ELSM AND GLSR TECHNIQUES OF ARRAY ALGEBRA IN SHAPE MATCHING AND MERGE OF MULTIPLE DEMs

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ABSTRACT:

Two techniques of array algebra are introduced. The first technique, Entity Least Squares Matching (ELSM), automates tie point mensuration by using fast array algebra polynomials for real-time registration of overlapping Digital Elevation Models (DEMs). The dense raw DEMs (produced by interferometric SAR, automated stereo image matching, medical imaging etc.) may contain several blunders and missing data. ELSM is designed as a real-time measuring and adjustment system: 1) The LSM tie point matching of small windows gets speeds of over 10,000 automated transfers/sec among 4 overlapping DEMs. It is robust against blunders because the parameters of its entity model are point invariant, 2) The global entity model is given the structure of Kronecker or R-products for their "fast" solution and evaluation by array algebra, 3) Every post can be considered as a tie point in a new vertical model, strip or block adjustment of array algebra with an automated blunder elimination, and 4) The technique can be expanded into data fusion and compression of 3-D and 4-D arrays in medical and video images. The second technique of Global Least Squares Rectification (GLSR) is introduced for rigorous merging and regridding in output space. It uses the refined ELSM or other orientation data, forming the geometric data base for automated site modeling and scene simulation. A similar process is applicable for multi-image ortho rectification or least squares object reconstruction. The merged DEM has a better quality and resolution than the raw input DEM arrays oriented by ELSM. The fast GLSR solution of array algebra eliminates most blunders of the raw input data. An experimental prototype achieved speeds of over 100,000 merged DEM posts/second.

1. INTRODUCTION

Current automated photogrammetric and SAR mapping techniques can produce high resolution DEM arrays at high through-put rates. But this is true only for the raw (unedited) data, often requiring some refined orientation and "feathering" to merge overlapping DEMs into seamless mosaicks of a reference datum. The reported automated rates of Global Least Squares Matching (GLSM) with optical and SAR stereo images are getting over 10,000 match points/sec in modern computers, (Rauhala, 1992), (Hermanson et.al., 1993). Similar raw DEM collection rates are available from interferometric SAR (IFSAR) sensors.

The bottleneck of the overall DEM production is its validation and manual edit. This is especially true for the new flood of high density micro DEM or automated feature mensuration of the often poorly visible or defined layers of the terrain surface and its canopy. The visible parts of these layers are captured by the typical 2x2 pixel point density of GLSM. The problem is worse with IFSAR DEMs capturing some canopy while penetrating the other. As the elevation variations of this micro topography start approaching the measuring error, some new automated techniques are required to classify the measurements according to these surface layers. The array algebra based "fast" technologies offer new enabling capabilities for this task.

A controlled experiment in a GLSM study of automated low density (macro) DEM extraction with four experienced cartographers illustrated this problem of canopy layers, (Hermanson et.al., 1993). The human eye-brain stereo mensuration process failed to estimate a unique elevation value for the terrain surface at over 5% of the posts. The techniques of ELSM and GLSR reduce the problem into an automated weeding of outliers and undesired layers on the more uniquely defined (but more spurious) surface of the visible parts of terrain and its occluding top portion of the canopy layer. The automated edit and merge constraint of multiple overlapping DEM arrays consists of the fact that the same (interpolated) elevation should be achieved within the limits of random errors after the elimination of a systematic orientation or deformation error. A significant residual error is caused by a) an outlier, b) a valid change of the terrain and its top canopy, or c) an ambiguous elevation at the horizontal post location. Use of three or more overlapping DEM arrays can also eliminate the blunder if the other data sets agree within the limits of random errors.

The automated edit and merge problem is made more challenging by requiring simultaneous refinement for the orientation parameters and a removal of the systematic deformations of the data. The merged output DEM is required to be a seamless mosaick over a large area of interest. The process is split into two phases. The mensuration of "tie points" by an automated matching of the local terrain shape from all overlapping DEMs is made using the LSM technique and its entity adjustment or global edit model. Therefore, the technique is called

ELSM. The adjusted orientation and deformation parameters of the ELSM solution act as "fast address generators" in mapping the input DEMs into the common output space. GLSR treats the transformed elevations at the arbitrary horizontal locations of the unknown merged DEM grid as the observables of a simultaneous but fast least squares adjustment of finite elements, (Rauhala, 1986), (Rauhala et. al., 1989). The continuity or regularization constraints contribute to an automated fill-in, smoothing and editing of raw DEM data.

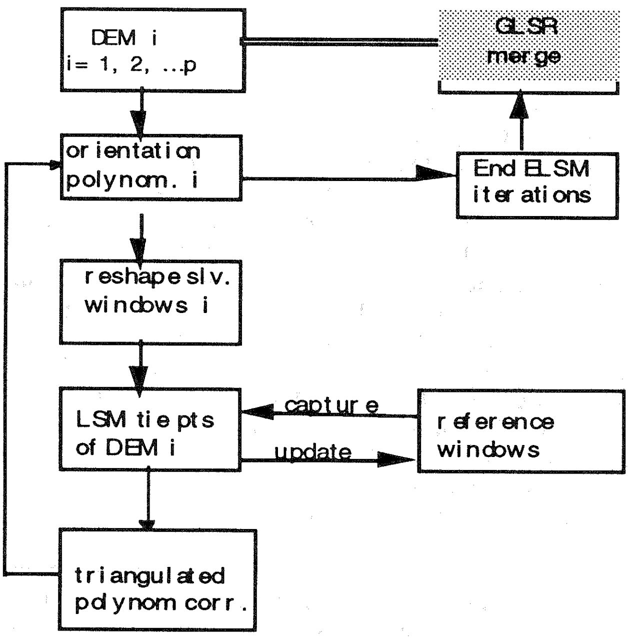

Figure1 showing the flow chart of the ELSM and GLSR

2. DESIGN OF ELSM

This section discusses DEM tie point mensuration technique of ELSM and the associated real-time orientation. The orientation solution is updated after each iteration of the LSM tie point measurements in all overlapping DEMs, thereby feeding the initial values of the next iteration as shown in the flow chart of Figure 1. All points of a given DEM share a set of "entity orientation" parameters mapping the input DEM arrays into a common reference datum. This is different from the traditional image-to-image registration model or a regular tie point mensuration where each match location also depends on its point variant object space coordinates. For instance, GLSM has typically 512x512 or over 250,000 point variant shift and illumination bias parameters (total of over 750,000) to cover a 1kx1k reference image area at 2x2 pixel node spacing. ELSM employs only few orientation (and shear error) parameters in object space. This is achieved as a modification of the fast image space bundle adjustment and automated feature recognition, called FELSM, where the linear features act as the tie and control entities, (Rauhala and Mueller, 1995):

• Least Squares Matching (LSM) produces estimates (and their 3x3 weight matrix) for three small parameter corrections dh,dx,dy at the "tie point" locations of a regular sampling grid on the overlap area. At each sample location, a window of 3x3-7x7 slave array values are matched with the DEM reference window to derive the "observed" LSM values of dh, dy and dx.

• The spacing of the LSM tie point grid is typically sparser than the used window size. Thus, not all DEM values are used in the least squares estimation of the corrections to the orientation polynomials. The LSM starts by evaluating the predicted values dx^o, dy^o of the horizontal shifts of the orientation polynomials. The predicted or reshaped elevation values $g(x+dx^o,y+dy^o)$ of the slave window are interpolated in the slave DEM and corrected by the vertical orientation polynomial dh(x,y). The differences of predicted "slave" DEM values, g, from reference values f(x,y) are used in LSM to get small local corrections dh,dx,dy and their 3x3 weight matrix (normal matrix scaled by the locally minimized square sum of residuals of LSM).

• The local normals of LSM can be considered as a (weighted) observation equation in the adjustment of small corrections for the global orientation model used in predicting the values of g. These two adjustment processes can be combined to express the observed differences g-f directly with the unknown orientation polynomials, resulting in the nonlinear ELSM observation equations

$$dh(x,y) + g[x + dx(x,y)\, ,\, y + dy(x,y)] = f(x,y) + v(x,y). \qquad (1)$$

Each 2-D global shift function dx,dy and the linear vertical bias dh is given an array polynomial of n1,n2 terms. The polynomial parameters have the separable array structure

$$df(x,y) = \underset{1,\,n_1}{[1,x,x^2, \ldots x^{n1-1}]}\ \underset{n_1,n_2}{\mathbf{A}}\ \underset{n_2,\,1}{[1,y,y^2 \ldots . y^{n2-1}]^t}$$

$$= \sum_i \sum_j x^{i-1}\, a_{ij}\, y^{j-1}$$

$$= \sum_i x^{i-1} a_{i,1} + y \sum_i x^{i-1} a_{i,2} \ldots + y^{n2-1} \sum_i x^{i-1} a_{i,n2}\, . \qquad (2)$$

Variables x,y are the horizontal geographic coordinates of the output space (merged DEM).

• There are some practical benefits of choosing three separable sets of 2-D polynomial coefficients **A** in (2) as the global modeling parameters of ELSM. One is their convenient and compact arrangement into a 3-D $n_1,n_2,3$ array for a given DEM. The orientation parameters of p overlapping DEMs form the 4-D array **A** of dimensions n1,n2,3,p. This arraying of parameters is not only convenient but results in computational savings in their least squares fitting to gridded observed values. Another practical benefit of the separable model is that the evaluation of these "address generator" polynomials at a grid of x,y variables is "fast". The recursive evaluation of one 2-D polynomial takes n1-1 or n2-1 (depending on the order of summations in (2)) additions per point vs n1*n2 multiplications and additions per point if evaluated at random locations. These savings are realized in the reshaping process of the LSM tie point grid, enabling the speed of over 10,000 tie points/sec. ELSM and GLSR exploit these "fast" properties as discussed next.

• The fast solution of the 4-D parameter array **A** enables the real-time ELSM triangulation of the orientation parameters using the gridded LSM "tie point values" of local dh,dx,dy estimates and their full 3x3 weight matrix. A brute-force solution of the orientation parameters and the m merged elevation parameters is prohibitive as its operation count of each iteration would be about op= $(3pn_1n_2 + m)^3$. In the example of p= 4 DEM models, $n_1=n_2=4$ and m=1,000, op = $1{,}192^3$. ELSM exploits the fast solution techniques of the 4-D array parameters **A**, allowing m to become so large that the valid tie points overpower the effect of outliers in the estimation of the orientation parameters. This is achieved by an array reformulation of the adjustment of independent models.

• Merged DEM values are considered as the vertical coordinate parameters of an independent model adjustment. The observables g are processed in the post wise order of p values at a given post. This allows the elimination of the unknown (merged) elevation parameter resulting in the reduced normals of the orientation polynomials. After all posts are processed, the solution of the orientation normals has the count of $p(3n_1n_2)^3 = 4\ (3x4x4)^3$ in each iteration of <u>array relaxation</u>. This idea makes the array algebra applicable for practical problems that otherwise would prevent the use of the "fast" solutions, (Rauhala, 1986). ELSM exploits the array relaxation such that the effect of the covariance terms among the p sets of orientation parameters is moved to the right hand side of the normals. A fast convergence is achieved when the number of tie points is increased such that the effect of outliers is overpowered by the "good" DEM values.

• Back substitution of orientation parameters for the "merge" or the solution of the vertical coordinate parameters has two "fast" solutions. As in the traditional model adjustment, the coordinate parameters are merely weighted averages of the transformed coordinates of each model. By coinciding the tie point density with the 3x3-7x7 post window size of LSM, the merge takes place as a by-product of updating the reference window value in the final ELSM iteration of the orientation polynomials. The second, more general, solution with the finite element constraints is achieved by GLSR.

• The solution of the array parameters **A** can be further speeded up by sacrificing some rigor in the stochastic (statistical) error model. This sacrifice is minor in comparison to reducing the functional math model of dh,dx,dy to three averaged shifts or to the 7-parameter transformation of absolute orientation, (Rosenholm and Torlegard, 1988). The very fast solution consists of preserving the rigor of the stochastic model in the partial solution along the x-direction. The full (point variant) 3x3 LSM weight matrix of the local tie point observations makes the three polynomials correlated. This weight matrix is applied in the 1-D partial solutions of each line, each requiring the matrix inversion of order $3n_1$ or about $27\ n_1^3$ operations. This is followed by the "corner turning" (partial solution over the second index of the 4-D array) or unweighted polynomial regression along the y-direction, involving only one matrix inversion of the order n_2. This very fast solution was discarded after some practical experiments. Its quality and robustness could not compete with the adopted rigorous baseline.

By reducing the values of n_1,n_2 in (2) into 1-2, the traditional orientation models are recovered as special cases of the adopted baseline. Higher degree polynomials also compensate for the systematic deformations or shear errors. Similar polynomials are used in the generic math models of softcopy workstations to approximate the rigorous nonlinear models of image geometries. These systems can handle the real-time transforms from the input-to-output space based on the raw support data. Thus, their small corrections d**A** can be considered as the main parameters of the ELSM triangulation. The refined support data are found by adding the estimates of the ELSM polynomials d**A** to the polynomials of the raw support data. The input of GLSR can then be taken from the original input space (whose output served as the input of ELSM). This requires that ELSM should be able to handle up to the bi-cubic polynomials of $n_1=n_2=4$ of the typical softcopy systems employing fast image rectifiers.

3. TEST RESULTS OF ELSM

Two test areas were processed. Each area had three input DEMs produced by the automated DEM technique of Global Least Squares Matching (GLSM) from three SAR stereo pairs. A manually measured ground truth DEM of 5 m spacing was used as the reference in 4-5 pull-in iterations. The averaged posts of the LSM sample windows were used as the reference in the last two iterations. In practice, the ground truth DEM is replaced by some information for the absolute orientation of the relative ELSM solution where any input DEM can serve as a temporary reference.

The ELSM algorithm outputs the statistics of each iteration, including the weighted unit error of each iteration and the solution of parameter corrections. Residual statistics are summarized after the final iteration for each input DEM. letting the operator to "see" the locations of outliers or disagreements among all data sets. The analysis of the tests confirmed the high parallax mensuration accuracy of 0.2-0.4 pixel of a similar experimental EO test of the GLSM technology, (Hermanson et.al, 1993). This resulted in the ELSM sigma r.m.s. difference of 1-2 m for the individual SAR DEMs of a good 2-ray stereo geometry.

Both test areas had such a poor imaging geometry (equivalent to a poor base-to-height ratio in EO stereo) for two of the input DEMs that their contributions in the merged solution of the object space were negligible. Their outliers were clearly visible in the residual analysis. The merged solution can be converted into "raw edited" shift values of new GLSM processes to produce the "refined edited" merge. The resulting solution can then feed the reference-to-reference image registrations, thereby coupling all 2p images with the tie points of about 2x2 pixel spacing. The resulting dense tie, feature and DEM point mensuration of automated edit enables the system concept of multi-ray image mapping, (Rauhala, 1986, 1989).

A practical ELSM operations strategy was developed through thousands of tests with various factors affecting the quality and speed of convergence of the ELSM orientation solution. The solution speed in fitting the 4-D parameter array to the LSM estimates of local dh,dx,dy values was so high that it could match the LSM speed of 10,000 tie points/sec in a SUN Sparc 5 computer. Thus, all input DEM posts could be afforded in the orientation solution, making it as robust as possible against the outliers. The solution typically converged in 3-7 iterations and 2-6 seconds of user time. Three slave DEMs and a Ground Truth reference DEM of about 200x200 posts were oriented and merged in both test areas.

The ELSM algorithm was debugged using a simulated reference 256x256 DEM of 2-D sine curve. An initial sequential (vs. simultaneous merge) baseline of GLSR was used to create the regridded slave DEMs of known bi-linear address polynomials from the simulated reference DEM. They were recovered (within the computer round-off error) by ELSM in 3-4 iterations. Shifts or translations up to 10 posts and shift variations of 5-10 posts due to the higher order terms were recoved without resorting to the pyramidal pull-in process. The convergence and range of pull-in were improved by allowing the high order terms to adjust beyond $n_1=n_2=2$. The bi-quadratic case of $n_1=n_2=3$ was an optimal model of the real test data with large blobs of locally diverged spots at the deep SAR shadows. Since the LSM weight matrix is divided by the large residual error of such blobs, their effect on the global solution was found negligible.

Practical insights were gained in using the minimum residual estimation theory of nonlinear array algebra beyond least squares. Also explored was the use of the nonlinear perturbation theory (use of multiple initial values for each parameter) by including the high order partials in the linearized normals of LSM, (Rauhala, 1992). An improved convergence was noticed with the simulated error free data. The outliers of the real data made some LSM sample normals (weight matrix) negative definite opening new possibilities for automated edit of the nonlinear ELSM, FELSM and GLSM technologies.

The discussed initial test provided insights on the DEM shape matching and merge problem, showing the robustness of ELSM even with the very poor test cases. The dissimilar merge of IFSAR data and GLSM DEMs produced from the stereo SAR and/or EO is getting feasible. The goal of such tests is to reduce or automate the cumbersome manual DEM validation and fill-in. The stereo SAR and IFSAR DEM techniques often lack the capability of manual mensuration and fill-in, making the full automation a high priority in the future production of high density DEMs or Digital Elevation Canopy Model (DECM). The envisioned merge of dissimilar stereo models allows the manual edit performed in the existing EO workstations. The dissimilar merge can be expanded to 3-D and 4-D medical images or to a compression of digital video and other image sequences.

Some interesting questions and problems for further testing and development of ELSM are: How many models are typically needed before the plateau of diminishing returns is reached? The answer can be found in practical tests using IFSAR DEMs and by developing the chaining processes of GLSM, FELSM, ELSM and GLSR with the expanded ELSM strip and block triangulation of entire DEM sequences of a systematic overlap pattern. The ELSM strip adjustment (with a 5-D array of orientation parameters and a 3-D array of the merged elevation parameters) is also applicable for the "fast" image space bundle adjustment of image sequences in the fashion of differential photogrammetry, (Kubik, 1992). These new technologies open a new era of multi-ray softcopy workstations replacing the traditional feature and sparse DEM data base collection with the system concept of automated and user friendly image mapping. Their control networks can be established in an integration of some photogeodetic GPS ideas of (Brown, 1994) into the data base of control features and site models of FELSM, (Holm et.al., 1995), (Rauhala and Mueller, 1995). In the transition period, the DECM of GLSM can support the more traditional feature extraction by FELSM.

4. DESIGN AND RESULTS OF GLSR

The estimation of the orientation and elevation parameters of the merged grid is split into two phases. ELSM of the first phase couples the tie point mensuration into a real-time adjustment of the orientation (and deformation) parameters. The second phase of GLSR considers these parameters known by mapping the input DEMs into the output space. The resulting horizontal locations form irregular grids which have to be merged into a single unknown regular grid by GLSR. The unknown elevation values at the merged grid are estimated in GLSR while automating the edit of the input data. An automated blunder elimination is feasible at post locations having two or more observations in their close neighborhood. The results of the GLSR prototype prove that GLSR is practically feasible. The speed of 100,000 posts in a second was reached in the initial sequential algorithm where each input DEM was transformed and regridded to support the debugging and testing of the ELSM program.

GLSR is an evolutionary "fast transform" product of array algebra. Some starting ideas of array algebra are today getting the attention of applied mathematics and engineering, (Fausett and Fulton, 1994), (Van Loan and Pitsianais, 1992), (Rauhala, 1974, 1976, 1980, 1986, 1992, 1995). Array algebra is an expansion of the Kronecker or tensor products to more general R-products and estimation theory of general "fast" matrix and tensor operators. It provides tremendous savings for a solution of large systems of linear and nonlinear equations.

GLSR applies array algebra to the least squares solution of the Finite Element DEM and array rectification:

• The unknown elevations of a regular (merged) output grid are arranged into a 2-D parameter array

• Two types of linear observation equations, sample and continuity equations, are applied for a simultaneous (indirect) solution of the parameter array. The sample equations consist of

+ 3-D coordinate transformation of the input DEM to the output system using the fast grid evaluation rules of the separable polynomials (2)

+ use of the bi-linear interpolation coefficients in the observation equations to express the transformed elevation as a linear function of 2x2 closest unknown post values

• The continuity or regularization equations consist of expressing the second derivatives of the unknown grid as linear functions of the 3x3 neighboring posts. Their normal equations couple each post into the 5x5 closest posts. They, in turn, are coupled to their 5x5 neighbours such that a simultaneous network solution is required. Unique estimates are found for any output post by an automated fill-in due to the interpolation and smoothing effect of the continuity constraints.

• Many problems of engineering and applied math only have the boundary values known with infinite weights. In GLSR, the properly weighted observed values are scattered more or less uniformly over the entire grid such that about p input values exist for each output node. This abundance of the "boundary values" and two or more weighted continuity constraints per node make the system very robust, enabling the automated blunder elimination. The merged DEM can be made denser than the input DEMs to reflect its improved resolution and accuracy.

• After the accumulation of all sample and continuity equations, their combined normals are solved by the fast array algebra techniques. The traditional sparse solution is often over 1,000 times slower, making this rigorous multi-image formulation of indirect ortho-rectification practically feasible. The solution simplifies the high speed memory management while exploiting the parallel disk technology for fast processing of very large arrays.

An early prototype of GLSR was modified for a sequential "slave DEM generator" in ELSM simulations of error free data. In the first stage of its slave DEM regridding from a (simulated 256x256) reference DEM, one input (reference) line is visited and the three known polynomials dh,dx,dy are evaluated at each input post. Two sample equations are formed using the output x-location as the variable for the transformed elevation and y-coordinate. After one input line is processed, the 1-D continuity equations are added on the 1-D normals of finite elements. Their fast solution for the elevation and y-coordinates at the regular output x locations is stored until all lines are processed. A similar 1-D process is repeated along the y-direction by considering the intermediate elevation solution as the new observed values. The result is a regular elevation grid in the output space. This prototype can be reversed for the simultaneous merge of all (p) slave DEMs. The multi-image ortho rectification is achieved by replacing the ELSM address polynomials with the point variant image-to-image registration solution of multi-image GLSM.

The initial merge results of the ELSM technique (using weighted averaging vs. the rigorous GLSR) indicate that more than 3 DEMs are needed for a robust blunder detection and automated DEM validation. With the exception of one good stereo model, the models contained large spots of invalid data due to the artifacts of SAR shadows and other problems. The operational single model GLSR for the error free ELSM simulation data had the speed of over 100,000 posts/second in a SUN Sparc5 computer. The high speeds of ELSM, FELSM and GLSR enable new technologies for machine vision and image understanding by exploiting the geometric power of their photogrammetric math models. The array algebra provides the foundation to formulate the problems in terms of the rigorous nonlinear or linear estimation theories and the "fast" numerical analysis.

5. SUMMARY

Two evolutionary applications of array algebra were introduced. Entity Least Squares Matching (ELSM) employed the basic ideas of the LSM theory from the mid 1970's to the problem of automated tie point mensuration or shape matching among overlapping DEMs. The output of LSM feeds the global or entity math model which is more robust than those used in the GLSM image-to-image registration or in the image space bundle adjustment of FELSM using linear features. The entity model of the object space merge and mosaicking is point invariant after a fast elimination of the coordinate parameters of the independent model adjustment. The reduced normals of orientation parameters are formed and solved so fast by array algebra that their updated values feed, in turn, the next iteration of the real-time nonlinear LSM tie point mensuration. This makes the LSM corrections of dh,dx,dy from the predicted locations robust against outliers. The LSM reshaping bottleneck is removed by the fast evaluation rules of the orientation polynomials of Kronecker products, resulting in the speed of over 10,000 point transfers/sec among three slave DEMs and one reference DEM in a SUN Sparc5 computer.

Global Least Squares Rectification (GLSR) performs the back-substitution of the orientation parameters for each input DEM. The transformed coordinates in the object space are treated as the observed values of the finite element DEM technique. They are complemented by the weighted continuity constraints coupling 5x5 neighboring nodes in the combined normals. A fast solution of the normals is made feasible by array algebra for virtually unlimited array sizes, as reported in the photogrammetric community since the early 1970's. The applied math and other engineering fields empoying the least squares finite elements are awakening today to exploit array algebra. The reported speed of GLSR was over 100,000 posts/second. By slight modifications, GLSR gets applicable for ortho rectification or object reconstruction from multiple images registered by GLSM at the typical resolution of 2x2 pixel node spacing. The resulting high resolution DEM of automated edit captures the visible portions of the terrain and its occluding canopy features in the fashion of the emerging SAR DEMs.

One of the first practical goals of ELSM and GLSR is the automation of the validation and edit of the high density DEMs and feature data bases of the new image mapping systems enabled by GLSM and IFSAR technologies. The tests showed the feasibility for merging data of varying quality while assessing their consistency in real-time with the full 3x3 LSM weight matrix in the global adjustment. The main reason for the outliers is that different parts of the terrain and its canopy layers are visible from different sensor views. The percentage of the screened data, not passing the automated edit, is in the order of 1-5% of the raw data. The interactive edit can also be speeded up by the fact that the outliers are clustered on local features such that the new techniques of FELSM, ELSM, GLSR and GLSM get applicable also in the local repair work.

ELSM can be expanded to 3-D and 4-D arrays and for dissimilar sensor fusion with applications limited only by the imagination of the users. An expansion of the single model ELSM mode into entire strips or blocks of DEM and image sequences is feasible in the theory of array algebra. The orientation parameters form 5-D or 6-D arrays and the coordinate parameteres are arranged into 3-D or 4-D arrays. Some of the parameters can be eliminated by the principle of differential photogrammetry and back-substituted to the "absolute" bundle adjustment for image or point variant self-calibration. The resulting quality of the 2x2 pixel dense feature geometry is approaching that of the photogeodetic control points.

The reported ELSM simulations pioneered some practical uses of the nonlinear estimation theory of minimum residuals and perturbation, published at the 1992 ISPRS. The new general theory of nonlinear estimation recovers the known solution algorithms as special cases. It uses the high order partials of Taylor series in the linearized solution of the normals minimizing any arbitrary power (vs. the power=2 of least squares) of the residual object function. The perturbation theory applies an entire grid of initial values of the nonlinear parameters for a simultaneous solution. The range and rate of the convergence is improved by using the 3-D, 4-D etc. arrays of high order partials and the estimate of the uncertainty grid of the initial values. The direct (one iteration) solution of large systems of nonlinear equations is getting practically feasible but lots of experimentation is needed to gain insights on these advanced concepts. The reported technology driven work took the first steps on this fertile new boundary of modern math and engineering sciences.

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