PRACTICAL WIND WAVE MODELING†

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This paper and the corresponding presentation at the workshop discuss practical numerical wind wave modeling. It reviews the history, common numerical models and approaches as well, as problems and unresolved issues that can be the subject of further research.

Keywords: Wind waves, numerical modeling, operational forecasting

1. Background and history

When winds blow over the surface of water, they generate capillary and gravity waves that are generally referred to as wind waves. These waves range in length and height from a few centimeters to lengths of up to a kilometer, and heights of over 30 m. Waves that are actively generated by the local winds are generally referred to as wind waves. When the wind subsides, waves propagate freely over the ocean [1]. The latter waves are generally referred to as swell.

Much of the basic theory for regular gravity waves has been developed in the first half of the 19$^{\text{th}}$ century, with several landmark publications [2–4]. The first practical operational forecasting of wind waves is generally associated with the preparations for D-day in World War II [5]. Numerical wave prediction at the National Weather Service (NWS) in the US and at its predecessors dates back to 1956. A review of the evolution of numerical wave prediction at the NWS can be found in [6]. Note that at the NWS predictions are made by forecasters and model results are not considered as forecasts but as guidance for forecasting.

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Initially, wave models considered only a single representative wave described with a representive height and period. However, it has long been understood that waves at sea represent a stochastic process, and that a single representative wave height does not adequately describe the wave field at sea. Using work in other fields concerning (radio) waves [7], a spectral description of wave was introduced. In this description, the mean wave energy distribution over wavenumber (or frequency) and direction is considered in a wave energy spectrum. Numerical spectral wave models are based on the a form of the spectral wave energy or action equation [8]

$$\frac{DF(f,\theta)}{Dt} = S_{in}(f,\theta) + S_{nl}(f,\theta) + S_{ds}(f,\theta) + \dots \quad , \tag{1}$$

where F is the wave energy spectrum, expressing the wave energy distribution in terms of the wave frequency f and direction θ. The left hand side of this equation represent effects of (conservative) linear wave propagation along great circles [1], and the right hand side represents sources and sinks of wave energy. Hasselmann [8] identified the main source and sink processes to be in the input of wave energy by wind (S_{in}), nonlinear interactions between waves (S_{nl}) and dissipation due to wave breaking or 'whitecapping' (S_{ds}). The ellipsis in Eq. (1) represents additional (usually shallow water) processes that have been added since the original publication. Such processes include bottom friction, other wave-bottom interactions and shallow water nonlinearities.

There are many variations to Eq. (1), with different spectra (energy versus action), different descriptions of spectral space (wavenumber versus frequency), and additional source terms. The most critical distinction between wave models, however, is not which flavor of the basic equations is used, but how the nonlinear interactions S_{nl} are treated. The critical role of the nonlinear interactions was established in the JONSWAP experiments [9], and in many theoretical studies. The interactions represent the lowest order process that shifts energy to longer waves, and stabilizes the shape of wind sea spectra. The nonlinear interactions represent a six-dimensional Boltzmann integral in spectral space, which is prohibitively expensive to compute in practical wave models.

The first spectral wave models did not attempt to explicitly compute the nonlinear interactions. Instead, spectral shapes and energy growth rates were prescribed for wind seas. This lead to a proliferation of so-called first and second generation (1G and 2G) wave models. The SWAMP study represents a comprehensive comparison of these 1G and 2G wave models [10].

A major finding of this study was the recognition that the nonlinear interactions S_{nl} needed to be accounted for explicitly in wind wave models in order to be able to consistently account for arbitrarily varying wind conditions, particularly to avoid the need for an arbitrary separation between wind seas and swell that is essential in 1G and 2G models. This study was the catalyst for the development of third generation (3G) wave models, where S_{nl} is accounted for explicitly in Eq. (1), and where the spectral shape is allowed to develop dynamically without prescribed constraints.

Third generation wave models became feasible with the development of the Discrete Interaction Approximation (DIA, [11]) to the nonlinear interactions. The DIA is the center piece of the first 3G wave model (WAM). The WAM model [12,13] was in essence developed as a community model. Since the development of WAM, a limited number of other 3G wave models have been developed, such as WAVEWATCH III, SWAN [6,14–16] and others (see Section 3). These third generation models have replaced most 1G and 2G models, although several 2G models are still used operationally at various forecast centers.

2. Describing wind waves

As mentioned in the introduction, wind waves are generally described with an energy density spectrum. An example of such a spectrum in polar representation is given in Figure 1. The direction relative to the center of the figure identifies the direction in which the waves propagate, the distance to the center identifies the wave frequency, in this case ranging from 0 in the center of the figure to 0.25Hz at the outer grid circle in the figure. To capture a wide range of energies, a logarithmic scaling of energy is used with a factor of 2 increments between consecutive contours.

There are two coherent energy distributions in spectral space, identifying two separate wind wave systems. The highest peak in spectral energy density travels in NNW directions (upper left part of plot). The coherent pattern of energy in spectral space is widely distributed over directions and frequencies. This is generally representative for wind seas, which typically have somewhat chaotic appearance with short wave crests. Another indication that this is an actively generated wind sea is that it is lined up with the wind direction (arrow in the center of the figure).

The second wave field that can be identified travels in ENE directions, at significantly lower frequencies than the wind sea. The distribution of energy over frequencies is much narrower. This is a typical signature of a swell system. Note that the second system here is uncharacteristically broad

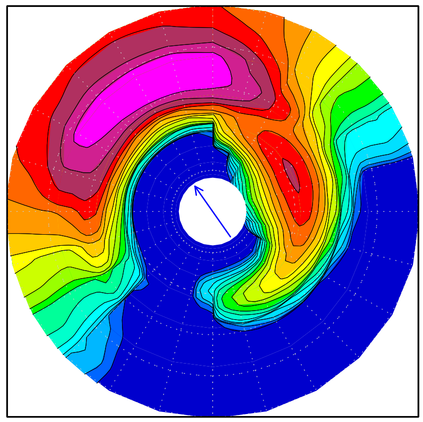

Fig. 1. An example of a wave energy spectrum from a wind wave model. Polar representation with low frequencies in the center of the grid (ranging from 0 to 0.25Hz, grid lines at 0.05Hz intervals), and directions representing direction in which wave energy travels. Logarithmic scaling of shading with contours at factor 2 intervals of spectral energy density. Vector in the center depicts wind direction.

in terms of energy distribution over directions. In nature swell energy is generally distributed over directional ranges of 30° or less, resulting in long wave crests.

A spectrum as shown in Fig. 1 contains too much information for systematic use in most forecast applications. Therefore, the wave field is typically described with mean wave parameters. The most popular parameter is the significant wave height H_s. Traditionally, this wave height is defined as the mean height of the 1/3 of the highest waves in a time series. Because the human eye filters out the smaller waves, this is closely related to the visually observed 'mean' wave height. Statistics of the waves are closely related to the energy in the spectrum, and in models the significant wave height H_s is computed as

$$H_s = \sqrt{4E} \quad , \quad E = \int \int F(f, \theta) \, df \, d\theta \quad , \tag{2}$$

where E it the total energy in the spectrum F, and where the spectrum is defined in terms of a frequency f and a direction θ. This relation is consistent with the standard expected distribution of wave heights. Assuming that the waves represent a stationary Gaussian process, the distribution of individual wave heights is described well with the Rayleigh distribution [7].

With this distribution, a wave height H_Q corresponding to a probability of exceeding of this wave height Q can be computed as

$$H_Q = \left[\ln\left(Q^{-1/2}\right)\right]^{1/2} H_s \quad . \tag{3}$$

This implies that one in a hundred waves ($Q = 0.01$) will be higher than $1.52H_s$, one in a thousand waves will be higher than $1.86H_s$, and one in ten thousand waves will be higher than $2.16H_s$. Considering that a typical storm consists of several thousand waves passing each given point, this implies that the highest expected wave height at any location in a storm is roughly $H_{\max} \approx 2H_s$.

The Rayleigh model for extreme wave heights in essence assumes linear wave behavior with symmetric crests and troughs of waves. Nonlinearity in waves tends to steepen crests and increase crest heights, and flatten troughs and reduce troughs depths. With this extreme wave heights (small Q) become somewhat probable [17,18], but wave heights $H > 2H_s$ remain rare.

Waves whose height exceeds the expected maximum height of roughly $2H_s$ are generally called rogue or freak waves. Such waves have been the subject of much recent research (e.g., Rogue Waves 2008 conference[a]). Because such waves have been associated with accidents at sea they are a hot topic for research, and attempts are made to forecast their probability. There are several issues with Freak Wave forecasting.

The science on Freak Waves is not yet mature. Much of the theoretical work is done with unidirectional waves, but the findings from these studies do not appear to be relevant for real sea states (several presentations at Rogue Waves 2008).

How do you forecast rogue waves? One could attempt to predict the highest wave in a storm based on advanced statistical approaches, but these would not be real ‘rogue’ waves. One can envision a system comparable to severe weather forecasting, with warning boxes for areas with heightened probability of rogue waves. However, if these waves are still very unlikely in the forecast, then what is the value of identifying an area as a warning zone?

Considering this, much work remains to be done on freak or rogue waves. However, it may be more useful in the context of practical wave conditions, to concentrate more on considering new output parameters of a wave model

[a]October 13-15, 2008, Brest France, http://www.ifremer.fr/web-com/stw2008/rw

and new forecast products from forecast centers with a direct (predictable) implication for safety at sea, such as wave steepness and wave breaking probability.

3. Tools

Numerical models are the main tools supporting operational wave prediction at the NWS and other meteorological centers world wide. Such models are based on equations like Eq. (1). This equation represents a forced and damped system. The quality of forecasts based on such a system is dominated by the quality of the forcing, as well as the quality of the model. Unlike with weather and ocean circulation models the estimate of the initial conditions is of less importance. Therefore, unlike with weather and ocean models, good wave guidance can be achieved without any assimilation of wave observations.

Equation (1) is a hyperbolic equation, governed by characteristic velocities. There are two basic techniques to solve this equation. One is using the property of these equations that spectral energy density or energy of wave packages is conserved along characteristic in physical and spectral space. Models based on these properties are generally called ray models. The other technique is based on direct discretization of Eq. (1). Such models are generally referred to as grid models.

Ray models are particularly interesting for predicting wave conditions for a limited number of stationary or moving targets [19]. However, such models represent older technology, since they by design can only be 1G or 2G models. Grid models are generally used for providing forecast guidance, with their inherent capability to provide a full development of the wave fields in space and time. The most popular grid models in present operational use are 3G models such as WAM, SWAN and WAVEWATCH III [6,14–16]. These model all represent a classical Eulerian discretization of Eq. (1) on a regular grid.

The range of wave model approaches, however, is much broader. There are grid models solving the left side of Eq. (1) using ray approaches. Some models use partial ray approaches for propagation on unstructured grids [20,21, TOMAWAC]. Other models use full ray approaches on unstructured grids [22, CREST]. Some models use finite element methods on unstructured grids [23]. Unstructured grid approaches are expected to become more important in the near future for coastal applications, particularly for coupled modeling of wind waves and storm surges.

Third generation wave models of the WAM type represent the state

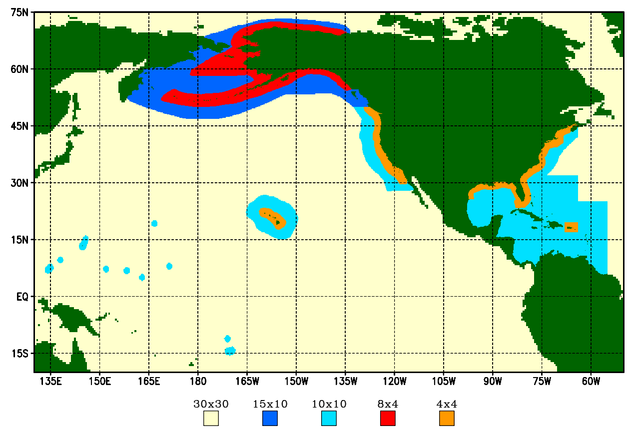

Fig. 2. Grid resolutions for mosaic wave model operational at the NWS. Grid resolution in minutes longitude×latitude

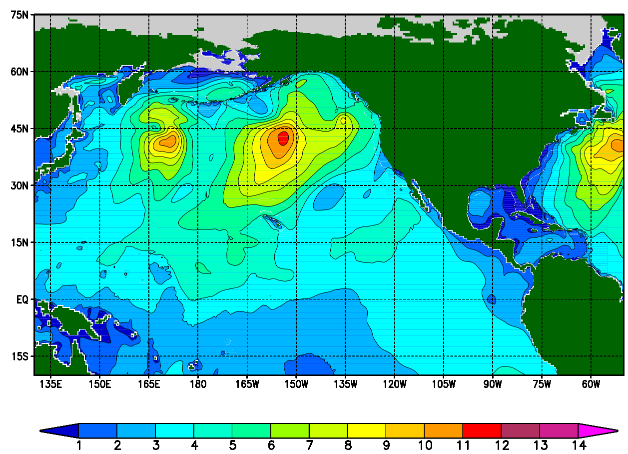

Fig. 3. Composite significant wave heights H_s in meters for the eight grids in Fig. 2 for January 17, 2006, 0000 UTC. Grey shading indicates ice covered areas.

of the art in operational wave modeling at most meteorological centers. A recent development is the inclusion of so-called mosaic approach to wave modeling in the WAVEWATCH III model [24]. In this approach, full two-way interactions between grids with various resolutions are considered. Effectively, this makes a mosaic of grids a single wave model.

The NWS presently provides model guidance for a mosaic of eight grids [25]. The NWS runs additional wave models for the Great Lakes (not

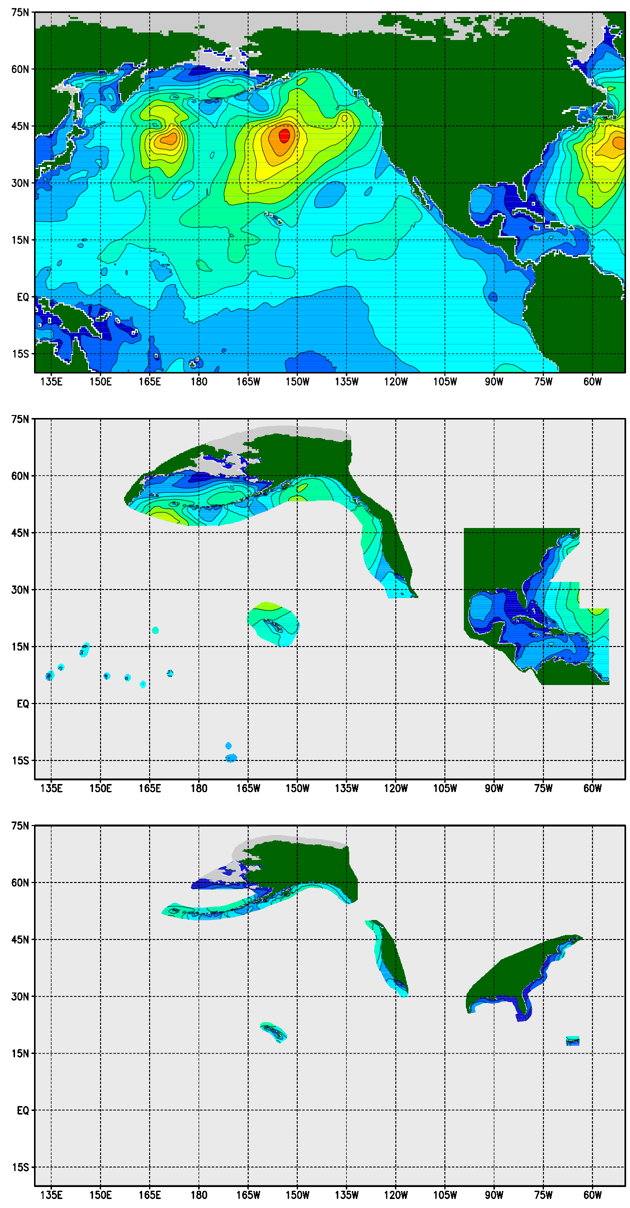

Fig. 4. Wave heights H_s from 30', 10' and 4' grids (top, middle and bottom panels, respectively), corresponding to Fig. 3.

illustrated here). The layout of the eight grids in the mosaic is dictated by resolution requirements of forecaster in the NWS, and is illustrated in Fig. 2. The grid resolutions range from typically 56 km in the deep ocean, to 7.5 km along the entire US coast. The mosaic approach provides consistent solutions for all grids, as is illustrated in Fig. 3, which shows an overlay of wave heights for all eight grids. Wave heights for the three resolution classes are presented separately in Fig. 4.

4. Problems and issues

Operational wave modeling has been very successful in the past decades [26]. Nevertheless, there are many unresolved scientific issues, and there is still much room for improvement in operational wave modeling. An in depth review of scientific issues with present wave models can be found in [27]. A brief review of outstanding issues for operational wave modeling will be given in the following paragraphs.

4.1. *Propagation*

Wind wave propagation in deep water is a simple problem from a physical perspective, with for most of the world's ocean, waves propagating along great circles closely following simple linear theory [1]. However, numerically, using a discrete spectral wave model, this is a rather difficult problem.

Numerical modeling of accurate advection of swell energy over large distances is daunting by itself, which is complicated by the the fact that the discretization of the local spectra of wave energy leads to a disintegration of a continuously dispersing swell field into discrete swell fields [28]. This phenomenon is known as the Garden Sprinkler Effect (GSE). Presently, the most advanced numerical propagation schemes used are third order total variance diminishing schemes [6,29,30]. Whereas these schemes are accurate, there is always room for improvement. Note that spectral resolutions also need to be considered as part of this problem also. For instance, a spectral resolution of 10% in frequencies as commonly used in wave model barely if at all resolves the spectral peak of observed spectra. The GSE can be alleviated [31], but is not yet truly solved.

Many more issues occur in shallow water. Linear kinematic effects such as wave refraction and shoaling (with or without mean current present) are well understood and are included in many models. Practical applications, however, may need new numerical approaches such as unstructured grids (see previous section). Whereas such approaches are available, they have not yet been used in an operational environment.

In shallow water, nonlinear propagation effects potentially become important [32]. Such effects have not yet been considered in operational wave models. Furthermore, spatial scales of the wave field evolution become much smaller in coastal areas, tentatively violating the assumption of scale separation between individual waves and the evolution of the wave field that is one of the basic precepts behind Eq. (1). This implies that effects such as diffraction, phase coupling and nonlinearities that are included in coastal phase-resolving wave models, need to be included in typical operational models based on Eq. (1). Some progress in this aspect has been made in the SWAN model, but such capability is not yet used in operational models (partially due to operational attainable resolutions).

4.2. *Wave physics*

Wave physics understanding and approaches are much less developed than the wave propagation and kinematic aspects discussed in the previous subsection. An extensive and comprehensive review of our understanding of wave physics is given in a recent review produced by the WISE group [27]. Here, only general concepts will be addressed. For a full review and references see the latter paper.

The basic physics package in a wave model consists of the "deep water" physics, that is, input (S_{in}), nonlinear interactions (S_{nl}) and dissipation or whitecapping (S_{ds}) [Eq. (1)]. Of these, the nonlinear interactions are arguably well understood. However, exact computations are prohibitively expensive for operational use, and sufficiently economical parameterizations have eluded us for two decades. Some progress has been made in recent years, and systematically increasing computing power makes more complicated approaches more feasible. Models for input have been available for decades. The main difficulty in the input source term is that the momentum transfer occurs close to the surface in very hostile conditions in nature, making accurate observations very difficult. The lack of accurate observations in nature is a main stumbling block for progress in understanding and hence modeling of the input source term. The dissipation source term has long been used as the 'closure term' in the balance equation, used mainly to tune models. Only recently, substantial progress has been made in understanding whitecapping, which in turn has lead to more physics based source term formulations for wave models.

The availability of altimeter wave data over the last two decades, and the corresponding capability to do global wave model validation [33], as well as recent analysis of SAR data[b], have shown that long distance swell dissipation on long time scales is a fourth process that needs to be considered explicitly as part of the "deep water" physics. This is in fact the first systematic addition to Eq. (1) for deep water since its inception nearly half a century ago.

When moving into restricted water depth, the first concern is that the above physics are properly applicable in restricted water depths. Generally, it is assumed that this is achieved by using the proper wave kinematics and wave geometry in the general deep water parameterizations. In fact, this assumption is rarely if ever validated. In addition, wave-bottom interactions are added to the right side of Eq, (1). Many processes can be included in this interaction [34], but the major focus has been on bottom friction. In this context, active wave-sediment interactions pose a main problem, with active ripple formation on the bottom greatly modifying the physical roughness of the bottom. Recent research is also focusing on wave-mud interactions, particularly around river deltas (e.g., Mississippi delta).

In severely restricted water depths, including the surf zone where the wave height and water depth become of the same order of magnitude, additional physical processes become important. One of these is depth-limited wave breaking. Robust parameterizations of this process have been available for decades [35]. A major challenge is to unify parameterizations of wave breaking at arbitrary water depths. Furthermore, it is generally believed that nonlinear interactions between three waves (triads) dominate the "deep water" interactions between four waves (quadruplets) in extremely shallow water. Only crude parameterizations for the triad interactions are available for operational wave models. Additional areas of research for extremely limited water depth include considering phase information of spectral components, and bispectral model. In general, it appears that elements of time domain and phase resolving wave models need to be included in spectral operational wave models for shallw water applications.

4.3. *The bigger picture*

The previous two sections deal with wave modeling in an isolated sense. In a bigger picture view of wave modeling, coupling, data assimilation and ensemble forecasting are an important part of (operational) wave modeling.

Historically, environmental modeling has focused on isolated problems, like weather, ocean, storm surge or waves. Increasingly, it is understood that

[b] F. Ardhuin, WISE 2008 presentation

progress in many of these fields will depend on properly accounting for the interactions between such systems, and hence on the coupled modeling of such systems. For waves, prime benefits are expected from coupling to current models (e.g., Gulf Stream impact on waves) and storm surges (actual water depths in severely depth limited conditions). Waves in turn provide direct forcing for inundation (storm surge) models, and more physical parameterizations of boundary layer processes in both the atmosphere and the ocean. Whereas some of these coupling activities have been addressed for several decades, systematic coupling between systems in an operational environment has only started to be addressed recently.

As mentioned above, wave models do not represent an initial value problem, and data assimilation is not critical in order to prduce a good wave forecast. Consequently, data assimilation has not been getting much attention for wind waves. Nevertheless, data assimilation can improve short term forecasts (12 to 24h) in general, and potentially can improve swell prediction on the Pacific Ocean up to two weeks into a forecast. It should be noted that, compared to best practices in atmospheric data assimilation, data assimilation in wave modeling is rather primitive. This implies that much work can be done in wave data assimilation, particularly by using specific wave physics in new data assimilation methods [36,37].

Finally, probabilistic methods are becoming more and more important in weather forecasting. In such an approach, multiple model runs are made with different initial conditions to address the uncertainty in the model guidance, and to get the best consensus. Wave model ensembles based on such weather model ensembles have been produced at meteorological centers for up to a decade, but the forecast benefits and probabilistic reliability of the ensembles has not yet received much attention.

5. Summary and conclusions

The present paper represents a brief review of the state of the art of operational ocean wave modeling. It also identifies outstanding issues with such models, and as such provides many topics for further research in this field.

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