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Statistical Analysis As a Flood Routing Tool

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A western Indian symbol for rain. It also symbolizes man's dependence on weather and environment in the West.

U. S. DEPARTMENT OF COMMERCE
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WEATHER BUREAU

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STATISTICAL ANALYSIS AS A FLOOD ROUTING TOOL

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TABLE OF CONTENTS

STATISTICAL ANALYSIS AS A FLOOD ROUTING TOOL

The use of statistics as an aid to flood routing analysis is referred to by Linsley, Kohler and Paulus [1]. It is pointed out that coefficients of the Muskingum flood routing technique can be derived statistically. Requirements of their equation (10-24d) on page 229 do, however, specify that the coefficients derived sum to unity in order that a rational solution can be achieved. This requirement precludes use of any parameters which are not linear (i.e., polynomial arguments), for such parameters cannot maintain volumetric continuity except at a single level of discharge.

Can a statistical analysis provide a rational solution of a linear analysis with dependability? A hypothetical case of statistical analysis of the flow from two tributaries to a downstream point indicates some of the problems which statistics may encounter. Consider a 100-square mile basin with two major gaged tributaries (Figure 1).

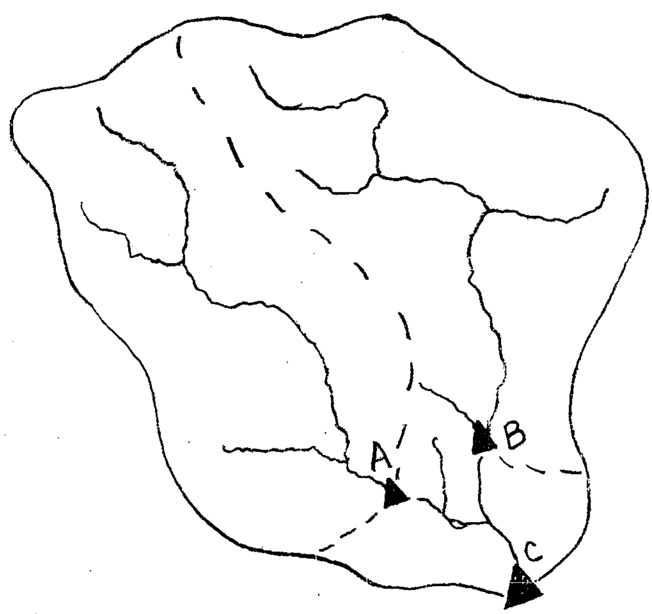

Figure 1. Hypothetical case, with tributaries gaged at A and B; combined flow gaged at C.

The tributaries are gaged at A and B. The combined flow plus five additional miles of local inflow is gaged at C. The flow at all gaging points is listed for successive time intervals as:

A	B	C
2.0	1.8	4.0
4.2	3.9	8.5
8.5	8.5	17.5
7.3	6.9	14.9
4.2	3.9	8.5
3.0	2.7	6.0

Correlating A and B against C with standard correlation techniques yields a more perfect prediction of C than can be obtained from any routing. The correlation coefficient R square is .999 and the prediction equation is:

$$C = 2.76147A - .77314B - .09684.$$

However, the coefficients for A and B do not stand the scrutiny of component analysis. For example if a reservoir were built above A, the prediction equation could yield negative flow at C or if unusually heavy rain should occur above B, the flow could also be negative at C. Statistical routing results in a dilemma in that an increase in flow at B not accompanied by a commensurate increase at A results in a flow reduction at C.

This case is intended to demonstrate the basic problems with statistical routings. Although computers make such techniques easy to apply, unless each input component has a logical positive coefficient an invalid analysis will result. The high degree of interrelationship between independent variables makes it highly unlikely that the component will be evaluated in a physically realistic manner.

A second case is taken from the Manual of Hydrology [2]. This case also illustrates the inconsistency of a statistical approach even though for the data set investigated the statistical fit again appears superior to the storage routing method. There are two tributary and one mainstem inflows to this section (Figure 2). Data used, and the results of the storage analysis are indicated in Tables 1-3.

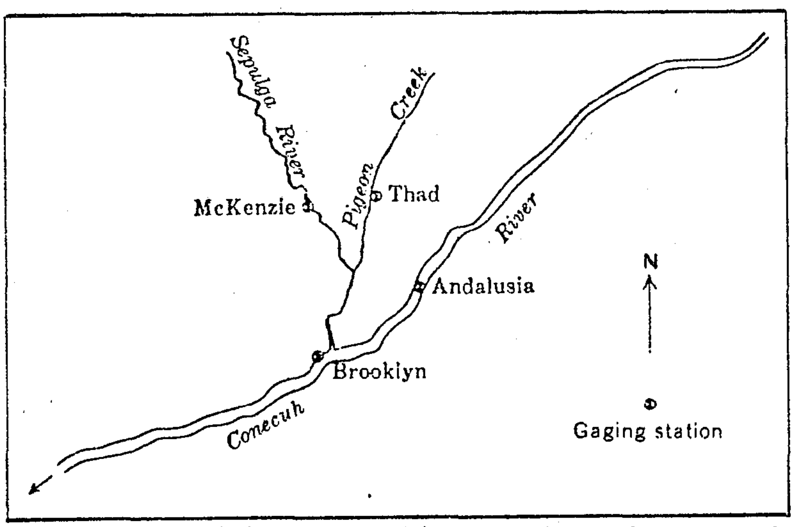

Figure 2. Andalusia-Brooklyn Reach on Conecuh River, From: Manual of Hydrology - Flood Flow Techniques.

$[O_2 = C_o I_2 + C_1 I_1 + C_2 O_1]$

$[C_2 = 0.20(4570) + 0.52(2710) + 0.28(942) = 2{,}590$ cfs - Local Inflow to Brooklyn, March 18]

	Local Inflow to Brooklyn		Thad to Brooklyn		McKenzie to Brooklyn		Andalusia to Brooklyn		Total Routed Outflow	Measured Outflow
	$\Gamma=0.20$, $C_o=.20$, $C_2=.28$	$K=1.1$, $C_1=.52$	$\Gamma=0.2$, $C_o=.01$, $C_1=.41$	$K=2.4$, $C_2=.58$	$\Gamma=0.2$, $C_o=.01$, $C_1=.41$	$K=2.4$, $C_2=.58$	$\Gamma=0.2$, $C_o=.06$, $C_1=.41$	$K=1.9$, $C_2=.50$		
	Inflow	Outflow	Inflow	Outflow	Inflow	Outflow	Inflow	Outflow		
1944										
March 16	500	500	552	552	698	698	2,510	2,510	4,260	4,180
17	2,710	912	766	554	1,030	702	3,410	2,610	4,810	5,600
18	4,570	2,500	947	645	1,480	844	4,170	2,940	7,020	7,270
19	5,890	4,280	1,630	778	2,210	5,520	7,000	3,720	14,300	9,620
20	6,070	5,480	2,520	1,150	3,400	4,140	9,600	5,040	15,800	17,000
21	4,020	5,490	2,280	1,720	4,550	3,840	10,100	6,840	17,900	18,300
22	6,630	4,950	2,560	1,960	5,080	4,140	12,300	8,600	19,600	19,400
23	9,330	6,700	7,250	2,260	8,420	4,570	21,000	11,000	24,500	25,400
24	5,370	7,800	8,390	4,370	16,400	6,270	29,800	16,500	34,900	34,100
25	2,650	5,510	8,390	6,060	16,900	10,500	29,800	23,200	45,300	44,600
26	2,650	3,450	5,419	7,010	10,300	13,100	29,500	26,500	50,100	51,200
27	1,590	2,660	3,190	6,330	5,680	11,900	24,000	27,700	48,600	51,900
28	500	1,670	2,440	5,000	4,120	9,270	14,600	23,300	41,200	41,700
29	5,860	1,900	3,799	3,940	6,630	7,130	18,400	20,200	33,200	35,800
30	4,520	4,480	5,860	3,900	15,500	7,010	19,300	19,400	34,800	36,100
31	1,840	3,970	10,000	4,760	18,800	10,600	18,500	19,300	38,600	35,100
April 1	1,840	2,440	5,950	6,920	11,500	14,000	22,000	19,100	42,500	38,500
2	2,320	2,100	3,300	6,490	6,210	12,900	22,000	20,600	42,100	44,600
3	1,500	2,100	2,120	5,140	2,690	10,100	14,200	20,800	38,100	40,100
4	1,120	1,590	1,570	3,870	1,930	6,080	10,100	17,300	29,700	30,000
5	786	1,180	1,220	2,900	1,480	4,850	7,950	13,600	22,500	20,600
6	786	896	998	2,190	1,170	3,430	6,340	10,700	17,200	13,700
7	666	793	830	1,690	925	2,480	5,410	8,460	13,400	10,300
8	500	668	718	1,330	800	1,830	4,210	7,190	11,000	8,530
9	500	547	671	k,070	742	1,400	4,170	5,700	8,700	7,710

Table I. Example of Routing for Andalusia-Brooklyn Reach, Conecuh River. From: Manual of Hydrology: Flood Flow Techniques.

To obtain a comparison with a statistical approach, a correlation of observed outflow versus the inflows at time zero, one time period ago, two time periods ago and three time periods ago was prepared.

The statistical analysis was allowed to eliminate parameters which did not meet statistical tests for a valid analysis. Thus all parameters which were retained are considered statistically valid. The resulting coefficients and their individual sums are given in Table 2.

Flow at	Inflows			
Time Now	0.5	-1.686	.471	.728
One Time Period Ago	1.05	-1.771	1.431	0.0
Two Time Periods Ago	0.0	0.0	0.0	.324
Three Time Periods Ago	0.0	0.0	.762	.231
Sums	1.05	-3.457	2.664	1.283

Sum of all coefficients = 1.540

Table 2. Coefficients and Individual Sums from Statistical Analysis.

In addition, a constant of 720 has been provided by the analysis and must be added to the coefficient flow.

An error analysis of the two techniques indicates that the statistical method has provided an extremely accurate analysis. A comparison with the storage routing in the Manual of Hydrology [2] is shown in Table 3.

TABLE 3

ERROR ANALYSIS

Measured Outflow	Total Routed Outflow	Absolute Error in Routed Outflow	Statistical Analysis: Statistical Solution	Statistical Analysis: Absolute Error in Statistical Solution
4,180	4,260	80	4,430	250
5,600	4,810	790	4,870	730
7,270	7,020	250	7,750	480
9,620	14,300	4,680	11,590	1,970
17,000	15,800	1,200	14,490	2,510
18,300	17,900	400	17,550	750
19,400	19,600	200	20,880	1,480
25,400	24,500	900	25,570	170
34,100	34,900	800	34,830	730
44,600	45,300	700	44,000	600
51,200	50,100	1,100	50,900	300
51,900	48,600	3,300	52,410	510
41,700	41,200	500	42,640	940
35,800	33,200	2,600	35,370	430
36,100	34,800	1,300	35,750	350
35,100	38,600	3,500	35,220	120
38,500	42,500	4,000	38,790	290
44,600	42,100	2,500	44,200	400
40,100	38,100	2,000	39,960	140
30,000	29,700	300	28,980	1,020
20,600	22,500	1,900	20,720	120
13,700	17,200	3,500	13,590	110
10,300	13,400	3,100	10,810	510
8,530	11,000	2,470	8,530	0
7,710	8,700	990	7,480	230
	Σ ERROR	43,060	Σ ERROR	15,140

Thus, the statistical analysis has resulted in an overall error about 1/3 the size indicated by storage routing. Storage routing does, however, possess the capability of maintaining volume during steady or variable flow. Let us test the statistical procedure under steady flow conditions.

Utilizing the two techniques and applying them to steady flow conditions yields the following cases.

<u>Case 1</u>. Steady inflow of 2,000 cfs from each of the four inflow points.

<u>Statistical Analysis</u>

Inflow 1	1.05 × 2,000 =	2,100
Inflow 2	-3.457 × 2,000 =	-6,914
Inflow 3	2.664 × 2,000	5,328
Inflow 4	1.283 × 2,000 =	2,566
	Constant =	720
Total Flow from Statistical Analysis	=	3,790

Analyzing the same data by the Storage Coefficients in the Manual of Hydrology yields,

<u>Storage Routing</u>

Inflow 1	(.20 + .28 + .52) × 2,000 =	2,000
Inflow 2	(.01 + .41 + .58) × 2,000 =	2,000
Inflow 3	(.01 + .41 + .58) × 2,000 =	2,000
Inflow 4	(.06 + .44 + .50) × 2,000 =	2,000
Total Flow from Storage Routing	=	8,000

Thus, in Case 1 we have found that the statistical procedure has reduced the outflow to less than 1/2 what both reason and storage routing indicate.

Let us examine a second situation for more dramatic demonstration of the problem with application of statistical analysis.

<u>Case 2</u>. A steady reservoir release of 2,000 cfs from Inflow 2, coupled with a nominal steady flow of 200 cfs from each of the other three inflow sites.

<u>Statistical Analysis</u>

Inflow 1	1.05 × 200 =	210
Inflow 2	-3.457 × 2,000 =	-6,914
Inflow 3	2.664 × 200 =	533
Inflow 4	1.283 × 200 =	257
Total Flow from Statistical Analysis	=	-5914 cfs

Under these circumstances, statistical analysis indicates the flow in the river has reversed and is flowing at 5,914 cfs in the opposite direction.

Storage Routing

Inflow 1	(.20 + .28 + .52)	×	200	=	200
Inflow 2	(.01 + .41 + .58)	×	2,000	=	2,000
Inflow 3	(.01 + .41 + .58)	×	200	=	200
Inflow 4	(.06 + .44 + .50)	×	200	=	200

Total Flow from Storage Routing = 2,600 cfs

Thus we have again arrived at an impossible solution applying a statistical analysis while the storage analysis is in agreement with a logical evaluation of steady flow.

A fantastic array of irrational conditions can be produced by applying statistical analysis, including the fallacy that if there is no inflow from any site, the outflow will be 720 cfs indefinitely.

Does this mean that statistics has no place in flood routing? The answer is, not as a direct routing tool. Statistics can be utilized as mentioned by Linsley, Kohler, and Paulus [1], but restrictions must be followed. These restrictions are:

1. All inflow parameters must be linear.

2. Coefficients for each inflow point must be greater than zero.

3. The coefficient set for each inflow point must sum to unity.

4. The constant of regression must be zero.

A violation of any one of these rules is a violation of the law of continuity and will produce incorrect results. Since it is nearly impossible to impose these conditions on a direct statistical analysis, it should be clear that the function of statistics in flood routing is to serve as a mathematical tool for optimizing routing techniques which do not violate the law of continuity, not as a routing method by itself.

References

1. Hydrology for Engineers, pages 229 - 233. Linsley, Kohler, and Paulus. McGraw-Hill, 1958.

2. Storage and Flood Routing; Manual of Hydrology: Part 3, Flood Flow Techniques. Geological Survey. Water Supply Paper 1543-B.