleration

Diffusive Shock Acceleration

$$V_z \frac{df}{dz} - \frac{d}{dz}\left(K_{zz}\frac{df}{dz}\right) - \frac{1}{3}\frac{dV_z}{dz}\, p\frac{df}{dp} = Q\delta(z)\delta(p - p_0)$$

$$f(z < 0) = \frac{3Q}{(V_u - V_d)\,p_0}\left(\frac{p}{p_0}\right)^{-\beta} \exp\left(\frac{Vz}{K}\right)$$

$$f(z > 0) = \frac{3Q}{(V_u - V_d)\,p_0}\left(\frac{p}{p_0}\right)^{-\beta}$$

$$\beta = 3X/(X-1)$$

Fisk, 1971;………………..

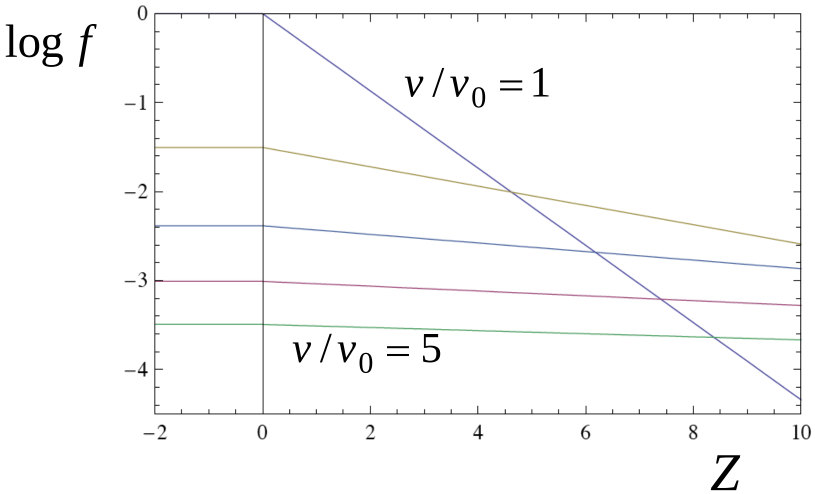
Axford, Leer and Skadron, 1977

Krymsky, 1977

Blandford and Ostriker, 1978

Bell, 1978

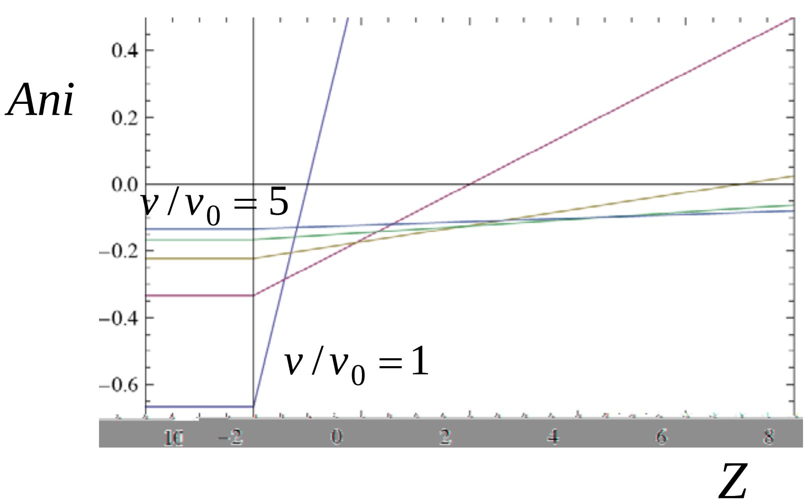
Planar Stationary DSA

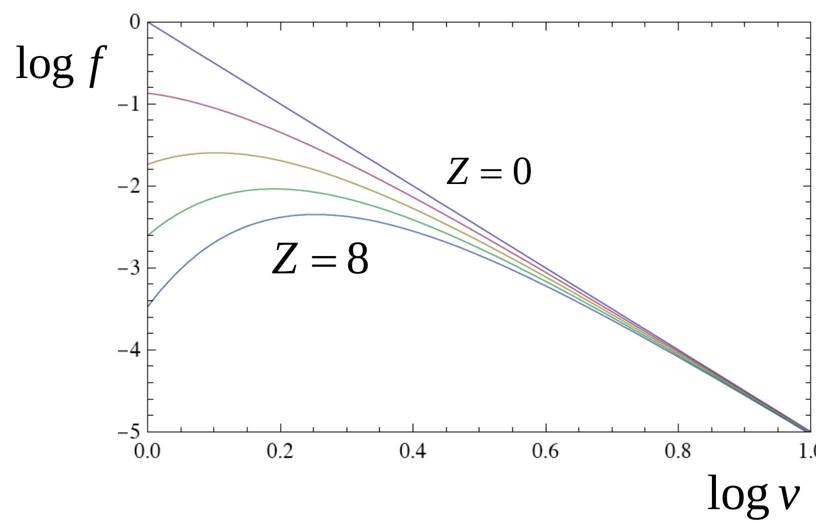


$$K_{zz} = K_0 (v / v_0)^2$$

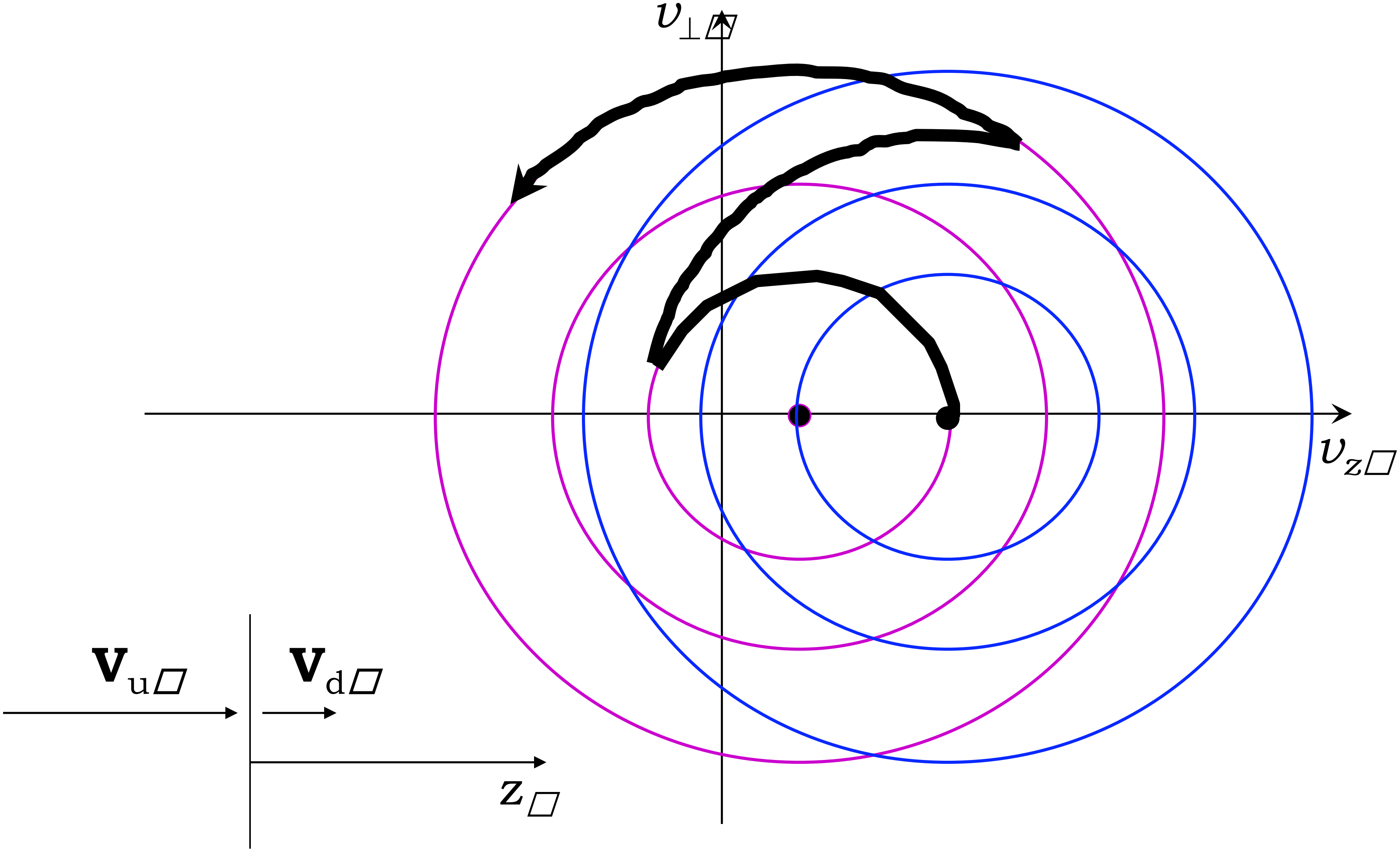
$$Z = Vz / K_0$$

$$\beta = 5$$

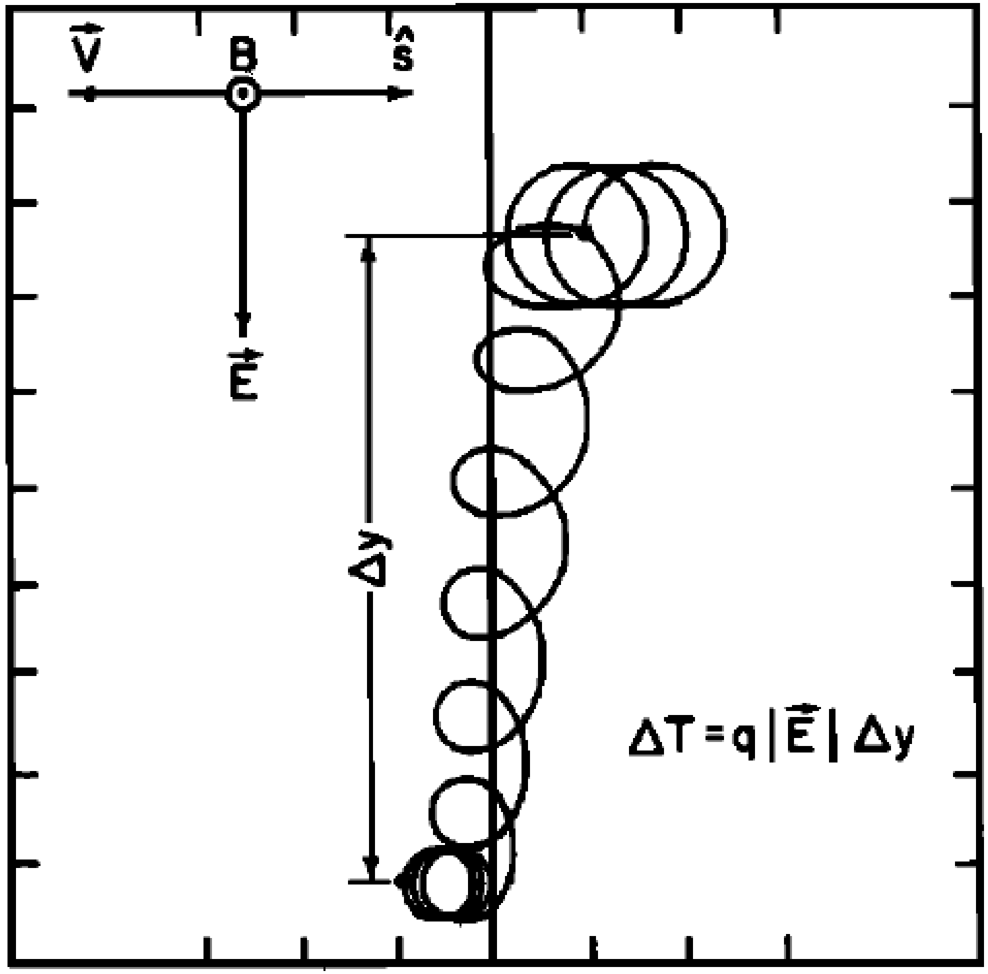




First-Order Fermi Acceleration

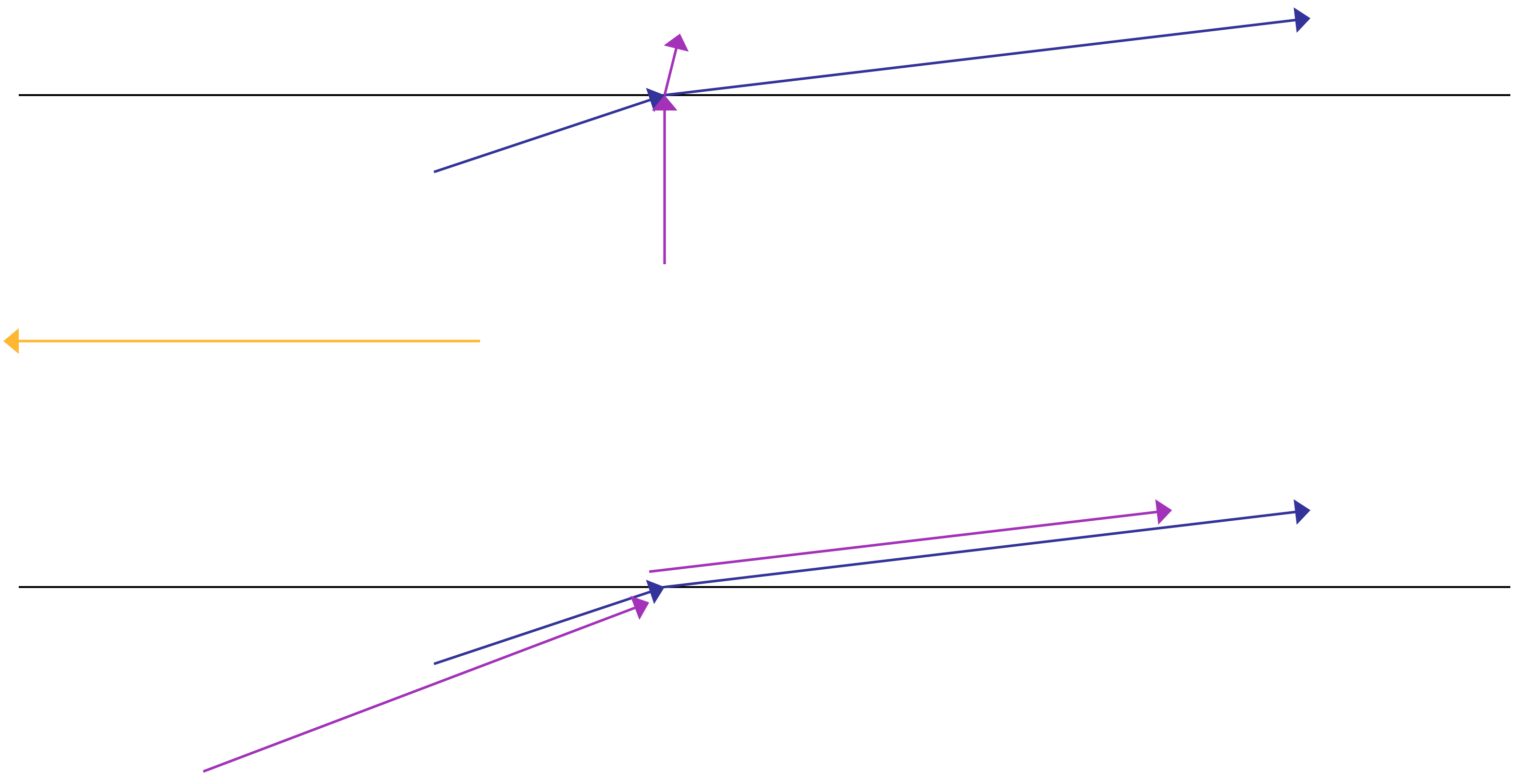


“Shock Drift” Acceleration



Pesses, 1981

Diffusive Shock Acceleration



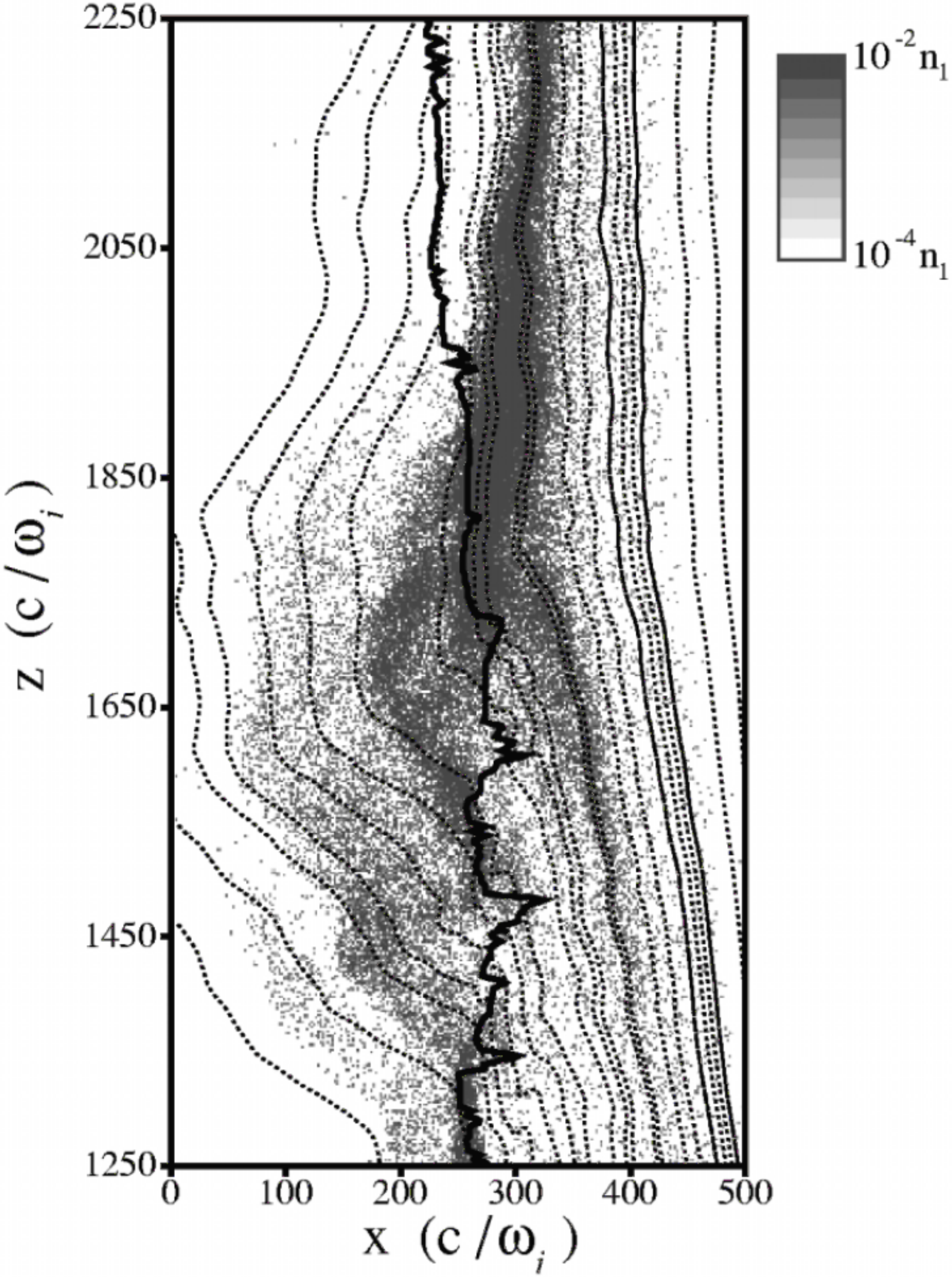
Anisotropy Limitation

$$\frac{|\mathbf{S}|}{vf} = \frac{V}{v}\left[1 + \frac{K_A^2 \sin^2\theta + (K_\parallel - K_\perp)^2 \sin^2\theta \cos^2\theta}{(K_\parallel \cos^2\theta + K_\perp \sin^2\theta)^2}\right]^{1/2}$$

$$K_\parallel >> K_\perp, K_A :$$

$$\Rightarrow \frac{|\mathbf{S}|}{vf} = \frac{V}{v\cos\theta}$$

Giacalone and Jokipii, 1999



Quasi-Perpendicular Shock Simulation: Be Careful!

Giacalone, 1999

Shock Modification

$$\partial/\partial t = \partial/\partial y = \partial/\partial z = \mathbf{V}_D = Q = 0$$

$$V \frac{dP_c}{dx} - \frac{d}{dx}\left(\overline{K} \frac{dP_c}{dx}\right) + \gamma_c \frac{dV}{dx} P_c \cong 0$$

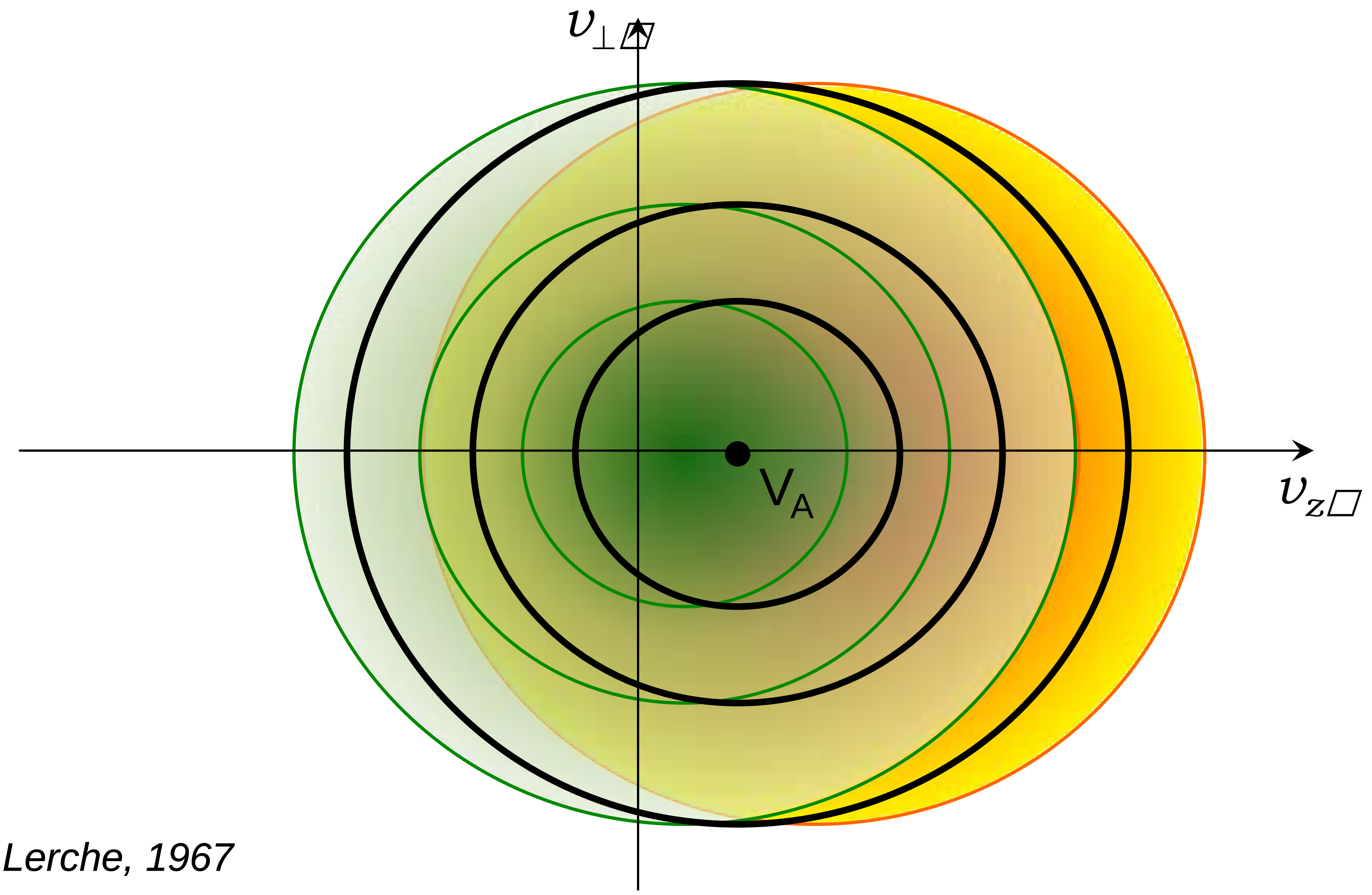
$$\frac{d}{dx}(\rho V) = 0$$

$$\rho V \frac{dV}{dx} = -\frac{d}{dx}(P_g + P_c)$$

$$V \frac{dP_g}{dx} + \gamma_g \frac{dV}{dx} P_g = 0$$

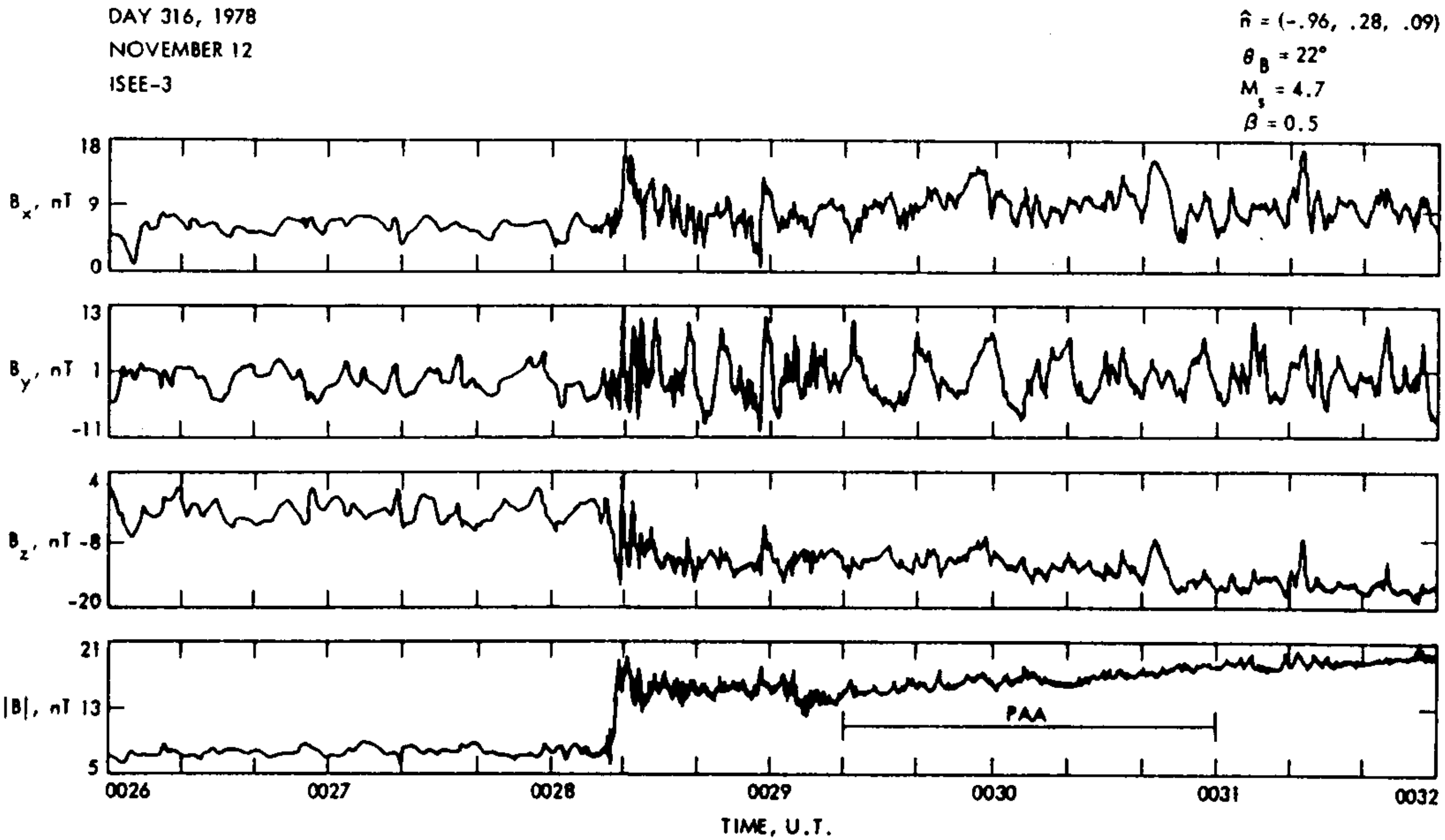
5. Wave Excitation at Shocks

Instability Mechanism

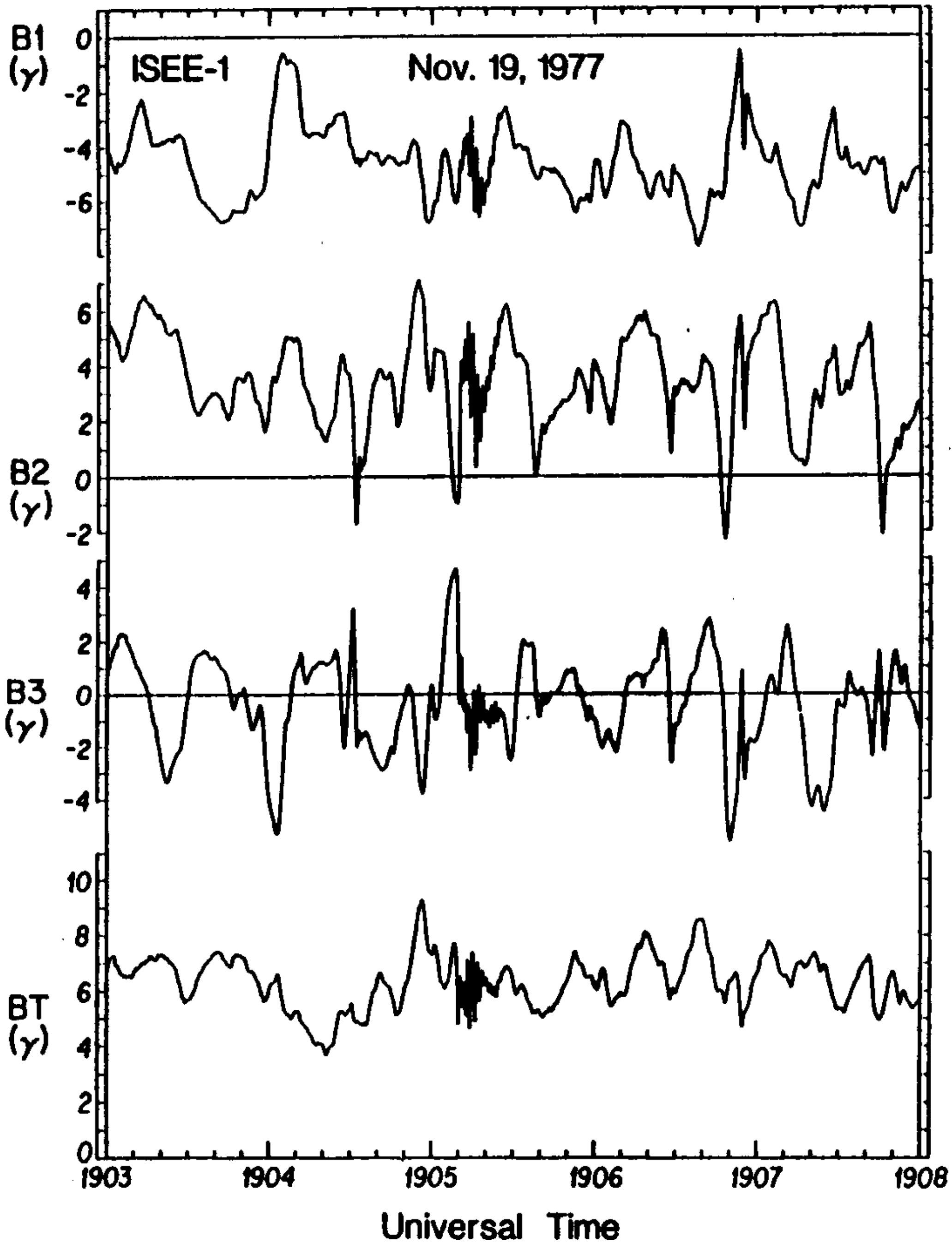


Lerche, 1967

Upstream Waves I



Tsurutani et al., 1983



Upstream Waves II

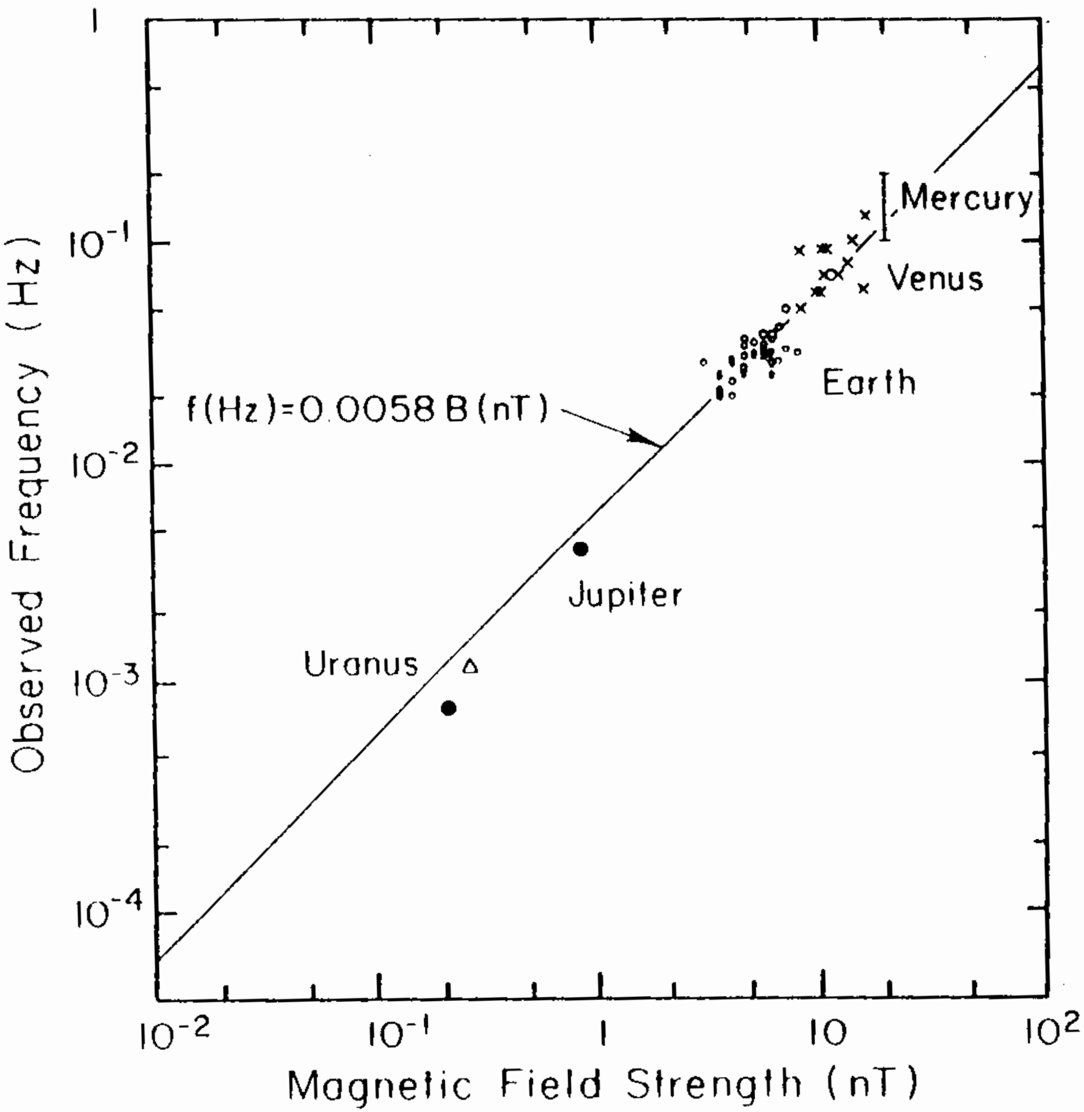
Hoppe et al., 1981

Cyclotron Resonance Condition

$$\omega - kv_z + \Omega = 0$$

$$kv_z \approx \Omega$$

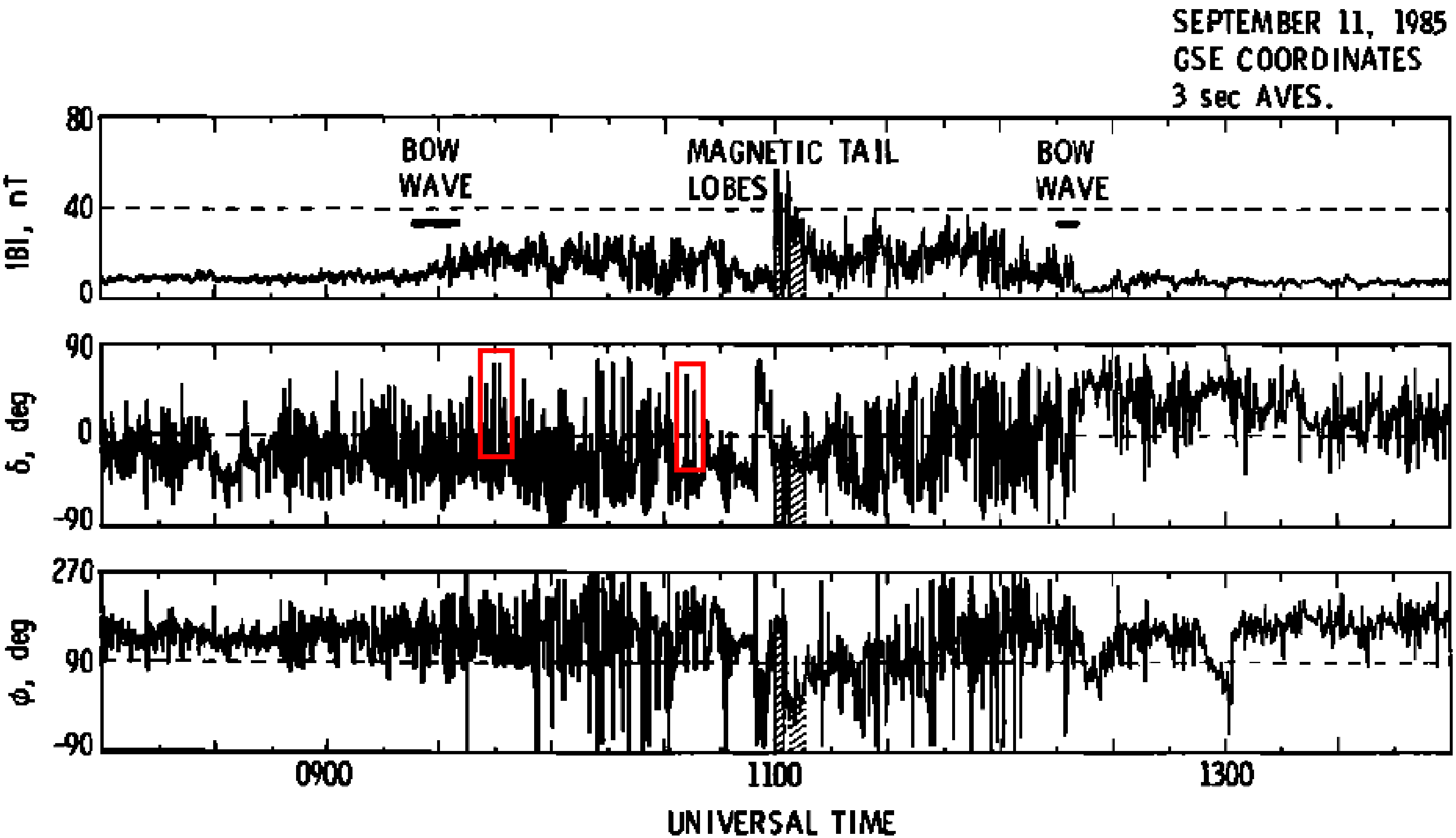
$$\omega_s \sim kV_{sw} \sim \Omega(V_{sw} / v_z) \propto B$$



Upstream Waves at Planetary Shocks

Russell et al., 1990

Pickup Ion Excited Waves at Comet G-Z



Tsurutani and Smith, 1986

Wave Excitation - I

$$-V\partial I_{\pm} / \partial z = 2\gamma_{\pm} I_{\pm}$$

$$I \cong I_{+} = I^{\circ}_{+}(k) + \frac{4\pi^2}{k^2}\frac{V_A}{V}|\Omega_p| m_p \cos\psi \int_{|\Omega_p / k|}^{\infty} dv v^3 \left(1 - \frac{\Omega_p^2}{k^2 v^2}\right)(f_p - f_{p,\infty})$$

$$f_{p,\infty} = \overline{n}_p (4\pi v_{p,0}^2)^{-1} \delta(v - v_{p,0}) + \overline{C} v^{-\gamma} S(v - \overline{v}_{p,0})$$

Wave Excitation - II

$$I = I_{+}^{\circ} + \frac{4\pi^2}{k^2}\frac{V_A}{V}|\Omega_p| m_p \cos\psi \int_{|\Omega_p/k|}^{\infty} dv v^3 (1 - \frac{\Omega_p^2}{k^2 v^2}) \cdot$$

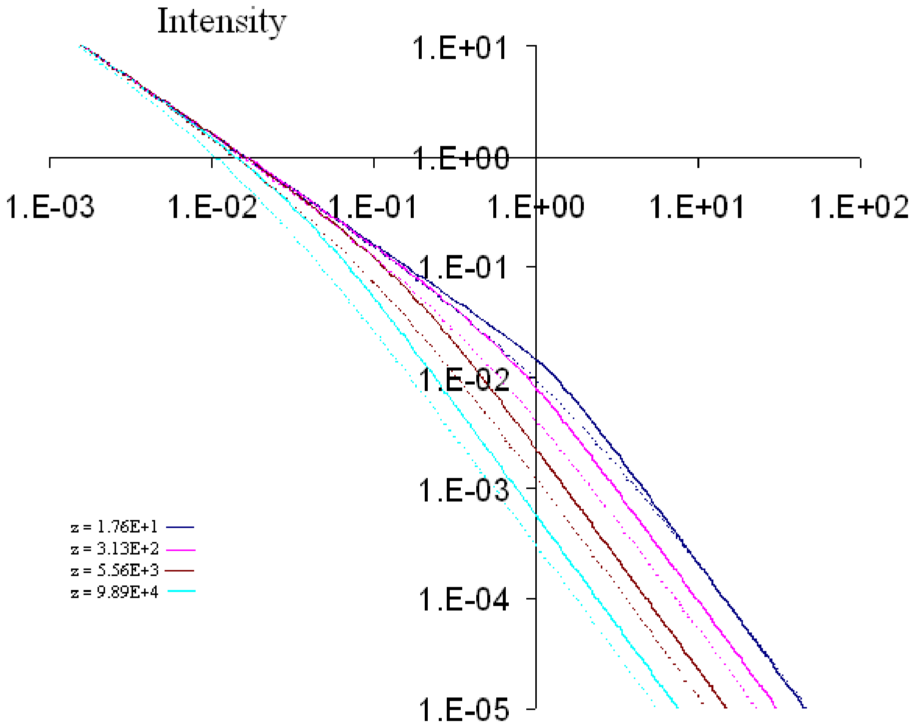
$$\cdot \left\{ \frac{\beta \overline{n}}{4\pi v_{0,p}^3} (\frac{v}{v_{0,p}})^{-\beta} S(v - v_{0,p}) - \frac{\overline{n}}{4\pi v_{0,p}^2} \delta(v - v_{0,p}) \right.$$

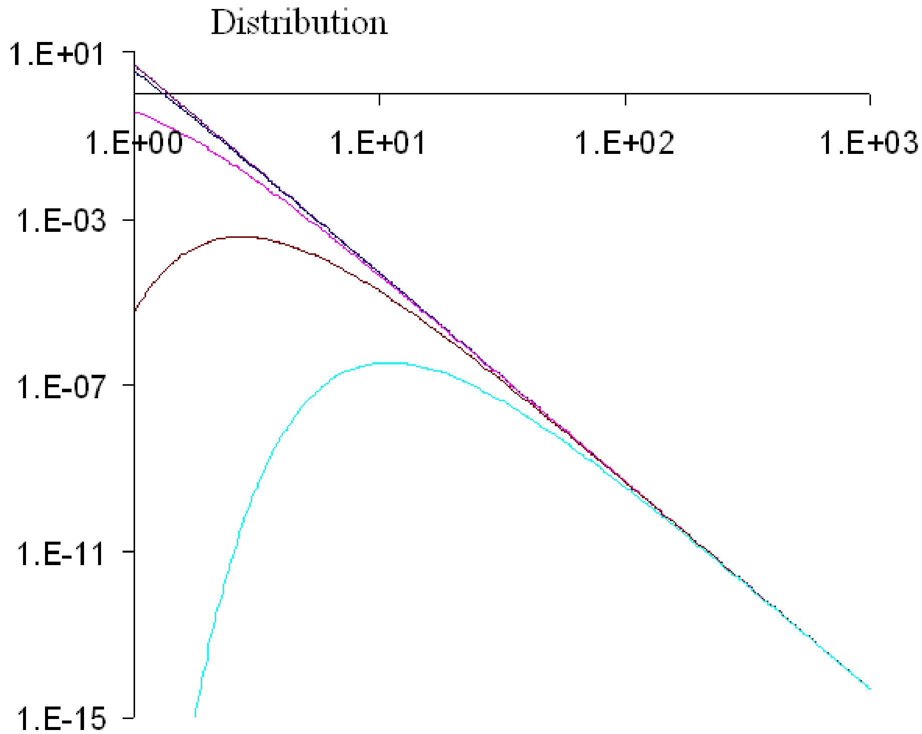
$$\left. + \frac{\overline{C}\,\overline{v}_{0,p}^{-\gamma}}{\beta - \gamma} \left[\gamma (\frac{v}{\overline{v}_{0,p}})^{-\gamma} - \beta (\frac{v}{\overline{v}_{0,p}})^{-\beta} \right] S(v - \overline{v}_{0,p}) \right\} \cdot$$

$$\cdot \exp\left\{ -V \int_0^z dz' \left[\cos^2\psi \frac{v^3}{4\pi} \frac{B_0^2}{\Omega_p^2} \int_{-1}^{1} d\mu \frac{|\mu|(1-\mu^2)}{I(\Omega_p \mu^{-1} v^{-1})} + \sin^2\psi K_{\perp} \right]^{-1} \right\}$$

Wave Excitation - III

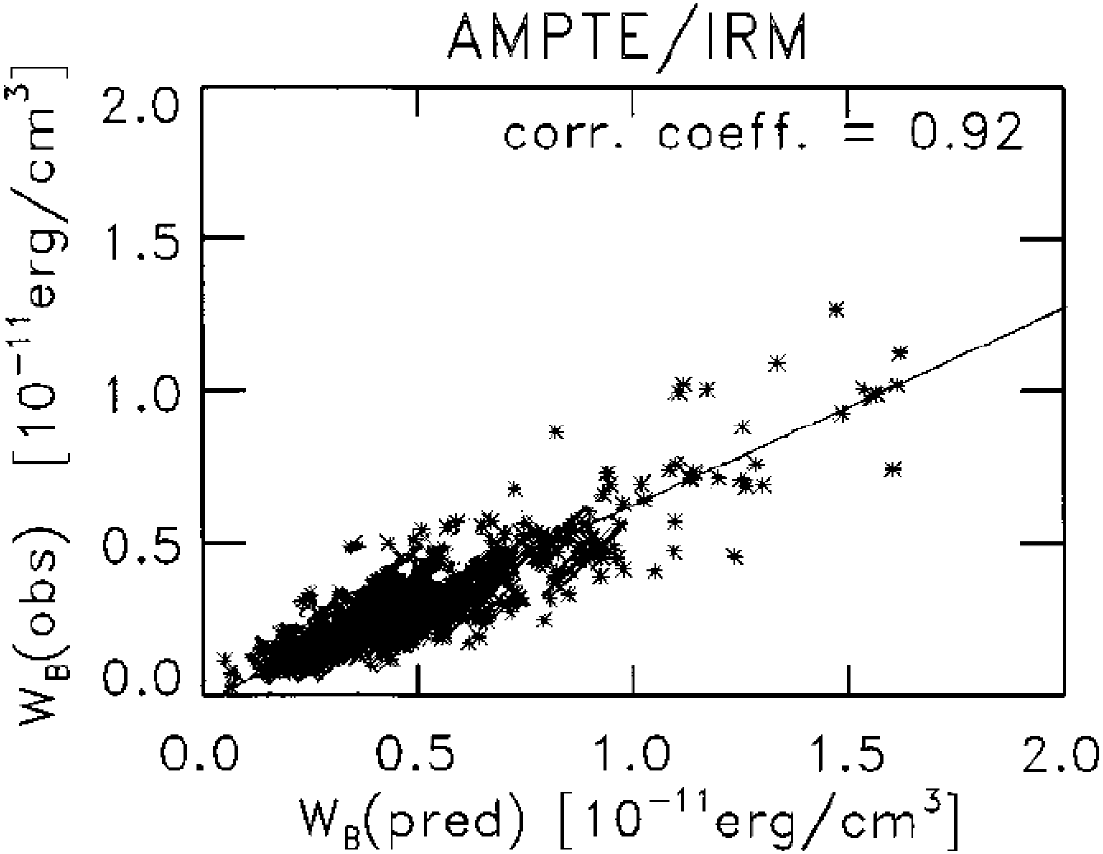
$\beta = 7; \; I_0(k) \approx 0$



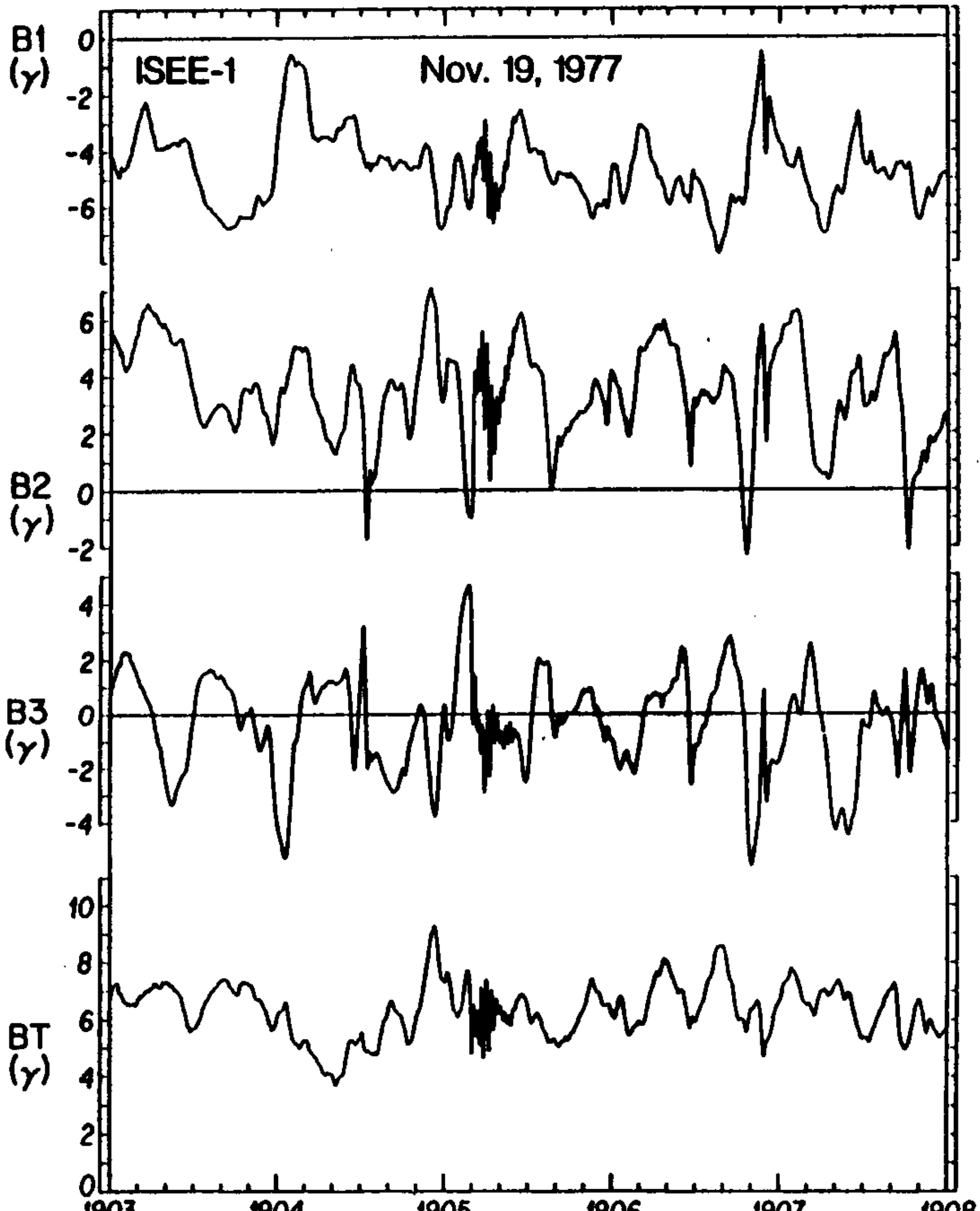


Waves Upstream of Earth's Bow Shock

$$W_B = \frac{1}{3}\frac{V_A(\hat{e}_b \cdot \hat{e}_g)}{V_{sw}(\hat{e}_z \cdot \hat{e}_g) - V_A(\hat{e}_b \cdot \hat{e}_g)} W_p$$

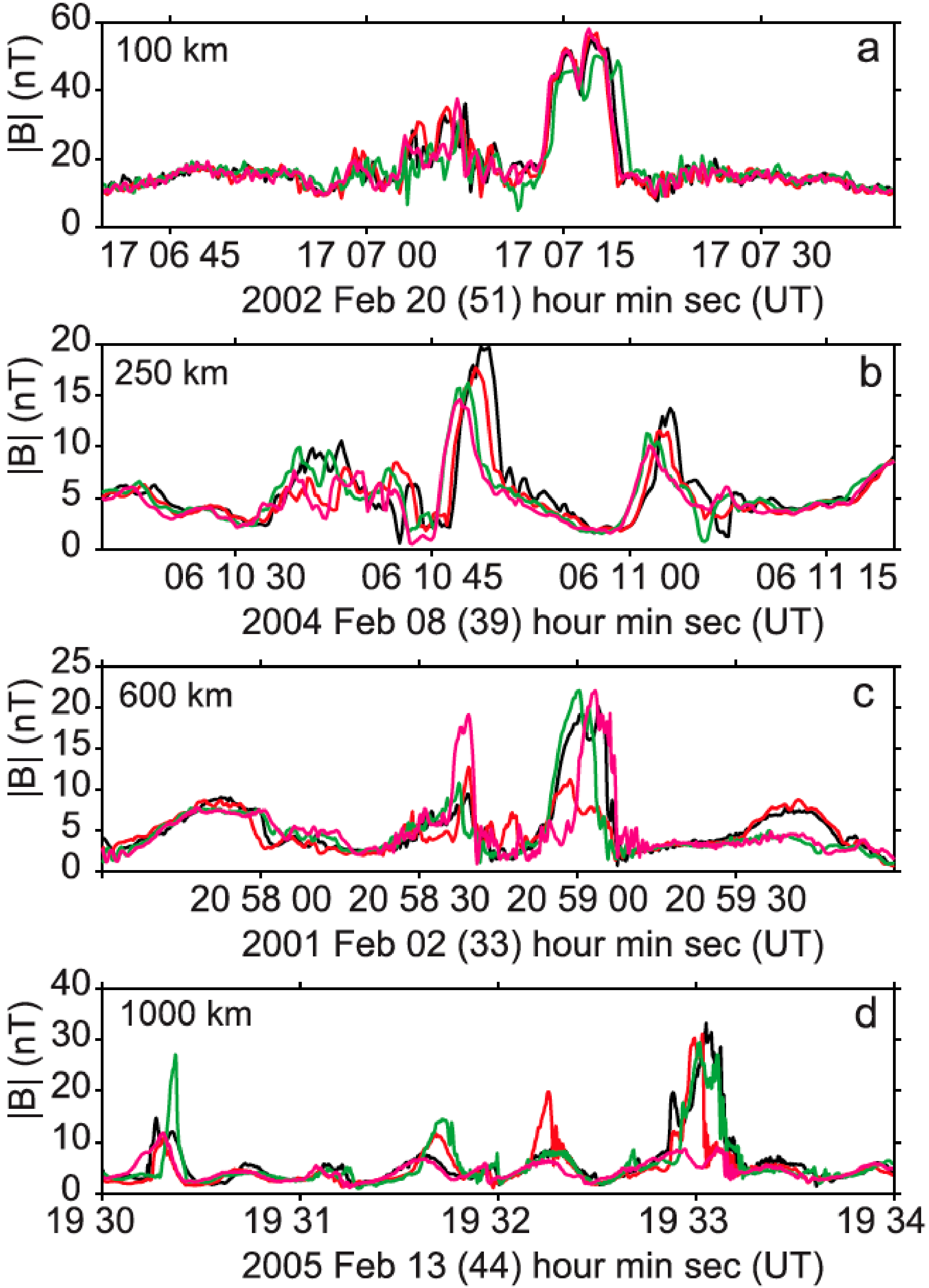


Gordon et al., 1999



Upstream Waves

Hoppe et al., 1981

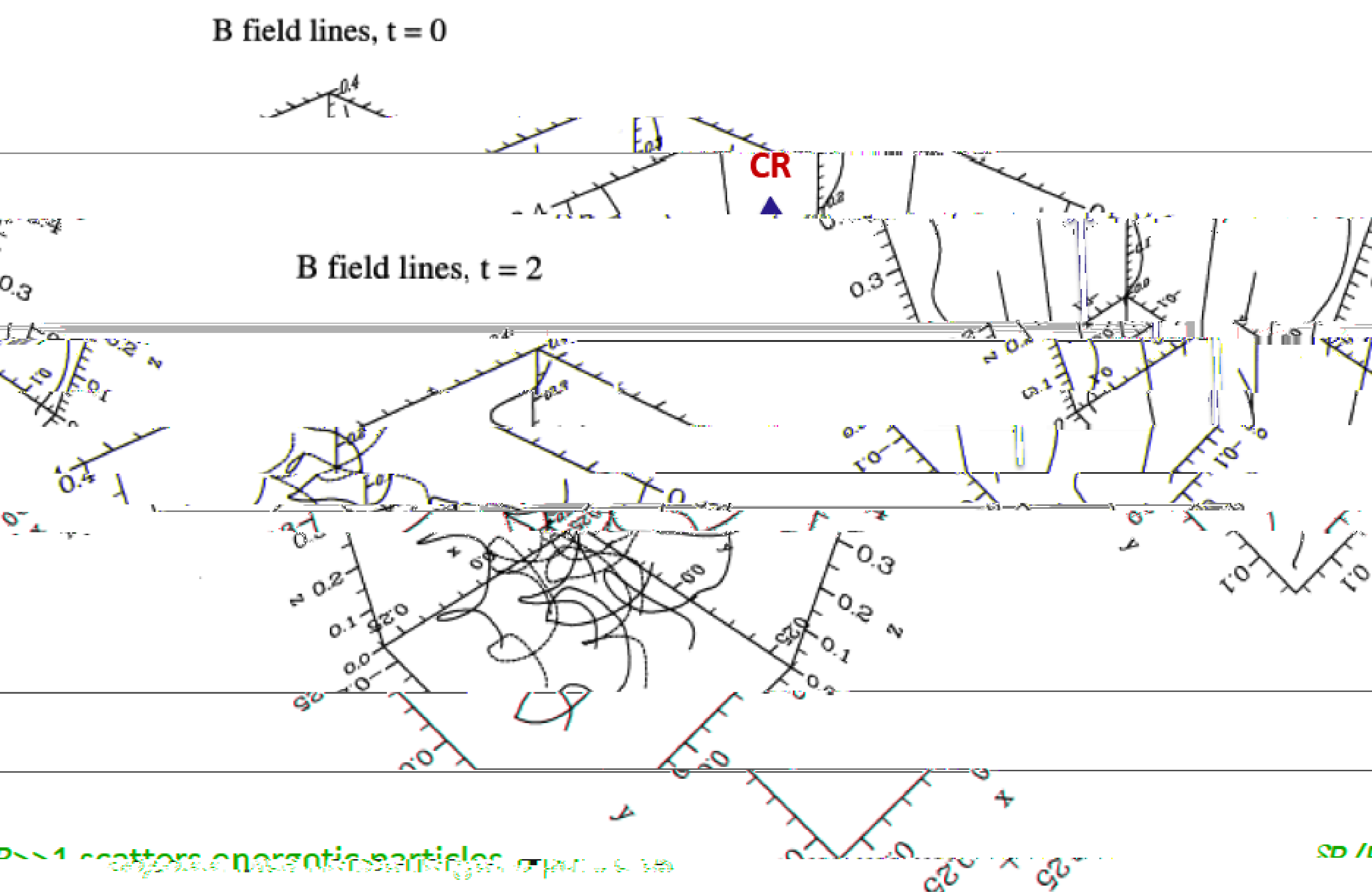


SLAMS

Lucek et al., 2008

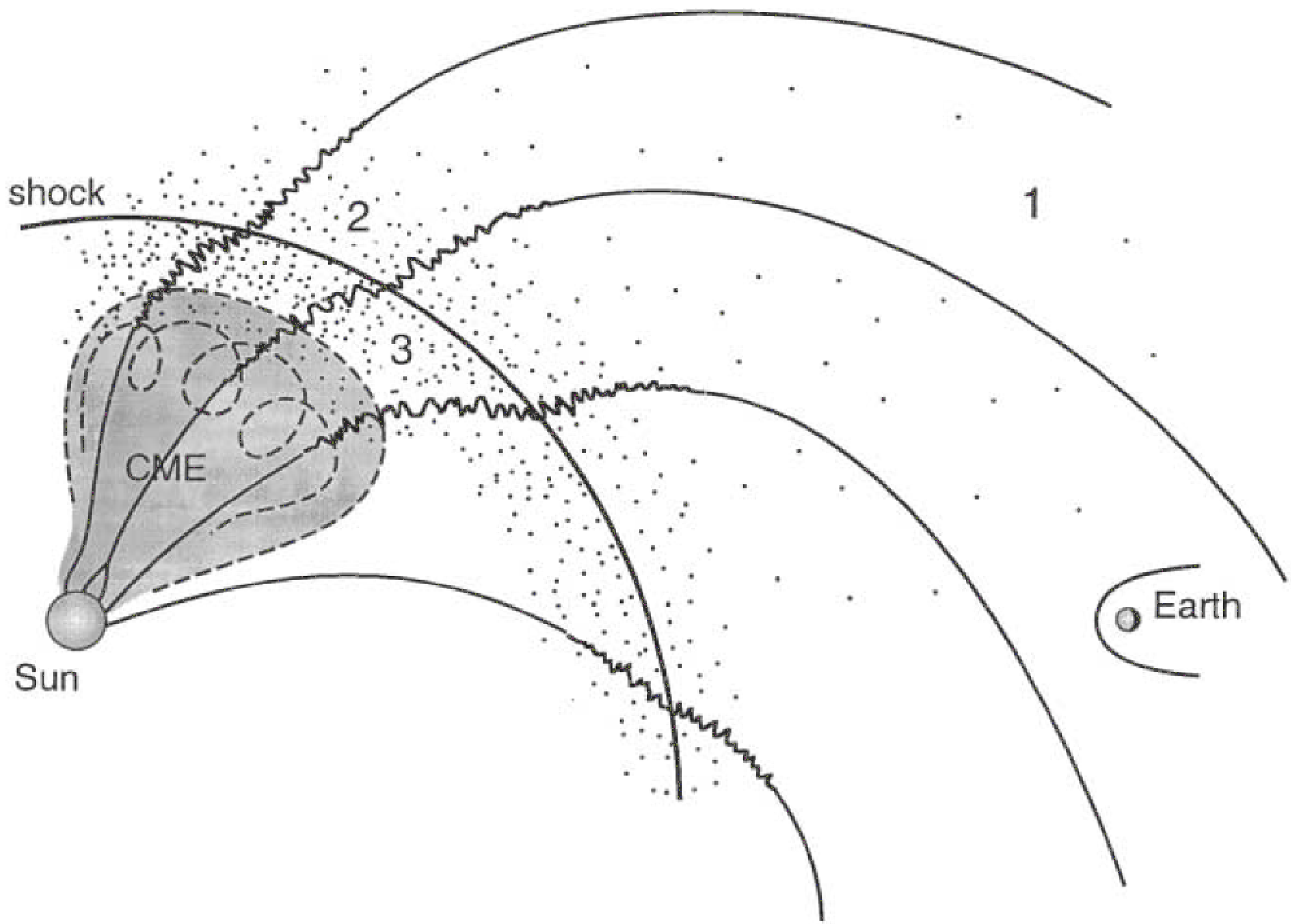
Streaming instability driven by cosmic rays

Lucek & Bell 2000



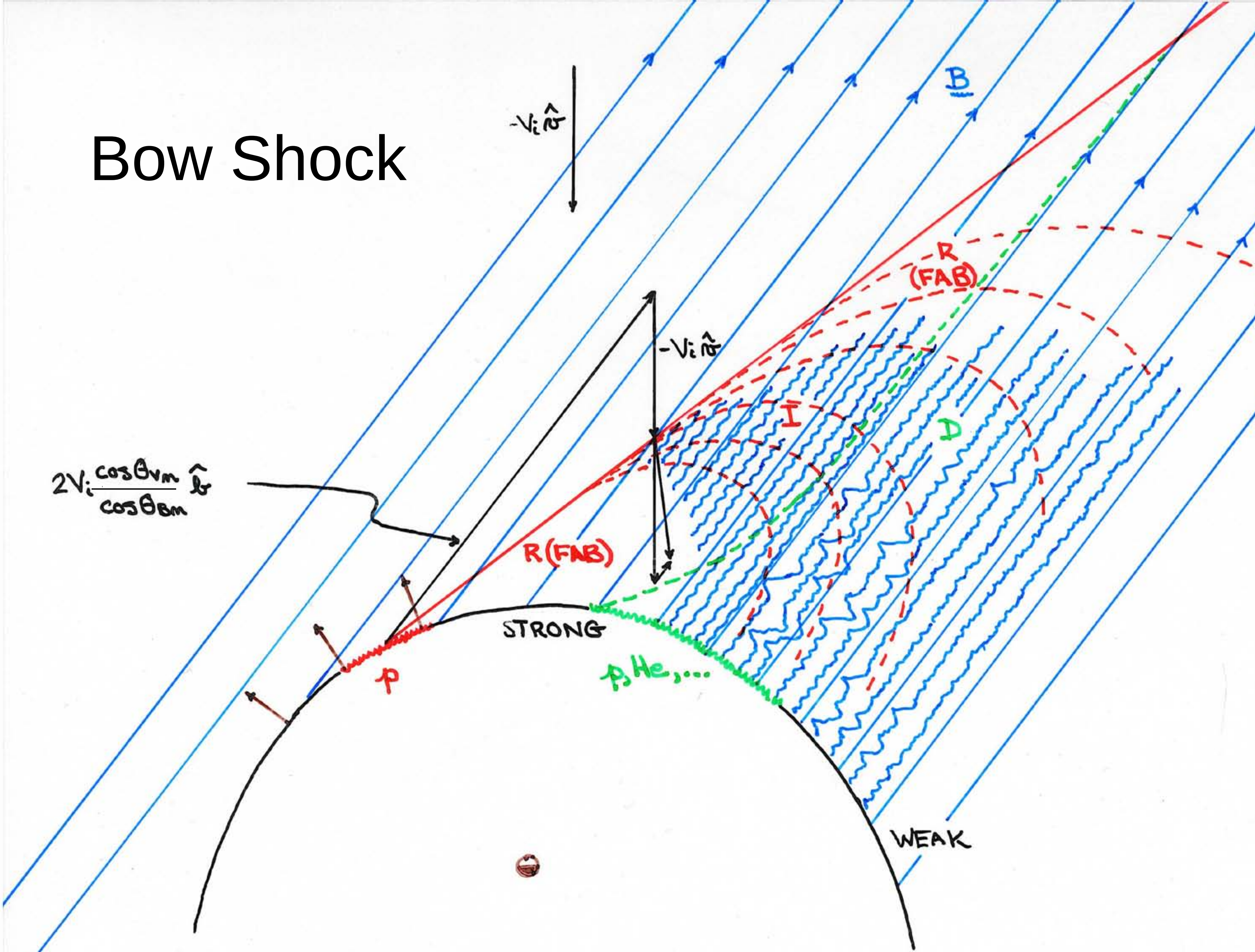
6. Applications of DSA

Acceleration at a CME-Driven Shock

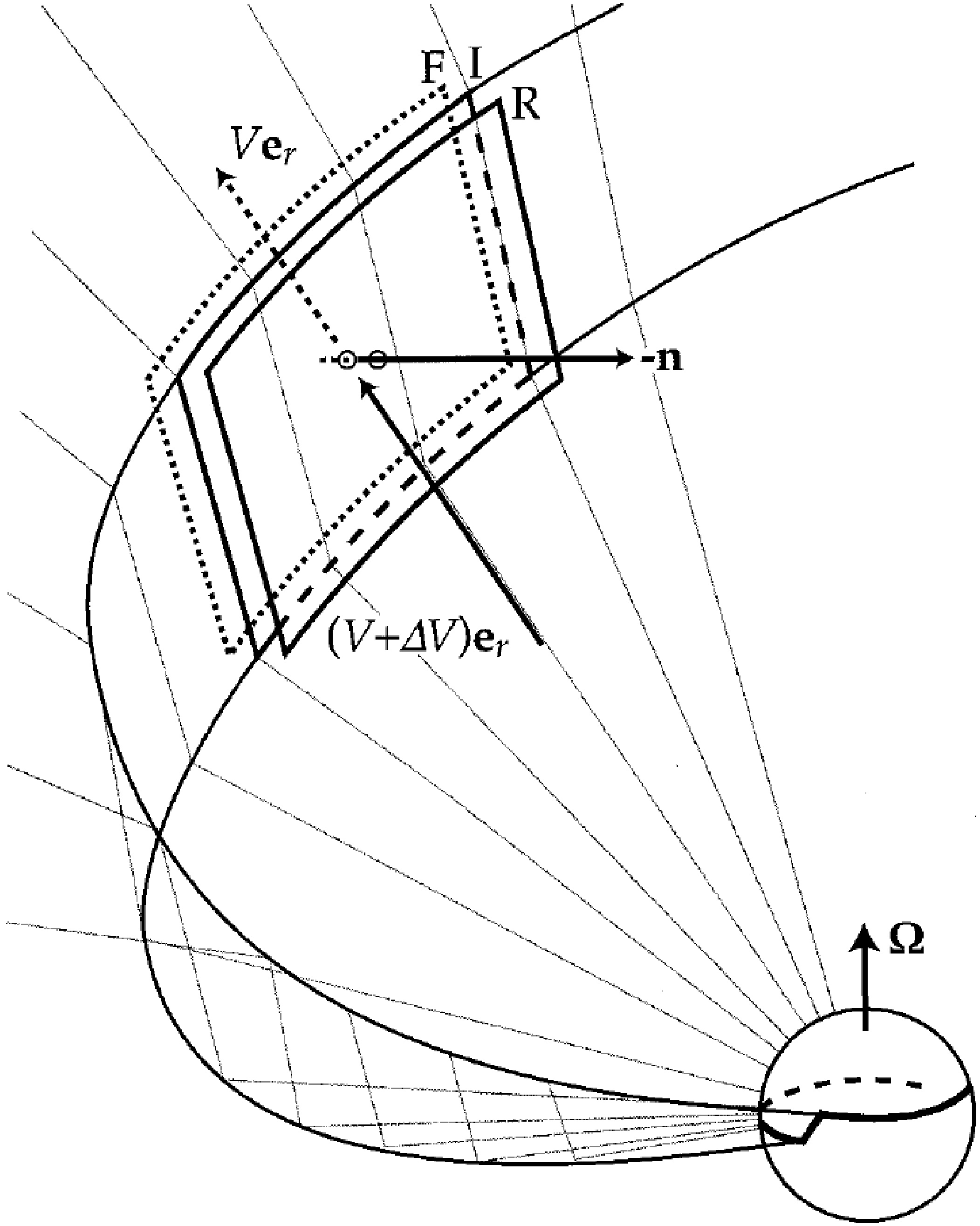


Lee, 2005

Bow Shock



$-V_i \hat{v}$
B
R
(FAB)
$-V_i \hat{v}$
I
D
$2V_i \frac{\cos\theta_{vn}}{\cos\theta_{Bn}} \hat{b}$
R (FAB)
STRONG
p
p, He,...
WEAK

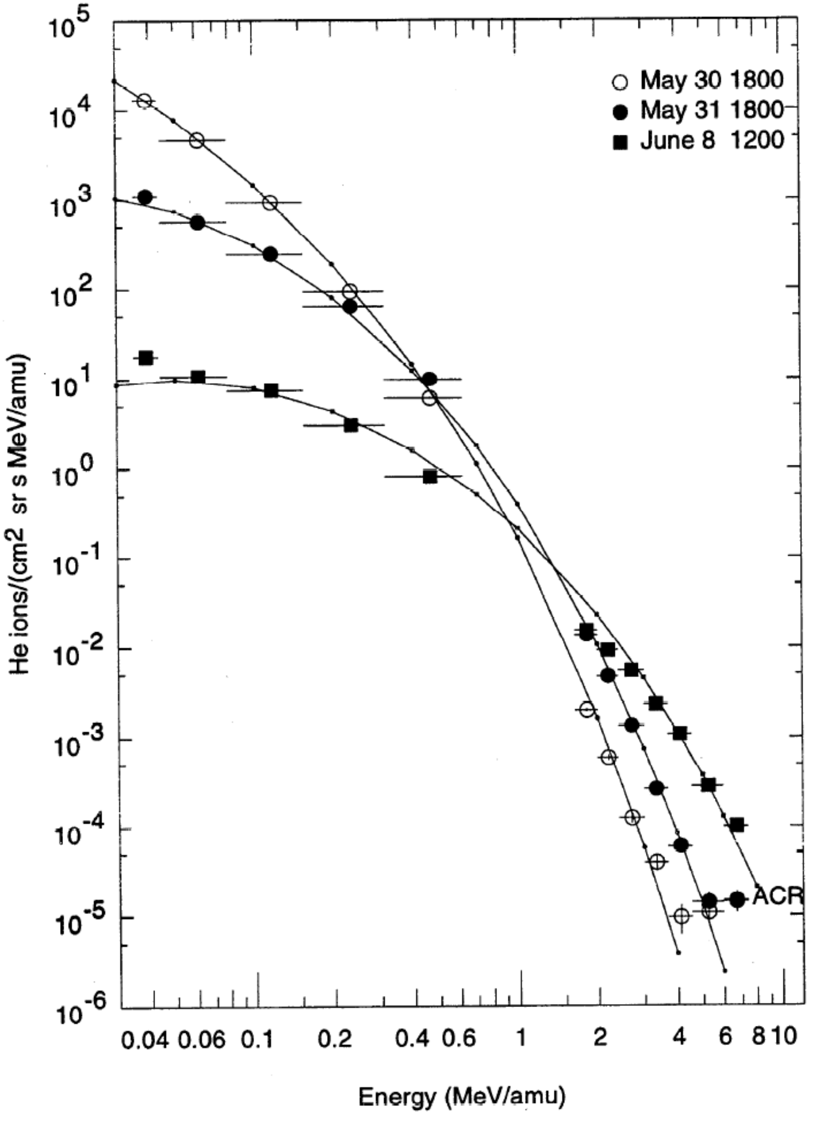


CIR Geometry

Corotating Ion Events

$$f \sim (r/r_s)^{(2/(R-1))+V/(\kappa_0 v)}$$
$$\times v^{-3R/(R-1)} \exp[-6\kappa_0 vR/(V(R-1)^2)]$$

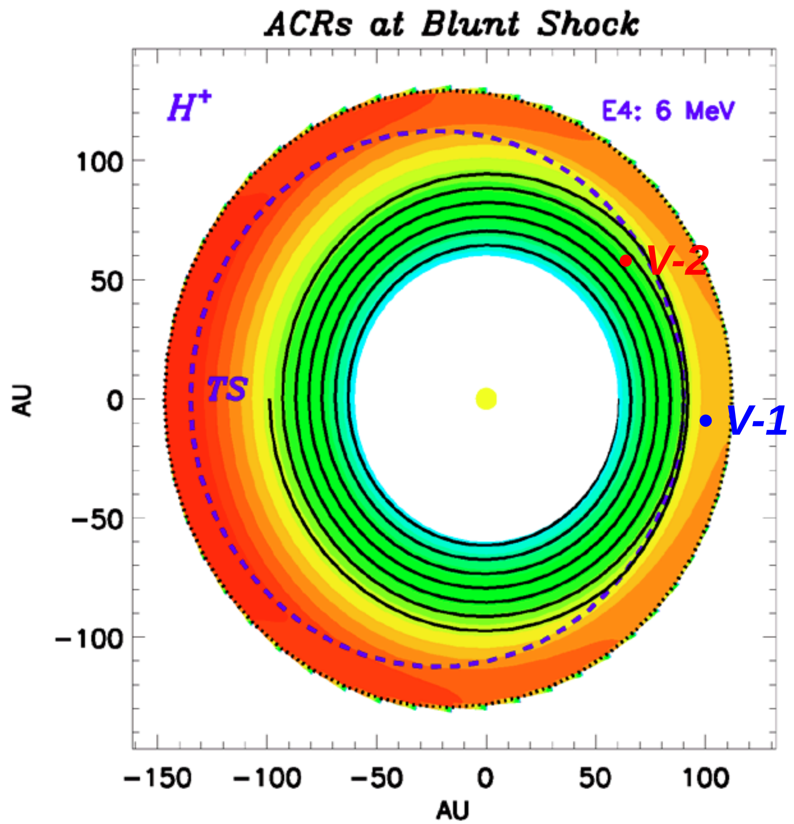
Fisk and Lee, 1980

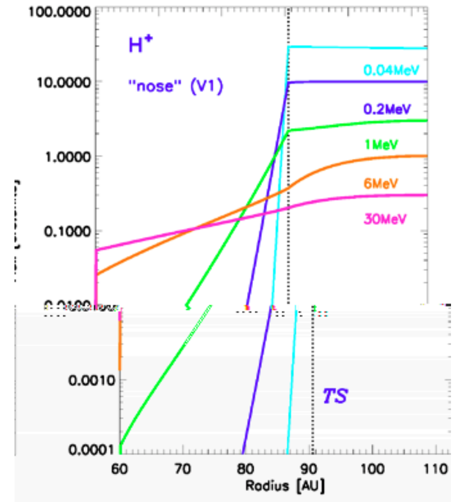


Reames et al., 1997

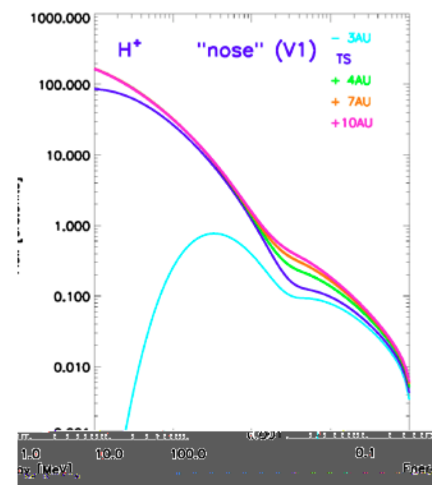
Blunt Shock:
2D Simulation for ACR energies

TS is offset circe,
small cross field diffusion: η=0.02

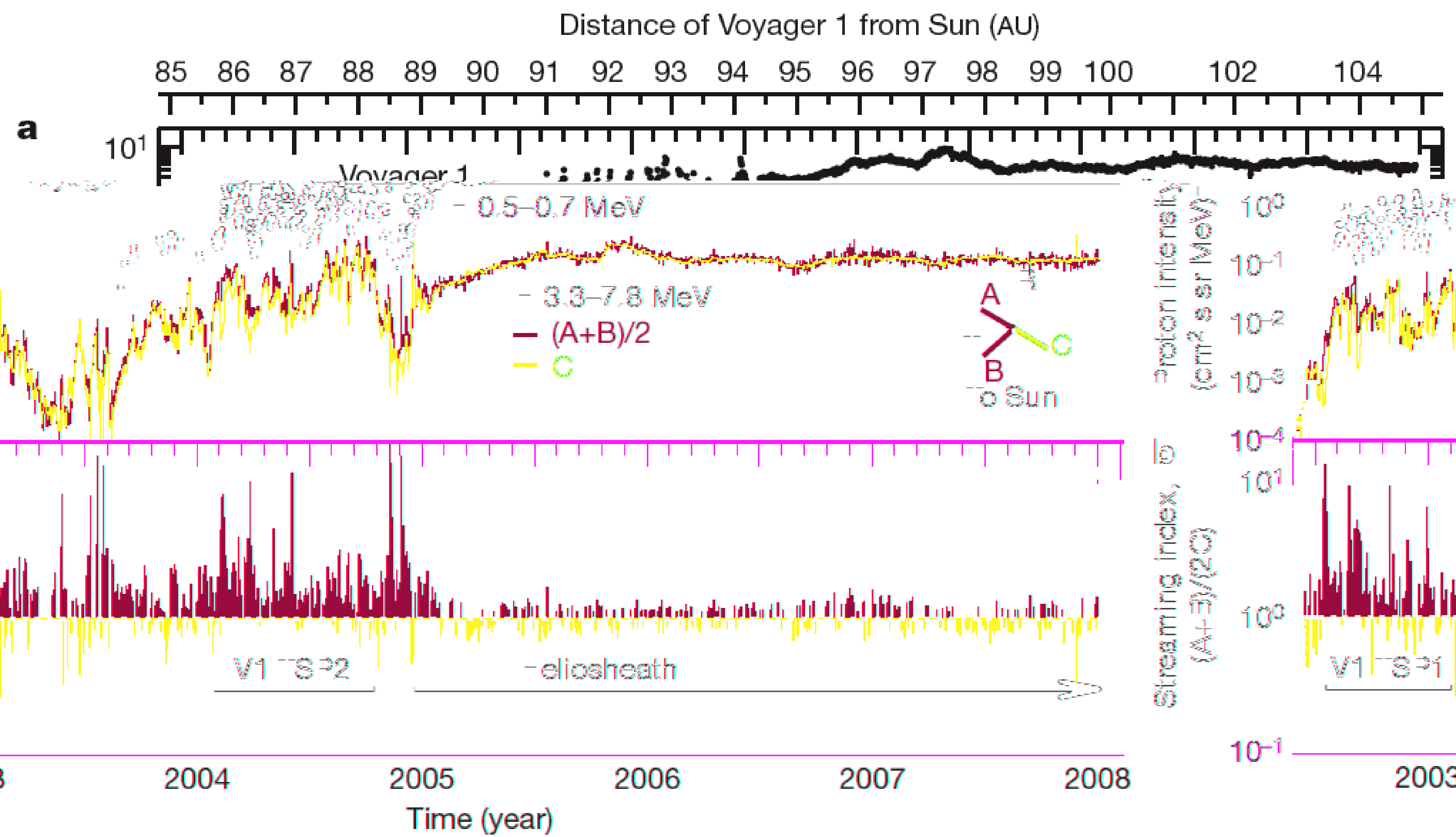




ACR flux increases into the Heliosheath



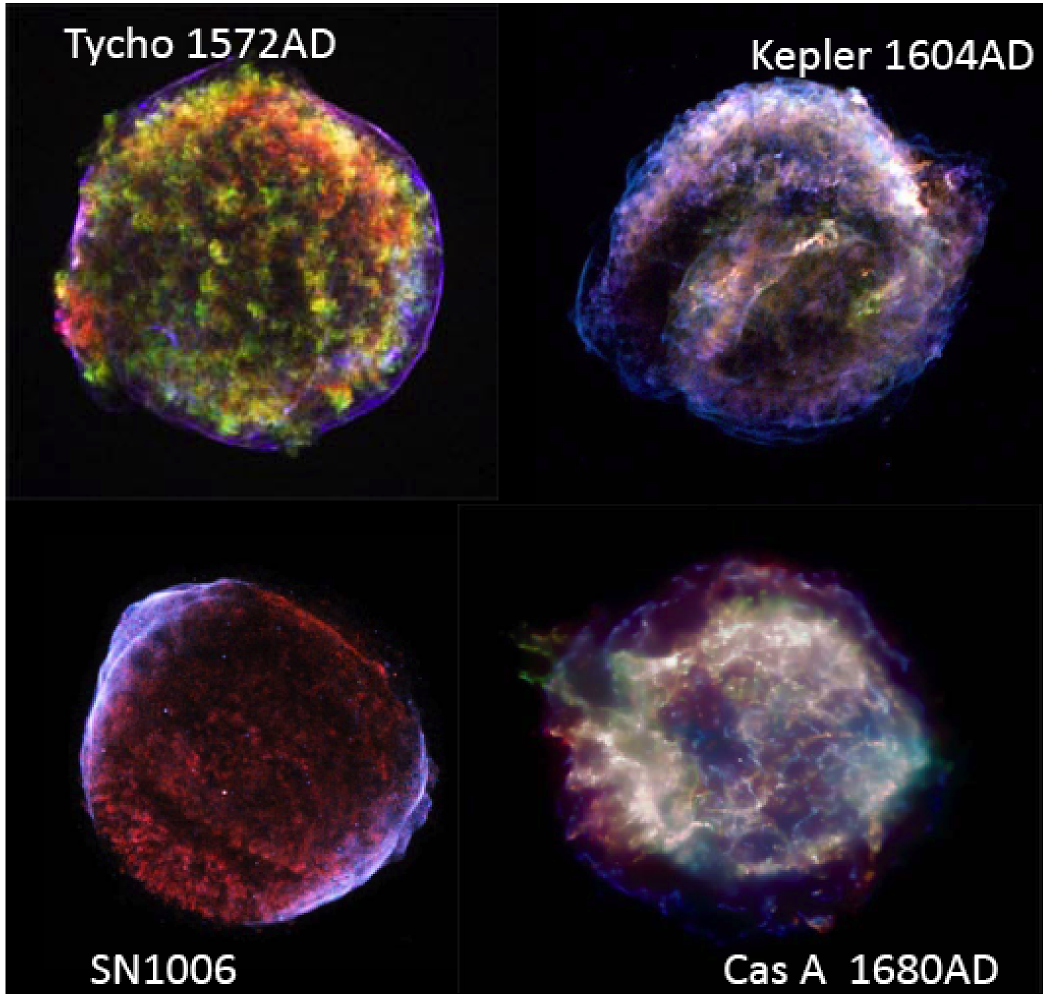
Spectrum gradually unfolds



Stone et al., 2008

Evidence for magnetic field amplification at shock

(Vink & Laming, 2003; Völk, Berezhko, Ksenofontov, 2005)



JMass Amherst/

NASA/CXC/Rutgers/ J.Hughes et al.

NASA/CXC/Rutgers/ J.Warren & J.Hughes et al.

NASA/CXC/NCSU/ S.Reynolds et al.

NASA/CXC/MIT/U M.D.Stage et al.