MATH 651
HOMEWORK IV
SPRING 2013

Problem 1.

(1) Show that for any three spaces, there is a bijection

$$\mathcal{C}(X \amalg Y, Z) \cong \mathcal{C}(X, Z) \times \mathcal{C}(Y, Z).$$

(2) We introduced the wedge sum (or one-point union) $X \vee Y$ of based spaces on Friday. Let $\mathcal{C}_*(X, Y)$ denote the set of based maps $X \longrightarrow Y$. Show that for any three based spaces X, Y, Z, there is a bijection

$$\mathcal{C}_*(X \vee Y, Z) \cong \mathcal{C}_*(X, Z) \times \mathcal{C}_*(Y, Z).$$

Problem 2. If X and Y are based spaces, there is an inclusion $X \vee Y \hookrightarrow X \times Y$. Define the **smash product** of X and Y to be

$$X \wedge Y := X \times Y/(X \vee Y).$$

Assume that for "all" spaces the natural map

$$\mathcal{C}(X \times Y, Z) \longrightarrow \mathcal{C}(X, \mathrm{Map}(Y, Z))$$

is a bijection. For *based* spaces Y and Z, denote by $\mathrm{Map}_*(Y, Z)$ the space of all based maps. Show that the above induces a bijection

$$\mathcal{C}_*(X \wedge Y, Z) \cong \mathcal{C}_*(X, \mathrm{Map}_*(Y, Z)).$$

Problem 3. Show that there is a homeomorphism $S^1 \wedge S^1 \cong S^2$. (Hint: recall that $S^2 \cong I^2/\partial I^2$.)

Problem 4. Find the fundamental group of $\mathbb{R}^n - \{0\}$ for any n (the answer depends on the value of n). Hint: Show that $\mathbb{R}^n - \{0\}$ is homeomorphic to a product of spaces.

Problem 5. Does the Borsuk-Ulam theorem also hold for the torus? That is, given a map $f : S^1 \times S^1 \longrightarrow \mathbb{R}^2$, must there be a point (x, y) such that $f(x, y) = f(-x, -y)$.

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