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EVENT: Start with the initial **nqthm** theory.

EVENT: For efficiency, compile those definitions not yet compiled.

EVENT: Add the shell *zn*, with recognizer function symbol *znp* and 2 accessors: *pos*, with type restriction (one-of numberp) and default value zero; *neg*, with type restriction (one-of numberp) and default value zero.

DEFINITION:

$zlessp(x, y) = ((pos(x) + neg(y)) < (neg(x) + pos(y)))$

DEFINITION: $zlesseqp(x, y) = (\neg zlessp(y, x))$

DEFINITION:

$zmax(x, y)$
= **if** $zlessp(x, y)$ **then** y
else x **endif**

DEFINITION:

$zmin(x, y)$
= **if** $zlessp(x, y)$ **then** x
 else y **endif**

DEFINITION: $zsub1(x) = zn(pos(x), 1 + neg(x))$

DEFINITION:

$pzdifference(x, y) = ((pos(x) + neg(y)) - (neg(x) + pos(y)))$

DEFINITION:

$m1(x, y, z)$
= **if** $zlesseqp(x, y)$ **then** 0
 else 1 **endif**

DEFINITION:

$m2(x, y, z) = pzdifference(zmax(x, zmax(y, z)), zmin(x, zmin(y, z)))$

DEFINITION: $m3(x, y, z) = pzdifference(x, zmin(x, zmin(y, z)))$

DEFINITION:

$tak0(x, y, z)$
= **if** $zlesseqp(x, y)$ **then** y
 elseif $zlesseqp(y, z)$ **then** z
 else x **endif**

DEFINITION:

$m(x, y, z) = cons(m1(x, y, z), cons(m2(x, y, z), cons(m3(x, y, z), nil)))$

THEOREM: tak0-satisfies-tak-equation

$tak0(x, y, z)$
= **if** $zlesseqp(x, y)$ **then** y
 else $tak0(tak0(zsub1(x), y, z),$
 $tak0(zsub1(y), z, x),$
 $tak0(zsub1(z), x, y))$ **endif**

THEOREM: m1-lesseqp-0

$(\neg zlesseqp(x, y))$
 $\rightarrow (m1(x, y, z)$
 $\not\leftarrow m1(tak0(zsub1(x), y, z), tak0(zsub1(y), z, x), tak0(zsub1(z), x, y)))$)

THEOREM: m1-lesseqp-1

$(\neg zlesseqp(x, y)) \rightarrow (m1(x, y, z) \not\leftarrow m1(zsub1(x), y, z))$

THEOREM: m1-lesseqp-2

$(\neg zlesseqp(x, y)) \rightarrow (m1(x, y, z) \not\leftarrow m1(zsub1(y), z, x))$

THEOREM: m1-lesseqp-3

$$(\neg \text{zlesseqp}(x, y)) \rightarrow (\text{m1}(x, y, z) \not\prec \text{m1}(\text{zsub1}(z), x, y))$$

THEOREM: m2-lesseqp-0

$$(\neg \text{zlesseqp}(x, y))$$

$$\rightarrow (\text{m2}(x, y, z)$$

$$\not\prec \text{m2}(\text{tak0}(\text{zsub1}(x), y, z), \text{tak0}(\text{zsub1}(y), z, x), \text{tak0}(\text{zsub1}(z), x, y)))$$

THEOREM: m2-lesseqp-1

$$(\neg \text{zlesseqp}(x, y)) \rightarrow (\text{m2}(x, y, z) \not\prec \text{m2}(\text{zsub1}(x), y, z))$$

THEOREM: m2-lesseqp-2

$$((\neg \text{zlesseqp}(x, y)) \wedge (\text{m1}(\text{zsub1}(y), z, x) = \text{m1}(x, y, z)))$$

$$\rightarrow (\text{m2}(x, y, z) \not\prec \text{m2}(\text{zsub1}(y), z, x))$$

THEOREM: m2-lesseqp-3

$$((\neg \text{zlesseqp}(x, y)) \wedge (\text{m1}(\text{zsub1}(z), x, y) = \text{m1}(x, y, z)))$$

$$\rightarrow (\text{m2}(x, y, z) \not\prec \text{m2}(\text{zsub1}(z), x, y))$$

THEOREM: m3-lessp-0

$$((\neg \text{zlesseqp}(x, y))$$

$$\wedge (\text{m1}(\text{tak0}(\text{zsub1}(x), y, z), \text{tak0}(\text{zsub1}(y), z, x), \text{tak0}(\text{zsub1}(z), x, y))$$

$$= \text{m1}(x, y, z)))$$

$$\rightarrow (\text{m3}(\text{tak0}(\text{zsub1}(x), y, z), \text{tak0}(\text{zsub1}(y), z, x), \text{tak0}(\text{zsub1}(z), x, y))$$

$$< \text{m3}(x, y, z))$$

THEOREM: m3-lessp-1

$$((\neg \text{zlesseqp}(x, y)) \wedge (\text{m1}(\text{zsub1}(x), y, z) = \text{m1}(x, y, z)))$$

$$\rightarrow (\text{m3}(\text{zsub1}(x), y, z) < \text{m3}(x, y, z))$$

THEOREM: m3-lessp-2

$$((\neg \text{zlesseqp}(x, y)) \wedge (\text{m1}(\text{zsub1}(y), z, x) = \text{m1}(x, y, z)))$$

$$\rightarrow (\text{m3}(\text{zsub1}(y), z, x) < \text{m3}(x, y, z))$$

THEOREM: m3-lessp-3

$$((\neg \text{zlesseqp}(x, y)) \wedge (\text{m1}(\text{zsub1}(z), x, y) = \text{m1}(x, y, z)))$$

$$\rightarrow (\text{m2}(\text{zsub1}(z), x, y) < \text{m2}(x, y, z))$$

EVENT: Disable zlessp.

EVENT: Disable m1.

EVENT: Disable m2.

EVENT: Disable m3.

EVENT: Disable tak0.

EVENT: Disable zsub1.

DEFINITION:

make-ordinal3 (x)
= cons (cons (1 + car (x), 0), cons (1 + cadr (x), fix (caddr (x))))

THEOREM: ordinalp-make-ordinal3
ordinalp (make-ordinal3 (x))

DEFINITION:

lex3 (x , y) = ord-lessp (make-ordinal3 (x), make-ordinal3 (y))

THEOREM: m-goes-down-1

$(\neg \text{zlesseqp } (x, y)) \rightarrow \text{lex3 } (m (\text{zsub1 } (x), y, z), m (x, y, z))$

THEOREM: m-goes-down-2

$(\neg \text{zlesseqp } (x, y)) \rightarrow \text{lex3 } (m (\text{zsub1 } (y), z, x), m (x, y, z))$

THEOREM: m-goes-down-3

$(\neg \text{zlesseqp } (x, y)) \rightarrow \text{lex3 } (m (\text{zsub1 } (z), x, y), m (x, y, z))$

THEOREM: m-goes-down-0

$(\neg \text{zlesseqp } (x, y))$
 $\rightarrow \text{lex3 } (m (\text{tak0 } (\text{zsub1 } (x), y, z), \text{tak0 } (\text{zsub1 } (y), z, x), \text{tak0 } (\text{zsub1 } (z), x, y)),$
 $m (x, y, z))$

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