MATH 651
HOMEWORK X
SPRING 2013

Problem 1. Recall that the **rank** of an abelian group is the maximal number of linearly independent elements in the group. If $A \cong \mathbb{Z}^r \times B$, where B is a finite group, then $\operatorname{rank} A = r$.

A chain complex is said to be **exact** if all homology groups are zero. An exact chain complex of the form

$$0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

is called a **short exact sequence**.

(a) What does exactness at A tell you about the map f? What does exactness at B tell you about g?

(b) Show that if A, B, and C are all finitely generated, then $\operatorname{rank}(B) = \operatorname{rank}(A) + \operatorname{rank}(C)$.

Problem 2. Show that a CW complex is locally path-connected.

Problem 3. Suppose that X and Y are CW complexes of dimensions n and k, respectively. Suppose that $H_n(X) \cong H_k(Y) \cong \mathbb{Z}$. Show that $H_{n+k}(X \times Y) \cong \mathbb{Z}$.

Date: March 9, 2013.