

MODE Object Areas Problem

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This whitepaper discusses the details of the MODE area bug discovered in March 2013. The source of the bug was the need for two distinct ways of counting how many grid points in an object are “turned on.” These two different ways are needed in several parts of MODE. One place where they’re needed is in the calculation of the convex hull, and in this whitepaper we’ll use the convex hull to illustrate the bug.

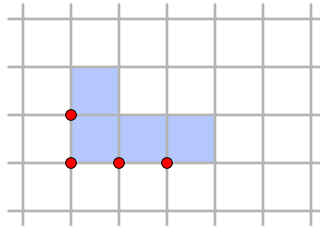


Figure 1

Suppose we have the object shown in Figure 1. The object consists of the grid points shown in red. But in order to associate an *area* with an object, we associate to each grid point in the object the grid square that has that point as its lower left corner. So although the object is represented internally to MODE as a collection of points, we think of the object as consisting of the collection of grid squares shaded in Figure 1.

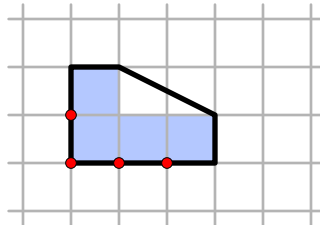


Figure 2

However, when MODE calculates the convex hull of an object, it needs as its input, not the collection of grid points of the object, but rather the collection of points that constitute the boundary of the grid *squares* of the object. See Figure 3.

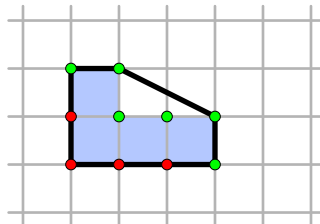


Figure 3

So, in addition to the collection of grid points of an object, MODE also keeps track of the collection of points that are potentially on the boundary of one of the object’s grid squares. These “extra” points are shown in green in Figure 3. And this was the source of the bug: when MODE was writing the `ascii` output file, it was writing the count of points in the object boundary, *i.e.*, the count of both the red and green points in Figure 3, and not just the count of red points.

How much difference in the area of an object does the bug make? For simplicity, consider the case of a square object n grid points on a side. The area of this object should be $A = n^2$. However, if the grid points are counted the wrong way, an extra top row and an extra right-hand column of grid squares will be counted, and so the area will be calculated as $A' = (n + 1)^2$. The fractional difference is

$$\begin{aligned}\frac{A' - A}{A} &= \frac{(n + 1)^2 - n^2}{n^2} \\ &= \frac{2n + 1}{n^2} \\ &= \frac{2}{n} + \frac{1}{n^2} \\ &\approx \frac{2}{n} \quad \text{if } n \text{ is large} \\ &= \frac{2}{\sqrt{A}}\end{aligned}$$

So the fractional difference scales roughly as $A^{-1/2}$. The larger the object, the smaller the relative effect.

Finally, note that since the bug essentially changes which grid squares in an object are “turned on,” it will have effects on some other derived parameters and attributes downstream from the area computation. Examples are intersection areas and percentile intensity values.