

# THE CONTRIBUTION OF INFORMATION THEORY TO DEVELOPMENT OF MAPPING THEORY OF DIGITIZED AIRPHOTOS

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## ABSTRACT:

Information theory is one of the contemporary scientific theories that is of great significance during the photogrammetry. It is exactly due to the application of many views from Information theory that promotes going deep into research of digital photogrammetric methodology and lays a theoretical basis guiding photogrammetry from an engineering technology mainly based on the optical and mechanical instruments to an information technique using computer as the key equipment. There are several examples demonstrating the practical application of Information theory in photogrammetry in recent years, including measurement of digital image information, data interpolation, data compression and communication in digital mapping of airphotos etc.

## 1. INTRODUCTION

Digital image processing is a technology to handle the digitized imagery which is widely used in remote sensing. Because the sensors on satellite probe the spectral information of surface features by means of the automatic scan, transmit the electromagnetic signals through communication channel and record them in digital form on the magnetic media, it is most suitable for computer manipulation. Besides, the satellites transit over the same area periodically during their orbiting, hence the data volume returned to ground from aerospace is of a huge amount that there are not other alternative facilities to treat the voluminous information in time except for computers.

The aerial photographs with the perspective geometry taken by a camera are the primitive information source for photogrammetry at present. On the airphotos there are recorded the characteristics of the spectral intensity between the visual and near infrared bands ( $0.4 - 1.1\mu$ ) which ground objects viewed by the camera reflect at the moment of exposure. However, what has been formed on the films sensitive to light is the two-dimensional images with the continuous variation of tone within a constant frame. It is not possible of com-

puter processing directly before the conversion of airphoto image from analogous data to digital ones. Image processing systems in the early time were of bulk and sophisticated equipment with a higher cost. Mapping through photogrammetry was implemented mostly by the optical and mechanical instruments. Recently owing to the emergence of the portable devices for digital record and the microcomputers with satisfactory performance as well as dramatic outdown of the prices it becomes most prosperous development direction to digitize the airphoto imagery for the automatic process in computers.

The aerial imagery plays a role of the geographic information carrier. The manipulation of image information includes the procedures from the simple visual interpretation to the quantitative and qualitative analyses using the arithmetic programs based on the rigorous mathematic models. It is involved to carry and handle the image data in the whole technological process of digital mapping. As Information theory is a subject which applies the contemporary statistics to research into data communication with high efficiency and reliability, it certainly has an undoubted significance to guide

the manipulation of image information. Moreover, the quantitative analysis and the reasoning process relied on that theory can simplify the discussion of many complicated problems. Therefore, it has been naturally riveted much attention by the foreseeing photogrammetrists.

The following exposition will focuses on the digitized image of the panchromatic black and white film which is quite popular in photogrammetric operations. Because any color film is able to be formed into three separated monochromic images with a basic color through the relevant filters, it can be treated in a similar way to the panchromatic film.

## 2. AN AVERAGE INFORMATION CONTENT OF THE DIGITIZED IMAGE

A photographic picture is a continuous variation of the tone between bright and dark on a plane, which is different sharply from the general maps of lineal graphics. Digitizing a photograph can just adopt scanning it line by line and one pixel after another. There are lots of available devices for doing that work. Here we do not intend to describe them in detail. The result of scan is a fact that the image with continuous tone variation has been converted into a grey value matrix arranged in rows and columns. Mathematically, it is equivalent to transform a two-dimensional function  $f(x,y)$  from the continuous distribution on certain planar domain into the discrete values with a limited number,  $g(m,n)$ , namely

$$g(m,n) = f(m\Delta x, n\Delta y) \quad (1)$$

where  $m=0,1,2,\dots,M$  and  $n=0,1,2,\dots,N$ ;  $\Delta x$  and  $\Delta y$  represent respectively the lateral and longitudinal intervals between the proximal scanned points;  $M$  and  $N$  are the numbers of row and column resulted in image discretization in which there are often taken  $\Delta x = \Delta y$  and  $M=N$ .

The scanned image point is called the pixel as well. The grey value of each pixel  $g(m,n)$  is fur-

ther quantified, that is, it is changed from the original value into the nearest integer according to a selected grey unit or level with equal interval, then the continuous image data are perfectly converted into separated digital ones. Figure 1 is an example of digitized image parcel.

As a rule the number of grey scale division,  $S$ , is selected as  $S = 2^k$  ( $k$  is a plus integer.) because the hardware design of computers adopts the binary coded system and the measure unit of information takes a logarithm to the base 2 which is called bit in most cases. When  $k=1$ ,  $S=2$  implies the grey scale has only two levels, that is black and white or zero and one. With  $k=8$  it follows  $S=256$ , the grey scale includes all integers between 0 and 255, in which 0 shows the darkest pixel and 255 the the brightest. Assume the grey level of each pixel to be a message, the entire frame of digitized imagery may be regarded as a batch of messages. Therefore, the maximum probable amount of information content within that frame,  $H_{max}$ , would be as follows:

$$H_{max} = MN \log_2 S = MNk \quad (\text{bits}). \quad (2)$$

The equation (2) is based on an assumption under an ideal condition that all scanned pixels have the identical possibility to take any grey level on the scale and the choice of grey level for individual pixels is independent of each other. In fact, the grey value of every pixel is always related to those of its surrounding pixels and influenced by their variation, otherwise it is impossible of the image data interpolation. It is able to be aware of that the grey level variation in each row or column of a digital image belongs in the Markovian process of the discrete data from information theory, thus the probabilities of selecting any grey level for each pixel are not exactly equal to  $1/S$ . Alternatively there is a heterogeneous allocation of the grey levels on any actual photograph. One can be given an impression of the unbalanced grey distribution through making an examination of the histogram

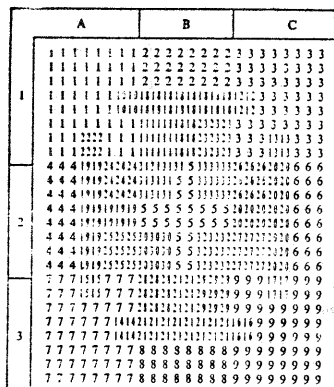


Fig.1. A piece of digital image.

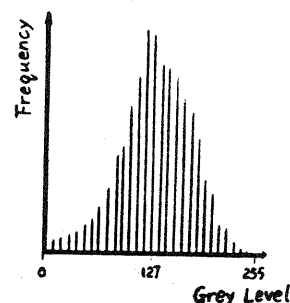


Fig.2. Histogram

composed of the pixel numbers per grey interval or the frequency distribution according to statistical method after digitization. Thus we should use the entropy to estimate the actual information content  $H_a$  from the following equation:

$$H_a = MN \left( - \sum_{i=1}^S P_i \log_2 P_i \right) \quad (\text{bits}) \quad (3)$$

where  $P_i$  is a frequency of the grey level  $i$  in the histogram.

In Information theory one explains the information as an extent of eliminating uncertainty in his mind after acceptance of a message. When all  $S$  kinds of situation for a pixel happen with the same opportunity it contains the maximum uncertainty. Therefore, the clearance of that uncertainty gives a maximal amount of information content as well.

Making a comparison between the actual entropy  $H_a$  and the maximum  $H_{max}$  we can derive the relative entropy  $H_r$  and the redundancy  $R$ :

$$H_r = \frac{H_a}{H_{max}} \quad ; \quad (4)$$

$$R = 1 - H_r \quad . \quad (5)$$

### 3. APPLICATION OF SAMPLING THEOREM

Each row of digital image virtually is a sample of a continuous strip with the variable tone on an aerial photograph. In the scanning digitizer it is implemented to sample a frame of image in accordance with the equal interval, in other words the digitizer records the signals from a continuous imagery in the constant cycle length  $t$ . This process means to convert a continual message  $f(t)$  into the time sampling ones  $f(n\Delta t)$  in which  $\Delta t$  becomes the sampling interval and  $n = 1, 2, \dots$

It is learned from Information theory that sampling a continual signal in the time domain,  $g(t)$ , must obey the sampling theorem. This theorem indicates if the frequency spectrum of a continual signal  $g(t)$  possesses a limited band width of  $W$ , there will exist the following relation:

$$g(t) = \sum_{k=-\infty}^{\infty} g\left(\frac{k}{2W}\right) \frac{\sin \pi(2Wt-k)}{\pi(2Wt-k)} \quad (6)$$

Based on the theorem it is able to know the intensity value at any moment for the signal with a bandwidth  $W$  provided that there are given the values at  $t=k/2W$ . ( $k$  is an integer.) As to image digitization it is necessary to observe the sampling theorem during recording pixels in order to recover all the changeable grey values between scanned points of the original imagery afterwards.

Theoretically, the sampling cycle should be  $\Delta t = 1/2W$ , but one often makes  $\Delta t \leq 1/2W$  in practice. When the sampling duration  $T$  is known the number of sample point should not be less than  $2WT$ . Having taken the interval between the neighbouring pixels,  $\Delta x$ , corresponding to  $\Delta t$  into account we are in a position to realize a complete recovery of the original imagery from the sampled analog data provided that the selected sampling interval enables the highest frequency of the sequential images to have two pixel records at one cycle, or to sample a frame of image in terms of a half of the shortest cycle. This is an important theoretical basis on which the resampling or interpolation of data relies.

In Information theory  $\frac{\sin x}{x}$  is called the sampling function which corresponds to the output of an ideal low-pass filter with the cut-off frequency  $W$  while a Dirac function acts on that filter (See Figure 3b). Figure 3a shows the sampling values of  $g(t)$  at  $t=k/2W$  ( $k=0, \pm 1, \pm 2, \dots$ ), that is  $g(k/2W)$ , and two curves of the sampling function for  $k=0$  and  $k=5$  respectively. It is clear from the graph that the curve of  $g(t)$  is just made of a superposition of the individual sampling functions. In Figure 3c there is an illustration of recovering the original signal from the sampled data.

During airphoto digitization the interval between the adjacent pixels should not be far less than the amount calculated by the sampling theorem,

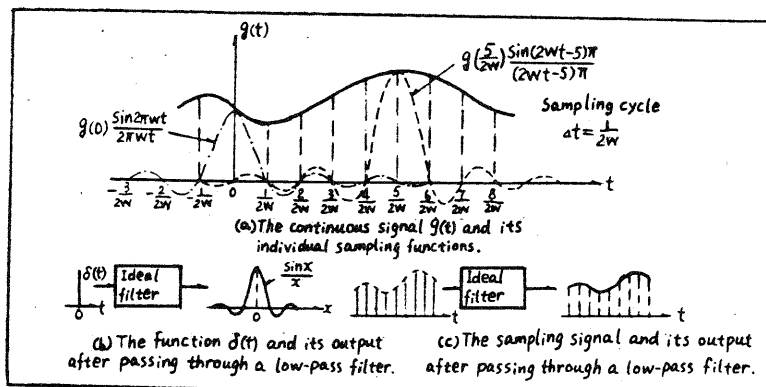


Fig. 3. The illustration of sampling theorem on time domain.

otherwise it may generate a great deal of redundant data which increase a burden of computer storage and operation. In addition, the shortest sampling interval of an image is restricted by the grain size of light-sensitive emulsion on the film base and the resolution of the available digitizer.

The continuous imagery is transformed into a series of the grey value of discrete pixels through sampling operation but the range of grey variation remains continuous. Consequently we need to quantify those values in order to turn the infinite possibilities taken by the grey variable into the finite grey levels with a constant interval. The method of quantization is based on the fidelity factor requirements of receiving the digital signals, namely one must select an appropriate quantizing unit under the conditions of decreasing the quantization errors as to ensure the discrete message to have enough levels to reflect the details of amplitude variation for a continuous message. As stated before, we choose an exponent to the base 2 as the scope of grey level and round the grey values off to the nearest levels against them. Currently many scanning digitizers use the scale quantized up to 256 levels. Because this division is proper for the representation of one byte in the computer storage it is very useful for digital image processing.

#### 4. THE OPTIMUM CODING OF IMAGE DATA

The volume of data generated from the photo digitization is of huge bulk. For instance, using a scanning record of the airphoto of size  $23 \times 23 \text{ cm}^2$  with 64 grey levels and selecting the pixel of  $0.1 \times 0.1 \text{ mm}^2$  make the information capacity per unit area amount to  $6 \times 10^7 \text{ bits/cm}^2$ , and there are about  $3.2 \times 10^7$  bits in a whole photo. With 256 grey levels and the pixel of  $0.025 \times 0.025 \text{ mm}^2$  the whole photo can contain  $6.8 \times 10^8$  bits. This causes the practical difficulty for transmission efficiency and storage rooms of data.

It is one of the major concerns about digital mapping as well to utilize the optimal image encoding for the effective storage and transmission of data. Information theory and some related considerations can render a method of the optimum coding and become a theoretical guidance to the best storage and transmission of digital message.

The ultimate goal of image encoding is to compress the data volume and improve the transmission efficiency. Usually a frame of imagery involves a lot of redundant data. The tactics of data compression are commonly based on the probability distribution of source signals or their grey levels and the to-

lerable distortion approved by the information receivers or users. In most cases compression will incur the information loss more or less. However, the approach to data compression devised according to Information theory enables that loss to be reduced to a minimum. The lost information is almost nonsignificant.

In the following we briefly describe a few kinds of digital image encoding which are widely used in various image processing systems.

Every line of digital image is actually an arrangement of the pixel grey levels such as  $x_1, x_2, \dots, x_m$ . When one carefully examines any row of that grey level array it is able to find out some pixel strings, long or short, in which each string is composed of the same grey level. Those sequential pixels with an equal grey value are known as a run length, thus a line of successive grey levels may be split up into several run lengths, for example  $k$  lengths ( $k < m$ ). Run length coding means to map a pixel series in a scan line  $x_1 x_2 \dots x_m$  onto a sequence of integer pairs  $(g_1, l_1), (g_2, l_2), \dots, (g_k, l_k)$  where  $g_i$  is a certain grey level and  $l_i$  indicates the number of times in successive occurrence of that grey level or the amount of same pixels. As a result the message made of  $m$  pixels on a scan line is able to be conveyed by only  $k$  integer pairs. When  $k$  is much less than  $m$ , it is accomplished to compress data remarkably.

In differential coding it is to hold the grey level difference between two successive pixels instead of the original quantity of each pixel. Since the changing range of possible differences is smaller than that of the originals the encoding may pick less number of bit. Usually it may realize some cutdown of data.

Fractal data compression is one of the most efficient image coding in recent years. For an image which consists of a single well defined object one hopefully recognises a large degree of deterministic self-similarity, and so constituent parts of the representation can be formed from transformations of the object itself. To ensure stability, such transformations have to be contractive, and they consist of translations, rotations and scalings whose parameters need to be found. It is said that a data reduction may arrived at more than 1000 times.

Huffman coding is a well-known compact one in the variable length coding. The Huffman encoding procedure of the binary data is as follows:

(1) Arrange  $N$  messages in terms of the order in which their probabilities appear from large to

Message	Natural code	Probability	Encoding process chart	Word	Length	
$g_1$	000	0.40		0	1	
$g_2$	001	0.18		0	001	3
$g_3$	010	0.10		0	011	3
$g_4$	011	0.10		0	0000	4
$g_5$	100	0.07		0	0100	4
$g_6$	101	0.06		0	0101	4
$g_7$	110	0.05		0	00010	5
$g_8$	111	0.04		0	00011	5

Fig. 4. An example of Huffman encoding.

small.

(2) Assign the digits 0 and 1 to the two messages with the smallest probabilities respectively, then calculate the probability summation of that two messages.

(3) Take the above-mentioned sum of probabilities as the probability of a new message, put it in the rest original messages and rearrange the reduced set in accordance with a sequence of declining probabilities.

(4) Repeat the previous steps (2) and (3) till the summation of two probabilities is equal to unit at last.

(5) Set out the latest assignment of digits, trace the way adverse to the encoding progress of individual messages and pick the allocated binary digits at all steps in succession. Finally the digital strings of 0 and 1 placed in such a way are taken as the code words corresponding to the individual messages respectively. Figure 4 illustrates an example of Huffman coding.

The principles of optimum coding which digital image has to observe are: (1) the uniqueness of decoding, and that any short code word does not form a prefix of the longer one; (2) the message with larger probability deserves a shorter code word, and the message with smaller probability a larger code word. Huffman coding meets the above-mentioned elements at all. It is preferable to utilize run length coding in cooperation with Huffman coding. Generally run length coding is conducted prior to the implementation of Huffman coding so that one can reach the coding efficiency as high as possible.

## 5. THE COMMUNICATION MODEL OF DIGITAL MAPPING

In Figure 5 there is showed a model of communication system that is discussed in Information theory. It is also appropriate for dealing with the other information transmission systems, so the diagram in Figure 5 is a general information processing model. We use the common flow scheme of digital mapping as an example to explain the actual meaning in each block of that diagram.

In the information source of digital mapping there are unrectified airphotos, specification parameters of camera, coordinates of ground controls and other initial data. Aerial imagery and geodetic coordinates belong to the different record types of geographic information. The image in airphotos is merely some approximate impression of the real scenery appearance of the earth surface. The original airphotos often involve various deformations due to both geometric and optical errors in the aerial photographic process. In those errors there are not only the systematic but also the accidental ones. This shows the information source consists of signals and noises from the beginning. One can not adopt a simple communication mode before data preprocessing.

Information destination is the output results of digital mapping. They usually include the topographic and thematic maps, the orthophotomaps and the other documentation of iconic representation which are rendered to users. These products contain the reliable records of the geographic information related to the aerial survey area. They may become a significant input part of the regional geographic

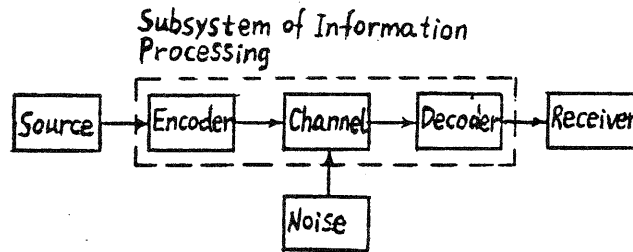


Fig.5. The general communication system

information system and improve the data base with updating.

Information channel is a physical medium that undertakes message transmission in a communication system. However, computers that play a channel part in digital mapping are not only sending the image information but also executing the various mathematic and logical operations on the digital imagery such as editing, computation of orientation parameters as editing, computation of orientation parameters, data interpolation, creation of digital terrain models, feature classification and so forth.

The encoding process in a communication system is not performed at once in most circumstances. The general mission of encoding lies in that of converting the messages out of the information source into the signals suitable for transmission and processing in the channels. The total encoding is usually divided into two parts, that is the source's and the channel's. The main objective of the former is to improve the efficiency of transmitting message and reduce the the redundancy, and the latter pays attention to enhance of the ability of coded signals to resist disturbances which is important for the distant transmission in particular. Image digitization itself is a general encoding of the analog-to-digital conversion which aims at transforming the photo imagery into the discrete pulse signals which are capable of acceptance and treatment by computers. The digitized airphoto imagery, however, does not serve the purpose of telecommunication as TV broadcast. Nevertheless some computers need reformatting the initial digital image before the routine manipulation.

Decoder fulfils the inverse task of encoding process. This kind of operations in digital mapping is mostly implemented through plotters and other-photoscopes. These instruments are able directly to accept the computer output by means of the on-line mode. It is also possible of that the content of data base created by computer on an external storage is postponed to some time and inputted into the drawing systems through a reading device under the convenient conditions.

The disturbing source in the block diagram represents a set of various noises imposed on the whole communication system. Disturbances may occur from

the heating noise in the electronic circuits owing to the ambient condition change as well as the man-made interference. Currently one of the most destructive troubles to computer information processing is the damage from computer viruses, especially the destruction to a great variety of programs on many microcomputers in widespread use. These troubles are probably incurred during the improper software reproduction. We need to take some special approaches to defend the useful information against crash.

#### 6. CONCLUSION

As photogrammetry and remote sensing are converging to a direction of the geographic information manipulation respectively, many advanced means of image processing from remotely sensed data are gradually accepted by photogrammetrists. This trend has promoted the development of automatic plotting systems with a fully digital photogrammetry and the research on their application. Moreover most methods of digital image processing have benefited from the guidance of Information theory fundamentals. Today the analytic tools and softwares devised by means of Information theory are increasingly applied in a variety of geographic information systems. At present the high spacial resolution of airphotos is still superior to that of satellite images, thus lots of tasks such as the large scale mapping, a thorough inventory of land use in the county level, the civil engineering survey, the regional cadastral investigation and so on will be undertaken by photogrammetry for a long time to come. The general development tendency of image processing suggests a demand of more sophisticated techniques. It is obvious that designing the new equipments and methods of photogrammetry to meet the requirements from users remains to depend on further understanding the essence of digital imagery information. The latent power of Information theory and other newly developing disciplines to the future progress of photogrammetry is worthy to be deeply tapped by us.

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