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A Neural Network Nonlinear Multi-model Ensemble for Predicting Precipitations over ConUS¹

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Abstract

A novel multi-model ensemble approach based on the neural network (NN) technique is formulated and applied for improving 24-hour precipitation forecasts over the continental US. The developed nonlinear approach allowed us to account for nonlinear correlation between ensemble members and “optimal” forecast represented by a nonlinear NN ensemble mean. The NN approach is compared with the regular multi-model ensemble, with multiple linear regression ensemble approaches, and with results obtained by human forecasters. The NN multi-model ensemble improves upon regular multi-model and multiple linear regression ensembles: (1) it significantly reduces high bias at low precipitation level; (2) it significantly reduces low bias at high precipitation level, and (3) it sharpens features making them closer to the observed ones. The NN multi-model ensemble performs at least as well as human forecasters supplied with the same information. The developed NN approach is a generic approach that can be applied to other multi-model ensemble fields as well as to single model ensembles.

1. INTRODUCTION

For numerical weather prediction (NWP) models the rainfall is one of the most difficult fields to predict accurately. Detailed knowledge of the atmospheric moisture and vertical motion fields is critical for predicting the location and amount of rainfall, but these are difficult quantities to predict and observe accurately. Precipitations are determined by cloud dynamics and microphysical processes involved. Clouds and convection are among the most important and complex phenomena of the atmospheric system. The processes that control clouds, and through which they interact with other components of the Earth system involve slow and fast fluid motions carrying heat, moisture, momentum and trace constituents, and influence other important physical processes through phase changes of water substances, radiative transfer, chemistry, production and removal of trace constituents, and atmospheric electricity. Numerical weather prediction (NWP) models cannot adequately represent the cloud dynamics and microphysical processes involved in rainfall generation because these processes occur on a subgrid scale, which means that they have time and space scales that are well below the resolution of the scales explicitly treated in NWP models. Therefore, NWP models must resort to parameterizations that treat convective clouds in a very simplified way.

The errors in quantitative precipitation forecasts (QPFs) can arise as a result of errors in the observations and limitations of the forecast model. To compensate for shortcomings in observing systems and model physics there has been a trend in recent years toward ensemble forecasting, the realization of a number of model integrations using perturbed initial conditions. Ensemble prediction systems (EPSs) have been extensively tested and used in operations at the European Centre for Medium-Range Weather Forecasts (ECMWF) and the U.S. National Centers for Environmental Prediction (NCEP) (Buizza et al. 2005, Palmer et al. 2007). Using this strategy one can estimate the probability of various events and possibly also the uncertainty associated with a particular forecast. The ensemble average has repeatedly been shown to give a more accurate forecast than a single realization of the forecast model (Zhang and Krishnamurti 1997; Du et al. 1997; Buizza and Palmer 1998). This technique is very computationally expensive and lower-resolution versions of the models are generally employed. A drawback with the single-model EPSs (assuming that errors result primarily from uncertainties in the initial conditions) is that any biases present in the model itself will also be present in the ensemble and may require calibration. The recent introduction of “stochastic” or “perturbed” physics attempts to account for uncertainties in the model subgrid-scale processes (Buizza et al. 1999, 2005; Krasnopolsky et al. 2008).

Multi-model ensemble (MME) (aka poor men's ensemble) is another approach that has been taken to address aforementioned issues. It combines forecasts from more than one NWP model. Hamill and Colucci (1997, 1998) combined ensembles from the NCEP Eta model and regional spectral model to generate improved short-range forecasts of probability of precipitation. Ebert (2001) exhaustively investigated advantages and problems of MME approach using a MME composed of seven operational NWP global and regional models.

In the case of MME, the ensemble is composed of output from different models and/or initial times, rather than a single model with perturbed initial conditions. Unlike EPSs that use singular vectors or breeding modes to generate optimal perturbations to the initial conditions, MME samples the uncertainty in the initial conditions via the different observational data, data assimilation systems, and initialization methods used by operational centers. MME also samples the uncertainty in model formulation due to the differences in model dynamics, the variety of model physical parameterizations, numerics, and resolutions. As the result, MME can be considered as an approach, in which all components of NWP system are perturbed not only initial conditions or model physics. Many authors (e.g., Speer and Leslie 1997, Du et al. 1997, and Ebert 2001) demonstrated superior performance of MME.

In this paper we introduced a new nonlinear MME based on a neural network (NN) technique. The purpose of this study is to examine improvements that a nonlinear NN based MME may introduce over a regular (linear) MME for the case of precipitation forecast. The next section reviews linear methods of combining ensemble members and calculating the ensemble prediction and introduces a nonlinear NN based MME. Section III describes the forecast and verifying data that we used in the study. Section IV describes approaches to improve prediction of precipitations that we investigate. Section V contains results and their discussion. The paper finishes with conclusions.

2. Calculation of the ensemble results

In MME as well as in EPS the final product is a combination of the ensemble members. The simplest and most common combination of the ensemble members is an ensemble mean (EM), which is calculated as a simple average of ensemble members:

$$EM = \frac{1}{N} \sum_{i=1}^N P_i , \quad (1)$$

where N is the total number of ensemble members and P_i is the i^{th} ensemble member generated by the model number i .

More sophisticated approaches (Krishnamurti et al. 1999, 2000) use the weighted ensemble mean (WEM),

$$WEM = \frac{1}{N} \sum_{i=1}^N W_i \cdot P_i , \quad (2)$$

here each ensemble member is subscribed a weight, W_i , based on some ad-hoc considerations. For example, Krishnamurti et al. (1999, 2000) used multiple linear regression technique to determine optimal weights, W_i , for combining the ensemble members based on a training dataset; a significant improvement was demonstrated using weighted ensemble mean over the simple ensemble mean.

The aforementioned approaches (both simple and weighted mean) implicitly assume a linear dependence between ensemble members and the best predicted value (the amount of precipitations in our case). However, in some cases this assumption may be incorrect. For example, for longer forecast times when bifurcation of the ensemble forecasts may occur, it can lead to misleading results. Also for fields (like precipitation fields) with high gradients and sharp, localized features the assumption of linearity may lead to significant problems in MME predictions (see more detailed discussion in the following sections). In such cases the dependence between the ensemble members and the best predicted value may be a complex nonlinear one.

In this study, we relaxed the linearity assumption and allowed for an arbitrary nonlinear dependence between the MME members and the best predicted value. A neural network (NN) technique is used to approximate this arbitrary nonlinear dependence using a training set. The NN technique is used because NN is a universal approximator that can approximate any continuous or almost continuous dependence given a representative data set for training (Cybenko 1989, Hornik 1991).

The nonlinear NN ensemble mean (NNEM), which we introduce here, is defined following Krasnopolsky (2007a, b); it is an analytical multilayer perceptron that can be written as:

$$NNEM = a_0 + \sum_{j=1}^k a_j \cdot \phi(b_{j0} + \sum_{i=1}^n b_{ji} \cdot X_i) \quad (3)$$

where X_i are components of the input vector X composed of the same N inputs (ensemble members) as those used for EM and WEM equations (1) and (2) plus optional additional input parameters (see Section IV), n is the number of inputs ($n \geq N$), a and b are fitting parameters (weights), and $\phi(b_{j0} + \sum_{i=1}^n b_{ji} \cdot X_i)$ is a so-called “neuron”. For the activation function ϕ we use a hyperbolic tangent and k is the number of neurons in (3). It is noteworthy to repeat that expression (3) is capable of approximating any nonlinear relationship between the ensemble members P_i and the ensemble forecast NNEM.

3. Forecast and verification data

At NCEP we applied different aforementioned MME techniques for calculating 24-hour precipitation forecast over the continental US (ConUS) territory (Lin and Krasnopolsky 2011). 24-hour precipitation forecasts over ConUS are available from eight operational models, including NCEP's own mesoscale and global models (NAM and GFS), the regional and global models from the Canadian Meteorological Center (CMC and CMCGLB), global models from the Deutscher Wetterdienst (DWD), the European Centre for Medium-Range Weather Forecasts (ECMWF), the Japan Meteorological Agency (JMA) and the UK Met Office (UKMO). Also NCEP Climate Prediction Center (CPC) precipitation analysis is available. CPC's 1/8 degree daily gauge analysis is used in the training of NNs and for the verification of model predictions.

Results indicate that all models demonstrate similar behavior: at lower levels of precipitation they are slightly wetter than the CPC analysis and at the higher levels (> 50 - 60 mm/day) they are dryer than the CPC analysis (for detailed discussion, see Lin and Krasnopolsky, 2011). Moreover, locations of highs and lows and details of precipitation features are different in the precipitation fields produced by different models. The model results (24h forecast) for three models (NAM, GFS, and ECMWF) together with the CPC verification analysis are shown in Fig.1.

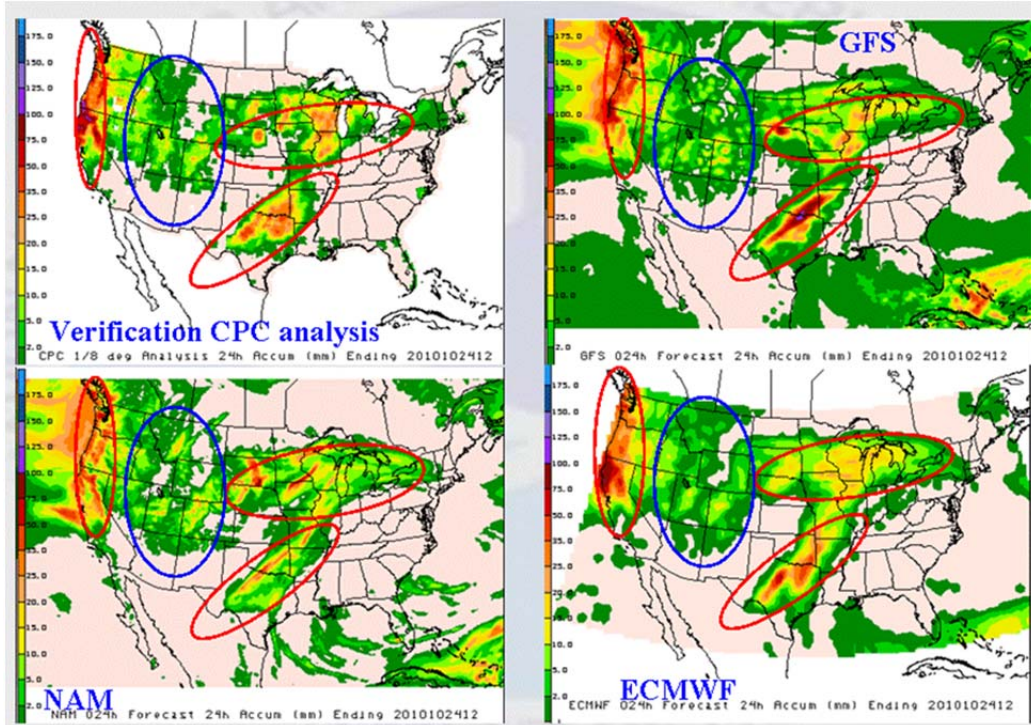


Fig.1. The model results (24h forecast) for three models (NAM, GFS, and ECMWF) together with the CPC verification analysis for October 24, 2010. Red and blue ellipses show high and low precipitation areas respectively. The figure illustrates the differences in model forecasts, especially for high and low precipitations.

All gridded data fields were interpolated to the same grid, the 40-km Lambert-conformal AWIPS Grid 212 that encompasses ConUS.

Fig. 2 shows a scatter plot which presents all aforementioned eight model predictions over the first six months of 2010 plotted vs. the CPC analysis. It demonstrates a tremendous spread in the MME results. The uncertainty of the forecast is especially large at higher levels of the precipitation amount. The

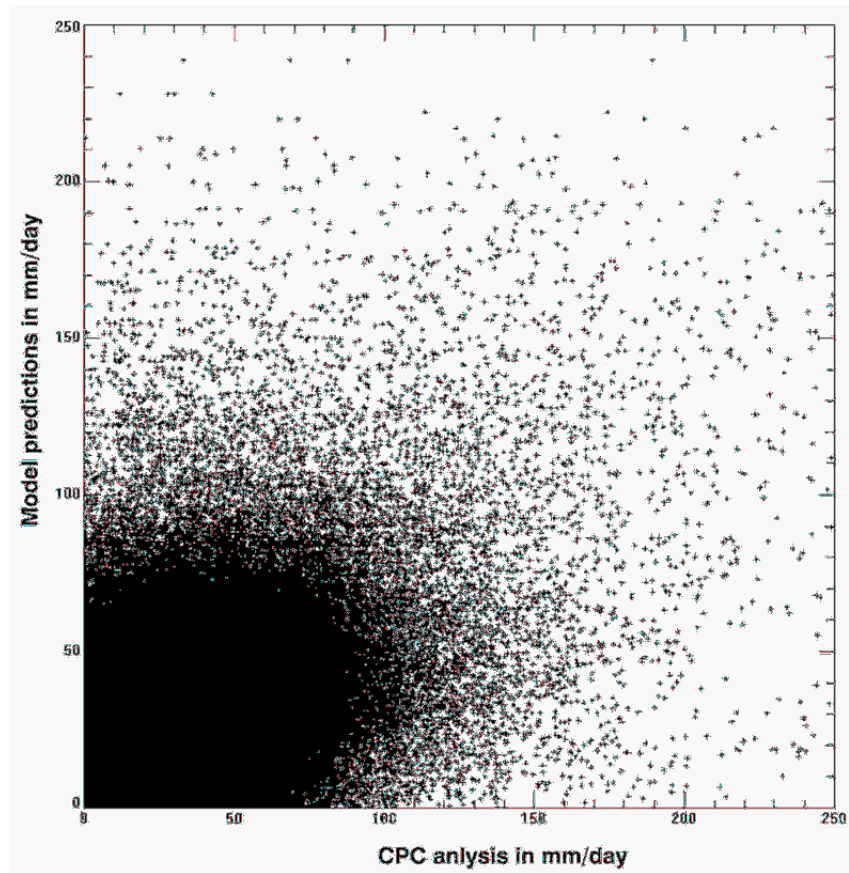


Fig. 2 Scatter plot showing 24h precipitation forecasts obtained by eight models over the first six months of 2010 vs. corresponding CPC analysis.

binned scatter plots of all eight model 24h predictions vs. CPC verification analysis is shown in Fig. 3. The models create an envelope with the spread increasing with the increase of the precipitation rate. All models have increasingly low bias at high levels of precipitations. Figures 2 and 3 illustrate very well the aforementioned problems.

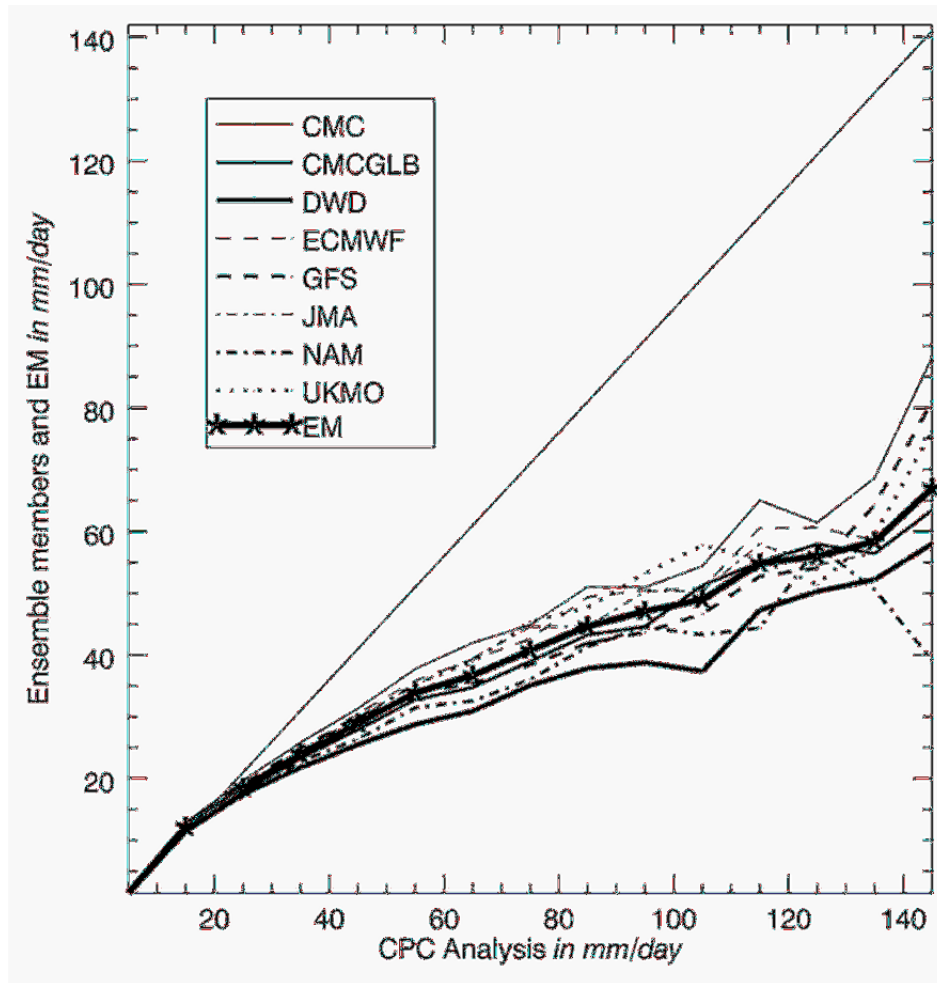


Fig. 3. Binned scatter plot for eight models (ensemble members) and EM (1)

In the next sections, we investigate possibilities of improving the multi-model ensemble technique for 24h precipitation forecast using linear (multiple linear regression) and nonlinear (NN) techniques to improve upon the standard linear ensemble (1).

4. Ensemble approaches to improve prediction of precipitations

Because of the model problems described above, the research community has been exploring various ways of making better precipitation forecasts. Among the approaches investigated at NCEP and in this paper we consider an eight member MME, which is averaged in three different ways calculating: (i) a regular *EM* (1), (ii) *WEM* (2) based on multiple linear regression, and (iii) a nonlinear NN ensemble, *NNEM* (3).

As can be seen in Fig. 3, the regular *EM* (1) goes inside (in the middle of) the envelope created by the models. *EM* provides a better placement of precipitation areas; however, it does not improve the situation significantly. Moreover, as it is illustrated in Figs. 5 and 6, *EM* (1) smoothes, diffuses features, reduces spatial gradients; it has high bias for low level of precipitations (large areas of false low precipitations) and low bias at high level of precipitations (highs are smoothed out and reduced). These problems are illustrated in the next section. They motivated us to search for improved techniques including nonlinear NN ensemble.

First we introduced and investigated an improved linear technique. To make just comparison with the NN ensemble, we redefined *WEM* (2) as a multiple linear regression using the same inputs as the NN ensemble. The multiple linear

regression ensemble mean (*WEM*) was created in the following way (Krasnopolsky and Lin 2011):

$$WEM = a_1 \cdot cjd + a_2 \cdot sjd + a_3 \cdot lat + a_4 \cdot lon + \sum_{i=1}^8 a_{i+4} \cdot P_i \quad (4)$$

where $\{a_i\}_{i=1,\dots,12}$ are regression parameters, $cjd = \cos(\frac{\pi}{183} \cdot jday)$,

$sjd = \sin(\frac{\pi}{183} \cdot jday)$, $jday$ is the Julian day, lat is the latitude, lon is the

longitude, and P_i are the ensemble members in a particular grid point of ConUS grid. Thus, the multiple linear regression (4) has totally 12 input parameters.

The NN ensemble mean (*NNEM*) is defined as in (3) where the input vector X is composed of the same $n = 12$ inputs as those used for *WEM* (4). $k = 7$ was selected after multiple trials to avoid over-fitting (Krasnopolsky and Lin 2011). Both *WEM* and *NNEM* have one output – 24h precipitation forecast. The same CPC analysis corresponding to the time of the forecast was used to train outputs in both cases. It is noteworthy that the regression parameters for *WEM* and NN weights for *NNEM* are the same for all grid points and do not depend on time. After *WEM* and *NNEM* are trained, they are used with the same set of regression coefficients (or weights for NN) in any grid point of the ConUS grid at any time. Thus, the results depend on time and location only through their input parameters.

5. Results and Discussion

The *WEM* and *NNEM* have been developed using 2009 data (more than 310,000 in/out records, Krasnopolsky and Lin, 2011). They have been validated on independent data for the first half of 2010, e.g., the results shown in Figs. 2, 3 and 4 have been calculated using these validation data. Fig. 3 shows the binned scatter plot for the amount of precipitation over the ConUS territory during the first six months of 2010. It shows the eight available models together with *EM* (1) results vs. CPC analysis. Our validation showed that, for precipitation fields, *WEM* (4) does not significantly improve upon the regular multi-model ensemble *EM* (1). In Fig. 4 these two ensemble means, *EM* and *WEM*, are shown by thick solid and dashed black lines correspondingly. As can be seen from the Fig. 3 and 4, all models, *EM*, and *WEM* are slightly wetter than the CPC analysis at lower precipitation amounts and significantly dryer than the CPC analysis at higher precipitation amounts. The linear ensembles, *EM* and *WEM*, do not change the situation significantly (see both panel of Fig. 4). Also the multiple linear regression ensemble, *WEM*, does not introduce any significant improvement upon *EM*.

There is a significant difference between linear ensemble averaging techniques (1 and 2) and the nonlinear one (3). *EM* (1) is always unique. *WEM* (2) provides always the unique solution for a given training set. Nonlinear ensemble

averaging, and NN ensemble mean $NNEM$ (3) in particular, provides multiple solutions for a given training set. For accurate training data, different solutions have different approximation errors, and the best solution with the smallest approximation error can be selected. For training data with the high level of uncertainty (noise), like our data shown in Figs. 2 and 3, multiple solutions have almost the same approximation accuracy close to the uncertainty of the data. It means that all these solutions provide equally valid nonlinear averaging of the MME.

In terms of the NN approach, we trained ten NNs (3) with the same architecture ($n = 12$ inputs, one output and $k = 7$ hidden neurons) but different initialization values for weights a and b (see eq. (3)). The training of these NNs, which is a nonlinear minimization of an error function, leads to ten different local minima of the error function with approximately the same value of the approximation error. However, because these ten NNs have different weights a and b (see eq. (3)), they produce very different results in the areas where the uncertainty of the data is higher (higher levels of precipitations).

The results of the application of different MME averaging procedures to the validation data set are shown in Fig. 4. It shows binned scatter plots for EM (1), WEM (4), and ten $NNEMs$ (3) ($NNEM_i$, $i = 1, \dots, m$ and $m = 10$). The left panel

displays the whole interval of precipitation values from 0 to 145 mm/day and the right panel magnifies the lower precipitation area from 0 to 50 mm/day.

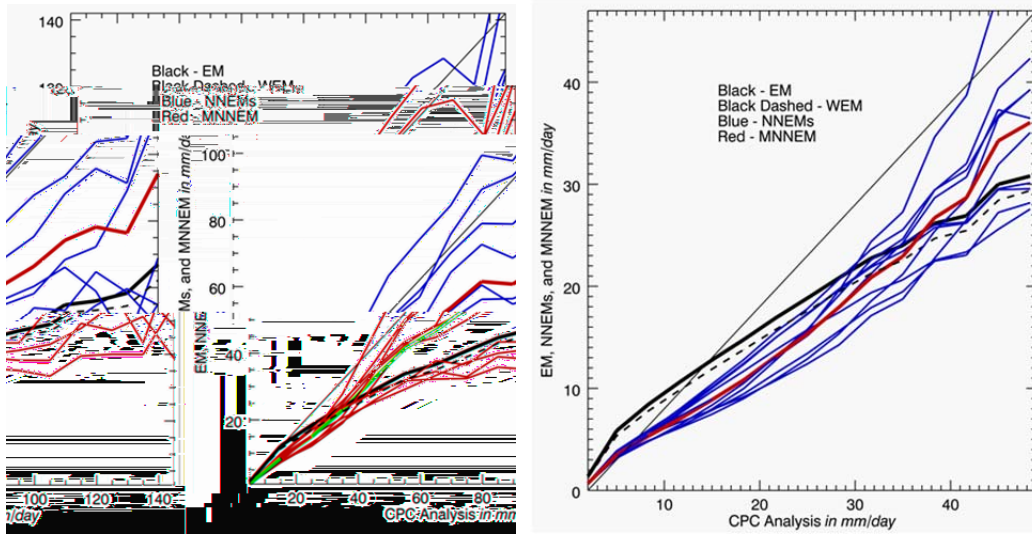


Fig. 4 Binned scatter plots for EM (1) (black solid), WEM (4) (black dashed), ten NNEMs (3) ($NNEM_i$, $i = 1, \dots, m$ and $m = 10$, all blue), and MNNEM (red) that is defined by eq. (5) below. The right panel shows the magnified lower precipitation area.

All ten NNEMs are in a good agreement at the lower levels of precipitation. They diverge significantly at the higher levels of precipitation. Their large spread reflects the uncertainty in the data that is the uncertainty of MME, i.e., the differences in predicting higher levels of precipitation by the different members of the MME (see Fig. 2). Also, it is noteworthy that in the training and validation data sets only less than 0.5% of the data records correspond to precipitation values greater than 50 mm/day and only a few records to precipitation values greater than 100 mm/day.

To improve statistical significance of nonlinear NN ensemble averaging (especially at higher precipitation values), we can consider the ten aforementioned NNs as an ensemble of averaging NNs and calculate the ensemble mean as,

$$MNNEM = \frac{1}{m} \sum_{i=1}^m NNEM_i \quad (5)$$

where $m = 10$, and each $NNEM_i$ is one of ten $NNEM$ (3). Now we can use $MNNEM$ as MME forecast. It is shown in Fig. 4 by a red solid line. $MNNEM$ produces a significant improvement relative to EM and WEM results at higher levels of precipitations (Fig. 4, left panel); it significantly reduces the low bias at higher precipitation levels (35 mm/day and higher). It also improves results at low precipitation levels, significantly reducing high bias at lower precipitation levels (from 0 to 10 mm/day). However, at medium precipitation levels from ~12 to 30 mm/day $MNNEM$ and the majority of NN members have lower bias than EM and WEM , which can be seen in Figs. 4 (right panel). Thus, the nonlinear NN ensemble averaging approach is flexible enough to negotiate the wetness at lower amounts of precipitations with the dryness at the higher amounts.

Using an ensemble of NN MME means ($NNEMs$) has an additional advantage. It allows us to calculate the uncertainty of MME forecast as the standard deviation of $NNEMs$,

$$\sigma = \sqrt{\frac{1}{m} \sum_{i=1}^m (NNEM_i - MNNEM)^2} \quad (6)$$

Figs. 5 and 6 demonstrate two case studies that show advantages of nonlinear NN ensemble forecast, *MNNEM*, as compared with the regular ensemble forecast, *EM*. Here we do not show *WEM* (4) results because visually they are not distinguishable from *EM* results. The CPC analysis for the time corresponding to the forecast is used for verification. Also, manual 24h forecast produced at the Hydrometeorological Prediction Center (HPC) by human forecasters is also presented for comparison. To produce the HPC forecast, a forecaster uses the model forecasts as well as all available observations and satellite data (including sequential satellite images) (Novak et al., 2011).

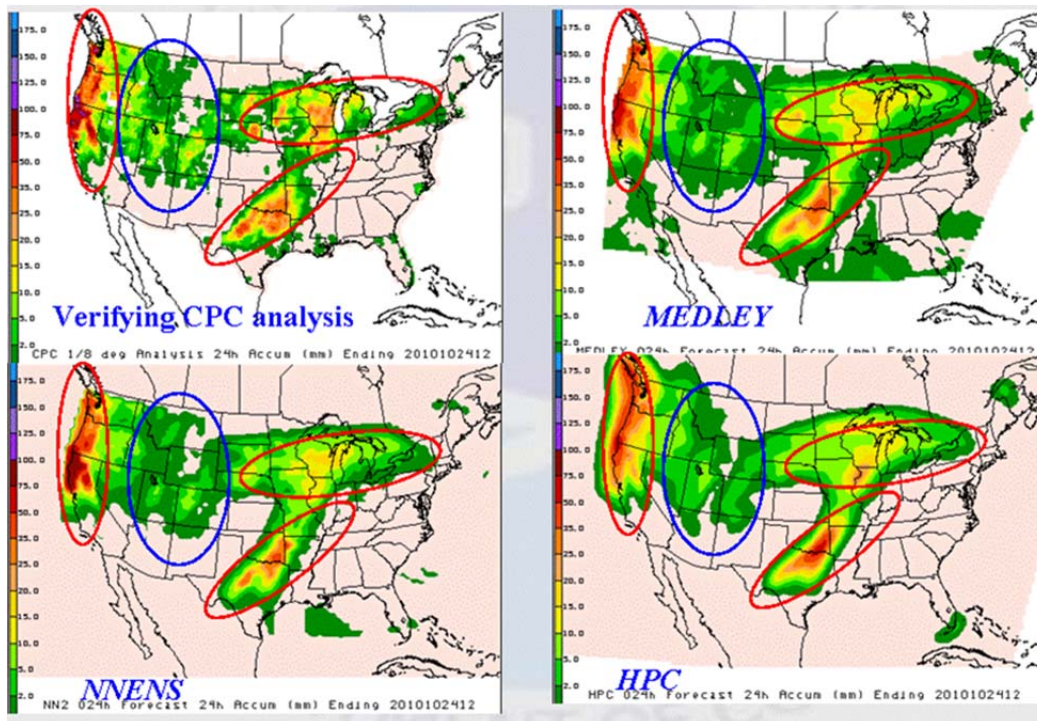


Fig. 5. Comparison of three 24h forecasts: *EM* (upper right), *MNNEM* (lower left), and *HPC* (lower right) vs. *CPC* analysis for October 24, 2010. Red ellipses show high precipitation areas and blue ellipses show low.

As figs. 5 and 6 demonstrate, the nonlinear NN averaging of MME improves positioning of precipitation features inside the precipitation fields. It removes significant areas of false low level precipitations produced by a standard *EM* (1) technique. It sharpens the features, enhances precipitation fronts and maximums. The *MNNEM* technique provides a forecast that is comparable with the HPC forecast while using much less resources and time.

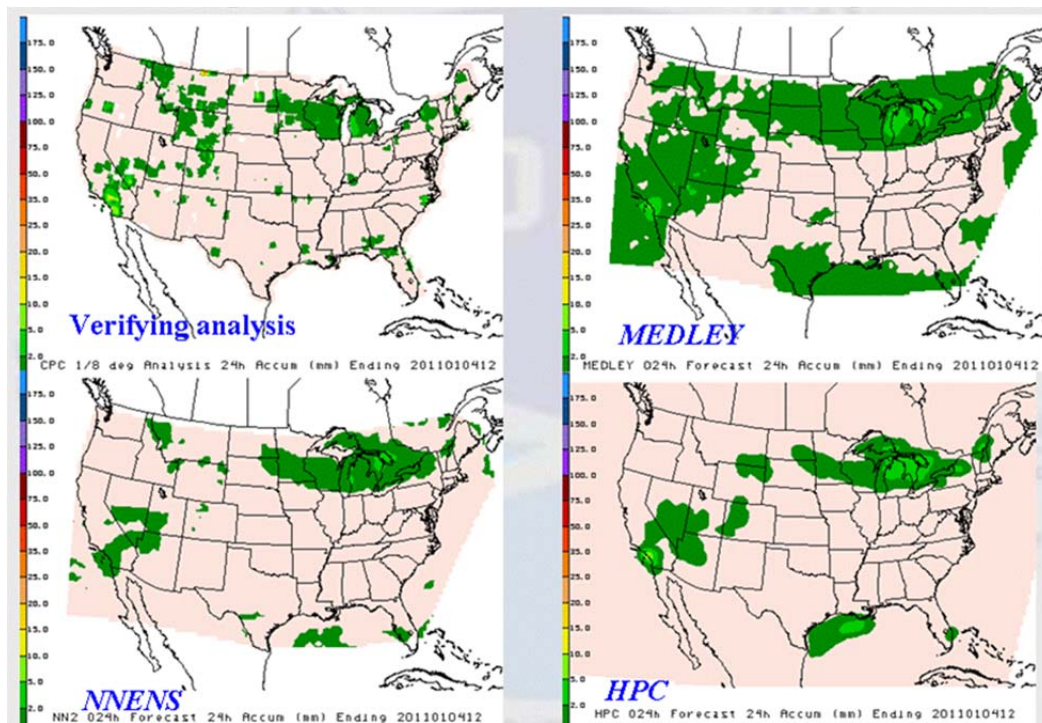


Fig. 6. The same as in Fig. 5 but for January 4, 2011.

In conclusion of the discussion, the statistical results that characterize the accuracy of positioning precipitation features are shown in Fig. 7. The statistics covers the period of eight months from November 15, 2010 to July 15, 2011. The ETS measures that fraction of observed events that are correctly predicted,

adjusted for correct predictions that are due to random chance. Possible ETS ranges from $-1/3$ to 1 (perfect forecast would have a score of 1 for every

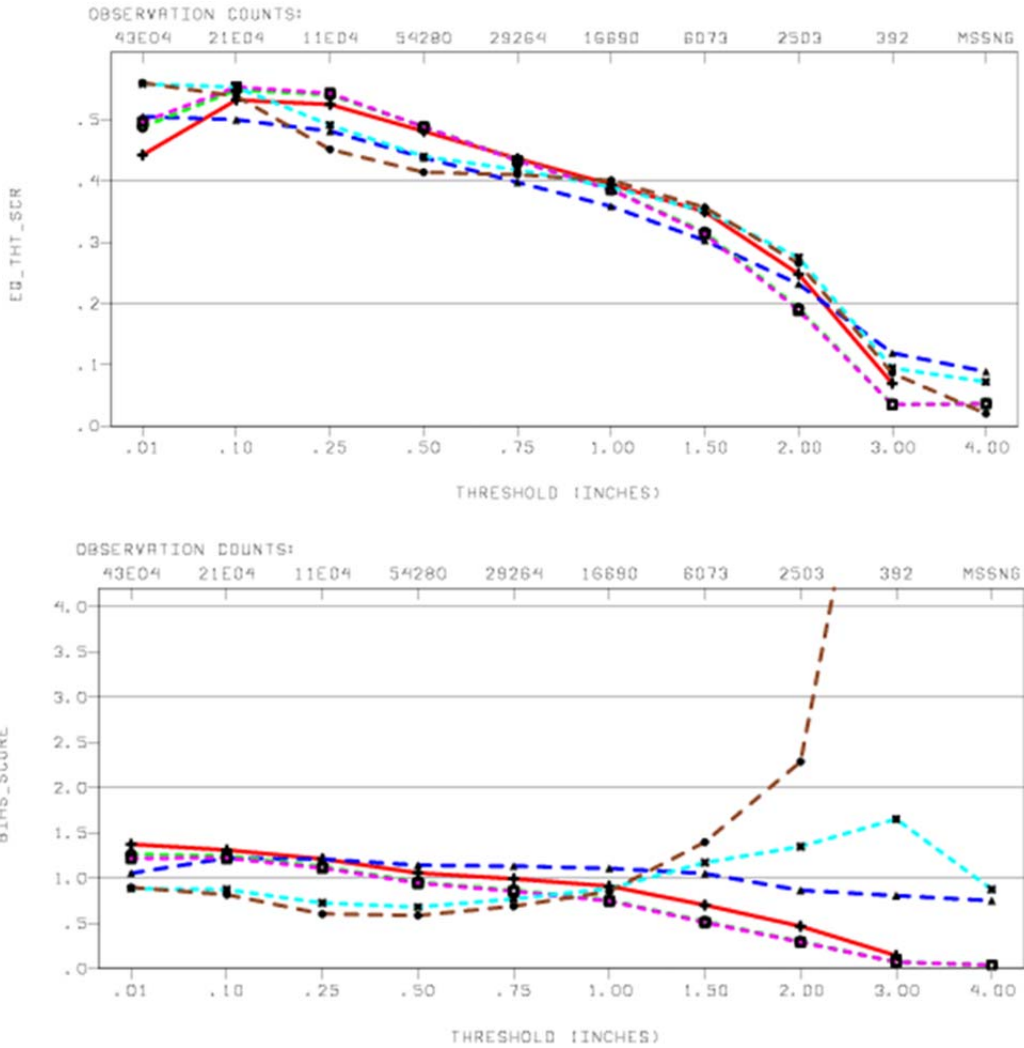


Fig. 7 EQ-THT score (upper panel) and BIAS score (lower panel) for the period of eight months from November 15, 2010 to July 15, 2011. Five different 24h MME forecasts are presented: EM (1) – solid red, WEM (4) – dashed pink, HPC forecast – dashed blue, MNNEM (5) – dashed light blue, and one of NNEM (one member of the NN ensemble (5)) forecasts – dashed brown.

precipitation threshold). Bias score is simply the ratio of areal coverage of forecast vs. observed precipitation exceeding a given threshold. An ideal forecast would have a bias score of 1 at every threshold.

Summarizing, the *MNNEM* forecast is comparable with the HPC forecast and significantly better than *EM* at the threshold values less than 0.1 inch/day and more than 1. inch/day, which is in a good agreement with the statistics presented in Fig. 4.

6. CONCLUSIONS

In this paper we introduce a nonlinear NN ensemble approach to improve 24h multi-model ensemble precipitation forecast. This straightforward application of NNs to the problem produced promising results. We showed that our NN ensemble improves upon simple linear ensemble:

1. It significantly reduces high bias at low precipitation level
2. It significantly reduces low bias at high precipitation level
3. It sharpens features making them closer to observed ones.

It is noteworthy that the NN multi-model ensemble forecast works at least as well as HPC forecast produced by human analysts without using any additional information that is available to the analyst and it is less time and resource consuming.

The presented study is actually a pilot study; we implemented NN in a simple way, supplying it with the same information which the linear multi-model ensemble uses. The flexibility of the NN approach allows us to introduce in future studies more sophisticated NN approaches. For example, we are planning to introduce information available to a human analyst (and HPC analyses itself) as additional input to our NN. We are also planning to implement a field-wise approach taking inputs from several neighborhood points etc.

The nonlinear NN averaging approach that we developed in this paper is a generic approach. Although here we applied it to precipitation fields, it is clear that it can be applied to other fields as well. Also here we applied this approach to calculate the MME mean; it can be applied as well to calculate nonlinear ensemble mean in a single model EPS.

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