Comments on "The Goddard Coastal Wave Model. Part I: Numerical Method"*

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1. Introduction

Lin and Huang (1996a, henceforth denoted as LH) discuss numerical methods for third-generation wind wave models. One of the justifications for this study is the concern that numerical errors might be mistaken for physical properties. Having advocated such studies, we greatly appreciate the effort of LH. Unfortunately, LH have not adequately discussed and referenced previous work, as will be shown in section 2. They present a fourth-order scheme for a "transport equation." The derivation of this equation appears to include an error, making the scheme valid for uniform and steady depths and currents only (section 3). Because the alternative "conservation equation" is generally more suitable for wave models (e.g., LH), we will not discuss the effects of this error in detail. Lin and Huang furthermore present two second-order schemes for the conservation equation. Unfortunately, the discussion of these schemes is incomplete and inconsistent with the discussion of previous work (section 4).

2. Previous work

Lin and Huang start with a brief discussion of the need for accurate wave modeling in the coastal zone, following a logical line of evolution from deep-water deep-ocean models to smaller scales. The third-generation WAM model (WAMDI Group 1988; Komen et al. 1994) is accepted as the state of the art. To isolate numerical errors, source terms are neglected, and pure propagation is considered.

Lin and Huang consider two types of equations: a transport equation [their Eq. (5)] and a conservation equation [their Eq. (33)]. Indeed, valid equations of both types can be derived and have been used in wave models. From a numerical point of view it is therefore irrelevant that we disagree with LH that their Eq. (5) is used in WAM. The corresponding discussion will be deferred to the ensuing comments on Lin and Huang (1996b).

Lin and Huang discuss previous numerical work in sections 1 and 2b(1), stating that only two propagation approaches have been used in WAM [first-order upwind (WAMDI Group 1988) and modified ICN (Tolman 1991)]. They do not mention the leapfrog scheme (WAMDI Group 1988), the SHASTA scheme (Tolman 1992), the third-order upwind scheme (Bender and Leslie 1994; Bender 1996), and the ULTIMATE QUICKEST (UQ) (Tolman 1995). Moreover, if source terms are neglected altogether, older models (e.g., SWAMP 1985[1]) and studies considering pure propagation (e.g., Neu and Won 1990) become equally relevant. To assess the relevance of these schemes for the study of LH, we will discuss the schemes as previously applied in third generation wave models in some detail. We will also use this discussion to point out some erroneous references and claims in LH.

The simplest scheme available is the first-order upstream scheme. As demonstrated by LH, this scheme includes a significant second-order truncation error resulting in an unacceptably strong numerical diffusion. Contrary to the claim on p. 840, however, the scheme as applied in WAM is conditionally stable when applied to either a transport or conservation equation (we could not find sup-

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[1] Several models in this study use advanced propagation schemes. Unfortunately, details are often difficult to trace.

port for this claim in the quoted paper). This numerical diffusion can only be reduced by adopting second- or higher-order schemes. In such schemes numerical dispersion errors generally become dominant. These errors manifest as spurious oscillations. Such oscillations are unacceptable in a wave model as they result in the prediction of negative wave energy. Even if the dispersion error is negligible, spurious oscillations related to the Gibbs phenomenon can occur (e.g., Boris and Book 1975).

The first published higher-order scheme in a WAM type model is the leapfrog scheme (WAMDI Group 1988). To avoid negative wave energy due to spurious oscillations, this scheme was augmented with strong diffusion. The resulting algorithm is nearly as diffusive as the first-order scheme (WAMDI Group 1988), and can therefore righteously be ignored in LH.

The second scheme is the modified ICN scheme (Tolman 1991), as implemented in the original version of the WAVEWATCH model (although LH on pp. 833–835 repeatedly suggest that Tolman works with WAM). The main drawback of the ICN scheme is that the underlying algorithm is unstable. Tolman presents several modifications to this scheme that effectively stabilize it. Whereas the modified ICN scheme is a significant improvement over the first-order scheme, it still includes significant diffusion and requires a tuneable parameter to assure stable interactions with strong source terms. Whereas the basic critique of LH on the ICN scheme is correct (i.e., its underlying instability), they seriously misrepresent it because (i) their Eq. (34) is not ''the'' ICN scheme because it ignores the predictor-corrector nature of this scheme [Tolman 1991, Eq. (22)], and because the quoted equation is an adaptation used in physical space only. The modified ICN scheme for the spectral spaces is significantly different (Tolman 1991, p. 787). (ii) Lin and Huang fail to mention that within the full algorithm the minimum absolute value of the tunable parameter α is 0.1 [Tolman 1991, Eqs. (25) and (26)]. (iii) When applied to a conservation equation this scheme conserves total action or energy (as will be discussed below). Lin and Huang's rendition of this scheme in their Fig. 3 more than doubles the total energy.

Not being satisfied with the modified ICN scheme, Tolman (1992) replaced it with a version of the SHASTA scheme of Boris and Book (1973). This scheme is specifically designed for conservation-type equations, is stable, and exhibits small dispersion and diffusion errors (Boris and Book 1973, Tables I–III). The flux corrected transport (FCT) algorithm included in SHASTA furthermore guarantees positive definite model behavior. The implementation of SHASTA furthermore does not include tuneable parameters like α in the modified ICN scheme. The SHASTA scheme thus presents a major improvement over the modified ICN scheme and should therefore have been included in the discussion of previous work and in the comparison with the new numerical schemes of LH.

Unfortunately, LH appear to be under the erroneous impression that the the ICN and SHASTA schemes are either similar or that Tolman (1992) applies an FCT algorithm to the ICN scheme (p. 840, right column). They furthermore misrepresent the FCT method as ''. . . nothing but a numerical diffusion.'' (p. 840, right column). For a comprehensive description of the FCT method (including its drawbacks) reference is made to Boris and Book (1975). Figure 6 of Tolman (1992) shows that the replacement of the first-order scheme by SHASTA effectively removes the numerical diffusion from the model without introducing spurious oscillations. It is therefore surprising that LH claim that Tolman (1992) ''. . . recently pointed out problems with the numerics of WAM, *but offered no remedy.*'' (pp. 833–834, italics added).

Recently (Tolman 1995), the SHASTA scheme in WAVEWATCH has been replaced by the more accurate UQ scheme (Leonard 1979, 1991). Higher-order schemes have also been introduced in WAM (Bender and Leslie 1994; Bender 1996). These latest developments occurred (partially) in parallel to the work of LH.

3. Transport equation

In section 2a, LH derive a new fourth-order accurate scheme for the transport equation. The derivation of the new scheme appears correct up to their Eq. (24). This intermediate version of the scheme still includes partial derivatives $\partial(c_g + u)/\partial x$. Subsequently, LH effectively assume that[2] $\partial(c_g + u)/\partial x = 0$, which they claim is justified ''Because the transport equation does not include $\partial(c_g + u)/\partial x$.'' However, the absences of these derivatives in the transport equation by no means implies that they are zero. Consequently, the resulting scheme is formally valid for conditions with $\partial(c_g + u)/\partial x = 0$ only, and is therefore formally not applicable to shallow coastal areas with currents. Because the transport equation is generally inferior to the conservation equation (see LH and section 4) and rarely (if ever) used in shallow water wave models, we have not investigated the effects of this apparent derivation error in detail.

4. Conservation equation

In section 2b LH discuss numerical solutions of the conservation equation. Before addressing particular schemes, LH make the general comment that the conservation of total action is a serious problem for the conservation equation (p. 840). This is not the case, as can be illustrated with the simple one-dimensional conservation equation

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x} cA = 0, \qquad (1)$$

many numerical schemes can be expressed as for which

[2] LH, unnumbered equation before Eq. (25).

$$A_j^{n+1} = A_j^n + \frac{\Delta t}{\Delta x}(\mathcal{F}_{j-(1/2)} - \mathcal{F}_{j+(1/2)}), \quad (2)$$

where n and j are time and space counters, Δt and Δx are the corresponding discrete increments, and $\mathcal{F}_{j-1/2}$ is the flux cA at the ''cell interface'' between grid points with counters $j - 1$ and j. If the fluxes $\mathcal{F}$ are equally applied to adjacent grid points, the scheme will conserve total action perfectly. Cahyono (1994) compares about 20 such schemes. Furthermore, the first-order scheme of WAM, and the ICN[3] and UQ schemes of WAVEWATCH can be formulated as

$$\mathcal{F}_{j-(1/2)} = \frac{1}{2}(c_{j-1} + c_j)A_u, \quad (3)$$

$$\mathcal{F}_{j-(1/2)} = \frac{1}{2}(c_{j-1}A_{j-1} + c_jA_j), \quad (4)$$

$$\mathcal{F}_{j-(1/2)} = \frac{1}{2}(c_{j-1} + c_j)f(\mu_{j-(1/2)}, A_{j-1}, A_j, \mathcal{CU}_u), \quad (5)$$

respectively, where the suffix u identifies upstream, and $\mathcal{CU}$ represents a three-point estimate of the curvature of A. The SHASTA scheme has a geometric design equivalent to (2) and also conserves total action (Boris and Book 1973). Thus, in contrast to the claim of LH, it is fairly easy to guarantee the conservation of total action if a conservation equation is used. In fact, this guaranteed conservation of total action is probably the single most important reason for using a conservation-type equation instead of a transport-type equation in a numerical wave model.

In section 2b(2) LH present two new second-order schemes A and B and a single stability analysis for both schemes. The presentation of these schemes is incomplete because (i) the stability analysis is not representative for scheme A and (ii) LH do not recognize that both schemes can produce negative energy. To illustrate this in a simple way, we will consider idealized conditions where the CFL number $\mu = (c_g + u)\Delta t/\Delta x$ is constant throughout the domain (e.g., linear propagation of low-steepness swell in deep water without currents). If the schemes fail in these conditions, they cannot be expected to give good results in more complicated situations.

In the above idealized conditions, schemes A and B reduce to [replacing ''factor'' in (35) with f],

$$A_j^{n+1} = (1 - \mu + 0.5f\mu^2)A_j^n + (\mu - f\mu^2)A_{j-1}^n + 0.5f\mu^2A_{j-2}^n, \quad (6)$$

$$A_j^{n+1} = (1 + 0.5\mu)^{-1}[(1 - 0.5\mu)A_j^n + 0.5\mu(A_{j-1}^{n+1} + A_{j-1}^n)], \quad (7)$$

respectively. A conventional stability analysis for Eq. (6) results in

$$\Lambda = 1 + \mu(e^{-ik\Delta x} - 1) + 0.5f\mu^2(e^{-ik\Delta x} - 1)^2, \quad (8)$$

which after some straightforward manipulations can be expressed as

$$\Lambda = 1 - \mu + \mu C\cos(k\Delta x) + i\mu C\sin(k\Delta x),$$
$$C = 1 - f\mu[1 - \cos(k\Delta x)], \quad (9)$$

where k is the wavenumber of a Fourier component of the propagated distribution. A quick (though not necessarily complete) impression of the stability of this scheme can be obtained by considering the worst possible numerical resolution $k\Delta x = \pi$, for which

$$\Lambda = 1 - 2\mu + 2f\mu^2. \quad (10)$$

This implies that scheme A can at best be stable for $0 < \mu < f^{-1} \approx 1.4$ (for typical values of f given by LH). In conditions where μ varies in space, stability requirements can only become more stringent.

A conventional stability analysis of Eq. (7) indeed results in an amplification factor identical to LH Eq. (38) with μ constant. Thus, scheme B indeed is unconditionally stable in conditions with constant μ. However, for nonlinear waves or variable depths and currents where μ varies, stability requires that $a \geq 0$ in LH Eq. (38) (LH, p. 843), which in turn implies the stability requirement

$$\mu_j \geq \mu_{j-1}\cos(k\Delta x). \quad (11)$$

For well-resolved wave fields with $\cos(k\Delta x) \rightarrow 1$ and decreasing CFL numbers $\mu_j < \mu_{j-1}$ stability is therefore not guaranteed. Although this potential instability might not be important in practical models, scheme B cannot be considered ''unconditionally stable'' either.[4]

Furthermore, both schemes require $\mu \geq 0$. As long as the sign of μ does not change in the computational domain, this is not a problem, because a stable computational direction can be assigned. For oscillating currents perpendicular to the wave propagation direction or in conditions of wave blocking, μ will change sign in the computational domain. We invite Lin and Huang to explain how schemes A and B can be applied in such conditions.

Positive definite behavior of the schemes (6) and (7) requires that all numerical coefficients are positive. In (6) the second term thus requires that $\mu < f^{-1}$, which corresponds to the stability criterion. In (7) the multiplication factor for A_j^n requires that $\mu \leq 2$ to avoid negative solutions.

Considering the above, LH misrepresent previous work and have not presented a complete analysis of their new schemes. In doing so, LH present schemes A and B as the only acceptable schemes for shallow water wave models using a conservation equation. This position is not supported by facts. First, LH apply con-

[3] Predictor only.

[4] If this instability ever proves important in a model, it will be an unconditional instability because it cannot be removed by reducing the time step.

flicting standards in the discussion of their schemes. Both schemes incorporate computational dissipation and diffusion as a function of μ (p. 843, in contrast to the claim of LH on p. 834). Whereas such errors are said to be unacceptable for all previous work, they are considered acceptable in the context of schemes A and B. Second, LH fail to identify the nonconservative nature of their schemes as a disadvantage not shared by many other schemes and instead present an artificial and subjective filter as a major advantage and innovation. Third, LH claim that large time steps allowed by their schemes is a major advantage (p. 834). In fact, scheme A requires $\mu < 1.4$ for stability, and scheme B requires $\mu < 2$ for positive definite solutions.[5] Lin and Huang furthermore appear to suggest that $\mu \leq 0.5$ for reasons of accuracy (p. 843). Such CFL numbers and therefore time steps are similar to those of other advanced higher-order schemes (e.g., Fletcher 1988).

5. Discussion and conclusions

Lin and Huang have presented several numerical methods for solving the spectral wave transport and conservation equations. The present paper shows that LH have presented an incomplete picture of their schemes and have misrepresented previous work. Their second-order schemes for the conservation equation nevertheless appear a significant improvement over the conventional first-order scheme and are probably preferable over the modified ICN scheme of Tolman (1991). Unfortunately, LH do not present the more appropriate comparison with the SHASTA scheme as used by Tolman (1992). This scheme and more recent higher-order schemes as mentioned in section 4, as well as advanced methods for solving conservation equations in other disciplines (e.g., Falconer and Cahyono 1993; Cahyono 1994), might well outperform the new schemes by LH. This, however, can only be established in a thorough side-by-side validation study, which is out of the scope of the present discussion.

It should nevertheless be noticed that the performance of LH's new scheme depends upon the application of an artificial and subjective filter to assure conservation of total action. Most other advanced schemes do not need this filter, as they implicitly conserve total action. It therefore remains to be seen if the new schemes should be used at all, even if they would prove to give quantitatively similar results in idealized test cases.

6. Postscript

From this discussion and the accompanying reply by Dr. Lin it is obvious that we disagree on many points. In our opinion, this disagreement is largely based on erroneous claims by Dr. Lin. For many claims, we have not been able to find support in the references provided by Dr. Lin. Furthermore several statements appear to be erroneously attributed to us, and we have not been able to reproduce most of the results that Dr. Lin attributes to ''our'' models and schemes. We have chosen not to discuss this in our comments, as it would seriously distract from our reservations with the original papers. Considering the above, we urge the reader not to take any statements in the comments and replies at face value, but to independently check all arguments, references, and results.

[5] Based on an analysis for constant μ. Actual requirements might be more stringent.

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