MATH 651
HOMEWORK V
SPRING 2013

Problem 1. Show that an injective covering map must be a homeomorphism.

Problem 2. Show that the map $q : (0, 2) \longrightarrow S^1$ defined by $q(x) = e^{2\pi i x}$ is **not** a covering map.

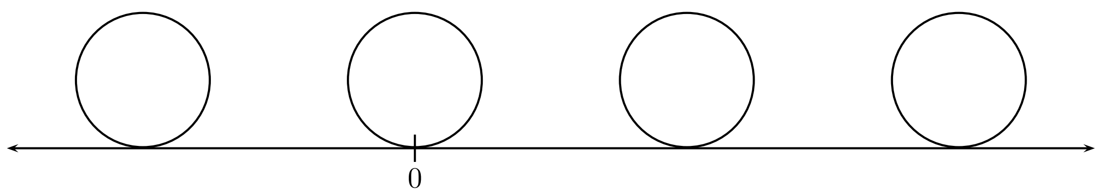
Problem 3. Suppose that $q : E \longrightarrow B$ is a covering and $b_1, b_2 \in B$. (As usual, assume that E is connected and locally path-connected.) Show that there is a bijection $q^{-1}(b_1) \cong q^{-1}(b_2)$.

Problem 4. A subspace $A \subseteq X$ is said to be a **deformation retract** of X if there is a retraction $r : X \longrightarrow A$ and a homotopy $h : i \circ r \simeq \mathrm{id}_X$ such that $h(a, t) = a$ for all $a \in A$.

(1) Show the torus with a point removed deformation retracts onto ∞ $(= S^1 \vee S^1)$. (Hint: Think of the torus as a quotient of I^2.)
(2) Show that $\mathbb{R}^2$ with two points removed deformation retracts onto ∞.
(3) What do parts (1) and (2) tell you about the fundamental groups of the punctured torus and the twice punctured plane?

Problem 5.

(1) Describe a covering of $S^1 \vee S^1$ by the space E given in the picture below:



(2) Take the point labelled as 0 as the basepoint for E. What is the image under your covering map p of the loop around the circle at 0? What about the loop (at 0) around the circle at 1?
(3) Show that the two loops in E described in part (2) are not homotopic. Use this to show that $\pi_1(S^1 \vee S^1)$ is not abelian.

Date: February 9, 2013.