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A Method for Determining Equivalent Depths of the Atmospheric Boundary Layer Over the Oceans

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The concept of an equivalent depth of the atmospheric boundary layer is discussed in the framework of vertically integrated boundary layer equations. A method is presented by which these depths may be computed from sea level pressure analyses and ocean surface wind speed measurements. An explicit representation that yields a realistic value for the equivalent depth from the equator to the pole is given in terms of these parameters. For this study, the equivalent depths were calculated from 3 days of altimeter wind speed data taken from Seasat. This study shows that over the mid-latitude and polar regions and in the range of surface wind speeds between 5 m/s and 15 m/s, the equivalent depths are calculated to be about 700–800 m, comparable to the heights of the atmospheric boundary layer; in the polar region, both scale heights are also quite comparable in values, ranging between 200 and 600 m; over the tropics, however, the conventional scale heights, as is well known, are unreasonably large, whereas the calculated equivalent depths are quite realistic and well defined. It is therefore concluded that the method presented in this study may be used to infer heights of the atmospheric boundary layer over the global oceans. In addition, the calculated equivalent depths together with the measured surface wind speeds provide an estimate of the spin up time for the boundary layer flows. During the 3-day period, the spin up times are found to vary between 4.5 hours in the polar regions to about 39.4 hours in the mid-latitudes.

1. INTRODUCTION

In a previous study, *Yu* [1987] proposed a technique to deduce wind directions from the Seasat altimeter and scatterometer wind speed measurements. This technique is based on simple vertically integrated Ekman boundary layer dynamics and uses the sea level pressure analyses together with satellite-measured wind speeds. Implied in this technique is a parameter, hereinafter referred to as "equivalent depth of the atmospheric boundary layer," $\hat{h}$. The equivalent depth is defined as $\hat{h} = \int_0^h A\,dz/A_0$, where h is the height of the atmospheric boundary layer above which the effect of the surface stress becomes negligible and A_0 refers to the surface value of the quantity A. Thus the equivalent depth is a characteristic length scale which together with the surface value of any quantity A enables one to estimate the integral value of that quantity within the atmospheric boundary layer.

For numerical modeling of atmospheric boundary layer flows, the equivalent depth of the boundary layer is an important scaling parameter in determining the momentum flux F across the air-sea or air-land interface. Customarily, F is represented by either a bulk aerodynamic drag law, i.e., $F = \rho_0 C_D |\mathbf{V}|\mathbf{V}$, or a linear Rayleigh friction formulation, i.e., $F = \rho_0 R\mathbf{V}$, where $\mathbf{V}$ is the surface wind vector, C_D is the surface drag coefficient, ρ_0 is air density, and R is the coefficient of Rayleigh friction. In either case, in the framework of vertically integrated boundary layer equations, the net dissipative body force acting on the atmospheric boundary layer due to momentum fluxes across its bottom is the quantity F scaled by some depth representing the thickness of the layer [e.g., *Bannon*, 1979]. In an oceanic context, the depth values used to scale the momentum fluxes at the top and bottom of an oceanic column are usually taken as either the entire depth of the column or the depth of the upper ocean layer above the thermocline. In the atmospheric context, the scale value conventionally used for the height of atmospheric boundary layer is $h = bu_*/f$ (see, for example, *Blackadar and Tennekes* [1968]), where u_* is the surface friction velocity and f is the Coriolis parameter. The value of the numerical constant b varies according to atmospheric stability; typically $b = 0.25$ during neutral stability conditions. This depth scale is perhaps reasonable for studies of middle and higher latitudes, but it certainly becomes unreasonably large and hence not valid for low latitudes where the boundary layer has a finite depth defined typically by the top of the inversion layer. Moreover, if one applies a vertically integrated atmospheric boundary layer system to study the balance of wind and mass fields right above the ocean surface [*Yu*, 1987], the surface momentum flux parameterized by the quantity F should be scaled by the equivalent depth of the layer and not by some predetermined depth of the boundary layer as the h defined above. Therefore under a given synoptic wind condition, the behavior of the bulk boundary layer dynamics depends on an internal parameter, namely, an effective surface drag coefficient, $\hat{C}_D = C_D/\hat{h}$, which is the ratio of the surface drag coefficient and some scale depth representing the characteristics of the layer. One would expect that in the real atmosphere there is some intrinsic depth scale which would make this internal parameter remain well defined throughout the entire range of latitudes. The method discussed in this study will permit one to calculate the values of such a depth scale from a given set of wind speed measurements and sea level pressure analyses and will help in establishing the variability in the depths of the atmospheric boundary layer under various geophysical conditions.

There have been many studies to determine the drag coefficient as a function of wind speed and atmospheric stability (see, for example, *Charnock* [1955] and *Wu* [1969]). These studies at least established the variability in the values of the surface drag coefficient that can be expected over a reasonable range of geophysical parameters. However, so far there have been no studies attempted to determine the equivalent depth of the boundary layer. On the other hand, there have been numerous investigations on the height of the atmospheric

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boundary layer (see, for example, *Blackadar and Tennekes* [1968], *Monin* [1970], *Clarke* [1970], *Melgarejo and Deardorff* [1974, 1975], *Yu* [1978], *Brost and Wyngaard* [1978], and many others). These studies show that over land the atmospheric boundary layer depth may vary from about 100 m during the stable conditions at night to about 2 km during the unstable regimes of the day.

Over the oceans there are fewer observational studies on determining the marine boundary layer depth. *Betts* [1975, 1976] and others analyzed the Barbados Oceanographic and Meteorological Experiment (BOMEX) data and found that the top of the atmospheric mixed layer occurred between 500 m and 1500 m over the tropical oceans. In the middle and higher latitude oceans, *Rogers et al.* [1985] and *Yuen* [1985] show that the atmospheric boundary layer heights are of the order of 1 km or so, corresponding to the bases of the cloud layer. We shall show in this study that the two scale heights discussed here, namely, the equivalent depth and the atmospheric boundary layer height, are approximately equal in middle and higher latitudes but differ substantially in the tropics.

This study presents a method for determining the equivalent depths of the atmospheric boundary layer over the oceans. The method may be equally applicable over land. Section 2 discusses the computational procedure which may apply to all sources of wind speed data including reports from ships and buoys over the oceans and surface reports over land. For this study we shall deal exclusively with the spaceborne wind speed measurements. Section 3 presents results calculated by using 3 days of altimeter wind speed data taken from Seasat. It will be shown that the equivalent depths thus computed are generally of the same order of magnitude as the height of the atmospheric boundary layer in mid-latitude regions. Further, the $\hat{h}$ values are quite realistic and remain bounded in the tropics. In view of the fact that Geosat now operationally provides altimeter wind speed measurements over the global oceans, the method discussed here permits one to routinely compute the equivalent depths of the atmospheric boundary layer. The method should be particularly useful for initialization of atmospheric mixed layer and trade wind models over the global oceans [e.g., *Albrecht*, 1979; *Albrecht et al.*, 1979; *Davidson et al.*, 1984, *Suarez et al.*, 1983; *Rogers et al.*, 1985; *Yuen*, 1985; *Yu*, 1986]. In addition, if one considers a linear time dependent boundary layer problem, the equivalent depths together with values of the surface drag coefficient and wind speed provide an estimate of the spin up time needed to establish a quasi-steady state for the boundary layer flows. Such information should be useful in assessing the effect of transient motions in the boundary layer.

2. PROCEDURE FOR DETERMINING THE EQUIVALENT DEPTH

The method is based on Ekman boundary layer dynamics which assume a balance between the pressure gradient, Coriolis, and frictional forces in the atmospheric boundary layer:

$$-fv = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial F_x}{\partial z}$$
$$fu = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial F_y}{\partial z} \qquad (1)$$

where P is atmospheric pressure; u, v are wind components in the east-west and north-south directions, respectively; and F_x and F_y represent fluxes of momentum for the u and v velocity component, respectively. If we integrate (1) from $z = z_*$ (a small height of typically about 10 m above the ocean surface) to h, the top of the marine boundary layer, we have

$$F_x\big|_{z=z_*}^{z=h} = \int_{z_*}^{h}\left(\frac{1}{\rho}\frac{\partial P}{\partial x} - fv\right)dz$$
$$F_y\big|_{z=z_*}^{z=h} = \int_{z_*}^{h}\left(\frac{1}{\rho}\frac{\partial P}{\partial y} + fu\right)dz \qquad (2)$$

we shall assume that the momentum fluxes vanish at the top of the marine boundary layer (i.e., at $z = h$, $F_x = F_y = 0$) and that the momentum flux at the low boundary (i.e., $z = z_*$) can be represented by a quadratic law in terms of the surface drag coefficient C_D and surface wind speeds, i.e.,

$$F_x\big|_{z=z_*} = C_D|S|u \qquad F_y\big|_{z=z_*} = C_D|S|v$$

where S is the surface wind speed, i.e., $S = (u^2 + v^2)^{1/2}$. Now, one can express the right-hand side of (2) without loss of generality as

$$\int_{z_*}^{h}\left(\frac{1}{\rho}\frac{\partial P}{\partial x} - fv\right)dz = \hat{h}_u\left(\frac{1}{\rho}\frac{\partial P}{\partial x} - fv\right)_{z=z_*}$$
$$\int_{z_*}^{h}\left(\frac{1}{\rho}\frac{\partial P}{\partial y} + fu\right)dz = \hat{h}_v\left(\frac{1}{\rho}\frac{\partial P}{\partial y} + fu\right)_{z=z_*} \qquad (3)$$

where $\hat{h}_u$ and $\hat{h}_v$ are the equivalent depths of the atmospheric boundary layer for the u and v momentum equation, respectively. Note that these equivalent depths are scalar quantities, and as such their values must be the same at a given point on Earth's surface. The reader is referred to the appendix for a detailed derivation which rigorously shows there is only one equivalent depth at any given point on Earth's surface. For the following analysis, we shall set $\hat{h} = \hat{h}_u = \hat{h}_v$.

From (2) and (3), (1) may now be rewritten as

$$-fv = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \hat{C}_D|S|u$$
$$fu = -\frac{1}{\rho}\frac{\partial P}{\partial y} - \hat{C}_D|S|v \qquad (4)$$

where $\hat{C}_D$ is now an effective surface drag coefficient. It is defined as $\hat{C}_D = C_D/\hat{h}$. All the quantities in (4) are evaluated at the height of $z = z_*$ over the ocean surface. Note that under barotropic and well-mixed (convective) conditions where the equivalent depth $\hat{h}$ should be expected to be equal to the height of marine boundary layer h, the wind and pressure fields in (4) represent a balance of forces in a depth-average fluid system affected by bottom friction [*Bannon*, 1979]. It follows from (4) that

$$\hat{C}_D{}^2S^4 + f^2S^2 = \left(\frac{1}{\rho}|\nabla p|\right)^2 \qquad (5)$$

Hence

$$\hat{C}_D = \left[\left(\frac{1}{\rho}|\nabla p|\right)^2 - f^2S^2\right]^{1/2} \Big/ S^2 \qquad (6)$$

From (6), one can uniquely determine the effective surface drag coefficient from given pressure gradient and wind speed measurements and does not require any a priori specification of the surface drag coefficient. From (6), one can write

$$\hat{h} = C_D S^2 \Big/ \left[\left(\frac{1}{\rho}|\nabla p|\right)^2 - f^2 S^2\right]^{1/2} \tag{7}$$

Equation (7) establishes a relationship between the equivalent depth of the boundary layer and wind speed, the surface pressure gradient, and a chosen value of the surface drag coefficient. Therefore if we know the pressure gradient from any conventional meteorological analysis and the wind speed from conventional or spaceborne measurements, we can compute the equivalent depth from (7) once a value of surface drag coefficient is specified. Moreover, it is seen from (7) that the equivalent depth of the boundary layer is well defined at the equator where $f = 0$, and there equation (7) becomes

$$\hat{h} = C_D S^2 \Big/ \left(\frac{1}{\rho}|\nabla p|\right)$$

An important point to note in connection with (7) is that if the wind speed is provided from a measurement, the other variable that is required to determine $\hat{h}$ is the surface pressure field. The latter is an integral of the mass through the depth of an atmospheric column and is not influenced by the details of boundary layer physics. Hence the surface pressure is probably the most reliable analysis field available on a global basis as compared with other meteorological variables from routine numerical weather prediction models.

It should be pointed out that the effective surface drag coefficient in (6) has another important implication related to the spin up time of the boundary layer. If one considers a transient system in the framework of vertically integrated boundary layer equations, one may write the governing equations as

$$\begin{aligned} \frac{\partial u_T}{\partial t} - f v_T &= -\frac{1}{\rho}\frac{\partial P}{\partial x} - \hat{C}_D |S| u_T \\ \frac{\partial v_T}{\partial t} + f u_T &= -\frac{1}{\rho}\frac{\partial P}{\partial y} - \hat{C}_D |S| v_T \end{aligned} \tag{8}$$

where the subscript T represents the transient velocity solution. If $(\partial P/\partial x, \partial P/\partial y)$ is regarded as a steady inhomogeneous term, then from (4), u and v represent the equilibrium steady state solution for the surface velocity field corresponding to (8). Let the departure of the time dependent velocity field from the equilibrium state be u' and v', i.e.,

$$u' = u_T - u \qquad v' = v_T - v$$

From (4) and (8), one may write

$$\frac{\partial W}{\partial t} + ifW + \hat{C}_D S W = 0 \tag{9}$$

where $W = u' + iv'$ and $i = \sqrt{-1}$. If we assume for simplicity that the effective surface drag coefficient is independent of time and that surface wind speed is given by some mean value $\bar{S}$ to linearize (9), the solution satisfying the condition that $(u_T, v_T) = 0$ at $t = 0$ is simply,

$$W = -W_0 \exp\left[-(if + \hat{C}_D \bar{S})t\right] \tag{10}$$

where W_0 represents the complex velocity at the equilibrium state. From (10), one can see that the transient component of velocity decays with an e-folding time of $t = (\hat{C}_D \bar{S})^{-1}$. This e-folding time may be approximately related to the equivalent depth from (6) and (7) resulting in

$$t = (\hat{C}_D \bar{S})^{-1} \doteq S \Big/ \left[\left(\frac{1}{\rho}|\nabla p|\right)^2 - f^2 S^2\right]^{1/2} = \hat{h}/(C_D S) \tag{11}$$

It is clear that the spin up time of the boundary layer to an imposed surface pressure gradient may be estimated if one knows the effective surface drag coefficient as given in (6). The damping time is directly proportional to the equivalent depth and inversely proportional to the magnitude of the surface wind speed. That is, the shallower the equivalent depth or the larger the surface wind speed, the less time it takes for an equilibrium state to be established. This is consistent with the results of *Shaeffer and Doswell* [1980]. By applying Ekman dynamics over land, and using a so called "antitriptic balance" approach, they argued that near the surface contact layer (of the order of 100-m depth), the large damping of the transient effect leads to a rapid establishment of the steady state Ekman solution.

From (7) we see that in order to compute the equivalent depth, one needs to specify the surface drag coefficient. *Garrat* [1977] made a thorough review of previously reported values of surface drag coefficients in relation to the ocean surface winds. He compiled observations of wind stress and wind profiles reported in the literature and found them to be consistent with *Charnock*'s [1955] relation between aerodynamic roughness (z_0) and friction velocity (u_*); that is, $z_0 = \alpha u_*^2/g$, with $\alpha = 0.0144$ and $g = 9.8$ m/s^2. He further argues that for practical purposes, Charnock's relation may be closely approximated (in the range of wind speed between 4 m/s and 21 m/s) by a neutral drag coefficient (referred to 10 m above the ocean surface) varying with the 10-m wind speed in a linear form,

$$C_D = (0.75 + 0.67S) \times 10^{-3} \tag{12}$$

In this study we shall adapt this formulation for surface drag coefficient over the oceans. On the basis of (7) and (12), we see that if the surface drag coefficient is a linear function of surface wind speed, the equivalent depth is proportional to the cubic power of wind speed at low latitudes. In higher latitudes it depends on the square of the wind speed and the ageostrophic contribution of the wind fields.

In general, equivalent depths and heights of the marine boundary layer are not equal, as can easily be seen from (3). Since the ageostrophic contribution due to surface friction is largest near the surface, one should, from (7), expect equivalent depths to be smaller than heights of the boundary layer. The only exception is under purely barotropic and convectively well-mixed conditions where the stress profile is linear and the equivalent depths should be the same as heights of the marine boundary layer. Over the oceans, where typically there exists a well-mixed layer capped by an inversion, one might expect the equivalent depth to be nearly the same as the height of the mixed layer. It should be pointed out that in deriving (7) we neglect the effect due to advection and to local (time) changes in the momentum balance. Both of these will undoubtedly affect values of the equivalent depths. On the other hand, the height of boundary layer, h, defined in our study to be the height at which turbulent stress vanishes, is not necessarily influenced by these effects.

As was discussed earlier, the height of the boundary layer may be estimated by the relationship $h = bu_*/|f|$. Since the surface friction velocity u_* is related to the surface wind speed and drag coefficient by the relationship $u_* = C_D^{1/2} S$ [e.g., *Hasse and Dunkel*, 1974], we shall also calculate heights of the atmospheric boundary layer using the altimeter wind speeds according to the formulation

$$h = bC_D{}^{1/2}\, S/|f| \qquad b = 0.25 \tag{13}$$

The numerical constant of $b = 0.25$ is adapted in this study to be a typical value for neutral stability conditions [*Blackadar and Tennekes*, 1968]. However, its value is still subjected to large degrees of variability under various conditions. For example, recent large-eddy simulation results from a three-dimensional baroclinic boundary layer model [*Mason and Thomson*, 1987] show that the height where the turbulent stress vanishes is of the order of $0.5u_*/f$ for neutral stability conditions. Under the steady, barotropic, stable conditions, the results of *Brost and Wyngaard* [1978] and many others have shown that b should be much smaller than 0.25. Nevertheless, the heights of the boundary layer calculated by (13) should serve as a reasonably good reference for our comparison with the values of the equivalent depth calculated by using the procedure proposed in this study. It should be noted that the formulation for the atmospheric boundary layer according to (13) is reasonable in mid-latitudes but is not valid in the tropics, where the height becomes unbounded as one approaches the equator. In order to apply (13), we shall make use of the β plane approximation for the tropical region, that is $|f| > f_0 = \beta y = 2.5 \times 10^{-5}\ \mathrm{s}^{-1}$, where $\beta = 2.2 \times 10^{-11}$ m/s and $y = 1200$ km are used in this study. This is equivalent to limiting values of the Coriolis parameter to be no less than the value at 10° N.

3. RESULTS FROM SEASAT WIND SPEED MEASUREMENTS

We shall apply (6), (7), (11), (12), and (13) discussed in the previous section to compute the values of the effective surface drag coefficient, the equivalent depth, the spin up time, the surface drag coefficient, and the height of the atmospheric boundary layer. For this study, wind speeds were taken from the altimeter wind speed measurements from Seasat, and the corresponding surface pressure gradients from the National Meteorological Center (NMC) analyses. These 3 days of altimeter data from SEASAT are identical to those used in a previous study of vector retrievals from the altimeter wind speeds reported by *Yu* [1987]. Further, the altimeter wind speed measurements from the satellite were taken at all points along the satellite tracks that fall within 1.5 hours before and after the surface pressure analysis time. On the average, the altimeter measures ocean surface winds every second or so, which can in principle result in a maximum of nearly 10,000 data points for a 3-hour window. Since the computational procedure laid out in equations (6), (7), and (11) depends on the use of NMC sea level pressure fields which are analyzed on a 2.5° by 2.5° longitude-latitude grid, in this study we also applied an averaging procedure as discussed by *Yu* [1987] to obtain the satellite altimeter wind speeds on the same grid. The averaging procedure is such that the weighting of each data point is inversely proportional to the distance between the data and the grid point to which the average will be assigned. Typically about 40 data points are used to generate an average grid point value. The total number of altimeter data after the averaging is about 300 for the Seasat period ($N = 276$ for September 17, $N = 279$ for September 18, and $N = 317$ for September 19, 1978) during the 3-hour window.

Following the previous discussions, we see that equivalent depths and effective surface drag coefficients are functions of three variables, that is, surface pressure gradient, wind speed, and latitude. Since the surface wind speed is somewhat correlated with the surface pressure gradient, we shall group the results into two independent categories, namely, latitude ϕ and surface wind speed S. Further, we shall classify the wind speed into two ranges: light wind speed range for $S < 5$ m/s and medium wind speed range for 15 m/s $> S >$ 5 m/s. During the 3-day periods, there were only a few observations with wind speeds greater than 15 m/s, and these were ignored. Similarly, we shall classify the latitudinal dependency into three regions: polar for $|\phi| > 60°$, mid-latitude for $60° > |\phi| > 20°$, and tropical for $|\phi| < 20°$. From Table 1, one can see that during the 3-day period, the wind speeds do not show a large variation within the two speed ranges. The mean Seasat altimeter wind speed for the light wind speed range is about 2 m/s in the polar region, and about 3–4 m/s in the mid-latitudes and tropics; for the medium wind speed range, the means are about 8–9 m/s from the polar region to the tropics. On the other hand, for each wind speed range, the analyzed NMC sea surface pressure gradients are much larger in the polar region than they are at the tropics. Further, the standard errors (the bracketed values in Table 1) for the NMC pressure gradients are about 10% of the means, which is much larger than those for the altimeter wind speeds. This relatively large uncertainty about the means in the NMC pressure gradient term clearly indicates a large variability within the three latitude categories used for this study and will undoubtedly contribute to larger variances in the other derived quantities.

Before discussing the calculated results of equivalent depths and heights of the atmospheric boundary layer, it is instructive to examine the calculated values of the surface drag coefficient and the effective surface drag coefficient and their dependency on wind speeds and latitudes shown in Table 1. The mean value of the surface drag coefficient is nearly constant ($C_D = 0.0013$) in the wind speed range of 15 m/s $> S >$ 5 m/s. For wind speeds of less than 5 m/s, the values of the surface drag coefficient decrease to about 0.0008–0.0010.

From (6), the values of the effective surface drag coefficient are directly proportional to the departure of surface wind speeds from geostrophy and inversely proportional to the square of surface wind speeds. Hence the calculated values of the effective surface drag coefficient are larger in the light wind category than they are in the medium wind speed range. Moreover, values of the effective surface drag coefficients are calculated to vary between $2.02 \times 10^{-6}\ \mathrm{m}^{-1}$ in the tropics to $8.87 \times 10^{-6}\ \mathrm{m}^{-1}$ in the polar region (see Table 1). The large values in the polar region may be explained by much larger values of the NMC surface pressure gradient term as discussed earlier. It may be pointed out that the NMC surface pressure analysis is produced by updating the forecast model's first guess by surface ship observations of winds and pressures over the oceans through a multivariate optimum interpolation analysis scheme [*Dey and Morone*, 1985]. If the observed altimeter wind speeds represent the true state over the ocean surface, the large values of the effective surface drag coefficient, which are indicative of large departure from the geostrophy in the polar region, would seem to suggest that the first guess of surface pressure from the NMC forecast model is far from being in geostrophic equilibrium with the observed altimeter surface winds. To a lesser degree, the same consideration may also be applied in the tropics. On the other hand, one can

TABLE 1. Means and Standard Errors of the Means (in Parentheses) Calculated for Equivalent Depth $\hat{h}$, Atmospheric Boundary Layer Depth h, Surface Drag Coefficient C_D, Effective Surface Drag Coefficient $\hat{C}_D$, Ekman Spin up Time t, NMC Sea Level Pressure Gradient, and Altimeter Wind Speed S for 3 Days of Altimeter Data From Seasat

Parameter	Sept. 17, 1978, 0000 UT $S < 5$ m/s	Sept. 17, 1978, 0000 UT $15 > S > 5$ m/s	Sept. 18, 1978, 0000 UT $S < 5$ m/s	Sept. 18, 1978, 0000 UT $15 > S > 5$ m/s	Sept. 19, 1978, 0000 UT $S < 5$ m/s	Sept. 19, 1978, 0000 UT $15 > S > 5$ m/s
			Polar Region ($\lvert\phi\rvert > 60°$)			
N	35	40	51	53	63	35
$\hat{h}$, m	190 (44)	579 (88)	156 (30)	341 (56)	110 (13)	197 (36)
h, m	113 (12)	599 (30)	134 (11)	588 (22)	106 (8)	632 (22)
C_D, $\times 10^{-3}$	0.885 (0.014)	1.332 (0.022)	0.971 (0.012)	1.307 (0.017)	0.877 (0.009)	1.355 (0.018)
$\hat{C}_D$, 10^{-5}/m	0.809 (0.043)	0.571 (0.058)	0.846 (0.027)	0.734 (0.041)	0.887 (0.013)	0.839 (0.028)
t, hours	38.9 (10.3)	14.4 (2.2)	21.6 (2.2)	9.9 (1.9)	22.6 (1.7)	4.5 (0.7)
S, m/s	2.01 (0.21)	8.69 (0.32)	2.36 (0.17)	8.48 (0.25)	1.89 (0.13)	9.03 (0.27)
$(1/\rho)\lvert\nabla P\rvert$, 10^{-4} m/s^2	9.329 (0.849)	14.506 (1.094)	11.432 (0.706)	16.679 (0.795)	14.680 (0.845)	17.665 (0.483)
			Mid-latitude Region ($60° > \lvert\phi\rvert > 20°$)			
N	26	105	31	98	28	119
$\hat{h}$, m	359 (78)	831 (52)	483 (81)	712 (58)	312 (70)	720 (51)
h, m	370 (32)	912 (29)	311 (23)	885 (35)	393 (24)	854 (31)
C_D, $\times 10^{-3}$	0.968 (0.012)	1.331 (0.013)	0.974 (0.013)	1.323 (0.012)	0.994 (0.009)	1.304 (0.010)
$\hat{C}_D$, 10^{-5}/m	0.656 (0.069)	0.371 (0.034)	0.565 (0.071)	0.476 (0.038)	0.698 (0.062)	0.437 (0.033)
t, hours	28.5 (5.5)	22.8 (1.6)	39.4 (6.1)	18.6 (1.6)	22.5 (4.6)	19.7 (1.5)
S, m/s	3.25 (0.18)	8.67 (0.19)	3.35 (0.19)	8.55 (0.17)	3.64 (0.14)	8.26 (0.15)
$(1/\rho)\lvert\nabla P\rvert$, 10^{-4} m/s^2	4.266 (0.558)	8.524 (0.611)	4.039 (0.647)	8.910 (0.570)	4.627 (0.647)	8.747 (0.489)
			Tropics ($\lvert\phi\rvert < 20°$)			
N	35	33	15	31	21	50
$\hat{h}$, m	172 (27)	630 (78)	384 (97)	888 (69)	272 (71)	851 (65)
h, m	996 (59)	2214 (116)	915 (90)	2593 (118)	850 (77)	2451 (84)
C_D, $\times 10^{-3}$	0.976 (0.012)	1.259 (0.012)	0.971 (0.021)	1.307 (0.016)	0.959 (0.016)	1.288 (0.011)
$\hat{C}_D$, 10^{-5}/m	0.751 (0.039)	0.360 (0.046)	0.555 (0.089)	0.202 (0.027)	0.693 (0.068)	0.237 (0.025)
t, hours	15.4 (2.3)	18.7 (2.4)	31.3 (7.0)	24.4 (2.4)	22.1 (4.1)	23.0 (1.8)
S, m/s	3.37 (0.18)	7.60 (0.18)	3.30 (0.32)	8.32 (0.23)	3.12 (0.23)	8.03 (0.17)
$(1/\rho)\lvert\nabla P\rvert$, 10^{-4} m/s^2	1.719 (0.167)	2.860 (0.282)	1.118 (0.162)	2.465 (0.306)	1.767 (0.277)	2.279 (0.192)

similarly postulate that in mid-latitudes, where there are more surface observations to update the model first guess of surface pressure, the observed altimeter wind speeds are in a closer balance with the analyzed NMC surface pressure fields.

The equivalent depths calculated for the mid-latitude region are found to be comparable to but slightly less than heights of the atmospheric boundary layer. That is, the values of equivalent depths are about 700–800 m in the range of surface wind speeds between 5 m/s and 15 m/s for the Seasat period (Table 1). These values should be compared with heights of the boundary layer, which are about 800–900 m. When the surface wind speed is less than 5 m/s, the equivalent depths decrease to about 300–500 m, which is quite comparable to the values of about 300–400 m for boundary layer. Note that in each category, the standard errors for both the equivalent depths and the atmospheric boundary layer heights are calculated to amount to about 15–20% of the mean values.

The equivalent depths calculated for the polar region are smaller than those in mid-latitudes and tropics. Similarly, the heights of the atmospheric boundary layer calculated according to (13) exhibit their minimum values in the polar region. In the light wind category, (i.e., $S < 5$ m/s) the calculated equivalent depths are quite comparable to those calculated by the conventional scale height formulation, and the values are found to vary between 100 and 200 m. In the medium wind speed range (i.e., 15 m/s $> S >$ 5 m/s), the equivalent depths are found to vary between 200 and 600 m, whereas the heights of the atmospheric boundary layer are about 600 m for all cases. This large variation in the values of equivalent depths calculated for the polar region is clearly caused by the large values of the effective surface drag coefficients as explained previously.

In light of the results discussed above, it would certainly suggest that the method presented in this study enables one to infer the height of the atmospheric boundary layer over the oceans. The method depends on two important meteorological parameters, namely, surface wind speed and pressure fields. As was stated earlier, the surface pressure field is an integral of the mass through the depth of an atmospheric column and is not influenced by the details of the boundary layer physics. Hence the surface pressure field is probably the most reliable analysis field available on a global basis. Similarly, the surface wind speed field can be expected to have reasonably good accuracies from the remotely sensed scatterometry and altimetry measurements [*Jones et al.*, 1982; *Fedor and Brown*, 1982]. The conventional boundary layer height formulation, on the other hand, is typically represented by the ratio of surface friction velocity and Coriolis parameter as is used in (13), which becomes undefined near the equator in the tropics. For this study, the surface friction velocities are estimated directly from the altimeter wind speeds and the surface drag coefficient formulation in (12). In general, however, the surface friction velocity is not a directly measured quantity but may be derived by the use of wind profile data and the Monin-Obukhov surface layer similarity theory [e.g., *Businger et al.*, 1971]. The surface friction velocity thus derived tends to be not as dependable as the surface wind speed itself. Moreover, the numerical constant, $b = 0.25$, used in equation (13) is not a universal constant. For these reasons, the method proposed in this study which requires surface pressure field in addition to the surface wind speed field may be more useful and dependable. The main advantage of using this method is that at the equator the equivalent depths are well defined, and the values in the tropics in general are about 650–900 m in the wind speed between 5 m/s and 15 m/s (Table 1). These values are quite realistic over the tropical oceans where the height of the boundary layer should be expected to correspond to approximately the top of lifting condensation level or where the cloud base should correspond to the top of the tropical atmospheric mixed layer. This is consistent with the results reported by *Betts* [1976], which based on the BOMEX tropical data show that the top of the mixed layer occurred at a height of between 500 and 1500 m. On the other hand, the use of (13) for the boundary layer depths leads to unacceptable values, which can be as large as about 2500 m.

Once the effective surface drag coefficients are calculated, one can estimate the Ekman spin up (or *e*-folding) time if we know the mean wind speed as discussed in (11). However, since the mean wind speeds are not available, the observed altimeter wind speeds are used to calculate the *e*-folding time in this study. The *e*-folding times thus estimated are found to have a large variability, varying between a minimum of 4.5 hours in the polar region to a maximum of 39.4 hours in the mid-latitude oceans. Further, the relatively large standard errors associated with the means suggest larger uncertainties in the estimates of the spin up times, especially in the light wind category. As expected, the spin up times are much larger when the surface wind speeds are less than 5 m/s, reaching as large as 40 hours. In the wind speed range of 5 m/s to 15 m/s, the spin up times are less than 20 hours in general; further, the spin up times are smaller in the polar region than they are in other latitudes. This indicates that under the same synoptic forcing of surface pressure fields in the medium wind speed range, steady state Ekman boundary layer flows are more rapidly established in the higher latitudes than they are in the tropics and mid-latitudes.

4. SUMMARY

This study discusses a method by which the equivalent depths of the atmospheric boundary layer may be computed from sea level pressure analyses and ocean surface wind speed measurements. An explicit representation for the equivalent depth has been given in terms of the sea level pressure gradient, wind speed, and surface drag coefficient. The equivalent depths were calculated from a 3-day period of altimeter wind speed data taken from Seasat. For comparison, a conventional scale height formula is used to calculate the heights of the atmospheric boundary layer. This study shows that over the mid-latitudes and in the range of wind speeds between 5 m/s and 15 m/s, the equivalent depths, calculated to vary between 700 and 800 m, are found to be quite comparable to the heights of the atmospheric boundary layer which are calculated to vary between 800 and 900 m. When the surface wind speeds decrease to less than 5 m/s, both the equivalent depths and the atmospheric boundary layer decrease to about 300–500 m in the mid-latitudes. In the polar region, both scale heights are found to be smaller and quite comparable in values, ranging between 200 and 600 m. In the tropics, however, use of the conventional scale height for the atmospheric boundary layer gives unreasonably large values, whereas the equivalent depths are calculated to be about 600–1000 m. These values compare favorably with the observed heights of the atmospheric mixed layer over the tropical oceans. It is therefore concluded that the method presented in this study may be used to infer heights of the atmospheric boundary layer over the global oceans. The method should be particularly useful for initialization of atmospheric mixed layer and trade wind models. Further, based on the measured altimeter wind speed data, the spin up times for the boundary layer flows are found to be shorter in the polar regions than they are in the mid-latitudes and the tropics in the surface wind speed range of 5 m/s to 15 m/s. This indicates that steady state Ekman boundary layer flows can be more rapidly established in the polar region than they are in the lower latitudes.

APPENDIX

Let us consider a coordinate system (X', Y') such that the X' axis is in the direction of surface wind vector **V** and the surface geostrophic wind **G** is oriented as is shown in Figure 1. Note that the (X, Y) coordinate system is of the conventional east-west and north-south direction.

If the angle between X (true east) and X' axes is β, then one can immediately write the following relationships:

$$x = x' \cos \beta - y' \sin \beta$$

$$y = x' \sin \beta + y' \cos \beta$$

$$u = u' \cos \beta - v' \sin \beta = u' \cos \beta$$

$$v = u' \sin \beta + v' \cos \beta = u' \sin \beta$$

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial x'} \cos \beta - \frac{\partial P}{\partial y'} \sin \beta$$

$$\frac{\partial P}{\partial y} = \frac{\partial P}{\partial x'} \sin \beta + \frac{\partial P}{\partial y'} \cos \beta$$

Note that u' and v' are wind components in the (X', Y') system, and v' is identical to zero, since wind is aligned with the X' axis.

Now, Ekman balance equations in integrated form in the (X, Y) system are

$$\hat{h}_u\left(\frac{1}{\rho}\frac{\partial P}{\partial x} - fv\right) = -C_D|S|u$$
$$\hat{h}_v\left(\frac{1}{\rho}\frac{\partial P}{\partial y} + fu\right) = -C_D|S|v \qquad (14)$$

where $\hat{h}_u$ and $\hat{h}_v$ are equivalent depths for the u and v components of wind, respectively, and C_D is the surface drag coef-

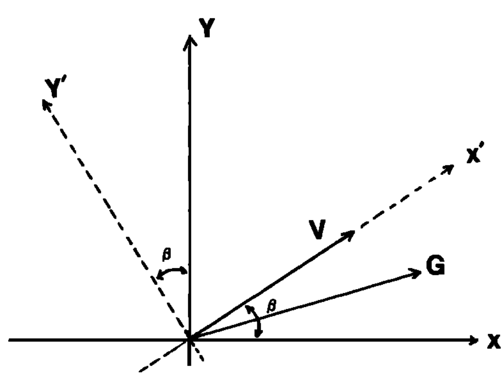

Fig. 1. Diagram of (X', Y') coordinate system.

ficient as defined in section 2. Upon substituting the relationships of (u, v) and $(\partial P/\partial x, \partial P/\partial y)$ between the (X, Y) and (X', Y') systems into (14), one gets

$$-f\hat{h}_u u' \sin\beta = -\frac{1}{\rho}\hat{h}_u\left(\frac{\partial P}{\partial x'}\cos\beta - \frac{\partial P}{\partial y'}\sin\beta\right) - C_D|S|u'\cos\beta \tag{15a}$$

$$f\hat{h}_v u' \cos\beta = -\frac{1}{\rho}\hat{h}_v\left(\frac{\partial P}{\partial x'}\sin\beta + \frac{\partial P}{\partial y'}\cos\beta\right) - C_D|S|u'\sin\beta \tag{15b}$$

From (15a) one obtains, by equating the coefficients for the cosines, $\hat{h}_u = -C_D|S|u'/(1/\rho\ \partial P/\partial x')$. Similarly, from (15$b$) one also obtains, by equating the coefficients for the sines, $\hat{h}_v = -C_D|S|u'/(1/\rho\ \partial P/\partial x')$. Thus $\hat{h}_u = \hat{h}_v$; i.e., the equivalent depths for the two momentum equations must be identical.

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