MATH 651
HOMEWORK I
SPRING 2013

Problem 1. Show that if X, X', Y, and Y' are spaces such that $X \simeq Y$ and $X' \simeq Y'$ then $X \times X' \simeq Y \times Y'$.

Problem 2. Show that homotopy equivalence is an equivalence relation on the collection of topological spaces.

Problem 3. (The universal property of quotients) Let $A \subseteq X$ be a subspace. We then have the quotient map $q : X \longrightarrow X/A$, where X/A is equipped with the quotient topology. Show that this is the universal example of a map from X which is constant on A. In other words, show that if $f : X \longrightarrow Z$ is any continuous map which is constant on A there is a unique continuous map $\varphi : X/A \longrightarrow Z$ such that $\varphi \circ q = f$.

Problem 4. Assume the identity map of X is null. Show that id_X is homotopic to *every* constant map $X \longrightarrow X$.

Problem 5. Show that if Y is contractible then any two maps $X \longrightarrow Y$ must be homotopic. Conclude that a contractible space is path-connected.

Problem 6. Recall that a subspace $A \subseteq X$ is said to be a **retract** of X if there exists a map $r : X \longrightarrow A$ such that $r(a) = a$ for every $a \in A$. Show that if X is contractible and A is a retract of X, then A must also be contractible.

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