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TECHNOLOGICAL DIFFUSION, CONVERGENCE AND GROWTH

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Abstract

We construct a model that combines elements of endogenous growth with the convergence implications of the neoclassical growth model. In the long run, the world growth rate is driven by discoveries in the technologically leading economies. Followers converge toward the leaders because copying is cheaper than innovation over some range. A tendency for copying costs to increase reduces followers' growth rate and thereby generates a pattern of conditional convergence. We discuss how countries are selected to be technological leaders, and we assess welfare implications. Poorly defined intellectual property rights imply that leaders have insufficient incentive to invent and followers have excessive incentive to copy.

In the neoclassical growth model, per capita output grows in the long run only because of exogenous technological progress. The interesting insights about growth involve the convergence behavior along the transition path. Because of diminishing returns to capital, economies grow faster when they start further below their steady-state positions. Thus, if the determinants of the steady-state positions are held fixed, then poorer places are predicted to grow faster in per capita terms.[1] This result—often described as conditional convergence—receives strong empirical support if the variables held constant include aspects of government policy.[2]

The recent endogenous growth theory, initiated by Romer (1987, 1990) and extended by Grossman and Helpman (1991, Chs. 3, 4) and Aghion and Howitt (1992), explains long-term growth from a model of technological progress. The private research that underlies commercial discovery is motivated along Schumpeterian lines by the flow of profit that accrues to an innovator. Since the profit flow depends on some form of monopoly power, the resulting equilibrium tends not to be Pareto optimal.

The strong point of the recent theories is that they endogenize the rate of technical change, a variable that is unexplained in the neoclassical growth model. Thus, the long-term growth rate becomes an endogenous variable that depends on the underlying parameters and disturbances in the model. However, the new theories are less attractive in that they tend to lose the prediction of conditional convergence.

The present analysis links the long-term growth implications of the recent theories with the convergence implications of the neoclassical growth model. In the long run, growth depends on the discovery of new products or technologies in a few leading

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[1]The main references for the neoclassical model are Ramsey (1928), Solow (1956), Swan (1956), Cass (1965), and Koopmans (1965). For an exposition, see Barro and Sala-i-Martin (1995, Chs. 1,2).

[2]See, for example, Barro and Sala-i-Martin (1995, Ch. 12). This kind of empirical analysis also holds constant the initial stocks of human capital in the forms of education and health. These stocks affect an economy's rate of convergence to the steady state; see Mulligan and Sala-i-Martin (1993), Caballe and Santos (1993), and Barro and Sala-i-Martin (1995, Ch. 5).

economies. The rates of invention and growth reflect the forces described by Romer (1990).

For the behavior across economies, the key element is that imitation is typically cheaper than invention. Most countries therefore prefer to copy rather than invent. Moreover, the relatively low cost of imitation implies that the typical follower grows relatively fast and tends to catch up to the leaders. (This result holds in a conditional sense; that is, for given government policies and other variables that affect the return from the introduction of new technologies.) As the pool of copiable material decreases, the costs of imitation tend to rise and the follower's growth rate tends to fall. Hence, a pattern of conditional convergence emerges in this model of the diffusion of technology. This similarity with the neoclassical model applies because the increasing cost of imitation is analogous to the diminishing returns to capital.

In the long run, all economies grow at the rate of discovery in the leading places. Thus, the rate of discovery plays the role in this model that the exogenous rate of technical change plays in the neoclassical model. The comparison of growth rates across countries reflects the conditional convergence behavior related to the costs of copying inventions. Thus, the cross—country implications are similar to those of the neoclassical model.

I. Setup of the Model

There are two countries, denoted by i=1,2. The production function in each country is of the Spence (1976)/Dixit and Stiglitz (1977) type:

$$(1) \qquad Y_i = A_i \cdot (L_i)^{1-\alpha} \cdot \sum_{j=1}^{N_i} (X_{ij})^{\alpha},$$

where $0<\alpha<1$, Y_i is output, L_i is labor input, X_{ij} is the quantity employed of the jth type of nondurable intermediate good, and N_i is the number of types of intermediates available in country i. The technology shown in equation (1) can be accessed by all agents in country i, and production occurs under competitive conditions. The output in country 1 is physically the same as that in country 2. The total quantities of labor in each country, L_1 and L_2, are constants.

The productivity parameter, A_i, can represent variations across countries in the level of technology; that is, differences in output that arise for given values of N_i, L_i, and the X_{ij}'s. In practice, however, the main source of differences in the A_i is likely to be variations in government policies, as reflected in infrastructure services, tax rates, the degree of maintenance of property rights, and the rule of law. The effects of these policies on outcomes are analogous to those from pure differences in the levels of technology. Thus, the measures of government policy used in empirical studies, such as Barro and Sala–i–Martin (1995, Ch. 12), are empirical counterparts of the A_i.

Trade is assumed to be balanced between the two countries; that is, domestic output, Y_i, equals total domestic expenditures. These expenditures are for consumption, C_i, production of intermediates, X_{ij}, and R&D aimed at learning about new varieties of intermediates. An agent can learn by inventing a new type of good or by imitating a product that is known in the other country.

Units of C_i or X_{ij} each require one unit of Y_i. The invention of a new variety of product requires a lump-sum outlay of η_i units of Y_i. The assumed constancy of η_i means that the returns from the discovery of new types of products are constant. One reason that diminishing returns may not apply, offered by Romer (1993), is that the world may possess an infinite number of potentially useful ideas. In this case, increases in cumulated knowledge need not exhaust the opportunities for further learning. In addition, past learning may make future learning easier, a force that could create increasing returns (as well as possible externalities). In any event, the main results

about the diffusion of technology would not change if discoveries of new types of products did not involve precisely constant returns. The costs of imitation are considered later.

Suppose, to begin, that country 1 is the technological leader, whereas country 2 is the follower. Specifically, $N_1(0) > N_2(0)$, and all of the varieties of intermediates known initially in country 2 are also known in country 1. Assume, for now, that all discoveries of new types of products occur in country 1. Country 2 imitates the intermediate goods known in country 1 but does not invent anything.

II. Innovation in Country 1

The setup for country 1 is similar to that described in Romer (1990), Rivera–Batiz and Romer (1991), Grossman and Helpman (1991, Ch. 3), and Barro and Sala–i–Martin (1995, Ch. 6). An inventor of an intermediate of type j is assumed to retain a perpetual monopoly over the use of this good for production *in country 1*. (It is straightforward to allow the good to become competitive with an exogenous probability p per unit of time.) If intermediate j is priced in country 1 at P_{1j}, then the flow of monopoly profit to the inventor is

$$(2) \qquad \pi_{1j} = (P_{1j} - 1) \cdot X_{1j},$$

where the 1 inside the parentheses represents the marginal cost of production for the intermediate.

The production function in equation (1) implies that the marginal product of intermediate j in the production of output is

$$\partial Y_1 / \partial X_{1j} = A_1 \alpha \cdot L_1^{1-\alpha} \cdot (X_{1j})^{\alpha-1}.$$

The equation of this marginal product to P_{1j} yields the demand function for intermediate j from all producers of goods in country 1:

$$(3) \qquad\qquad X_{1j} = L_1 \cdot (A_1 \alpha / P_{1j})^{1/(1-\alpha)}.$$

Substitution of the result for X_{1j} into equation (2) and maximization of π_{1j} with respect to P_{1j} yields the monopoly price:

$$(4) \qquad\qquad P_{1j} = P_1 = 1/\alpha > 1.$$

The monopoly price is the same at all points in time and for all types of intermediates.

The result in equation (4) implies that the total quantity produced of intermediate j in country 1 is

$$(5) \qquad\qquad X_{1j} = X_1 = L_1 \cdot A_1^{1/(1-\alpha)} \cdot \alpha^{2/(1-\alpha)}.$$

This quantity is the same for all intermediates j and at all points in time (because L_1 is constant). Substitution of the result from equation (5) into the production function in equation (1) implies that country 1's total output is

$$(6) \qquad\qquad Y_1 = A_1^{1/(1-\alpha)} \cdot \alpha^{2\alpha/(1-\alpha)} \cdot L_1 N_1.$$

Hence, output per person, $y_1 \equiv Y_1/L_1$, rises with the productivity parameter, A_1, and the number of varieties, N_1. The variable N_1 represents the state of technology in country 1. Increases in N_1 lead to equiproportionate expansions in output per worker.

Substitution from equations (4) and (5) into equation (2) implies that the flow of monopoly profit to the owner of the rights to intermediate j is

$$(7) \qquad \pi_{1j} = \pi_1 = (1-\alpha) \cdot L_1 \cdot (A_1)^{1/(1-\alpha)} \cdot \alpha^{(1+\alpha)/(1-\alpha)}.$$

Since the profit flow is constant, the present value of profits from date t onward is

$$V_1(t) = \pi_1 \cdot \int_t^\infty \exp[-\int_t^S r_1(v) \cdot dv] \cdot ds,$$

where $r_1(v)$ is the real interest rate at time v in country 1.

If there is free entry into the R&D business and if the equilibrium quantity of R&D is nonzero at each point in time, then $V_1(t)$ must equal the constant cost of invention, η_1, at each point in time. This condition implies that $r_1(v)$ is constant over time and given by

$$(8) \qquad r_1 = \pi_1/\eta_1,$$

where π_1 is given in equation (7). The rate of return, r_1, is the ratio of the profit flow, π_1, to the lump-sum cost, η_1, of obtaining this profit flow.

Consumers in country 1 are of the usual Ramsey type with infinite horizons. At time 0, these consumers seek to maximize

$$(9) \qquad U_1 = \int_0^\infty e^{-\rho t} \cdot [(C_1^{1-\theta} - 1)/(1-\theta)] \cdot dt,$$

where $\rho > 0$ is the rate of time preference and $\theta > 0$ is the magnitude of the elasticity of the marginal utility of consumption. (The intertemporal elasticity of substitution is $1/\theta$.) The number of consumers—that is, population—is constant over time.

Maximization of utility, subject to a standard budget constraint, leads to the usual formula for the growth rate of consumption:

$$(10) \qquad \dot{C}_1/C_1 = (1/\theta)\cdot(r_1-\rho).$$

Since r_1 is constant from equation (8), the growth rate of C_1 is also constant.

In the full equilibrium of this model, N_1 and Y_1 always grow at the same rate as C_1 (see Barro and Sala–i–Martin [1995, Ch. 6]). If γ_1 denotes this common growth rate, then

$$(11) \qquad \gamma_1 = (1/\theta)\cdot(r_1-\rho) = (1/\theta)\cdot(\pi_1/\eta_1 - \rho),$$

where π_1 is given in equation (7). Thus, all of the quantities in country 1—including the number of known products, N_1—grow at the constant rate γ_1. The parameters are assumed to be such that $\gamma_1 \geq 0$ holds in equation (11); that is, $\pi_1/\eta_1 \geq \rho$ applies. Otherwise, the solution would violate the constraint that N_1 cannot be decreasing, and the free-entry condition for R&D would not hold with equality.[3] Since L_1 is constant, equation (6) shows that growth of N_1 at the rate γ_1 is consistent with growth of Y_1 at the rate γ_1.

III. Imitation in Country 2

A. Setup of the Model

The form of the production function, equation (1), is the same in country 2 as in country 1. Country 2 is technologically behind initially in the sense that $N_2(0) < N_1(0)$. The parameters A_2 and L_2 and the innovation cost η_2 may differ from their counterparts in country 1. The copying and adaptation of one of country 1's intermediates for use in country 2 requires a lump-sum outlay $\nu_2(t)$, where $\nu_2(0) < \eta_2$, so that imitation is initially more attractive than innovation for country 2.

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[3]The transversality condition must also hold. This condition requires $r_1 > \gamma_1$, which entails $\rho > (1-\theta)\cdot\pi_1/\eta_1$. Hence, the transversality condition must be satisfied if $\theta \geq 1$.

Since the cost of innovation is constant, the discoveries of new types of products do not encounter diminishing returns. As mentioned before, this assumption can be rationalized from the idea that the number of potential inventions is unbounded.

Imitation differs from innovation in that the number of goods that can be copied at any point in time is limited to the finite number that have been discovered elsewhere. Specifically, country 2 can select for imitation only from the uncopied subset of the N_1 goods that are known in country 1. As N_2 increases relative to N_1, the cost of imitation is likely to rise. This property would hold, for example, if the products known in country 1 varied in terms of how costly they were to adapt for use in country 2. The goods that were easier to imitate would be copied first, and the cost ν_2 that applied at the margin would increase with the number already imitated. This property is captured here by assuming that ν_2 is an increasing function of N_2/N_1:

$$(12) \qquad\qquad \nu_2 = \nu_2(N_2/N_1),$$

where $\nu_2' > 0$.[4]

For $N_2/N_1 < 1$, the imitation cost ν_2 tends to be less than η_2 because copying is typically cheaper than discovery. But ν_2 can exceed η_2 when $N_2/N_1 < 1$ if the remaining pool of uncopied inventions comprises goods that are difficult to adapt to country 2's environment. In other words, it would be cheaper in some circumstances for a technological follower to start from scratch rather than adapt one of the leader's goods. Figure 1 shows, however, a simpler case in which $\nu_2(N_2/N_1) < \eta_2$ applies for $N_2/N_1 < 1$ and $\nu_2(N_2/N_1)$ approaches η_2 as N_2/N_1 approaches 1. The main results still hold if $\nu_2(N_2/N_1) > \eta_2$ holds for a range of values $N_2/N_1 < 1$.

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[4]This formulation assumes that ν_2, which represents the cost of copying the marginal good, depends only on the state variable N_2/N_1. A more complicated approach would treat the cost of copying a newly discovered product as a random draw from some probability distribution.

Suppose that an agent in country 2 pays $\nu_2(t)$ to imitate the jth variety of intermediate from country 1. We assume that this agent retains a perpetual monopoly right over the use of the intermediate for production in country 2.[5] The monopoly price is then $P_{2j}=P_2=1/\alpha$, the same as that for country 1 in equation (4). The formulas for quantity produced, X_{2j}, total output, Y_2, and flow of profit, π_{2j}, therefore parallel the expressions for country 1 from equations (5)–(7):

$$(13) \qquad X_{2j} = X_2 = L_2 \cdot (A_2)^{1/(1-\alpha)} \cdot \alpha^{2/(1-\alpha)},$$

$$(14) \qquad Y_2 = (A_2)^{1/(1-\alpha)} \cdot \alpha^{2\alpha/(1-\alpha)} \cdot L_2 N_2,$$

$$(15) \qquad \pi_{2j} = \pi_2 = (1-\alpha) \cdot L_2 \cdot (A_2)^{1/(1-\alpha)} \cdot \alpha^{(1+\alpha)/(1-\alpha)}.$$

The ratio of the per-worker products, y_i, for the two countries is

$$(16) \qquad y_2/y_1 = (A_2/A_1)^{1/(1-\alpha)} \cdot (N_2/N_1).$$

Thus, the ratio depends on the relative value of the productivity parameters, A_2/A_1, and on the relative value of the number of known varieties of intermediates, N_2/N_1. The corresponding ratio for the profit flows, π_2/π_1, depends on A_2/A_1 and also on L_2/L_1.

The effect from the relative labor endowments is a scale benefit. The relevant scale variable is the total of complementary factor input, L_i, that the intermediates work with in country i. Market size, *per se*, does not matter in this model because final

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[5]Producers in country 2 are assumed to be unable to circumvent the local monopoly by importing intermediate good j from country 1. Even if this intermediate could be purchased from abroad at a price below the monopoly level, the idea is that someone must make the lump-sum outlay ν_2 to learn how to use the good effectively in the environment of country 2.

goods are homogeneous and tradable internationally. The scale benefit from L_i arises because the cost of invention or imitation is assumed to be a lump-sum amount for the entire economy (country 1 or country 2).[6]

The present value of profits from imitation of intermediate j in country 2 is

$$(17) \qquad V_2(t) = \pi_2 \cdot \int_t^\infty \exp[-\int_t^S r_2(v) \cdot dv] \cdot ds,$$

where $r_2(v)$ is the rate of return in country 2 at time v. A gap in rates of return between the two countries, $r_2(v) \neq r_1$, is possible because international lending has been ruled out.[7] If there is free entry into the imitation business in country 2 and if the equilibrium amount of resources devoted to imitation is nonzero at each point in time, then $V_2(t)$ must equal the cost of imitation, $\nu_2(t)$, at each point in time:

$$(18) \qquad V_2(t) = \nu_2(N_2/N_1).$$

Substitution of the formula for $V_2(t)$ from equation (17) and differentiation of both sides of equation (18) with respect to t yields

$$(19) \qquad r_2 = \pi_2/\nu_2 + \dot{\nu}_2/\nu_2.$$

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[6]Becker and Murphy (1992), Quah (1994), and Alesina and Spolaore (1995) assume that increases in scale also entail costs. These costs involve coordination among agents, the processing of ideas, and heterogeneity in the preferences for public goods. In these settings, a group or country tends to have an optimal size, and an increase in scale beyond this point does not convey a net benefit.

[7]If international lending were permitted, then all current investment would flow to the R&D activity that offers the highest rate of return. Investments in more than one kind of R&D could coexist if the model were modified to include an inverse relation between the rate of return to R&D and the current amount of R&D spending.

Hence, if ν_2 were constant, then r_2 would be constant and equal to π_2/ν_2, the ratio of profit to the lump-sum cost of obtaining this profit. This result would parallel the formula for r_1 in equation (8). However, if ν_2 varies over time, then r_2 includes the capital-gain term, $\dot{\nu}_2/\nu_2$. With free entry, the monopoly right over an intermediate good must equal the cost of obtaining it, ν_2. If ν_2 is rising (because N_2/N_1 is increasing in equation [12]), then the expanding value of the monopoly right implies a capital gain at the rate $\dot{\nu}_2/\nu_2$. This gain adds to the "dividend rate," π_2/ν_2, to get the full rate of return in equation (19).

Consumers in country 2 are assumed to maximize Ramsey utility functions of the form specified in equation (9). Therefore, the growth rate of C_2 is related to r_2 in the usual way:

$$(20) \qquad \dot{C}_2/C_2 = (1/\theta)\cdot(r_2-\rho).$$

This result parallels the one for country 1 in equation (10). The preference parameters, ρ and θ, are assumed to be the same in the two countries.

B. Steady–State Growth

In the steady state, N_2 grows at the same rate, γ_1, as N_1, so that ν_2 remains constant in accordance with equation (12). The ratio N_2/N_1 therefore equals a constant, denoted $(N_2/N_1)^*$. Assume for now that the parameters are such that $0<(N_2/N_1)^*<1$. The subsequent analysis relates this inequality to the parameters A_i, L_i, and η_i.

In the steady state, the growth rates of Y_2 and C_2 equal the growth rate of N_2, which equals γ_1. Therefore, the steady-state growth rates of all the quantities in country 2, denoted by γ_2^*, equal γ_1.

Since C_2 and C_1 grow in the long run at the rate γ_1 and since the preference parameters, ρ and θ, are the same in the two countries, equations (8), (10), and (20) imply

$$(21) \qquad \overset{*}{r}_2 = r_1 = \pi_1/\eta_1,$$

where π_1 is given in equation (7). Thus, although the two countries do not share a common capital market, the adjustment of N_2/N_1 to the value $(N_2/N_1)^*$—which ensures $\overset{*}{\gamma}_2 = \gamma_1$—implies $\overset{*}{r}_2 = r_1$. In the long run, the process of technological diffusion equalizes the rates of return.

Since $\overset{*}{r}_2 = r_1$, equations (19) and (8) imply

$$\pi_2/\overset{*}{\nu}_2 = \pi_1/\eta_1,$$

where $\overset{*}{\nu}_2$ is the steady–state value of ν_2. The formulas for the profit flows from equations (15) and (7) therefore imply

$$(22) \qquad \overset{*}{\nu}_2 = \eta_1 \cdot (\pi_2/\pi_1) = \eta_1 \cdot (A_2/A_1)^{1/(1-\alpha)} \cdot (L_2/L_1).$$

The assumption, thus far, is that country 2 never chooses to innovate. This behavior is optimal for agents in country 2 if $\nu_2(t) < \eta_2$ applies along the entire path. Since ν_2 is an increasing function of N_2/N_1, the required condition (if N_2/N_1 starts below its steady–state value) is $\overset{*}{\nu}_2 < \eta_2$, which implies from equation (22)

$$(23) \qquad (A_2/A_1)^{1/(1-\alpha)} \cdot (L_2/L_1) \cdot (\eta_1/\eta_2) < 1.$$

In other words, country 2 has to be intrinsically inferior to country 1 in terms of the indicated combination of productivity parameters, A_2/A_1, labor endowments, L_2/L_1, and costs of innovating, η_1/η_2. If the inequality in (23) holds, then country 2 never has an incentive to innovate (because $\nu_2[t] < \eta_2$ applies throughout). Moreover, country 1 never has an incentive to imitate, because there never exists a pool of foreign goods to copy. Thus, if the inequality in (23) holds, then the equilibrium is the one already described in which country 1 is the perpetual leader and country 2 is the perpetual follower. We discuss in a later section the results when (23) does not hold.

Suppose now that the function for the cost of imitation in equation (12) takes the constant-elasticity form:

$$(24) \qquad \nu_2 = \eta_2 \cdot (N_2/N_1)^\sigma,$$

for $N_2/N_1 \leq 1$, where $\sigma > 0$. Note that ν_2 approaches η_2 as (N_2/N_1) approaches 1, the property assumed in Figure 1. The form in equation (24) is especially convenient for the dynamic analysis.

Equations (22) and (24) imply that the steady-state ratio of N_2 to N_1 is given by

$$(25) \qquad (N_2/N_1)^* = \left[(A_2/A_1)^{1/(1-\alpha)} \cdot (L_2/L_1) \cdot (\eta_1/\eta_2) \right]^{1/\sigma}.$$

The inequality in (23) implies $(N_2/N_1)^* < 1$.

Since $(N_2/N_1)^* < 1$, equation (16) implies that y_2 remains below y_1 in the steady state if $A_2 \leq A_1$. (Note that $A_2 > A_1$ can be consistent with the inequality in [23] if $L_2 < L_1$ or $\eta_2 > \eta_1$.) Thus, the follower country's per-worker output is likely to fall short of the leader's per-worker output even in the steady state. The potential to imitate therefore does not generally provide a strong enough force to equalize the levels of per-worker product in the long run.

Consumption, C_2, grows in the steady state at the constant rate γ_1, and the ratios C_2/Y_2 and C_2/N_2 remain constant. The levels of these ratios in the steady state can be ascertained from country 2's budget constraint: C_2 equals total output, Y_2 (from equation [14]), less the goods devoted to production of intermediates, $N_2 X_2$ (where X_2 is given in equation [13]), less the resources expended on imitation. The last amount is $\dot{\nu}_2 N_2$, which equals $\nu_2^* \gamma_1 N_2$ in the steady state. This budget condition can be used to determine $(C_2/N_2)^*$ and, hence, $(C_2/Y_2)^*$.

C. The Dynamic Path and Convergence

The dynamic behavior for country 2 can be studied by considering differential equations for the variables C_2 and N_2. (Since Y_2 is proportional to N_2, from equation [14], the dynamics of Y_2 are the same as those of N_2.) For tractability, the analysis uses the constant-elasticity form of the cost function from equation (24). The parameters are also assumed to satisfy the inequality in (23), so that $\nu_2^* < \eta_2$ and $(N_2/N_1)^* < 1$, as shown in Figure 1.

One differential equation comes from the formula for consumption growth in equation (20), together with the expressions for the rate of return, r_2, from equation (19) and the cost of imitation, ν_2, from equation (24). It is convenient to define the variable $\hat{N} \equiv N_2/N_1$, which will be constant in the steady state.[8] The formula for consumption growth can then be expressed as

$$(26) \qquad \dot{C}_2/C_2 = (1/\theta) \cdot \left[\pi_2/\nu_2 + \sigma \cdot \dot{\hat{N}}/\hat{N} - \rho \right].$$

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[8]This construction effectively filters out the growth of N_1 at the constant rate γ_1 from the solution for the growth rate of N_2. The growth rate γ_1 plays a role for country 2 that is analogous to that of exogenous technological progress in the neoclassical growth model.

As mentioned before, the change in N_2 is determined by the budget constraint: the resources devoted to imitation in country 2 equal total output, Y_2 (equation [14]), less consumption, C_2, less the quantity of intermediates, $N_2 X_2$ (where X_2 is given in equation [13]). The change in N_2 equals $1/\nu_2$ times the resources devoted to imitation, and the growth rate of $\hat{N}$ equals the growth rate of N_2 minus γ_1. The resulting formula for the growth rate of $\hat{N}$ is

$$(27) \qquad \dot{\hat{N}}/\hat{N} = (1/\nu_2)\cdot\left[\pi_2\cdot(1+\alpha)/\alpha - \chi_2\right] - \gamma_1,$$

where the new variable,

$$\chi_2 \equiv C_2/N_2,$$

will be constant in the steady state. Since Y_2 is proportional to N_2 (equation [14]), χ_2 is proportional to the consumption-output ratio, C_2/Y_2.[9]

Substitution for $\dot{\hat{N}}/\hat{N}$ from equation (27) into equation (26) yields an expression for $\dot{C}_2/C_2$ in which the only right-hand side variables are $\hat{N}$ and χ_2:

$$(28) \quad \dot{C}_2/C_2 = (1/\theta)\cdot\left\{(1/\nu_2)\cdot\left[\pi_2\cdot[1 + \frac{\sigma\cdot(1+\alpha)}{\alpha}] - \sigma\chi_2\right] - \rho - \sigma\gamma_1\right\}.$$

Equations (27) and (28) imply that the growth rate of $\chi_2 \equiv C_2/N_2$ is

$$(29) \quad \dot{\chi}_2/\chi_2 = (\frac{1}{\theta\nu_2})\cdot\left\{\pi_2 + (\theta-\sigma)\cdot[\chi_2 - \pi_2\cdot(1+\alpha)/\alpha]\right\} - (\frac{1}{\theta})\cdot(\sigma\gamma_1+\rho).$$

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[9]Equations (13) and (14) imply that value added (output net of intermediates), $Y_2 - N_2 X_2$, is proportional to N_2. Therefore χ_2 is also proportional to the ratio of C_2 to value added.

Equations (27) and (29) form a system of autonomous differential equations in the variables $\hat{N}$ and χ_2. The steady state of this system has already been discussed in the previous section. The dynamics can be described by means of a standard two-dimensional phase diagram in $(\hat{N}, \chi_2)$ space.

Equation (27) implies that the locus for $\dot{\hat{N}}=0$ is downward sloping in $(\hat{N}, \chi_2)$ space, as shown in Figures 2 and 3. (Recall that ν_2 is an increasing function of $\hat{N}$ from equation [24].) Equation (27) also implies that the $\dot{\hat{N}}=0$ locus is stable; that is, an increase in $\hat{N}$ reduces $\dot{\hat{N}}$ in the neighborhood of the locus.

Equation (29) implies that the slope of the $\dot{\chi}_2=0$ locus depends on the sign of $\theta-\sigma$. If $\theta > \sigma$, then the locus is upward sloping, as shown in Figure 2. This locus is unstable; that is, an increase in χ raises $\dot{\chi}$.

The directions of motion are shown by arrows for the four regions in Figure 2. The only path that avoids unstable behavior of $\hat{N}$ and χ_2 is the stable, saddle path, shown by the dashed arrows.[10] If country 2 begins with $\hat{N}(0) < \hat{N}^*$, then $\hat{N}$ and χ_2 each rise monotonically toward their steady-state values.

Figure 3 deals with the case in which $\theta < \sigma$. Equation (29) implies that the $\dot{\chi}_2=0$ locus is now downward sloping and stable. (We can show that the slope of this locus is always steeper in magnitude than that of the $\dot{\hat{N}}=0$ locus.) The key finding is that the stable, saddle path is again upward sloping; that is, $\hat{N}$ and χ_2 still rise monotonically during the transition from $\hat{N}(0) < \hat{N}^*$.[11]

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[10]The unstable paths can be ruled out as equilibria as in the neoclassical growth model (see Barro and Sala—i—Martin [1995, Ch. 2]). One new element in the present model is that N_2 cannot decline if people cannot forget how to use a class of intermediate goods in production. The analogue in the neoclassical model is that gross investment cannot be negative; that is, the fall in the capital stock cannot exceed depreciation.

[11]If $\theta = \sigma$, then the $\dot{\chi}_2=0$ locus is vertical. The stable, saddle-path is again upward sloping in

Since χ_2 and $\hat{N}$ rise monotonically toward their steady-state values, equation (27)

implies that $\dot{\hat{N}}/\hat{N}$ falls monotonically toward its steady-state value, 0. (The monotonic

rise of $\hat{N}$ implies a monotonic increase in ν_2.) Thus, during the transition, N_2 grows

faster than N_1—imitation is proportionately greater than innovation—but the growth

rate of N_2 falls steadily toward that of N_1. In the steady state, the rates of imitation

and innovation occur at the same rate, γ_1, and $\hat{N}\equiv N_2/N_1$ remains constant.

The follower's growth rate slows down during the transition because the imitation

cost, ν_2, steadily increases. This increase in ν_2 represents a form of diminishing returns,

in this case to imitation. In the standard neoclassical growth model, the diminishing

returns to capital accumulation play an analogous role.

The monotonic increase of $\hat{N}$ and monotonic decline of $\dot{\hat{N}}/\hat{N}$ imply a monotonic

decline of $\dot{C}_2/C_2$ in accordance with equation (26). Equation (20) therefore implies that

r_2 is monotonically decreasing; it falls steadily toward its steady-state value, r_1.

Since country 2's per-worker product, y_2, is proportional to N_2 (equation [14]),

the growth rate of y_2 exceeds γ_1 during the transition, but falls gradually toward γ_1.

Thus, the model exhibits the familiar convergence pattern in which the follower

country's per-worker output grows faster than that of the leader, but the differential in

the growth rates diminishes the more the follower catches up.

As mentioned before, the follower's per-worker output, y_2, is likely to fall short of

the leader's, y_1, in the steady state; that is, $(y_2/y_1)^*<1$. Equations (16) and (25) imply

that $(y_2/y_1)^*$ is an increasing function of A_2/A_1 and L_2/L_1 and a decreasing function of

η_2/η_1.

IV. Constant (or Slowly Rising) Costs of Imitation

The type of equilibrium discussed thus far depends on the assumption that the

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this case.

imitation cost, ν_2, rises to a sufficient degree as $\hat{N}$ increases. Specifically, in Figure 1, the condition is that ν_2 rise above $\overset{*}{\nu_2}$ for $\hat{N} \equiv N_2/N_1 < 1$. (The property that ν_2 approaches η_2 as N_2/N_1 approaches 1 is not important here.) Figure 4 deals with an alternative case in which ν_2 is constant and low, so that $\nu_2 < \overset{*}{\nu_2}$. The analysis would be similar if ν_2 were instead slowly rising, so that ν_2 approaches (from the left) a value below $\overset{*}{\nu_2}$ as N_2/N_1 approaches 1.

Intuitively, if ν_2 is small (in particular, below $\overset{*}{\nu_2}$), then the imitation process will carry on at a sufficient pace to exhaust eventually all of the available products discovered in country 1. That is, $\hat{N}=1$ will be reached at some finite date T. At this point, there will be an excess supply of persons willing to pay ν_2 to copy one of country 1's discoveries, which continue to flow in at the rate γ_1. Somehow, this excess supply has to be eliminated in the equilibrium. Moreover, for $t<T$, where $\hat{N}<1$, agents in country 2 realize that a state of excess supply will arise later, and their previous choices of rates of imitation must be consistent with this expectation.

A. The Steady State

It is easiest to begin at the end; that is, when $t>T$, so that $\hat{N}=1$ has already been attained. In this case, the natural conjecture from the previous analysis is that country 2 would be in a steady state in which N_2 grows at the rate γ_1, the growth rate of N_1, so that $\hat{N}=1$ applies forever. In this situation, the goods discovered in country 1 are immediately copied for use in country 2. Also, C_2 grows at the rate γ_1, so that $\chi_2 \equiv C_2/N_2$ remains fixed over time.

Suppose, however, that r_2 equals π_2/ν_2, the value implied by equation (19) when ν_2 is constant. In this case, $r_2 > r_1$ applies.[12] But $r_2 > r_1$ implies that C_2 would grow faster than γ_1, the growth rate of C_1, so that country 2 would not be in a steady state.

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[12] Recall that $r_1 = \pi_1/\eta_1$ and $\nu_2 < \overset{*}{\nu_2}$ in Figure 4. The result $r_2 > r_1$ follows from the expression for $\overset{*}{\nu_2}$ in equation (22).

The problem is that making copies at the low cost ν_2 is too good a deal to be consistent with the growth of C_2 and N_2 at the steady-state rate, γ_1. If the rate of return were π_2/ν_2, then agents in country 2 would want to devote enough resources to copying so that N_2 would grow at a rate faster than γ_1. But, since new goods are discovered only at the rate γ_1, there is insufficient copiable material available to support imitation at this fast a rate. Somehow the rate of return in country 2 must be bid down to r_1 to support the allocations that arise in the steady state.

If $N_2=N_1$ and imitators in country 2 expend the flow of resources $\nu_2\gamma_1N_1$, then N_2 would grow along with N_1 at the constant rate γ_1. However, if each individual in country 2 thinks that he can copy a good just by paying ν_2, then the amount spent on copying would exceed $\nu_2\gamma_1N_1$; that is, there would be excess demand for goods to be copied. We suppose in this excess–demand situation that the monopoly rights to the copied goods in country 2 are allocated in a random manner. Specifically, we assume that each person's probability of obtaining the property right is proportional to the amount spent on copying effort. In equilibrium, the total flow of resources expended by potential imitators must then be $\nu_2^*\gamma_1N_1$, where $\nu_2^*>\nu_2$ is the cost per good that drives the expected rate of return down to r_1 (see equations [21] and [22] and Figure 4).[13] This bidding up of the effective cost of copying to ν_2^* deters any further entry of potential imitators.

In the steady state, the effective cost of copying is $\nu_2^*>\nu_2$, and the expected rate of return to imitation is r_1. This rate of return is consistent with growth of C_2 and N_2 at the steady–state rate, γ_1. The steady-state solution is therefore the same as that shown in Figure 1, except that $(N_2/N_1)^*=1$ applies. (We continue to assume that $\eta_2>\nu_2^*$, as shown in Figure 4; that is, the inequality in [23] holds.)

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[13]This result holds if the risk involved in imitation success is diversifiable, so that potential imitators consider only the expectation of the return.

B. Transitional Dynamics

Consider now the situation when t<T, so that $N_2 < N_1$, and the copiable products are in plentiful supply. The rate of return in country 2 must then be

$$(30) \qquad \qquad r_2 = \pi_2/\nu_2,$$

which is constant. The growth rate of consumption is therefore also constant and given by

$$(31) \qquad \qquad \dot{C_2}/C_2 = (1/\theta)\cdot(\pi_2/\nu_2 - \rho).$$

This result corresponds to equation (26) with $\sigma=0$.[14]

The formula for $\dot{\hat{N}}/\hat{N}$ is the same as equation (27) and that for $\dot{\chi_2}/\chi_2$ is the same as equation (29) with σ set to zero:

$$(27) \qquad \qquad \dot{\hat{N}}/\hat{N} = (1/\nu_2)\cdot\left[\pi_2\cdot(1+\alpha)/\alpha - \chi_2\right] - \gamma_1,$$

$$(32) \qquad \dot{\chi_2}/\chi_2 = (1/\theta)\cdot(\pi_2/\nu_2)[1-\theta\cdot(1+\alpha)/\alpha] - \rho/\theta + \chi_2/\nu_2,$$

where $\chi_2 \equiv C_2/N_2$.

Equations (27) and (32) and be used, as before, to construct a phase diagram in $(\hat{N},\chi_2)$ space. Figure 5 shows the resulting diagram. Note that each locus is now a horizontal line. We can show readily (if $\pi_2/\nu_2 > r_1$) that the $\dot{\hat{N}}=0$ locus lies above the

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[14]In equation (24), $\sigma=0$ implies that ν_2 is independent of N_2/N_1. However, in the present case, $\nu_2 < \eta_2$ also applies.

$\dot{\chi}_2$=0 locus, as shown in the figure. We also have that $\hat{N}$ is falling for values above the $\dot{\hat{N}}$=0 locus and rising for values below it, whereas χ_2 is rising for values above the $\dot{\chi}_2$=0 locus and falling for values below it. These patterns imply that the stable, saddle path begins between the two horizontal loci and is then upward sloping. We have drawn the path so that it remains below the $\dot{\hat{N}}$=0 locus when $\hat{N}$ reaches 1, a configuration that is implied by the subsequent analysis.

Figure 5 implies a transition in which $\hat{N}$ and χ_2 increase monotonically. The rise in $\hat{N}$ means that $\dot{N}_2/N_2$ exceeds γ_1 along the path. The expansion of χ_2 implies from equation (27) that $\dot{N}_2/N_2$ declines steadily. Thus, the solution accords with the previous one in predicting that the follower grows faster (in terms of number of known products and output) than the leader, but the gap in the growth rates diminishes as the follower catches up. Note, however, that $\dot{C}_2/C_2$ is constant at a value that exceeds γ_1 (see equation [31]).

The tricky part of the solution concerns the behavior just at time T, when $\hat{N}$ reaches 1. Just to the right of this point, imitations effectively cost $\nu_2^* > \nu_2$, and the rate of return is r_1. Just to the left of this point, imitations cost ν_2, and the rate of return (from equation [30]) is $\pi_2/\nu_2 > r_1$. Anyone who pays ν_2 to imitate a good just before date T will, in the next instant, experience a sharp capital gain corresponding to the increase in the shadow price of an imitated product from ν_2 to ν_2^*. In fact, the rate of return to copying a good is infinite for an instant of time at date T. This curious behavior for the instantaneous rate of return supports the equilibrium for quantities when the cost of copying is small and constant.

Figure 6 shows the full path of the equilibrium for country 2's rate of return, r_2, and log of consumption, $\log(C_2)$. To the left of date T, the rate of return is constant at π_2/ν_2, and the slope of $\log(C_2)$ is the associated constant, $(1/\theta) \cdot (\pi_2/\nu_2 - \rho)$. To the

right of date T, the rate of return is constant at the lower value, $r_1 = \pi_1 / \eta_1$, and the slope of $\log(C_2)$ is the correspondingly smaller value, $(1/\theta) \cdot (\pi_1 / \eta_1 - \rho)$. At time T, the infinite rate of return (for an instant of time) supports a jump in the level of $\log(C_2)$. This jump is consistent with the economy's overall resource constraint, because the amount expended on imitation jumps downward at the same time by an equal amount.[15] Note that there is no jump at time T (or any other time) in the level of total output.

Suppose now that ν_2 were slowly rising, rather than constant, but that the value of ν_2 at $\hat{N}=1$ remains below ν_2^*. In this case, the behavior at time T still involves an infinite rate of return and a jump in the level of consumption. The main new results are that r_2 will fall steadily for $t<T$, and the growth rate of C_2 will therefore also decline in this range.

The bottom line is that cases of constant or slowly rising imitation costs agree qualitatively with the model from the previous section in the predictions about the follower's growth rates. In each case, a lower value of N_2/N_1 implies a higher growth rate of N_2 and, hence, of Y_2. This property extends also to the growth rate of C_2, except for the case in which the imitation cost, ν_2, does not rise at all until N_2 reaches N_1 at date T.

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[15]The change in the resources devoted to imitation involves two offsetting effects. First, the resource use falls because the growth rate of N_2 declines by a discrete amount. Second, the resource use rises because each unit now costs $\nu_2^* > \nu_2$. In the equilibrium (which involves an infinite rate of return at date T and, hence, an upward jump in consumption), the net effect must be a reduction in resource use for imitation. Also, the stable, shadow path shown in Figure 5 must remain below the $\dot{N}=0$ locus at date T in order to be consistent with the downward jump in $\hat{N}$ and the upward jump in χ_2 at date T. (The loci for $\dot{N}=0$ and $\dot{\chi}_2=0$ shift after date T—downward and upward, respectively—because ν_2 is replaced in the equations by the higher value ν_2^*.)

V. General Implications for Growth Rates in Follower Countries

The various models considered imply that the growth rate of output per worker in country 2 can be written in the form

$$(33) \qquad \dot{y}_2/y_2 = \gamma_1 + G[y_2/y_1, (y_2/y_1)^*],$$

where the partial derivatives of the function G satisfy $G_1 < 0$ and $G_2 > 0$, and $G(\cdot, \cdot) = 0$ when $y_2/y_1 = (y_2/y_1)^*$. Growth rates do not necessarily exhibit absolute convergence, in the sense described by Barro and Sala–i–Martin (1995, Ch. 1), because $\dot{y}_2/y_2 < \gamma_1$ can apply if $y_2/y_1 < 1$. If $(y_2/y_1)^*$ is small—for example, because A_2/A_1 is low—then $\dot{y}_2/y_2$ can be below γ_1 even if y_2 is substantially less than y_1. Country 2's growth rate, $\dot{y}_2/y_2$, exceeds γ_1 if $y_2/y_1 < (y_2/y_1)^*$.

The results exhibit conditional convergence, in the sense that $\dot{y}_2/y_2$ rises as y_2/y_1 falls for a given value of $(y_2/y_1)^*$. Also, for given y_2/y_1, $\dot{y}_2/y_2$ rises with $(y_2/y_1)^*$. For example, if the government of country 2 adopts policies that are more favorable to production and investment—such as lower tax rates or more effective enforcement of property rights—then the change amounts to an increase in A_2. Hence, $(y_2/y_1)^*$ increases, and the growth rate, $\dot{y}_2/y_2$, rises.

In the neoclassical growth model with labor-augmenting technological progress, as described in Barro and Sala–i–Martin (1995, Ch. 2), the formula for the growth rate of per capita output in a closed economy looks similar to equation (33). The differences are that γ_1 is replaced by the rate of exogenous technical change, denoted by x; y_2/y_1 is replaced by $\hat{y}$, the country's output per *effective* worker (a concept that takes account of the growth at rate x because of technological progress); and $(y_2/y_1)^*$ is replaced by $\hat{y}^*$, the steady-state level of output per effective worker. Thus, the growth formula in the standard model can be written as

$$(34) \qquad\qquad \dot{y}/y = x + H(\hat{y}, \hat{y}^*),$$

where the partial derivatives of the function H satisfy $H_1 < 0$ and $H_2 > 0$, and $H(\cdot, \cdot) = 0$ when $\hat{y} = \hat{y}^*$. The value $\hat{y}^*$ depends on elements included in the parameter A, such as government policies, and on the willingness to save. Higher values of A raise $\hat{y}^*$, whereas higher values of the preference parameters, ρ and θ, reduce $\hat{y}^*$.

One distinction between the two classes of models is that the intercept in equation (33) is γ_1, the growth rate of the leading economy (or economies), whereas that in equation (34) is x, the constant rate of exogenous technological progress. Operationally, γ_1 might be identified with the average growth rate of output per worker in a set of advanced countries.[16] The parameter x would not be directly observable and might vary over time or across countries.

If all followers have the same leaders—because the costs of imitation, ν_i, are the same in all cases—and if the rates of exogenous technical change are the same for all countries at a given point in time, then both models imply that the intercept is the same for all countries. In a single cross section, equation (33) would constrain the intercept to equal the observable value γ_1, whereas equation (34) would not impose this constraint. Thus, the diffusion model would, in this respect, amount to a restricted version of the neoclassical growth model.

In a panel setting, equation (33) would allow the intercept to vary over time, but only in line with the observable changes in γ_1. Equation (34) would fix the intercept, but only if we retain the version of the neoclassical growth model in which the rate of

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[16]Followers are influenced by the growth of N_1, not by the growth of the leader's output per worker, y_1, although the two growth rates coincide in the present model. In a setting that allows for short-term fluctuations of y_1 for given N_1, the growth rate of N_1 would likely be better estimated by a long-term average of the growth rate of y_1, rather than the current growth rate. Direct measures of N_1 would not generally be available, although patents or cumulated R&D spending would be possibilities.

technological progress, x, is constant (as well as the same for all countries). If the rate of technical change is exogenous, but not necessarily constant, then equation (34) would allow the intercept to vary over time in an unconstrained manner. In this case, the diffusion model would again amount to a constrained version of the neoclassical growth model.

With respect to the terms $G(\cdot)$ and $H(\cdot)$, the key aspect of equation (33) is that the growth rate depends on a country's characteristics expressed *relative* to those in the leading economy (or economies), whereas equation (34) involves the absolute levels of these characteristics. Suppose, for example, that the growth rate, γ_1, in the United States—the representation of the technological leader—is 2% per year. Equation (33) says that, for given γ_1, the growth rate of a typical follower, say Mexico, depends on the quality of its political and economic institutions (determinants of the parameter A_i) expressed relative to those in the United States. Equation (34) says that the characteristics of Mexican institutions matter for Mexican growth, but it is not necessary to condition these characteristics on the comparable attributes of the United States.

If all countries have the same leader, then, in a single cross section, the leader's characteristics merge into the overall intercept. In a panel context, changes in the leader's characteristics shift the intercept over time. The problem, however, is that the intercept can shift for other reasons. For example, in the neoclassical growth model, variations in the world rate of exogenous technological progress would be a source of these shifts.

Clear empirical distinctions between the diffusion model and the standard neoclassical growth model arise if countries differ in terms of their relevant leaders, for example, because the cost of imitation, ν_i, depends on physical distance or on the degrees of similarity in language or culture. In a cross section of countries, the growth rate γ_i then depends on country i's characteristics expressed in relation to those of a set

of potential leaders. The characteristics of these leaders would be weighted in accordance with measures, such as distance, that proxy for the cost, ν_i, of adapting technology. Results of Chua (1993) and Easterly and Levine (1994) on the growth effects of neighboring countries relate to this idea, although these studies focus on influences from physically adjacent places.

VI. Switchovers of Technological Leadership

We have considered thus far the case in which $(A_2/A_1)^{1/(1-\alpha)} \cdot (L_2/L_1) \cdot (\eta_1/\eta_2) < 1$, so that country 2 is intrinsically inferior to country 1 in terms of the underlying parameters. This inequality guarantees in Figures 1 and 4 that ν_2^* lies below η_2 on the vertical axis. For this reason, agents in country 2 never wish to innovate.

Suppose now that the inequality is reversed,

$$(35) \qquad (A_2/A_1)^{1/(1-\alpha)} \cdot (L_2/L_1) \cdot (\eta_1/\eta_2) > 1,$$

so that country 2 is intrinsically superior to country 1. Since $N_2(0) < N_1(0)$ still applies, country 2 again begins in a technologically inferior state. This situation would arise if, for example, country 2 had been inferior to country 1 for a long time, but a recent improvement in government policy—say an increase in A_2 relative to A_1—made country 2 intrinsically superior.

Return now to the case shown in Figure 1 in which ν_2 rises with N_2/N_1 and approaches η_2 as N_2/N_1 approaches 1. The inequality in (35) implies, however, that the value ν_2^* given in equation (22) now exceeds η_2. Thus, Figure 7 shows that N_2/N_1 reaches unity and correspondingly ν_2 reaches η_2 at a point where the cost of increasing N_2 is still below ν_2^*. This result means that agents in country 2 find it advantageous to

raise N_2/N_1 above unity by innovating at the cost η_2. Thus, once all of country 1's discoveries have been copied, country 2 switches to innovation.

The inventions in country 2 create a pool of products that can be imitated by country 1. Since the cost of copying is lower than η_1, agents in country 1 now find imitation preferable to invention. Country 1's role shifts accordingly from leader to follower.[17] Note that country 1's welfare will be enhanced by the presence of the technologically superior country 2.[18]

The initial model applies after the switchover with the roles reversed: country 2 is now the permanent technological leader, and country 1 is the permanent follower. Country 2's rate of return, r_2, and growth rate, γ_2 (of N_2, Y_2, and C_2), are constant after the switchover. The values of r_2 and γ_2 are given, respectively, by equations (8) and (11) if the subscripts in the formulas are changed from 1 to 2. The steady–state ratio of numbers of products, $(N_2/N_1)^*$, is still given by equation (25), but now exceeds unity.

Figures 2 and 3 describe the post–switchover dynamics for country 1 if $\hat{N}$ now equals N_1/N_2 and χ_1 replaces χ_2. The only difference from before is that $\hat{N}$ starts at unity, a value to the right of $\hat{N}^*$. The dynamic path therefore features steadily declining values of $\hat{N}$ and $\chi_1 \equiv C_1/N_1$. The steady fall in $\hat{N}$ means that country 2 continues to grow faster than country 1 during the post–switchover transition. As $\hat{N}$ falls, the cost, ν_1, for imitation in country 1 declines, and the rate of return and growth rates in

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[17]In the specification where $\nu_2(N_2/N_1)$ approaches η_2 as N_2/N_1 approaches 1 (as in Figures 1 and 7), country 1 switches all at once from leader to follower, and country 2 moves all at once from follower to leader. The switchover involves a transition with mixing of innovation and imitation within a country if $\nu_2(N_2/N_1)$ rises above η_2 before N_2/N_1 reaches 1. (An analogous cost function for imitation would apply to country 1.) In this revised formulation, country 2 would switch at some point from pure imitation to a mixture of imitation and innovation. Then, after a finite stock of country 2's discoveries had built up, the cost of imitation by country 1 would become low enough so that country 1 would shift to a mixture of imitation and innovation. Eventually, country 2 would move fully out of imitation, and country 1 would move fully out of innovation.

[18]Since final product is physically homogeneous, there is no possibility in this model of an adverse relative–price effect for country 1 because of the rise in productivity in country 2.

country 1 *increase*. In the steady state, country 1's rate of return reaches r_2, a constant, and its growth rate (of N_1, Y_1, and C_1) reaches γ_2, also a constant.[19]

The switch of technological leadership can occur only once in the model if the underlying parameters A_i, L_i, and η_i do not change. The switch occurs if the country that starts with the relatively low number of known products, N_i, is intrinsically superior in the sense of the inequality in (35). Thus, the present framework differs from models of leapfrogging, as explored by Brezis, Krugman, and Tsiddon (1993) and Ohyama and Jones (1993). In those settings, the changes in technological leadership reflect the effects of backwardness on the willingness to explore and adopt radically new ideas. In the present model, the countries that start out behind have a benefit from low costs of imitation, but have no advantages with respect to the discovery or implementation of leading-edge technologies.

In practice, the parameters A_i, L_i, and η_i would change over time; for example, because of shifts in government policies. These movements would occasionally create changes in the positions of technological leadership. (These changes would be lagged substantially from the shifts in the underlying parameters.) However, since backwardness does not enhance the discovery or implementation of new technologies and since the leaders are selected for the favorable values of their underlying parameters, there would be no tendency for a particular follower eventually to surpass a particular leader. In contrast, the probability that a leader would eventually be overtaken by *some* follower would likely be high.

These results seem consistent with the broad patterns of change in world technological leadership that are highlighted by Brezis, Krugman, and Tsiddon (1993).

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[19]The final possibility is that the parameter combination $A_i^{1/(1-\alpha)} L_i \eta_i$ is the same for the two countries. In this case, the equilibrium can be of the first type (where country 1 is the permanent leader and country 2 the permanent follower) or of the second type (where the leadership positions change). There could also be a mixture of invention and imitation in the two places. In the steady state, agents in both countries are indifferent between innovation and imitation.

They argue that Great Britain overtook the Netherlands as leader in the 1700s, the United States (and, in some respects, Germany) overtook Great Britain by the late 1800s, and Japan surpassed the United States in some sectors in recent years. The striking aspect of this pattern is not that changes in technological leadership occur, but rather that the positions at the top persist for so long. In particular, most countries have never been technological leaders. The empirical evidence therefore does not suggest any great benefits from backwardness, *per se*, in the discovery and use of the newest technologies.

VII. Welfare Considerations

Consider the model described in Figure 1 in which country 1 is always the technological leader, country 2 is always the follower, and the cost of imitation is increasing in N_2/N_1. One source of distortion in this model involves the monopoly pricing of the intermediates that have already been discovered in country 1 or imitated in country 2. This element is familiar from the models of Romer (1990), Rivera–Batiz and Romer (1991), and Barro and Sala–i–Martin (1995, Ch. 6). From a static perspective, the distortion reflects the excess of the price paid for each intermediate, $1/\alpha$, over the marginal cost of production, 1. This wedge can be eliminated by using a lump-sum tax in each country to subsidize purchases of intermediates at the rate $(1-\alpha)/\alpha$. Each user of an intermediate then faces a net price of one, the marginal cost of production.[20]

Another distortion in the present framework is that agents in country 1 have insufficient incentive to innovate because they do not take account of the benefit to

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[20]In a one–country version of the present model, this subsidy to the purchase of intermediates would be sufficient to achieve a Pareto optimum because a new invention does not affect the rentals of the existing monopolists. In other models, such as Aghion and Howitt (1992) and Barro and Sala–i–Martin (1995, Ch. 7), inventions destroy the rentals of the existing technological leaders. This element tends to make the privately chosen growth rate too high. The achievement of a Pareto optimum then requires an additional intervention, such as a tax on R&D. In Romer (1990), R&D has direct spillover benefits, so that a Pareto optimum necessitates a subsidy to R&D.

country 2 from an increase in the pool of copiable ideas. This effect would be internalized if each innovator in country 1 retained the international property rights over the use of his or her idea. In one setting, the inventor would subsequently adapt the intermediate for use in country 2. The initial R&D decision then considers the world market, which consists here of a combination of countries 1 and 2. The corresponding lump-sum cost of acquisition is the invention cost, η_1, plus the expense for adaptation of the discovery to country 2. (In some cases, adaptation will be too expensive to be worthwhile, and the results will be the same as those derived earlier for country 1.)

The innovator's adaptation of a new product to another country amounts to technology transfer through foreign direct investment. The results would be equivalent if an inventor from country 1 licensed the idea to an entrepreneur in country 2. In either case, the guarantee of intellectual property rights motivates researchers to consider the worldwide benefits of their inventions.

Another distortion arises in the model because agents in country 2 do not consider that the imitation of one of country 1's ideas raises the cost that will apply to future imitations. To isolate this effect, suppose that N_1 grows at the given rate γ_1 and that the effect from monopoly pricing in country 2 has been neutralized by a subsidy at the rate $(1-\alpha)/\alpha$ on the use of intermediates. This subsidy, financed by a lump-sum tax, implies that the net price of intermediates to users is one, the marginal cost of production. We can then compare the outcomes of a decentralized solution with those that would be determined by a social planner in country 2.

The social planner seeks to maximize the utility of the representative consumer in country 2, subject to the production function in equation (1); the specification of the cost of copying ν_2, assumed to be given by equation (24); and the growth rate of N_1 at the given rate γ_1. The optimal quantity of each intermediate, X_2, maximizes output, Y_2, net of the outlay on intermediates, and is given by

$$(36) \qquad X_2 = L_2 \cdot A_2{}^{1/(1-\alpha)} \cdot \alpha^{1/(1-\alpha)}.$$

The usual conditions for dynamic optimization lead to the following expressions for the growth rates of N_2 and C_2:

$$(37) \qquad \dot{N}_2/N_2 = (1/\nu_2) \cdot (\Psi - \chi_2),$$

$$(38) \qquad \dot{C}_2/C_2 = (1/\theta) \cdot (\Psi/\nu_2 - \rho - \sigma\gamma_1),$$

where $\chi_2 \equiv C_2/N_2$ and Ψ is defined as

$$(39) \qquad \Psi = (1-\alpha) \cdot L_2 A_2{}^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)}.$$

In a decentralized situation in which purchases of intermediates are subsidized at the rate $(1-\alpha)/\alpha$, Ψ turns out to equal the profit flow, π_2. (This amount exceeds the value for π_2 shown in equation [15].)

For the decentralized setting, the subsidy on purchases of intermediates implies that X_2 equals the social–planner's choice shown in equation (36). Since the values of X_2 are equal, the decentralized path for N_2 would coincide with the planner's path if the choices of χ_2 were the same. That is, the formula that determines $\dot{N}_2/N_2$ in the decentralized case is the same as equation (37). Differences in results arise only because of differences in the choices of consumption.

The growth rate of consumption in the decentralized solution turns out to be

$$(40) \qquad \dot{C}_2/C_2 = (1/\theta) \cdot \left[\Psi/\nu_2 - \rho - \sigma\gamma_1 + (\sigma/\nu_2) \cdot (\Psi - \chi_2) \right].$$

This expression differs from the social–planner's result in equation (38) only by the term that involves $\Psi - \chi_2$. It is possible to show that $\Psi > \chi_2$ applies in the steady state. Moreover, since χ_2 can be shown to be monotonically increasing during the transition (from the type of phase–diagram analysis used before), $\Psi - \chi_2$ must be positive throughout. It follows that the decentralized choice of $\dot{C}_2/C_2$ is greater than the social–planner's value at all values of N_2/N_1 (and, hence, ν_2). In other words, the decentralized solution involves lower levels of χ_2 and higher growth rates of C_2.

Equation (37) then implies that the decentralized choice of $\dot{N}_2/N_2$ is greater than the social–planner's choice at each value of N_2/N_1. This result implies that the steady-state value of N_2/N_1 in the decentralized solution exceeds the steady-state value chosen by the social planner.[21]

The growth rate is too high in the decentralized solution because the allocation of resources to imitation (and, hence, growth) is analogous to increased fishing in a congestible pond. Specifically, an agent that expends $\nu_2(N_2/N_1)$ to raise N_2 does not consider that this action will raise the cost faced by future imitators of products.

Viewed alternatively, private agents count the capital gain, $\dot{\nu}_2/\nu_2$, as part of the return to imitation, whereas this element does not enter into the social return. This kind of distortion would not arise if potential imitators in country 2 were somehow assigned well-defined property rights at the outset to the goods that could be copied from country 1. Alternatively, the distortion would not arise if the inventors in country 1 possessed these rights of adaptation to country 2.

We can make analogous welfare comparisons for the case discussed in section IV in which ν_2 is low and constant. In the steady state, the social planner's and decentralized solutions each feature $N_2/N_1 = 1$ with N_2 and C_2 growing at the rate γ_1. However, in the decentralized case, the competition among potential copiers drives the effective cost

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[21] The parameters are assumed to be such that N_2/N_1 remains below unity in the steady state.

of imitation up to $\nu_2^* > \nu_2$. This waste of resources implies that the steady-state level of $\chi_2 \equiv C_2/N_2$ is lower than in the social planner's setting. (This result holds even if the decentralized solution involves the appropriate subsidy for the use of intermediates in country 2.)

Recall that, when $N_2 = N_1$ was attained at time T in the decentralized case, C_2 jumped upward, and the resources devoted to copying jumped downward correspondingly. We can show that the solution for the social planner in country 2 entails no such jumps. The growth rate of C_2 falls discretely at time T, but the level of C_2—and, hence, the amount of resources spent on copying—do not jump.

For $t < T$, we can shown that the decentralized choice for $\dot{N}_2/N_2$ exceeds the social planner's value. (This result holds if the decentralized solution involves the appropriate subsidy on the use of intermediates in country 2.) The values for $\dot{C}_2/C_2$ are the same (and constant) in the two environments, but the decentralized path features lower levels of $\chi_2 \equiv C_2/N_2$ and correspondingly higher levels of resources devoted to copying, $\nu_2 \dot{N}_2$.

Again, the problem is the excessive incentive to secure property rights in the follower country. In the model with smoothly rising costs of copying, ν_2, this incentive is communicated by a stream of capital gains to holders of monopoly rights in country 2. In the model with constant ν_2, the inducement comes from the prospect of an infinite rate of capital gain for an instant at time T. Either way, the capital gains motivate imitation at too fast a rate.

VIII. Concluding Observations

Our analysis of invention and imitation combines features of endogenous–growth models with the convergence implications of the neoclassical growth model. In the long run, the world's growth rate is driven by discoveries in the technologically leading economies. Followers converge at least part way toward the leaders because copying is cheaper than innovation over some range. As the pool of uncopied ideas diminishes, the

cost of imitation tends to increase, and the followers' growth rates tend accordingly to decline. Therefore, the results exhibit a form of conditional convergence, a property found in the cross—country data on economic growth.

The outcomes deviate from Pareto optimality for reasons that involve the publicness of discoveries, imperfect competition, and limited specification of property rights. We stress the consequences from the absence of intellectual property rights across economies. In this context, the leading places tend to have insufficient incentive to invent, and the follower places tend to have excessive incentive to copy.

In the long run, the identities of the technological leaders and followers are also endogenous. In the present model, the private reward from innovation depends on its complementarity with domestic production possibilities. (It is not possible to invent things and retain control over their use in other places.) Therefore, the technological leader is selected in the long run in accordance with the attractiveness of the local environment for production and research (high parameter A_i and low parameter η_i) and with the scale of complementary domestic inputs (high L_i). We suggest that government policies on security of property rights, taxation, and infrastructure are ultimately key determinants of an area's attractiveness for production and research (the parameters A_i and η_i). However, our analysis takes these government policies as exogenous, because we lack a theory about the convergence or divergence of government policies across countries or regions.

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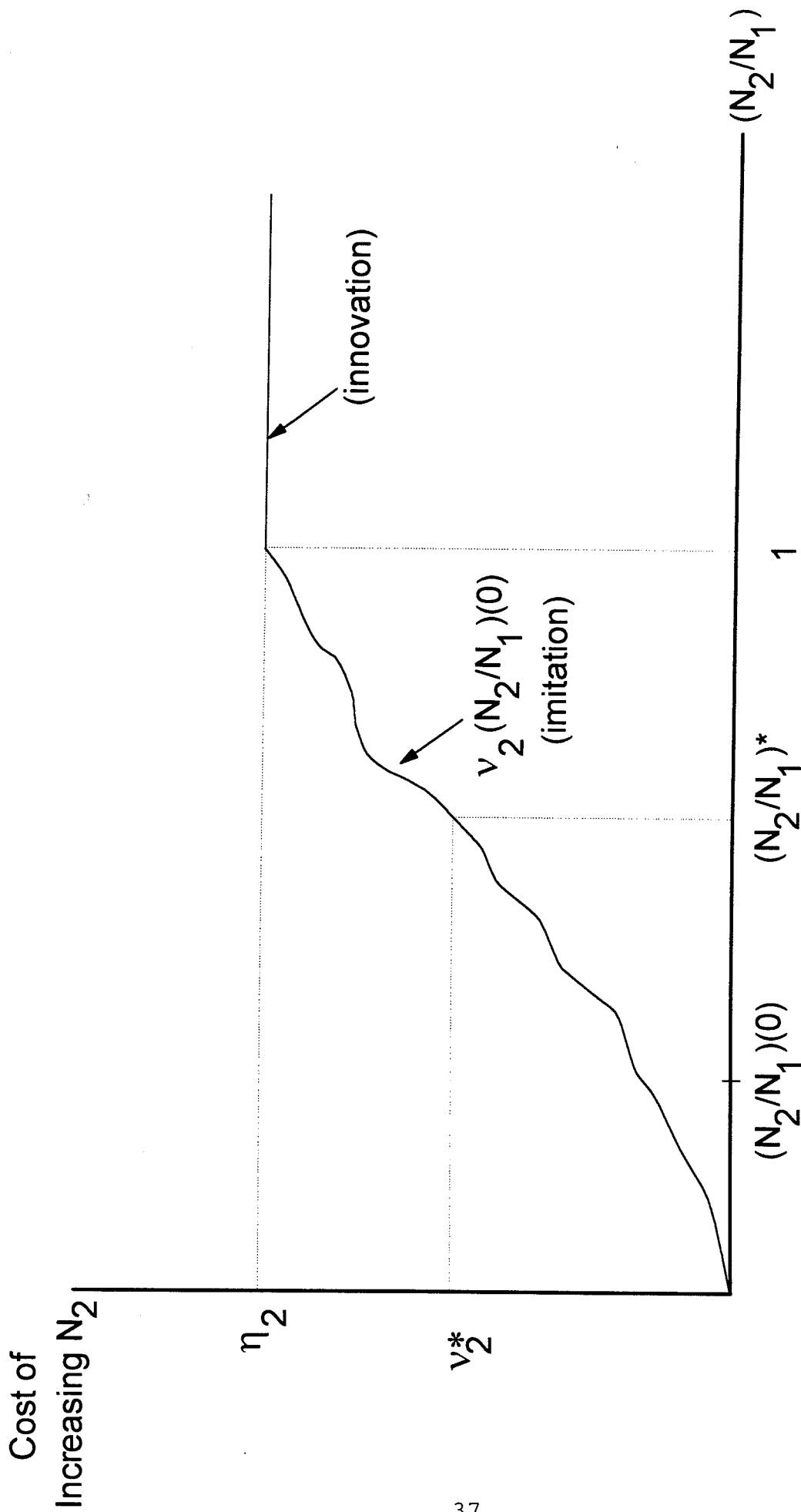Swan, Trevor W. (1956). "Economic Growth and Capital Accumulation," *Economic Record*, 32 (November), 334–361.

Figure 1. Cost of Technological Change in Country 2

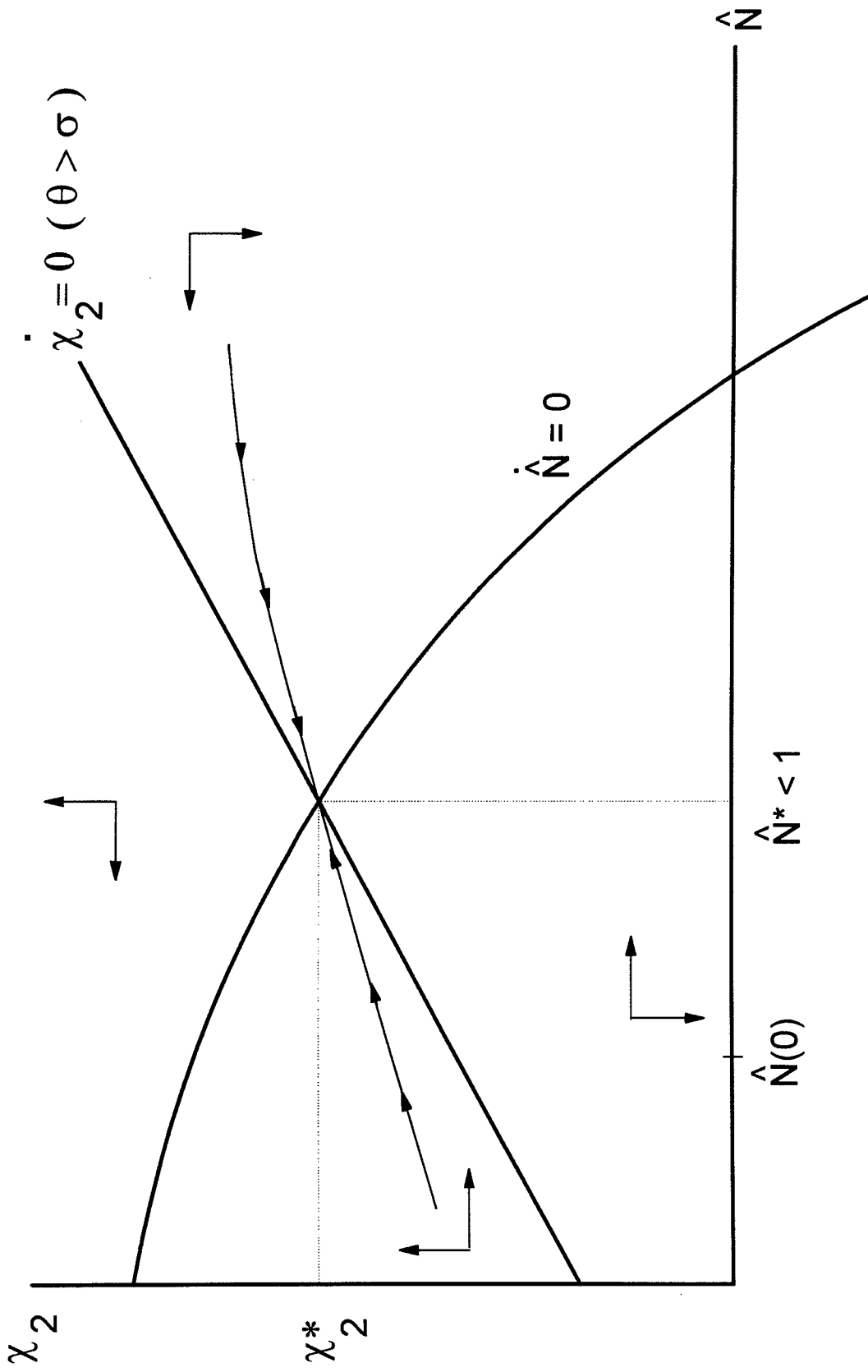(for the environment in which $\nu_2^* < \eta_2$)

Figure 2. Phase Diagram for Country 2 when $(\theta > \sigma)$
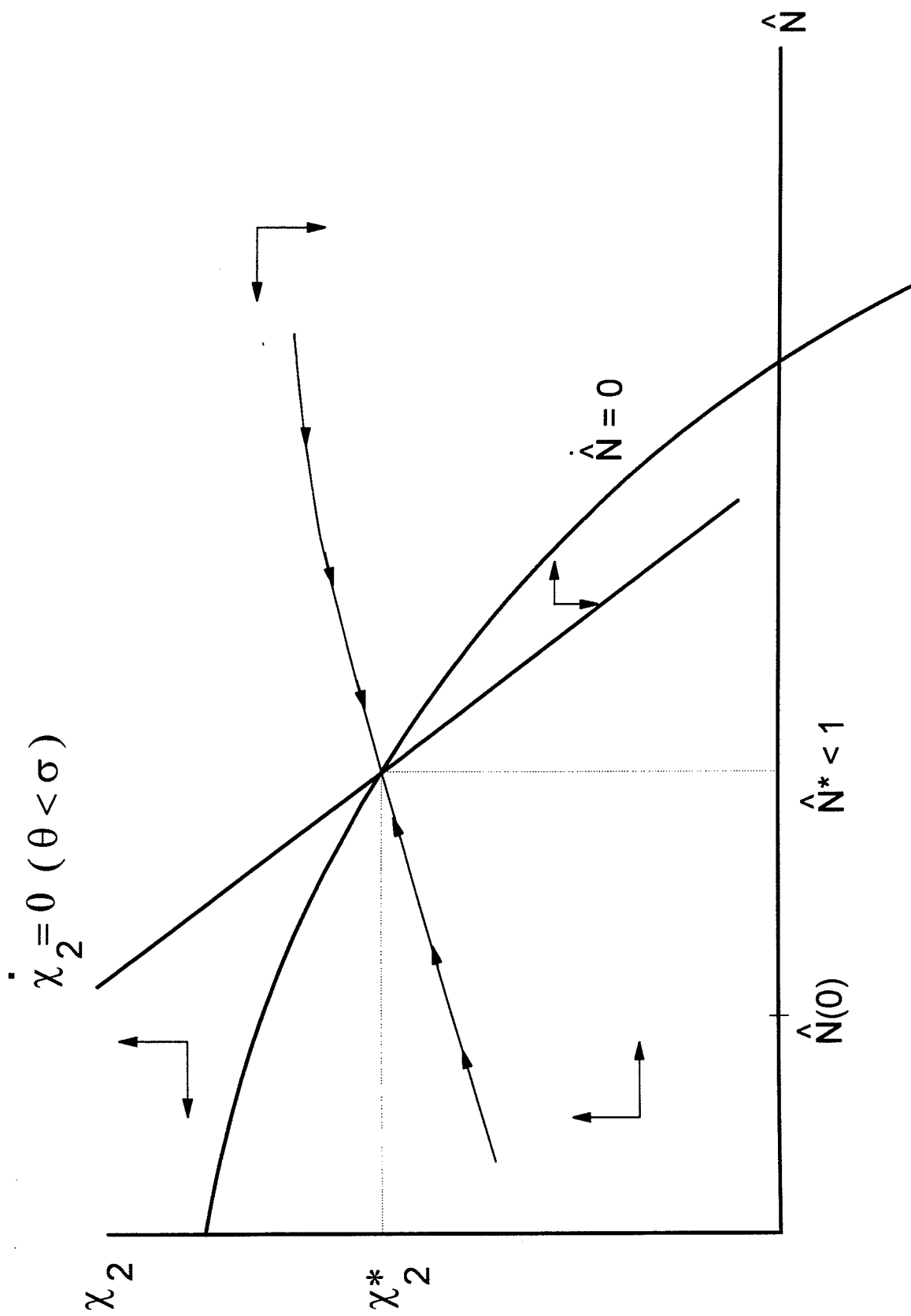
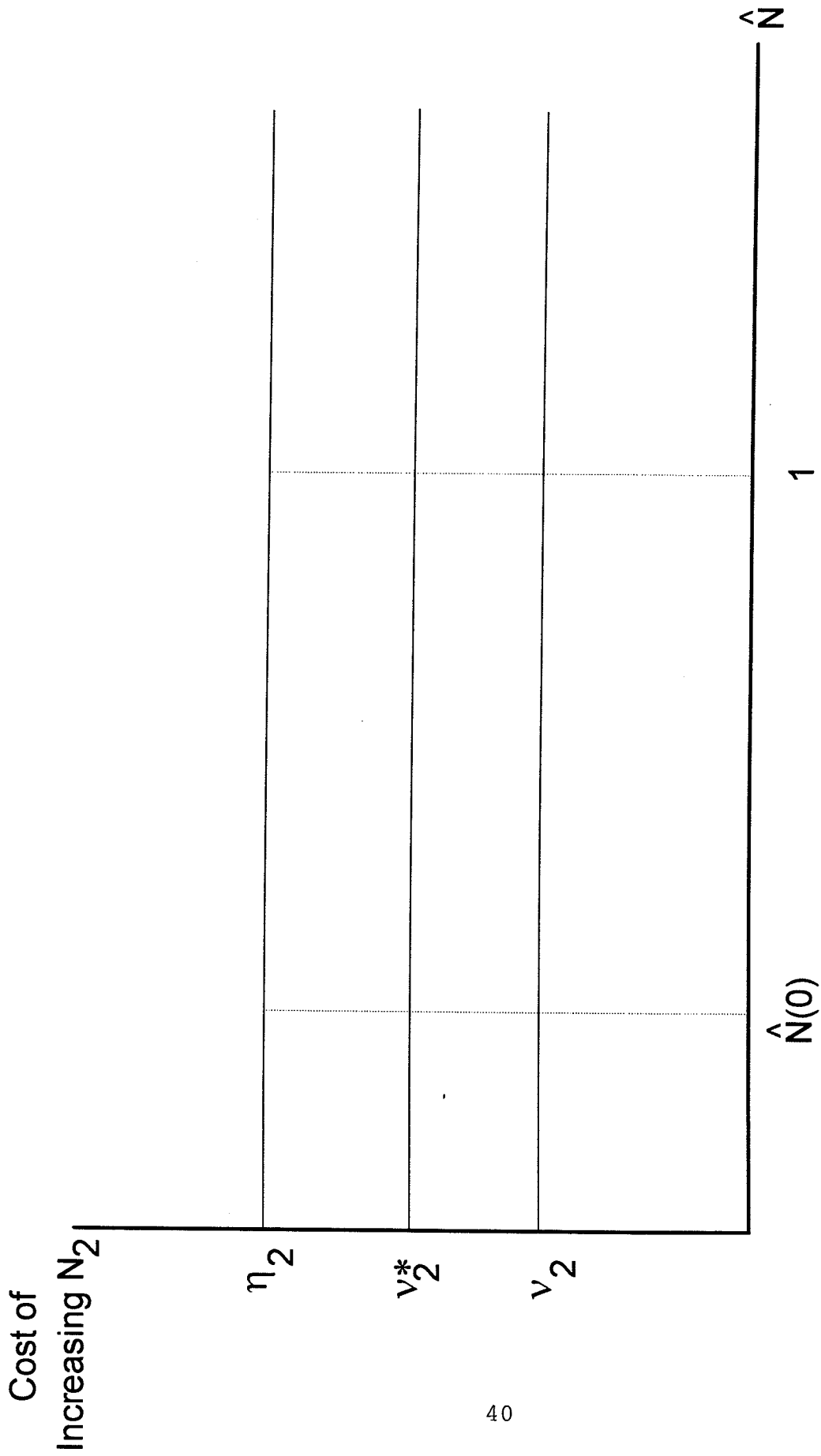Figure 3. Phase Diagram for country 2 when $(\theta < \sigma)$

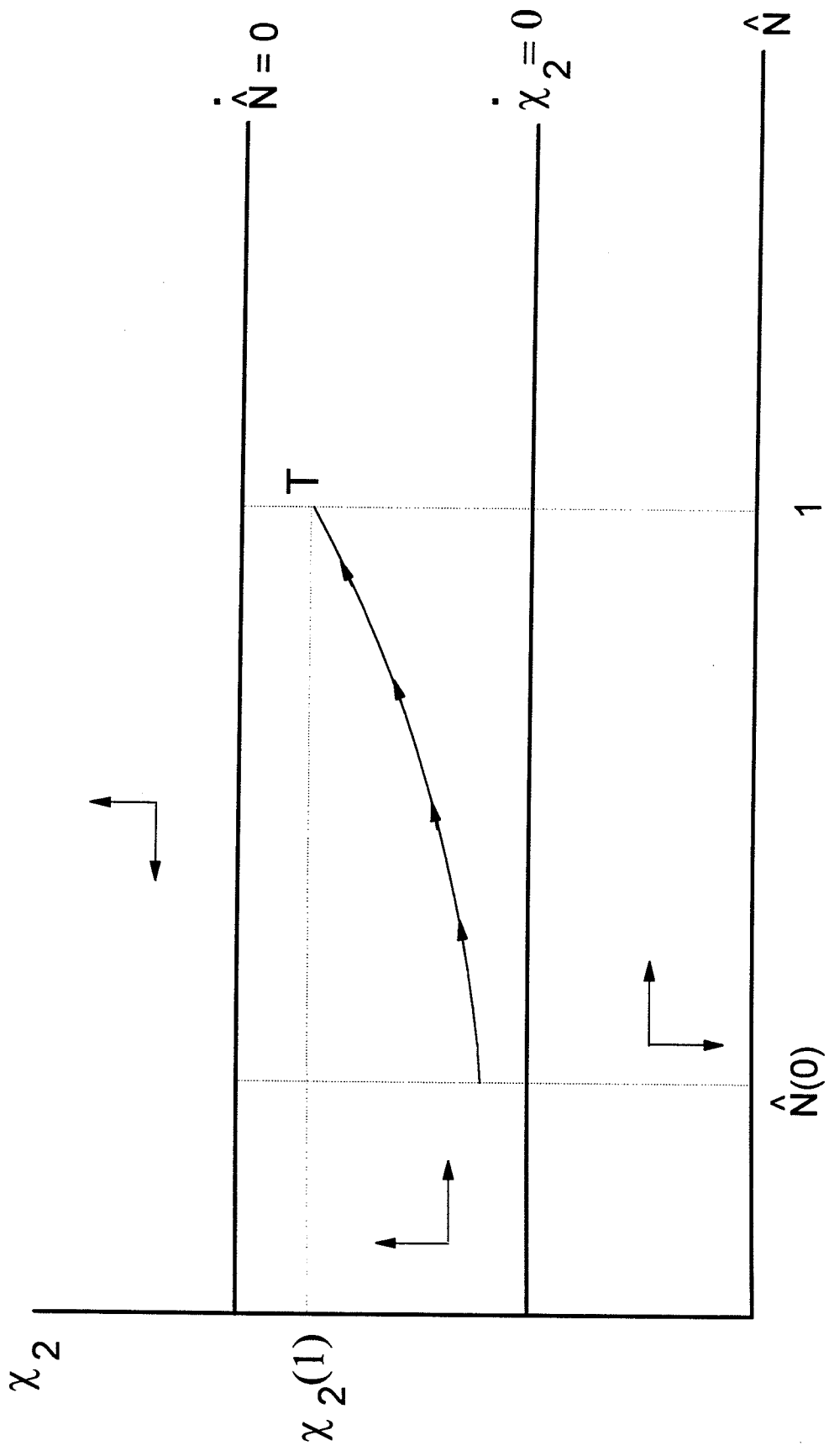Figure 4. Low and Constant Cost of Imitation in Country 2

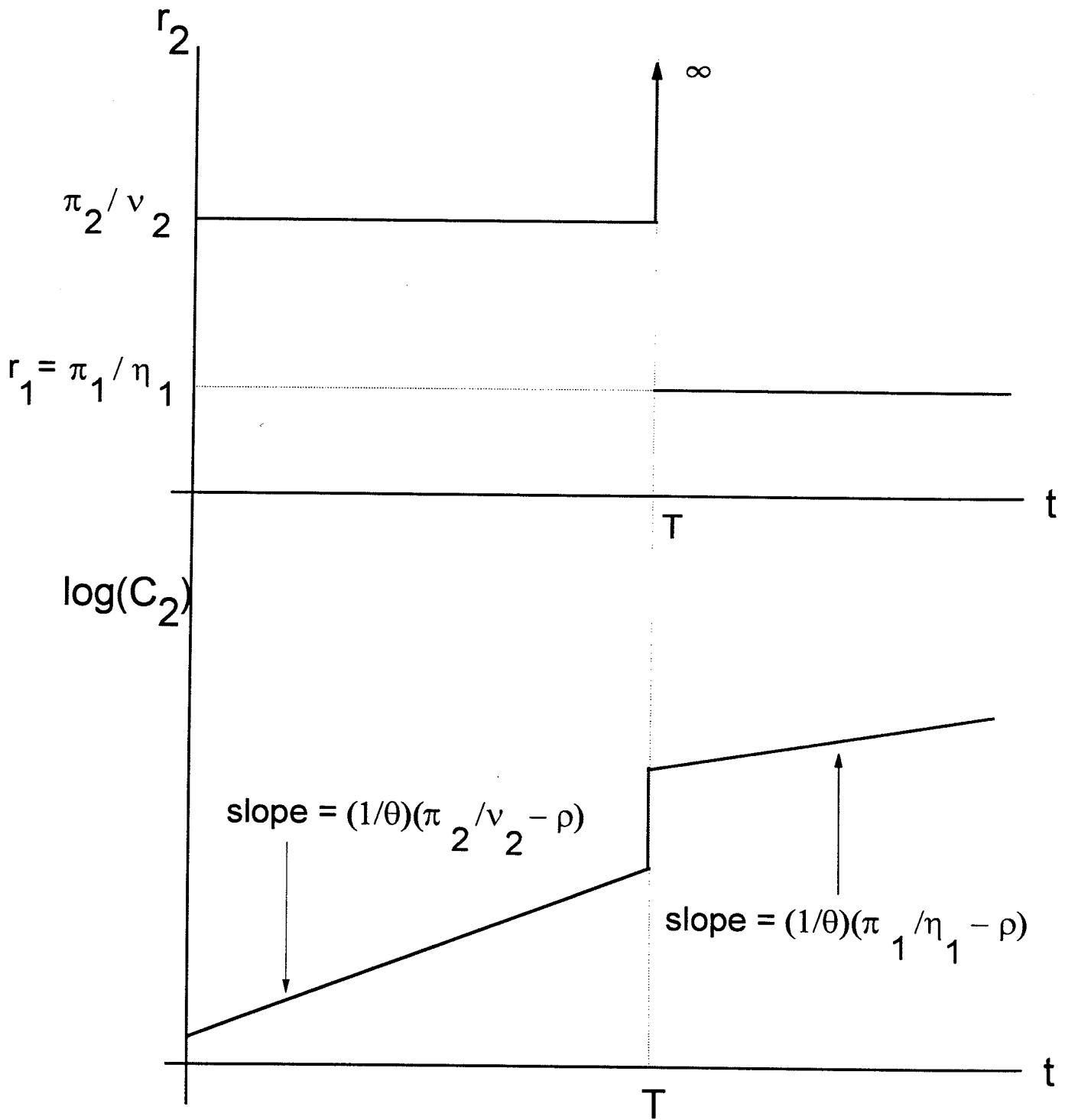Figure 5. Phase Diagram for Country 2 when ν_2 is constant ($\sigma = 0$)

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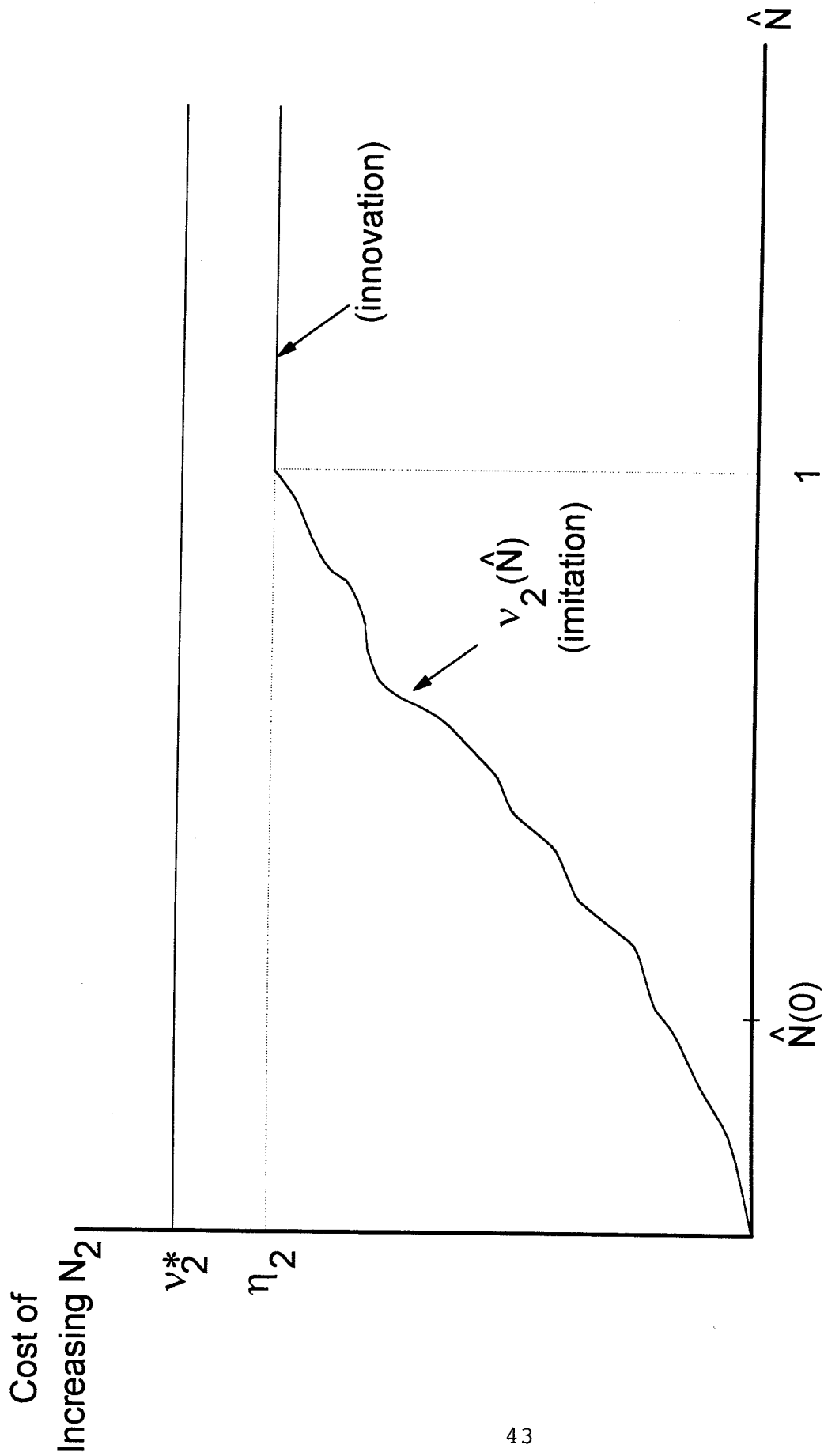Figure 6. Time Paths of r_2 and $\log(c_2)$ when v_2 is small and constant

Figure 7. Cost of Technological Change in Country 2

(for the environment in which $\nu_2^* > \eta_2$)