MATH 651
HOMEWORK III
SPRING 2013

Problem 1. Let X and Y be spaces, and denote by $\mathrm{Map}(X,Y)$ the space of continuous functions, equipped with the compact-open topology. Let $g : Y \longrightarrow Z$ be a continuous map to a third space Z. Show that composition with g defines a continuous function

$$\Phi : \mathrm{Map}(X,Y) \longrightarrow \mathrm{Map}(X,Z), \qquad \Phi(f) = g \circ f.$$

Problem 2. Delayed until next week.

Problem 3. Let γ and δ be loops in S^1 based at the standard basepoint $(1,0)$. Let $\tilde{\gamma}$ be a lift of γ to $\mathbb{R}$ such that $\tilde{\gamma}(0) = 1$. Similarly, let $\tilde{\delta}$ be a lift of δ, but take this to be the (unique) lift such that $\tilde{\delta}(0) = \tilde{\gamma}(1)$. Show, without using the theorem $\pi_1(S^1) \cong \mathbb{Z}$, that the path-composite $\tilde{\gamma} * \tilde{\delta}$ is a lift of the loop $\gamma \cdot \delta$.

Problem 4. Show that $\mathrm{id} : S^1 \longrightarrow S^1$ is not null.

Problem 5. For this problem, think of S^1 as the space of unit complex numbers. Let $f, g : S^1 \rightrightarrows S^1$ be the maps $f(z) = z^n$ and $g(z) = 1/z^k$. Determine the induced maps

$$f_*, g_* : \pi_1(S^1) = \mathbb{Z} \rightrightarrows \pi_1(S^1) = \mathbb{Z}.$$

Problem 6. Let $f : X \longrightarrow Y$ be a homotopy equivalence. Show that, for any choice of basepoint $x_0 \in X$, the map f induces an isomorphism

$$f_* : \pi_1(X, x_0) \xrightarrow{\cong} \pi_1(Y, f(x_0)).$$

For simplicity you may assume that X and Y are path-connected. (Warning: you cannot assume that the map $g : Y \longrightarrow X$ sends $f(x_0)$ back to x_0.)

Date: February 2, 2013.