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National Oceanic and Atmospheric Administration
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Thunderstorms and Hail Days Probabilities in Nevada

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Western Region

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U. S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE

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THUNDERSTORMS AND HAIL DAYS PROBABILITIES IN NEVADA

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WESTERN REGION
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THUNDERSTORM AND HAIL DAYS PROBABILITIES IN NEVADA

ABSTRACT

A computer program was developed to provide probabilities for selected number of thunderstorm days in a month and in a year. In addition, probabilities for selected number of hail days in a year were determined. Two distribution models were tested in the analysis: (a) Poisson and (b) negative binomial. The program determines which of these two models is appropriate. Furthermore, if the negative binomial model is selected, tests are conducted to determine whether estimation of the parameters is to be made by the method of moments or by the method of maximum likelihood. A procedure for estimating efficient estimates of the parameters utilizing reiterative process and the curvilinear model is described. Estimates by this procedure compare favorably with those obtained "by eye".

The program was applied to five locations in Nevada. Results show that for Nevada, the Poisson distribution fits the monthly thunderstorm days for the months November through April, while the negative binomial fits this variable better from May through October. The negative binomial model also fits the annual thunderstorm days in Nevada. Annual hail days distribution favored the Poisson distribution where the frequency was small. The negative binomial fitted the annual hail days distribution at Ely and Elko. Cumulative probabilities are presented for these variables at the five sites, including Elko, Ely, Las Vegas, Reno, and Winnemucca.

I. INTRODUCTION

Frequency of thunderstorms or hail in an area can be an important concern in planning for an installation of equipment or manpower. Thunderstorms also imply the possibility of flash floods, and, consequently, necessary precautions must be considered in the development of a watershed for its varied uses.

Climatological probabilities provide quantitative information on the chance of occurrence of these meteorological phenomena and can be useful in a decision where cost-benefit analysis is vital. The purpose of this study is to analyze the frequency of occurrence of thunderstorm and hail days in Nevada and to derive probabilities for these events.

A thunderstorm day is defined as the occurrence-day of at least one thunderstorm cloud (cumulonimbus) accompanied by lightning and thunder. It may or may not be accompanied by strong gusts of wind, rain, or hail. A hail day is a day when precipitation in the form of ice is produced by convective clouds. During the winter, smaller-sized frozen droplets fall, usually smaller in size than hail. These are called "small hail" and, for the purpose of this study, "small hail" and hail have not been differentiated.

II. PROCEDURE

Thom (6) has indicated that the Poisson or the negative binomial distribution can be potentially applied to rare events, such as tornado frequency, tropical cyclone frequency, hail frequency, etc. The Poisson distribution has the mean equal to the variance. If the variance increases above the mean, the distribution tends to fit the negative binomial. Generalized guidelines as to which of the two models is appropriate are available but, until the proper tests are conducted, one cannot objectively determine which model is appropriate. A test of hypothesis, using χ^2 distribution with n-l degrees of freedom, is used to determine whether the Poisson or the negative binomial distribution is desirable. It is given by:

$$\chi^2_{n-1} = \frac{n\Sigma x^2}{\Sigma x} - \Sigma x \qquad (1)$$

where: variable x is the number of event days and n is the sample size.

The Poisson probability function is given by:

$$f(x) = \mu^x \frac{e^{-\mu}}{x!} \qquad (2)$$

where: f(x) is the probability of having, for example, exactly x hail days for the period in question. μ is the population mean.

Expressed in natural logarithms, the Poisson density function is:

$$\ln P = x \ln \bar{x} - \ln x! - \bar{x} \qquad (3)$$

where: P is the probability of exactly x hail days and $\bar{x}$ is the sample mean.

The negative binomial probability function can be given by (1):

$$f(x) = \frac{(k + x-1)!}{x!\,(k-1)!} \left[\frac{p^x}{(1 + p)^{k + x}}\right] \qquad (4)$$

where: k and p are the parameters of the distribution. These parameters can be initially estimated by the method of moments:

$$k = \frac{\bar{x}^2}{s^2 - \bar{x}} \qquad (5)$$

and $$p = \frac{s^2 - \bar{x}}{\bar{x}}$$

where: $\bar{x}$ and s^2 are the sample mean and variance, respectively.

Expressed in natural logarithms, the density function for the negative binomial is:

$$\ln P = k \ln\left(\frac{1}{1 + p}\right) + \ln K + x \ln\left(\frac{p}{p + 1}\right) \qquad (6)$$

where: P is the probability of x event days for the period in question.

K is defined as:

$$K = \frac{(k + x - 1)!}{x\,!\;\;(k-1)\,!} \qquad (7)$$

The moments method of estimating the parameters p and k is not always efficient. Fisher (3) has provided equation 8, a method of testing whether the efficiency of the moments method is less than 90% by:

$$C = \left(1 + \frac{1}{p}\right)\;(k + 2) \qquad (8)$$

If $C < 20$, the method of maximum likelihood estimates should be used. If $C > 20$, the method of moments suffices.

The maximum likelihood procedure involves writing the likelihood function,

$$L = \prod_{i=1}^{n} f(x_i, p, k) \qquad (9)$$

and maximizing the logarithm of L, by taking the partial derivative of the logarithm of L with respect to p and k. When set to zero, and solving, the two parameter estimates are determined. Taking the partial derivative of equation (4) with respect to p, and setting to zero,

$$L_1 = \frac{\partial \log L}{\partial p} = \frac{\Sigma x}{p} - \frac{nk + \Sigma x}{1 + p} = 0 \qquad (10)$$

Substituting $\bar{x}$ for $\Sigma x/n$, the mean of the sample is found to be the product of the parameters. Thus $\bar{x} = k\,p$ is the first equation.

Taking the partial derivative with respect to k, setting to zero, and using Haldane's (4) equation, which does not involve gamma functions, we obtain:

$$L_2 = \frac{\partial \log L}{\partial k} = kn \log (1 + \frac{\bar{x}}{k}) - [(g_1 + g_2 + \ldots g_R) +$$

$$\frac{k}{k+1} (g_2 + g_3 + \ldots + g_R) + \frac{k}{k+2} (g_3 + g_4 + \ldots + g_R) +$$

$$\ldots + \frac{k\,(g_R)}{k + R - 1}] = 0 \qquad (11)$$

where g_1, g_2, ... g_R are the observed frequencies for the number of thunderstorm or hail days, $x = 1, 2, \ldots, R$ is the largest x. $\bar{x}$ = sample mean; n = number of years; k = parameter estimate. Thom (7) suggests solving this equation by trial and error or by plotting a few values of L_2 against k. The value of k at $L_2 = 0$ is the final estimate of the maximum likelihood estimator of the parameter k. The maximum likelihood estimator of p is solved by substituting k in $\bar{x}$ = kp which was previously obtained.

III. DATA

Two sources of records were utilized to summarize information needed for the analysis. These were the Local Climatological Data (8) and the Climatological Records Book for each location.

IV. COMPUTER PROGRAM

A FORTRAN IV program was developed for the analysis of thunderstorm and hail days that facilitates the solution to the estimation of probabilities for these events. In the program, values of L_2 (see Procedure) were calculated reiteratively by selecting values of k in equation 11 and solving for L_2. The program then searches for the transition of negative and positive values of L_2. Several values of L_2 are selected from both sides of the transition point and subjected to the second order polynomial (curvilinear) equation. The final value of k is determined by setting the derived curvilinear equation to zero and solving for k by the quadratic equation. This procedure was done after repeated trials of curve fitting and the curvilinear model was determined to fit the observed curve very well. The above procedure eliminates the tedious process of curve fitting by eye.

Sample sizes from 10 to 40 years are the suggested limits for this program. This restriction results from the insertion of the Chi-square values at the 0.05 level of significance to test the adequacy of the Poisson distribution. To minimize the program size, a relationship was established between the degrees of freedom and the Chi-square values. Values for this relationship can be found in an elementary statistics test. The resultant equation at the 0.05 level of significance is:

$$Y = 4.54921 + 1.41672D - 0.0036744D^2 \qquad (12)$$

where:

Y = Chi-square value at the 0.05 level

D = degrees of freedom

The program was designed for five specific locations. If more locations are required, cards 5, 11, 12, 35, and 38 should be changed accordingly.[2] Furthermore, a maximum of 55 thunderstorm or hail days has been set. If more days (up to 99) are necessary, cards number 2, 3, 18, 39, 67, 108, 126, in the main program and cards 3 and 4 in subroutine NEGBINO need be changed to the appropriate number of days. A blank card is inserted between each new station.

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[2] Card numbers refer to the numbers listed on the extreme left margin of the program, as for example, 2:.

Card format is as follows. Blanks are read as zeros.

Columns	Remarks
1-2	Blank
3-6	Station number
7	Blank
8-11	Year (for monitor purpose; not necessary in program)
13-16	January (01) and number of thunderstorms (00 to 55)
17	Blank
18-21	February (02) and number of thunderstorms (00 to 55)
22	Blank
23-26	March (03) and number of thunderstorms (00 to 55), etc.
72	Blank
73-74	Annual thunderstorm days (00 to 55)
75	Blank
76-77	Annual hail days (00 to 55)
78-80	Blank

V. RESULTS

Probability Models

Table I shows the summary of model selection for the five locations in Nevada. The results indicate that for the monthly distribution, model selection for estimating probabilities of selected number of thunderstorm days depends on the season, and hence, the climate of a particular region. The data suggest that for the period from November through April, the Poisson model is preferred in Nevada, while the negative binomial distribution is appropriate for the period May through October.

There were 11 cases where the selected model did not coincide with the majority model. However, seven of these cases involved maximum differences of less than .023 between the Poisson and negative binomial distribution. The maximum difference between these two models in the other four cases was .108 for zero number of thunderstorm days. In view of the few cases with these differences, the results of the computer selection were retained in the probability tables shown in Tables 2A through 6B, which also show the observed cumulative distribution. The observed and computed probabilities were compared and tested with the Kolmogorov-Smirnov test (5) and all results were within tolerance at the .10 level of significance.

For annual thunderstorm days, the negative binomial model was selected at all stations. For annual hail days, however, only Ely and Elko were associated with the negative binomial; whereas, Reno, Winnemucca, and Las Vegas were fitted with the Poisson distribution. As shown in Table 7, the means at Ely and Elko are larger than the other three sites. Furthermore, the variance is considerably larger than the mean at Ely and Elko. The selection of either of two models for probabilities of annual number of hail days in Nevada suggests that climatic difference is a factor in the selection of the distribution model. Therefore, each climatic region should be analyzed separately to determine the proper selection of the model that fits the data. Calculated cumulative probabilities from the model as well as observed cumulative frequencies for annual thunderstorm and annual hail days are shown in Tables 8 and 9, respectively. The Kolmogorov-Smirnov test showed that the selected models fitted the observed data at the .10 level of significance.

Illustration of reading these probability tables follows: The computed probabilities for "0" number of thunderstorm or hail days are the chance of none occurring at each of the sites. For example, in Table 9, the probability of no hail at Las Vegas is .875. The probability of exactly x number of hail days, for example, $x = 5$ days at Ely is .717 minus .596 or .121; the probability of less than 5 days is .717; the probability of greater than 5 hail days at Ely is 1.000 minus .717 or .283. Probabilities for other selected number of days and sites are determined similarly.

Computer Outputs

Sample outputs from the computer program are shown in Tables 10 and 11. Table 10 illustrates an example of the output for the negative binomial distribution, utilizing the maximum likelihood procedure for estimating the parameters k and p. Table 11 is an example of the output for annual hail days probabilities at Winnemucca.

Comparison of the computer program procedure used for estimating the parameter k, when L_2 (Equation 11) is zero and that for estimating k by graphical (eye) procedure is shown in Table 12. Estimate of the parameter by the method of moments is also included. Excellent agreement is indicated by the results between the computer and "by eye".

It is concluded that the procedure utilized in this study is both a reliable and a rapid method for calculating the parameters of the negative binomial distribution by the maximum likelihood method.

VI. ACKNOWLEDGMENT

This study started as a joint term paper with my wife, Winifred, in her Computer Programming course at the University of Nevada. Through this project we both learned the rudiments of computer programming with many of its frustrating moments. Dr. Young Koh, College of Agriculture Statistician, was helpful and aided our efforts when the program seemed impossible to debug. To these two, the author expresses sincere gratitude.

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TABLE I

SUMMARY OF MODEL SELECTION FOR THUNDERSTORM AND HAIL DAYS IN NEVADA

Period	Location				
	Ely	Reno	Elko	Winnemucca	Las Vegas
Jan	P*	None	P	P	P
Feb	N	P	P	P	P
Mar	P	P	P	P	P
Apr	P	P	P	P	N
May	N	N	N	N	P
Jun	N	N	P	N	P
Jul	N	N	P	N	N
Aug	N	N	N	N	N
Sep	N	P	N	N	N
Oct	N	N	N	P	N
Nov	P	N	P	N	P
Dec	P	N	P	P	P
Ann	N	N	N	N	N
Annual Hail	N	P	N	P	P

*P = Poisson; N = Negative Binomial

TABLE 2A

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT ELKO, NEVADA, FROM JANUARY THROUGH JUNE (1941 - 1970)

No.	JAN		FEB		MAR		APR		MAY		JUN	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.875	.867	.717	.733	.693	.700	.393	.500	.114	.033	.020	.067
1	.992	1.000	.955	.933	.947	.933	.760	.700	.273	.300	.102	.167
2	1.000		.995	1.000	.994	1.000	.931	.867	.432	.433	.258	.333
3					.999		.985	1.000	.572	.633	.460	.400
4							.997		.685	.767	.655	.500
5									.773	.833	.806	.733
6									.839	.867	.903	.967
7									.887	.867	.956	.967
8									.922	.900	.982	1.000
9									.946	.900	.993	
10									.963	.900	.998	
11									.975	.900		
12									.983	1.000		
13									.989			
14									.993			
15									.995			

TABLE 2B

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT ELKO, NEVADA, FROM JULY THROUGH DECEMBER (1941 - 1970)

No.	JUL		AUG		SEP		OCT		NOV		DEC	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.006	.033	.063	.100	.317	.333	.636	.633	.819	.800	.875	.867
1	.038	.067	.180	.167	.587	.533	.858	.833	.983	1.000	.992	1.000
2	.119	.167	.324	.300	.766	.767	.944	.967	.999		1.000	
3	.256	.400	.468	.467	.872	.867	.977	.967				
4	.429	.400	.597	.500	.932	.967	.991	1.000				
5	.604	.533	.704	.667	.964	.967	.996					
6	.752	.633	.789	.767	.982	.967						
7	.860	.800	.851	.867	.991	1.000						
8	.928	.933	.897	.900	.995							
9	.966	.967	.930	.933								
10	.985	1.000	.953	1.000								
11	.994		.969									
12	.998		.980									
13			.987									
14			.991									
15			.995									
16			.997									

TABLE 3A

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT ELY, NEVADA, FROM JANUARY THROUGH JUNE (1941 - 1970)

No.	JAN		FEB		MAR		APR		MAY		JUN	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.905	.900	.903	.900	.648	.667	.231	.300	.061	.033	.038	.067
1	.995	1.000	.963	.933	.929	.900	.569	.533	.185	.166	.118	.100
2			.984	1.000	.990	1.000	.817	.833	.340	.433	.228	.233
3			.992		.999		.938	.900	.495	.500	.350	.266
4			.996				.983	.966	.632	.600	.471	.400
5			.996				.996	1.000	.742	.633	.581	.600
6									.825	.867	.676	.667
7									.884	.933	.754	.800
8									.925	.933	.817	.900
9									.953	.933	.865	.933
10									.970	.967	.902	.933
11									.982	.967	.930	.933
12									.989	1.000	.950	.933
13									.993		.965	.967
14									.996		.976	.967
15											.983	.967
16											.988	.967
17											.992	.967
18											.995	.967
19											.996	1.000
20												

TABLE 3B

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT ELY, NEVADA, FROM JULY THROUGH DECEMBER (1941 - 1970)

No.	JUL		AUG		SEP		OCT		NOV		DEC	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.021	.033	.005	.000	.192	.133	.466	.467	.716	.700	.766	.800
1	.068	.100	.024	.000	.442	.500	.706	.700	.955	.967	.970	.933
2	.135	.167	.062	.100	.654	.700	.837	.833	.995	1.000	.997	1.000
3	.217	.233	.120	.100	.801	.733	.909	.900				
4	.305	.267	.196	.266	.891	.900	.949	.933				
5	.395	.367	.285	.266	.943	.967	.971	.967				
6	.480	.466	.379	.333	.971	.967	.984	1.000				
7	.560	.500	.474	.400	.986	.967	.991					
8	.632	.533	.563	.566	.993	1.000	.995					
9	.695	.633	.644	.633	.997		.997					
10	.750	.733	.716	.667								
11	.796	.766	.776	.800								
12	.835	.833	.827	.867								
13	.868	.866	.867	.934								
14	.894	.866	.900	.934								
15	.916	.966	.925	.967								
16	.934	.966	.945	.967								
17	.948	.966	.960	.967								
18	.959	1.000	.971	.967								
19	.968		.979	.967								
20	.975		.985	.967								
21	.981		.989	.967								
22	.985		.992	.967								
23	.988		.995	.967								
24	.991		.996	1.000								
25	.993											
26	.995											
27	.996											

TABLE 4A

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT LAS VEGAS, NEVADA, FROM JANUARY THROUGH JUNE (1940-1971)

No.	JAN		FEB		MAR		APR		MAY		JUN	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.967	.967	.792	.833	.875	.833	.642	.633	.380	.400	.407	.400
1	1.000	1.000	.977	.967	.992	1.000	.858	.867	.748	.700	.773	.767
2			.998	1.000	1.000		.942	.933	.926	.967	.937	.933
3							.976	.967	.983	.967	.987	1.000
4							.989	1.000	.997	1.000	.998	
5							.996					

TABLE 4B

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT LAS VEGAS, NEVADA, FROM JULY THROUGH DECEMBER (1941-1971)

No. Days	JUL C	JUL O	AUG C	AUG O	SEP C	SEP O	OCT C	OCT O	NOV C	NOV O	DEC C	DEC O
0	.032	.067	.110	.166	.381	.400	.581	.567	.847	.867	.967	.967
1	.118	.167	.262	.166	.661	.600	.750	.800	.988	.900	1.000	1.000
2	.251	.300	.416	.400	.825	.800	.838	.867	.999	1.000		
3	.406	.367	.553	.633	.913	.967	.891	.900				
4	.558	.500	.666	.633	.958	.967	.925	1.000				
5	.688	.633	.755	.700	.980	.967	.948					
6	.790	.700	.823	.800	.991	1.000	.963					
7	.864	.867	.874	.833	.996		.974					
8	.915	.900	.911	.933			.981					
9	.949	1.000	.937	.967			.986					
10	.970		.956	.967			.990					
11	.983		.970	.967			.993					
12	.990		.979	1.000			.995					
13	.995		.986				.996					
14	.997		.990									
15			.993									
16			.996									

TABLE 5A

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT RENO, NEVADA, FROM JANUARY THROUGH JUNE (1941 - 1970)

No. Days	JAN		FEB		MAR		APR		MAY		JUN	
	C	O	C	O	C	O	C	O	C	O	C	O
0	1.000	1.000	.967	.967	.936	.933	.670	.633	.230	.200	.191	.167
1			1.000	1.000	.998	1.000	.938	.967	.479	.467	.396	.433
2							.992	1.000	.673	.667	.570	.567
3							.999		.804	.800	.702	.700
4									.887	.900	.798	.800
5									.936	.900	.865	.867
6									.965	1.000	.911	.900
7									.981		.942	.933
8									.990		.962	.967
9									.995		.975	.967
10									.997		.984	.967
11											.990	1.000
12											.994	
13											.996	
14												
15												

TABLE 5B

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT RENO, NEVADA, FROM JULY THROUGH DECEMBER (1941 - 1970)

No.	JUL		AUG		SEP		OCT		NOV		DEC	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.085	.167	.256	.233	.380	.433	.807	.833	.945	.967	.945	.967
1	.228	.200	.449	.500	.748	.733	.925	.967	.991	.967	.991	.967
2	.389	.333	.594	.633	.926	.867	.966	.967	.998	1.000	.998	1.000
3	.540	.466	.700	.667	.983	.967	.984	.967				
4	.666	.567	.780	.767	.997	1.000	.992	.967				
5	.765	.733	.838	.800			.996	1.000				
6	.838	.900	.881	.867								
7	.891	.933	.912	.900								
8	.928	.967	.936	.900								
9	.953	.967	.953	.967								
10	.970	1.000	.965	.967								
11	.981		.975	.967								
12	.988		.981	1.000								
13	.992		.986									
14	.995		.990									
15			.993									
16			.995									
17			.996									
18												
19												
20												

TABLE 6A

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT WINNEMUCCA, NEVADA, FROM JANUARY THROUGH JUNE (1941 - 1970)

No.	JAN		FEB		MAR		APR		MAY		JUN	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.967	.967	.875	.900	.819	.800	.435	.367	.219	.233	.146	.167
1	1.000	1.000	.992	.967	.983	1.000	.797	.933	.421	.367	.328	.333
2			1.000	1.000	.999		.948	.967	.581	.633	.497	.467
3							.990	1.000	.701	.767	.635	.633
4							.998		.789	.800	.742	.667
5									.852	.833	.821	.800
6									.897	.867	.878	.867
7									.929	.933	.917	.967
8									.951	.967	.944	.967
9									.966	.967	.963	.967
10									.977	1.000	.976	1.000
11									.984		.984	
12									.989		.990	
13									.993		.993	
14									.995		.996	
15									.997			

TABLE 6B

COMPUTED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF MONTHLY NUMBER OF THUNDERSTORM DAYS AT WINNEMUCCA, NEVADA, FROM JULY THROUGH DECEMBER (1941 - 1970)

No.	JUL		AUG		SEP		OCT		NOV		DEC	
Days	C	O	C	O	C	O	C	O	C	O	C	O
0	.160	.200	.240	.233	.355	.333	.420	.500	.894	.933	.936	.933
1	.347	.333	.442	.400	.640	.667	.785	.733	.965	1.000	.998	1.000
2	.512	.433	.597	.567	.814	.800	.943	.833	.986			
3	.647	.567	.712	.667	.908	.900	.988	1.000	.994			
4	.750	.633	.795	.767	.956	.967	.998		.997			
5	.825	.800	.855	.833	.979	.967						
6	.879	.900	.898	.867	.990	1.000						
7	.917	.900	.928	.933	.996							
8	.944	.967	.950	.967								
9	.962	.967	.965	1.000								
10	.975	.967	.975									
11	.983	1.000	.983									
12	.989		.988									
13	.993		.992									
14	.995		.994									
15			.996									

TABLE 7

MEAN AND VARIANCE OF ANNUAL THUNDERSTORM AND ANNUAL HAIL DAYS AT FIVE LOCATIONS IN NEVADA (1941 - 1970)

	Thunderstorm		Hail	
Locations	Mean	Variance	Mean	Variance
Elko	24.23	39.47	2.67	6.09
Ely	31.97	97.69	4.27	7.24
Las Vegas	13.47	25.84	.13	.12
Reno	13.50	37.22	1.17	1.11
Winnemucca	15.43	47.08	2.40	3.14

TABLE 8

CALCULATED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF ANNUAL THUNDERSTORM DAYS AT FIVE LOCATIONS IN NEVADA (1941 - 1970)

No. Days	LOCATIONS									
	ELKO		ELY		LAS VEGAS		RENO		WINNEMUCCA	
	C	O	C	O	C	O	C	O	C	O
0	.000	.000	.000	.000	.000	.000	.001	.000	.000	.000
1	.000	.000	.000	.000	.001	.000	.003	.000	.002	.000
2	.000	.000	.000	.000	.002	.000	.009	.000	.007	.033
3	.000	.000	.000	.000	.007	.000	.021	.000	.017	.033
4	.000	.000	.000	.000	.018	.033	.042	.033	.032	.067
5	.001	0.33	.000	.000	.037	.033	.072	.067	.055	.100
6	.002	.033	.000	.000	.066	.067	.112	.167	.086	.133
7	.005	.033	.000	.000	.108	.167	.162	.167	.123	.133
8	.010	.033	.001	.000	.162	.167	.219	.233	.167	.133
9	.018	.033	.002	.000	.227	.200	.283	.300	.217	.167
10	.030	.033	.003	.000	.300	.367	.349	.367	.272	.233
11	.048	.067	.005	.000	.380	.367	.418	.433	.329	.267
12	.072	.100	.009	.000	.460	.433	.485	.533	.387	.333
13	.103	.133	.013	.000	.540	.500	.551	.600	.446	.400
14	.143	.133	.020	.000	.615	.567	.612	.600	.503	.467
15	.189	.133	.028	.000	.684	.667	.669	.633	.558	.467
16	.242	.200	.039	.000	.745	.733	.720	.633	.610	.567
17	.300	.300	.053	.000	.798	.800	.765	.667	.658	.667
18	.362	.300	.070	.133	.842	.833	.805	.767	.702	.700
19	.426	.333	.090	.133	.879	.867	.839	.799	.743	.733
20	.491	.433	.113	.200	.908	.867	.869	.799	.779	.733
21	.554	.533	.139	.200	.931	.933	.894	.867	.811	.833
22	.615	.567	.168	.233	.949	.967	.914	.899	.840	.867
23	.672	.700	.200	.267	.963	.967	.932	.966	.865	.900
24	.724	.700	.235	.333	.973	1.000	.946	.966	.886	.900
25	.770	.800	.272	.333	.981		.957	1.000	.905	.900
30	.924	.900	.475	.433	.998		.988		.964	1.000
35	.981	1.000	.667	.533			.997		.987	
40	.996		.814	.767					.996	
45			.906	.933						
50			.942	.967						
55			.942	1.000						

TABLE 9

CALCULATED (C) AND OBSERVED (O) CUMULATIVE PROBABILITIES OF ANNUAL HAIL DAYS AT FIVE LOCATIONS IN NEVADA (1941 - 1970)

No. Days	LOCATIONS									
	ELKO		ELY		LAS VEGAS		RENO		WINNEMUCCA	
	C	O	C	O	C	O	C	O	C	O
0	.160	.100	.044	.000	.875	.867	.311	.333	.091	.100
1	.370	.300	.147	.100	.992	1.000	.674	.633	.308	.367
2	.561	.567	.292	.100	1.000		.887	.867	.570	.600
3	.710	.733	.450	.233			.969	1.000	.779	.767
4	.815	.867	.596	.633			.993		.904	.867
5	.886	.933	.717	.800			.999		.964	.933
6	.931	.933	.809	.833					.988	.967
7	.959	.933	.876	.867					.997	1.000
8	.976	.933	.922	.900						
9	.986	.967	.952	.933						
10	.992	.967	.971	.967						
11	.995	1.000	.983	1.000						
12			.990							
13			.994							
14			.997							
15										

TABLE 10

SAMPLE PROGRAM OUTPUT SHOWING THE MAXIMUM LIKELIHOOD PARAMETER ESTIMATE AND PROBABILITIES FOR SELECTED NUMBER OF THUNDERSTORM DAYS AT ELY, NEVADA

```
  SEPTEMBER  THUNDERSTORM DAYS AT       ELY

 MEAN= 2.133   VARIANCE= 3.499   NO. OF YEARS=  30

MAXIMUM LIKELIHOOD METHOD OF PARAMETER ESTIMATE

K=      3.368      P=       .633

PERIOD=    9  MODEL IS NEGATIVE BINOMIAL

 TABLE 9.   CHANCE OF SELECTED NUMBER OF THUNDERSTORM DAYS
AT     ELY    NEVADA  (1941-1970) FOR THE SEPTEMBER  PERIOD.

-------------------------------------------------------------

THUNDERSTORM DAYS     PROBABILITY     CUMULATIVE PROBABILITY

-------------------------------------------------------------

          0                .1916             .1916

          1                .2502             .4417

          2                .2119             .6536

          3                .1470             .8006

          4                .0908             .8914

          5                .0519             .9433

          6                .0281             .9713

          7                .0146             .9859

          8                .0073             .9932

          9                .0036             .9968

-------------------------------------------------------------
```

TABLE II

SAMPLE PROGRAM OUTPUT SHOWING PROBABILITIES OF SELECTED NUMBER OF ANNUAL HAIL DAYS AT WINNEMUCCA, NEVADA, WITH THE POISSON DISTRIBUTION

```
     ANNUAL    HAIL DAYS AT    WINNEMUCCA

   MEAN= 2.400    VARIANCE= 3.145    NØ. ØF YEARS=   30

     PERIØD=    14  MØDEL IS PØISSØN

  TABLE14.   CHANCE ØF SELECTED NUMBER ØF HAIL DAYS
AT WINNEMUCCA NEVADA (1941-1970) FØR THE   ANNUAL   PERIØD.
  ------------------------------------------------------------

  HAIL DAYS     PRØBABILITY     CUMULATIVE PRØBABILITY
  ------------------------------------------------------------

      0              .0907              .0907

      1              .2177              .3084

      2              .2613              .5697

      3              .2090              .7787

      4              .1254              .9041

      5              .0602              .9643

      6              .0241              .9884

      7              .0083              .9967

  ------------------------------------------------------------
```

TABLE 12

COMPARISON OF PARAMETER K ESTIMATES BY METHOD OF MAXIMUM LIKELIHOOD (MXL), METHOD OF MOMENTS (MOM) AND "BY EYE" FOR THUNDERSTORM PROBABILITIES IN NEVADA

	ELKO			ELY			LAS VEGAS			RENO			WINNEMUCCA		
Period	MXL	MOM	EYE	MXL	MOM	EYE	MXL	MOM	EYE	MXL	MOM	EYE	MXL	MOM	EYE
May	2.228	1.819	2.226	4.013	4.067	4.017	---	---	---	2.277	3.447	2.273	1.377	1.573	1.366
Jun	---	---	---	3.499	3.197	3.497	---	---	---	1.784	1.739	1.779	2.042	2.560	2.040
Jul	---	---	---	3.047	3.927	3.037	---	6.750	---	3.060	4.522	3.064	1.855	2.361	1.849
Aug	3.315	4.735	3.316	5.831	5.474	5.831	2.180	2.614	2.174	1.035	1.109	1.037	1.227	1.652	1.222
Sep	1.833	2.133	1.833	3.368	3.333	3.373	1.704	2.169	1.700	---	---	---	1.960	2.138	1.956
Oct	.840	1.065	.840	.902	1.044	.896	.382	.271	.381	.259	.190	.247	---	---	---
Ann	---	24.233	---	---	15.548		---	14.652	---	7.282	7.682	7.282	6.236	7.526	6.241

APPENDIX

FORTRAN IV program for computing probabilities of thunderstorm and hail days.

```
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON XF(55),CAI(14),PEA(14),PC(55),CPROB(55),FIN(14)
      DIMENSION ARRAY(55,14),EVENT(55),CEST(60,2),XBAR(14),VAR(14),YEARS
     1(14),G(3,3),H(3),IN(14)
      DIMENSION IPERIOD(14,3),ISITE(5,3)
      DIMENSION OK(2),Z(3)
      DATA ((IPERIOD(KI,J),J=1,3),KI=1,14)/'   JANUARY  ','  FEBRUARY  '
     2,'   MARCH    ','   APRIL    ','    MAY     ','    JUNE    ','
     3JULY    ','   AUGUST   ',' SEPTEMBER  ','  OCTOBER   ','  NOVEMBER
     4  ','  DECEMBER  ','   ANNUAL   ','   ANNUAL   '/
      DATA ((ISITE(ID,L),L=1,3), ID=1,5)/'    ELY     ','    RENO    ','
     2    ELKO    ',' WINNEMUCCA ',' LAS VEGAS '/
C
C     EVALUATION OF X FACTORIAL
C
      XF(1)=0.
      XF(2)=1
      DO 300 I=3,55
      XXF=I-1
  300 XF(I)=XF(I-1)*XXF
C
C
      WRITE(108,510)
  510 FORMAT(2X,'PROBABILITY OF THUNDERSTORM AND HAIL DAYS',/)
      WRITE(108,505)
  505 FORMAT(//,'ABSTRACT:  THIS PROGRAM CALCULATES PROBABILITIES OF MON
     1THLY AND ANNUAL',/'THUNDERSTORM DAYS AND ANNUAL HAIL DAYS AT FIVE
     2LOCATIONS IN NEVADA:',/' ELY, RENO, ELKO , WINNEMUCCA AND LAS VEGA
     3S.  TESTS ARE CONDUCTED TO',/'DETERMINE WHETHER THE POISSON OR THE
     4NEGATIVE BINOMIAL MODEL SHOULD',/'BE USED.  EFFICIENT ESTIMATES OF
     5OF THE PARAMETERS ARE CALCULATED BY EITHER',/'THE METHOD OF MOMENT
     6S OR BY THE METHOD OF MAXIMUM LIKELIHOOD FOR',/'THE NEGATIVE BINOM
     7IAL.')
C
C     DO FOR 5 SITES
C
C     MAIN PROGRAM BEGINS
      DO 70 ID=1,5
      DO 1 I=1,55
      DO 1 J=1,14
    1 ARRAY(I,J)=0.
      READ(105,100)IID,IN
C     FIND FREQUENCY FOR EACH NUMBER OF EVENTS
      DO 2 I=1,14
      IC=IN(I)+1
  100 FORMAT(2X,I4,5X,12(3X,I2),1X,I2,1X,I2)
    2 ARRAY(IC,I)=ARRAY(IC,I)+1.
C
C     TEST FOR NEXT LOCATION
    5 READ(105,100, END=50) IDD,IN
      IF(IID.NE.IDD) GO TO 50
      DO 3 I=1,14
      IC=IN(I)+1
    3 ARRAY(IC,I)=ARRAY(IC,I)+1.
      GO TO 5
C     DO FOR MONTHLY THUNDERSTORM + ANNUAL THUNDERSTORM AND ANNUAL HAIL.
   50 DO 60 KI=1,14
C     XX=SUM OF X SQUARES; XB=SUMS OF X; CA=TOTAL N OR NUMBER OF YEARS
C     INITIALIZE
```

```
C
   55 XX=0.
      XB=0.
      CA=0.
C
C     SOLVE FOR SAMPLE MEAN AND VARIANCE
C
      DO 6 I=1,55
      XI=I-1
      EVENT(I)=ARRAY(I,KI)
      XX=XX+EVENT(I)*XI*XI
      XB=XB+EVENT(I)*XI
    6 CA=CA+EVENT(I)
      IF (XB.EQ.0) GO TO 35
      XBAR(KI)=XB/CA
      VAR(KI)=(XX-XBAR(KI)*XB)/(CA-1.)
      YEARS(KI)=CA
      GO TO 36
   35 WRITE (108,108)(IPERIOD(KI,J),J=1,3),(ISITE(ID,L),L=1,3)
  108 FORMAT('1',3X,3A4,5X,3A4,5X,'NO THUNDERSTORMS',///)
      GO TO 60
C
C     CALCULATE PARAMETERS K AND P(K=CAI, P=PEA) BY METHOD OF MOMENTS
C
   36 CAI(KI)=(XBAR(KI)**2)/(VAR(KI)-XBAR(KI))
      PEA(KI)=XBAR(KI)/CAI(KI)
      IF(KI.EQ.14) GO TO 125
      WRITE(108,101) (IPERIOD(KI,J),J=1,3),(ISITE(ID,L),L=1,3)
  101 FORMAT('1',2X,3A4,'THUNDERSTORM DAYS AT',2X,3A4,/)
      GO TO 475
  125 WRITE(108,130) (IPERIOD(KI,J),J=1,3),(ISITE(ID,L),L=1,3)
  130 FORMAT('1',3X,3A4,'HAIL DAYS AT',2X,3A4,/)
C
C     PRINT PERIOD, LOCATION, MEAN, VARIANCE AND YEARS
  475 WRITE(108,106) XBAR(KI ),VAR(KI),YEARS(KI)
  106 FORMAT(3X,'MEAN='F6.3,3X,'VARIANCE=',F6.3,3X,'NO. OF YEARS=',I4,/)
C
C     SHOULD POISSON OR NEGATIVE BINOMIAL MODEL BE USED?
C     IF THE CHI-SQUARE VALUE OF DEGREES OF FREEDOM AT THE .05 LEVEL IS
C     EXCEEDED, PROCEED TO TEST WHETHER PARAMETERS
C     K AND P BY METHOD OF MOMENTS IS EFFICIENT.  IF NOT EFFICIENT,
C     CALCULATE PARAMETERS BY MAXIMUM LIKELIHOOD METHOD.
C     IF .05 LEVEL IS NOT EXCEEDED, PROCEED TO POISSON
C
C
C     INITIALIZE
C
      N=0
      DO 491 I=1,55
  491 IF(EVENT(I).NE.0) N=I
C     WRITE(108,493)N
C 493 FORMAT('N=',I3)
      XXX=YEARS(KI)-1
C
C     POLYNOMIAL EQUATION RELATING DEG FREEDOM WITH CHI-SQUARE VALUES
      WHY=4.54921+1.41672*XXX-0.0036744*(XXX*XXX)
C
      CON=(YEARS(KI)*XX/XB)-XB
      IF(CON.GT.WHY)  GO TO 310
C
```

```
C     EVALUATION OF POISSON. DISTRIBUTION
C
      CCPROB=0
C
C     N CHANGED TO MAXIMUM EVENT NUMBER FOR DO LOOP
C
      N=55
      DO 325 IX=1,N
      IIX=IX-1
      IF(IIX.EQ.0.OR.IIX.EQ.1) GO TO 494
      P=IIX*DLOG(XBAR(KI))-DLOG(XF(IX ))-XBAR(KI)
      GO TO 498
  494 P=IIX*DLOG(XBAR(KI))-XBAR(KI)
  498 P=DEXP(P)
      CCPROB=CCPROB+P
      PC(IX)=P
      CPROB(IX)=CCPROB
  325 CONTINUE
      GO TO 400
C
C     ARE PARAMETERS EFFICIENT?
C     METHOD OF MOMENTS VERSUS MAXIMUM LIKELIHOOD
C     INITAL ESTIMATES OF PARAMETERS ARE MOMENTS ESTIMATORS
C
C     CAI=K; PEA=P
C
C     TEST FOR EFFICIENCY
C
  310 AYE=(1.+(1./PEA(KI)))*(CAI(KI)+2.)
      IF((N-2).EQ.0) GO TO 92
      IF(AYE.GT.20) GO TO 92
C
C     ESTIMATION OF PARAMETERS BY MAXIMUM LIKELIHOOD METHOD
C     FIND MINIMUM DIFFERENCE BY PARTIAL DIFFERENTIATION
C
C     DII=CA(LN((1+XBAR)/K))-((R(1)+R(2)+...+R(N)/K)+(R(2)+R(3++...+R(N
C     )/(K+1)+(R(N)/(K+Z-1)) WHERE:
C     CA=TOTAL NUMBER OF YEARS
C     N=EVENTS
C     Z=HIGHEST VALUE OF N OBSERVED
C     R(N)=OBSERVED FREQUENCY OF N EVENTS
C     XBAR=K*P
C     CAI=INITIAL MOMENTS ESTIMATE OF K
C
      INTE=CAI(KI)/.05
      IF(INTE.GT.30) INTE=30
      RA=INTE
      CAIII=CAI(KI)-.05*RA
      IF(CAIII.LT.0) CAIII=.01
   22 DO 16 JK=1,60
      CAIII=CAIII+.05
      DI=DLOG(1+(XBAR(KI)/CAIII)) *YEARS(KI)
      DII=0
C     YEARS(KI)=NUMBER OF YEARS
      XBB=YEARS(KI)
      CAII=CAIII
C     K IS NOW EQUAL TO CAII
      CEST(JK,2)=CAII
C
C      REDUCE NUMERATOR BY FREQUENCY OF ARRAY
```

```
C
      NM=N-2
      DO 15 I=1,NM
      XBB=XBB-EVENT(I)
C
C     INCREMENT DENOMINATOR K BY 1 TO K+I-1
C
      DII=DII+(XBB/CAII)
   15 CAII=CAII+1.
      DIF=DI-DII
      CEST(JK,1)=DIF
C     WRITE(108,23) CEST(JK,1),CEST(JK,2),DI,DII
C  23 FORMAT(2X,4F10.5)
   16 CONTINUE
      CALL FIND(CEST,MIDD)
      ISTAR=MIDD-4
      ILAST=MIDD+4
      IF(ISTAR.LT.1) ISTAR=1
      IF(ILAST.GT.60) ILAST=MIDD
C     WRITE(108,14) ISTAR,ILAST
C  14 FORMAT(2X,2F10.5)
C
C     SECOND ORDER POLYNOMIAL
C     ESTIMATION OF CONSTANT AND REGRESSION COEFFICIENTS
C
      DO 17 I=1,3
      H(I)=0
      DO 17 J=1,3
   17 G(I,J)=0
      DO 18 K=ISTAR,ILAST
      Z(1)=CEST(K,2)
      Z(2)=Z(1)**2
      Z(3)=CEST(K,1)
C     WRITE(108,12345) Z
12345 FORMAT(3F10.3)
      DO 18 I=1,3
C
C     SUMS
      H(I)=H(I)+Z(I)
      DO 18 J=1,3
C
C     CROSS PRODUCTS
   18 G(I,J)=G(I,J)+Z(I)*Z(J)
      DO 19 I=1,3
      DO 19 J=1,3
C
C     CORRECTION TERM
      KK=(ILAST-ISTAR)+1
   19 G(I,J)=G(I,J)-(H(I)/KK)*H(J)
C     WRITE(108,13) KK
C  13 FORMAT('NO. OF SAMPLES=' ,I3)
      DO 30 I=1,3
      H(I)=H(I)/KK
   30 CONTINUE
C
C     MATRIX INVERSION
C
      CALL MATINV (G,2,Y)
      DO 31 I=1,2
      OK(I)=0
```

```
      DO 32 J=1,2
   32 OK(I)=OK(I)+G(I,J)*G(3,J)
   31 CONTINUE
      R=H(3)-OK(1)*H(1)-OK(2)*H(2)
C
C     CALCULATION OF REAL VALUES FROM QUADRATIC EQUATION
C
      D=ABS((OK(1)**2)-(4*OK(2)*R))
      DA=DSQRT(D)
      AA=2*OK(2)
      X1=(-OK(1)-DA)/AA
      X2=(-OK(1)+DA)/AA
      IF(X1.GT.X2 .AND.X2.GT.0) X1=X2
      FIN(KI)=X1
   24 PEA(KI)=XBAR(KI)/FIN(KI)
   20 WRITE(108,21)
   21 FORMAT(2X,'MAXIMUM LIKELIHOOD METHOD OF PARAMETER ESTIMATE',/)
      WRITE(108,102) FIN(KI),PEA(KI)
  102 FORMAT(2X,'K=',F10.3,5X,'P=',F10.3)
      GO TO 90
C
  400 WRITE(108,110) KI
  110 FORMAT(5X,'PERIOD=',I5,2X,'MODEL IS POISSON',//)
      GO TO 420
   92 WRITE(108,112)
  112 FORMAT(2X,'MOMENTS METHOD OF PARAMETER ESTIMATE',//)
      FIN(KI)=CAI(KI)
      WRITE(108,114) FIN(KI),PEA(KI)
  114 FORMAT(2X,'K=',F10.3,5X,'P=',5X,F10.3,//)
C
   90 N=55
      CALL NEGBINO (N,KI)
      WRITE(108,201) KI
  201 FORMAT(/,2X,'PERIOD=',I5,2X,'MODEL IS NEGATIVE BINOMIAL',//)
  420 IF(KI.EQ.14) GO TO 330
C
C     PRINT PROBABILITY (INDIVIDUAL AND CUMULATIVE) FOR EACH EVENT
C     THUNDERSTORM DAYS TABLE
C
  200 WRITE(108,240) KI,(ISITE(ID,L),L=1,3),(IPERIOD(KI,J),J=1,3)
  240 FORMAT(2X,'TABLE',I2,'.','   CHANCE OF SELECTED NUMBER OF THUNDERS
     1TORM DAYS',/'AT',3A4,'NEVADA  (1941-1970) FOR THE',3A4,'PERIOD.',/
     2)
      WRITE(108,245)
  245 FORMAT(2X,'--------------------------------------------------------
     1-------',/)
      WRITE(108,260)
  260 FORMAT(2X,'THUNDERSTORM DAYS     PROBABILITY     CUMULATIVE PROBAB
     1ILITY',/)
      WRITE(108,245)
      DO 265 IX=1,N
      IIX=IX-1
      WRITE(108,270) IIX,  PC( IX),CPROB( IX)
  270 FORMAT(10X,I3,15X,F6.4,11X,F6.4,/)
      IF(CPROB(IX).GT..995) GO TO 267
  265 CONTINUE
  267 WRITE(108,245)
      GO TO 60
C
C     PRINT PROBABILITY (INDIVIDUAL AND CUMULATIVE) FOR EACH EVENT
```

```
C     HAIL DAYS TABLE
C
  330 WRITE(108,340) KI,(ISITE(ID,L),L=1,3),(IPERIOD(KI,J),J=1,3)
  340 FORMAT(2X,'TABLE',I2,'.','   CHANCE OF SELECTED NUMBER OF HAIL DAY
     1S', / 'AT',3A4,'NEVADA (1941-1970) FOR THE',3A4,'PERIOD.')
      WRITE(108,245)
      WRITE(108,360)
  360 FORMAT(2X,'HAIL DAYS     PROBABILITY     CUMULATIVE PROBABILITY')
      WRITE(108,245)
      DO 365 IX=1,N
      IIX=IX-1
      WRITE(108,370) IIX ,PC( IX),CPROB( IX)
      IF(CPROB(IX).GT..995) GO TO 367
  370 FORMAT(6X,I3,13X,F6.4,13X,F6.4,/)
  365 CONTINUE
  367 WRITE(108,245)
   60 CONTINUE
   70 CONTINUE
      STOP
      END
```

```
      SUBROUTINE NEGBINO (NN,MM)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON XF(55),CAI(14),PEA(14),PC(55),CPROB(55),FIN(14)
      DIMENSION FFIN(55),FF(55),FKAY(55)
C     NEGATIVE BINOMIAL PROBABILITY FUNCTION IN NATURAL LOGARITHM
C     LN(P)=K*LN(1./(1+P))+LN(FKAY)+IX*LN((P)/(P+1.))
C     P=PEA(MM) AND K=FIN(MM) ARE NEGATIVE BINOMIAL PARAMETERS
C     WHERE:FKAY=(K+X-1)FACTORIAL/X FACTORIAL*(K-1) FACTORIAL
C
      DO 20 II=1,NN
      IX=II-1
C     EVALUATION OF (K+X-1) FACTORIAL =FFIN(II)
C
      PP=1.
      IF(IX.EQ.0) GO TO 15
      IF(IX.EQ.1) GO TO 16
      FFIN(II)=FIN(MM)+IX
      FF(II)=PP*FFIN(II)
      FFIN(II)=1.
      DO 18 I=1,II-1
      FFIN(II)=FFIN(II)*(FF(II)-I)
   18 CONTINUE
      GO TO 20
   15 FFIN(1)=0
   16 IF(IX.EQ.1) FFIN(2)=FIN(MM)
   20 CONTINUE
C
C     EVALUATION OF X! STORED IN COMMON AS XF(II)
C
      CCPROB=0
      DO 40 II=1,NN
      IX=II-1
      IF (IX.EQ.0) GO TO 25
      A=XF(II)
C
C     SOLUTION TO FKAY
C
      FKAY(II)=FFIN(II)/A
      P=FIN(MM)*DLOG(1./(1.+ PEA(MM)))+DLOG(FKAY(II ))+IX*DLOG( PEA(MM)/
     1(PEA(MM)+1.))
      GO TO 23
   25 P=FIN(MM)*DLOG(1./(1.+PEA(MM)))
   23 P=DEXP(P)
C
C     INDIVIDUAL AND CUMULATIVE PROBABILITY
C
      CCPROB=CCPROB+P
      CPROB(II )=CCPROB
      PC(II )=P
   40 CONTINUE
  329 RETURN
      END
```

```
      SUBROUTINE MATINV(A,N,V)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3,3),V(1)
      NM1=N-1
      DO 776 L=1,N
      A11=A(1,1)
      DO 774 J=1,NM1
  774 V(J)=A(1,J+1)/A11
      V(N)=1./A11
      DO 779 I=1,NM1
      IP1=I+1
      AIP11=A(IP1,1)
      DO 775 J=1,NM1
  775 A(I,J)=A(IP1,J+1)-AIP11*V(J)
  779 A(I,N)=-AIP11*V(N)
      DO 776 J=1,N
  776 A(N,J)=V(J)
      RETURN
      END
```

```
      SUBROUTINE FIND(CEST,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION CEST(60,2)
      IL=1
      IF(CEST(1,1).LT.0.) IL=2
      DO 1 I=2,60
      IS=1
      IF(CEST(I,1).LT.0.) IS=2
    1 IF(IS.NE.IL) N=I; GO TO 20
      N=1
   20 RETURN
      END
```

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