MATH 651
HOMEWORK VIII
SPRING 2013

Problem 1. Compute the Euler characteristic of a Möbius band.

Problem 2. Suppose that two finite CW complexes X_1 and X_2 are homeomorphic. In other words, we have two cell complex structures on the same space, each having finitely many cells. Suppose furthermore that X_{12} is a common finite refinement of the two. In other words, every n-cell of X_1 is a union of n-cells of X_{12}, and similarly every k-cell of X_2 is a union of k-cells of X_{12}. (Here an n-cell means the image of D^n in the pushout.)

(a) If X_1 and X_2 are both graphs (i.e. 1-dimensional), show that $\chi(X_1) = \chi(X_2)$.

(b) Show that this formula still holds when the dimension (of both) is n for any $n \geq 1$.

Problem 3. Let X be the quotient of S^2 obtained by identifying the north and south poles to a single point. Put a cell complex structure on X and use this to compute $\pi_1(X)$.

Problem 4. Let X and Y be finite CW complexes.

(a) Use the cell structures on X and Y to put a CW structure on $X \times Y$. (Hint: It may help to show that $S^{m+n-1} \cong (S^{m-1} \times D^n) \cup_{S^{m-1} \times S^{n-1}} (D^m \times (S^{n-1}))$.)

(b) Use this to deduce a formula for $\chi(X \times Y)$ in terms of $\chi(X)$ and $\chi(Y)$.

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