

Review of Methodology for Establishing Rates for Yield Exclusion

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Executive Summary

A new method for establishing rates for coverage levels above 85% need to be established because of the new requirement to allow farmers to drop low yields from their APH yield calculation. The dropping of low yields increases effective coverage levels above nominal coverage levels. RMA wants to determine insurance rates for nominally selected coverage levels that represent effective coverage level risk. The two procedures proposed to determine effective coverage level rates include linear interpolation of rates when effective coverage levels are less than 85% and linear extrapolation of rates when coverage levels exceed 85%.

I used Monte Carlo methods to determine the direction and magnitude of errors in rates that are likely using the proposed interpolation and extrapolation methods. Because premium rates increase at an increasing rate for effective coverage levels below 85%, the proposed interpolation methods lead to an upward bias in rates. That is, interpolated rates are greater than actuarially fair rates. However, because the interpolation points are so close together, linear interpolation does an adequate job approximating the actual nonlinear relationship. Thus I conclude that the proposed method of using linear interpolation does a good job approximating the nonlinear relationship between premium rates and effective coverage levels.

The proposed linear extrapolation method is based on the change in premium rates at two points—the 80% and 85% premium rates—and uses this change in premium rate to determine premium rates for all other effective coverage levels that exceed 85%. This method is appropriate and will lead to small approximation errors only when the relationship between premium rates and effective coverage levels above 85% is linear. But the general relationship is nonlinear. For most any well-behaved and commonly used yield distribution, and for most risk levels, the relationship is convex (rates increase at an increasing rate). The lower the risk level, as represented by the 65% premium rate, the greater the convexity. This convexity implies that linear extrapolation will underestimate actuarially fair premium rates. Most Corn Belt corn and soybean farmers have low 65% premium rates. And most of the book of business for the crop insurance program is represented by Corn Belt corn and soybean farmers. Thus the proposed rule will result in a large underestimation of premium rates for effective coverage levels that exceed 85% in the most important crop insurance region, in terms of liability and premium.

For higher risk farmers, the relationship between premium rate and effective coverage level is less convex, approximately linear, or even concave (rates increase

at a decreasing rate), depending on the level of risk, the effective coverage level, and the form of the yield distribution. Simulations show that if the relationship is concave, which can occur when the yield distribution is skewed sharply to the right, then the proposed extrapolation method can lead to a large overestimation of premium rates. Otherwise, extrapolation errors are likely to be lower in magnitude than the errors made for low risk farmers.

Exacerbating the problems with underestimation of premium rates for low risk farmers is the likelihood of increased moral hazard for very high effective coverage levels. The likelihood that a farmer will take less care of a crop when it is insured for more than it is expected to be worth is likely much greater than when a farmer has a significant insurance deductible. Thus the underestimation of premium rates will likely be larger than estimated here.

An alternative method of determining premium rates for greater-than-85% coverage levels is to take advantage of the yield draws from the truncated normal distribution that are used by RMA to determine revenue insurance rates. They will give a much better approximation of how premium rates should behave for coverage levels above 85% for low risk farmers than the proposed extrapolation method. For high risk farmers the use of the draws may be less appropriate because the truncated normal distribution does not allow for a concave relationship between premium rate and coverage levels for reasonable effective coverage levels.

Research Report

(1) Description of the methodology used by the expert reviewer.

The method used in this review was to carefully think about and use Monte Carlo simulations to determine the likely relationship between premium rates and effective coverage levels. The likely relationship was then used to determine whether the methods proposed by RMA are appropriate for calculation for premium rates when yields are dropped from APH yield calculations.

The rest of the research report is organized as follows. First an overview of what determines the relationship between premium rates and coverage levels for yield insurance is provided. Next, a comparison of an “ideal” method based on knowledge of the yield distribution to RMA’s proposed methods that are based on linear interpolation and extrapolation is made. The report then provides a discussion of the relevance of this review’s findings and provides a suggestion for an alternative method that could be used if desired. Answers to the review questions are then provided.

Overview of Relationship between Coverage Levels and Premium Rates

Actuarially fair premiums for yield insurance when plotted against the coverage level will take the same general shape of put option premiums on futures prices when plotted against the strike price. When the strike price is far below the futures price, the option premium will increase at an increasing rate with increases in strike prices. Once the strike price approaches the futures price then the option premium increases linearly. Eventually the option premium will increase at a decreasing rate. The reason why option premiums increase at an increasing rate when the strike price is low in relation to the futures price is that initially the probability that the option will be exercised is quite low. As the strike price increases from low levels the probability increases at an increasing rate because the probability distribution of price increases rapidly as the strike price moves towards the mean of the distribution. Once the strike price approaches the mode of the price distribution the probability of exercising the option no longer increases rapidly. This is why the premium eventually becomes linear in the strike price. Eventually the probability of exercising the option increases at a decreasing rate so the option premium increases at a decreasing rate also. As with option premiums so too with yield insurance premiums, assuming that yields follow single-peaked probability density functions, which will be assumed throughout this review. This means that actuarially fair yield insurance premiums will increase at an increasing rate for low coverage levels but will eventually become linear at very high coverage levels, and then increase at a decreasing rate.

The proposed method of RMA to linearly interpolate premium rates for coverage levels between 50% and 85% means that the interpolation will overestimate premiums because at these coverage levels almost all yield insurance premiums

increase at an increasing rate with coverage levels. The upward bias will be small when the rate relationship is approximately linear between any two coverage levels. For coverage levels above 85% the proposed method to extrapolate premium rates based on the increase in rate between 80% and 85% will underestimate premium rates if the premiums are still increasing at an increasing rate between these two coverage levels. The method will be largely unbiased for coverage levels not too much about 85% if the relationship between 80% and 85% is linear. When the relationship between 80% and 85% is linear, then because the rate of increase in the actuarially fair premium rate will start to decline, the extrapolation method will lead to an overestimate of premium rate for a very high coverage level.

To clarify these concepts, Figure 1 shows how actuarially fair premiums increase when yields follow a beta distribution with two 65% premium rates: 0.0082 and 0.10. These two rates represent, respectively, low risk and high risk crops. Figure 2 shows the coverage level differentials (CLD) for these two representative distributions which is simply the ratio of a premium rate to the 65% coverage level premium rate.

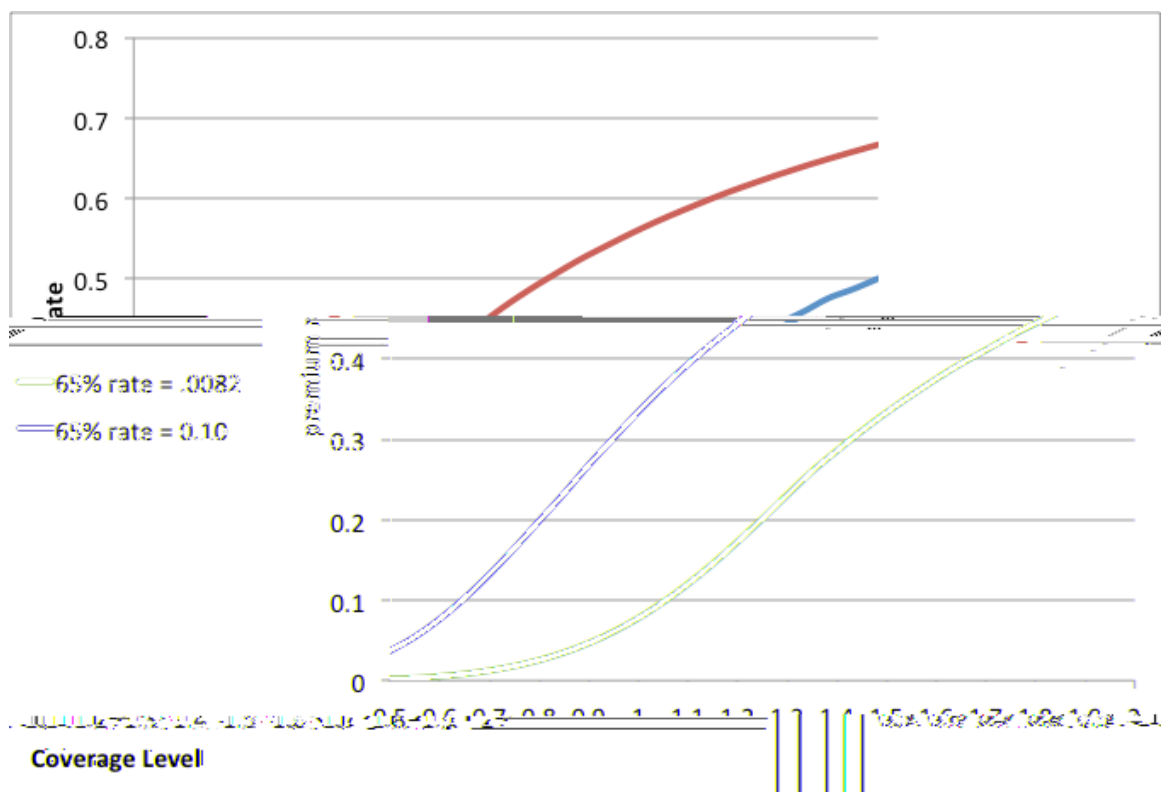


Figure 1. Simulated Premium Rates for Two Beta Distributions

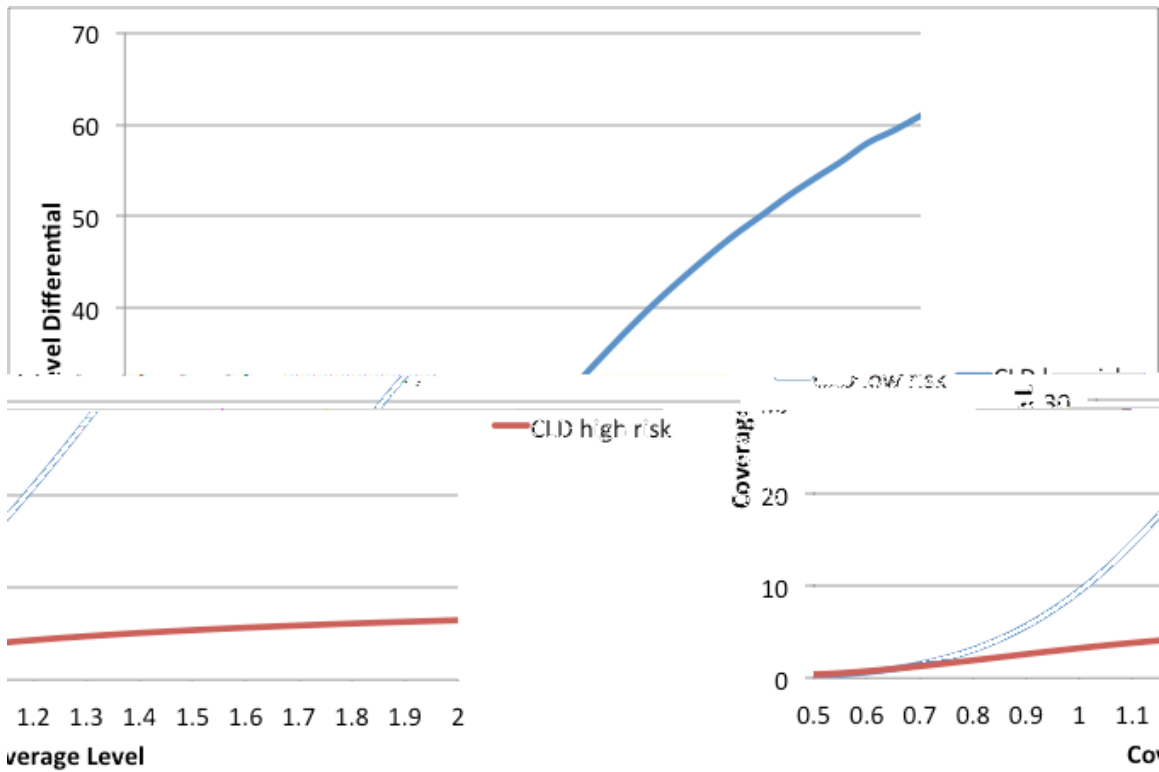


Figure 2. Coverage Level Differentials for High and Low Risk Yield Distributions

From these two figures it is straightforward to see the ramifications of extrapolating premium rates for coverage levels above 85% based on the rate of increase of premium rates between 80% and 85%. In Figure 1 it is apparent that premium rates are increasing at an increasing rate for the low risk yield distribution at the 85% coverage level. This implies that linear extrapolation will lead to an underestimate of what premium rates should be for effective coverage levels greater than 85%. It is apparent that premium rates are increasing at a decreasing rate for the high risk yield distribution. This implies that linear extrapolation will overestimate premium rates for greater than 85% effective coverage levels.

Figure 3 shows the actual coverage level differentials and the extrapolated differentials for the low risk yield distribution. Figure 4 shows the same graph for the high yield distributions. These two figures show that errors in both cases can be quite high. The RMA method extrapolation method underestimates premium rates for this low risk yield distribution and overestimates premium rates for this high risk yield distribution. The degree of overestimation for the high risk distribution may be capped by RMA's limitation that the increase in premium cannot exceed the increase in liability. Figure 5 shows the percent error in premium rate for the two distributions. For the high risk distribution the error is small until the effective coverage level increases above 100%. The percentage effort for the low risk distribution is significant in percentage terms at all coverage levels. However, the

absolute magnitude of the error is smaller than the percentage error for the low risk distribution for coverage levels close to 85% as shown in Figure 6.

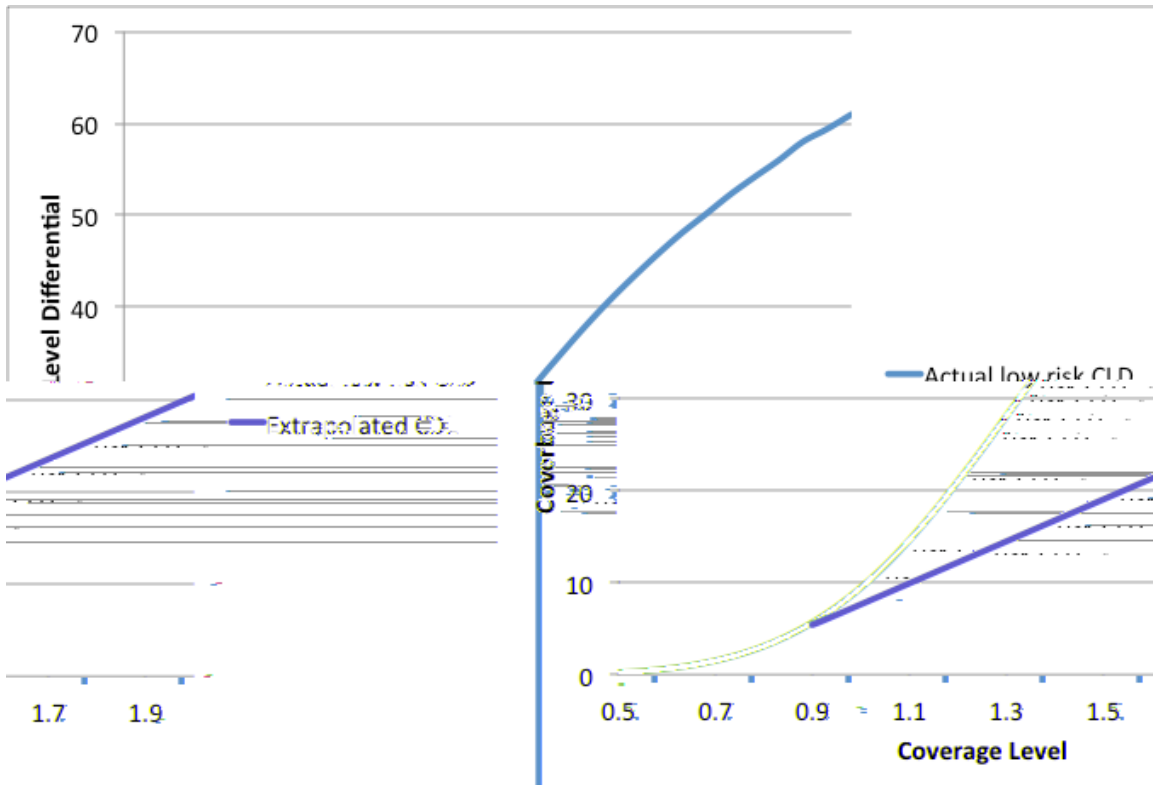


Figure 3. Actual and Extrapolated Coverage Level Differentials for the Low Risk Yield Distribution

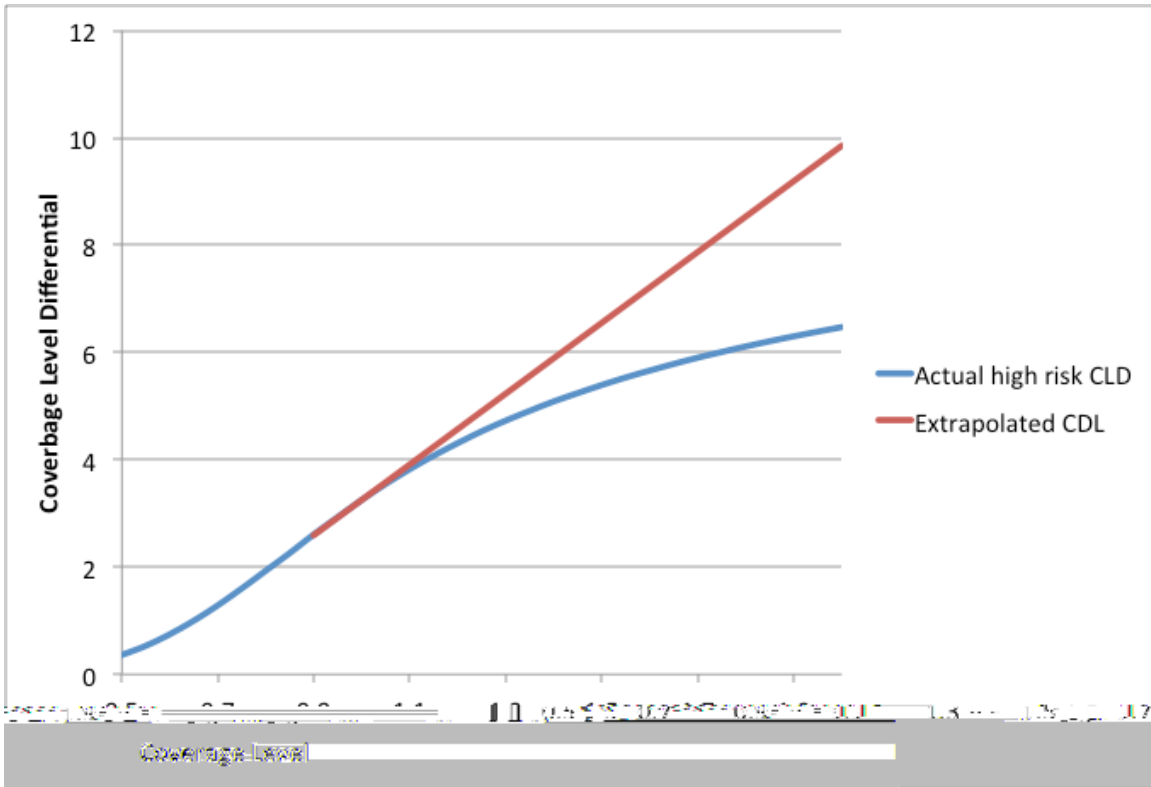


Figure 4. Actual and Extrapolated Coverage Level Differentials for the Low Risk Yield Distribution

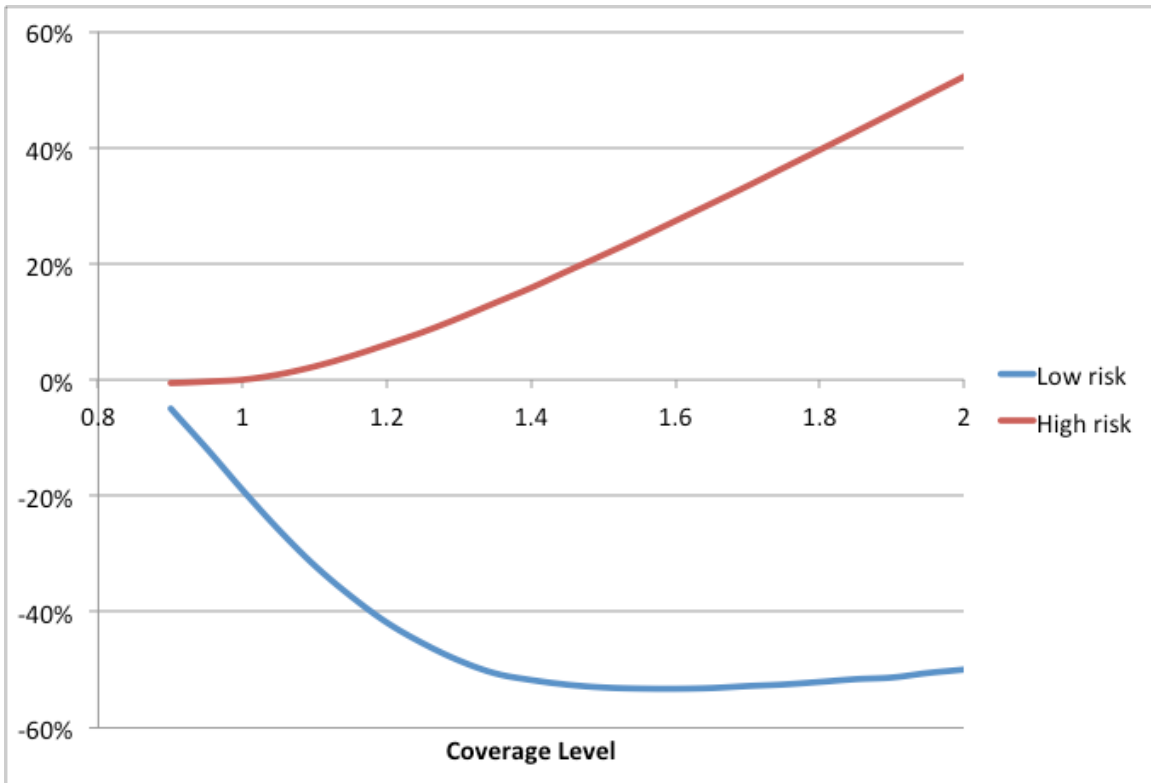


Figure 5. Percent Error in Premium Rate Using Proposed Extrapolation Method

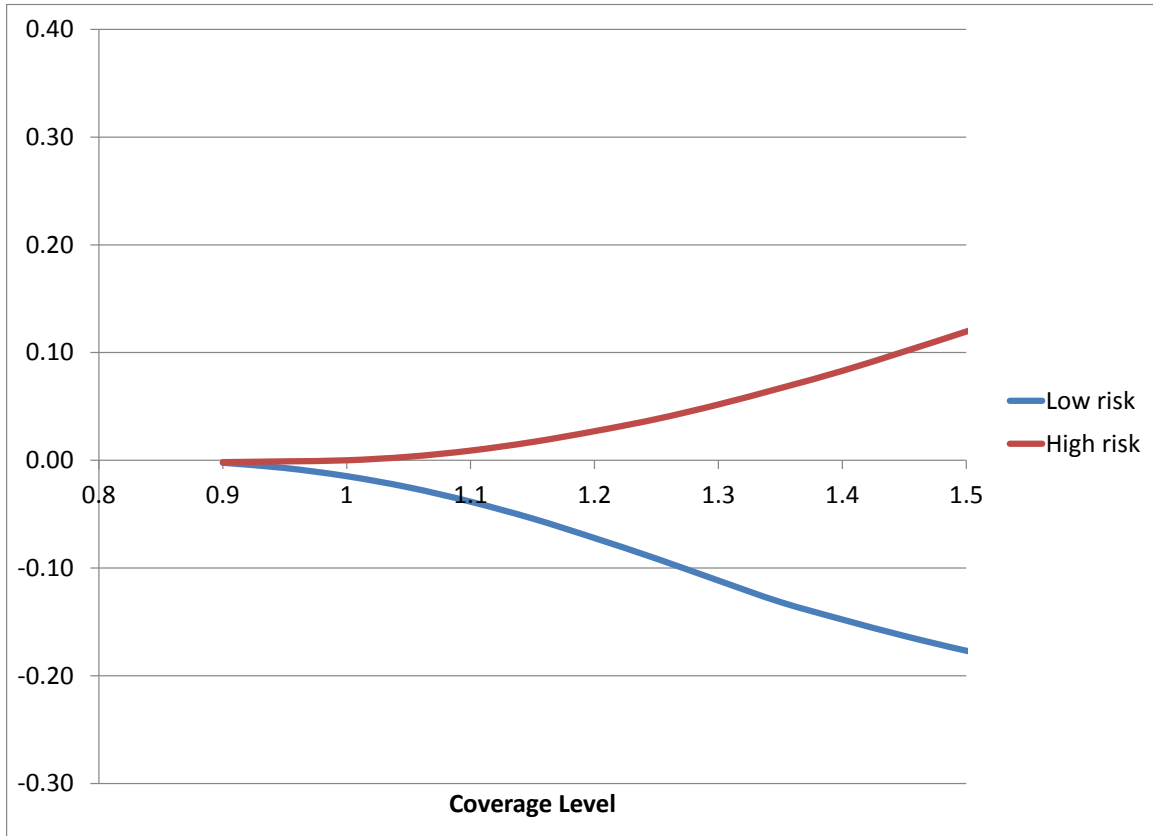


Figure 6. Extrapolation Errors Expressed in Levels

A Comparison of Ideal Methods and RMA Proposed Methods

To estimate the actual errors that would be made using RMA's proposed interpolation and extrapolation methods requires knowledge of the "true" yield distribution along with an assumption that the RMA equations used to determine 80% and 85% CDLs are consistent with this true yield distribution. However, there is no way of determining what the true yield distribution is, so an alternative method must be used. As demonstrated above the shape of the relationship between actuarially fair premiums and effective coverage level above 85% determines whether RMA's proposed extrapolation method leads to positive or negative errors and the magnitude of the errors. Because the curvature of this relationship varies according to the shape of the yield distribution, it is not possible to develop general predictions of the likely extrapolation errors for all 65% premium rates. However insight into the possible extrapolation errors that will be likely can be estimated by choosing different parametric forms for the yield distribution. In this review I use two distributions to obtain insight. The first is the truncated normal distribution. The second is the beta distribution. The advantage of using the truncated normal distribution is that it is the distribution used by RMA to determine revenue insurance premium rates for coverage levels up to 85%. The disadvantage of the

truncated normal distribution is that the general shape of the non-truncated part of the distribution is identical for all 65% premium rates. This implies that the general shape of the relationship between actuarially fair premiums and effective coverage levels cannot vary too much. As shown in Figure 1, this is not true for the beta distribution, hence it is used also.

The method used to gain insight into errors for these two distributions are to normalize all distributions so that they have a mean yield of 100 and a minimum yield of zero. The other parameters are chosen so that yield draws from the distribution generate a 65% premium rate that is equal to a target rate for analysis purposes. Actuarially fair premiums are calculated for each coverage level using Monte Carlo integration methods. The Monte Carlo premium rates for 80% and 85% for each distribution are used to extrapolate premiums rates for effective coverage levels greater than 85%. These extrapolated premium rates are then subtracted from the Monte Carlo-estimated actuarially fair rates to determine the direction and magnitude of errors.

The parameters of the distribution for six 65% premium rates are shown in Table 1. Non-representative very high premiums rates are included in this analysis because of the higher likelihood that farmers and crops with high rates will be most likely to drop yields from their APH calculation and subsequently have high effective coverage levels. The maximum yield increases as the 65% premium rate increases to represent the fact that low risk yield distributions are simply more stable. The beta distribution is left-skewed when risk is low and right-skewed when risk is high. Ignoring the truncation point, the truncated normal distribution is symmetric.

Table 1. Truncated Normal and Beta Parameters Used for Extrapolation Analysis

	65% Premium Rate					
	1%	2.5%	5%	10%	20%	30%
Truncated Normal						
mu	100	100	99.9	99.6	96.7	88.3
sigma	104.8	73.9	48.84	36.47	29.12	23.08
Beta						
p	6.83	4.62	3.15	1.90	1.13	0.69
q	3.70	3.25	2.82	2.29	3.38	3.45
Max yield	154.3	170.4	189.7	200.3	400	600
Min yield	0	0	0	0	0	0

The first step is to find the parameters of the truncated normal distribution that generate a 65% premium rate equal to a target premium rate and a mean yield equal to 100. This mean yield is held constant at 100 for all simulations. Premium rates for the 65% coverage level were varied across the levels shown in Table 1. After the parameters of the yield distribution were determined, it is straightforward

to simulate actuarially fair premium rates using Monte Carlo simulation and to then calculate the errors that are generated when premium rates for coverage levels

Figure 7 shows interpolation errors (in percentage terms) for the representative 65% base premium rates using the truncated normal distribution. The interpolation errors for the beta distribution are not shown because they are nearly identical to those shown in Figure 7. The errors are the difference between actuarially fair premium rates between the interpolation points (65%, 70%, 75%, 80%, and 85%) and the interpolated premium rates. The errors will equal zero at each interpolation point because it is assumed that the actuarially fair premium rates between the interpolation points are known.

The amount of interpolation error depends on the degree of curvature of the relationship between premium rate and coverage level. Because curvature decreases as 65% premium rates increase, so too does the percent error. But even at very low 65% premium rates, the amount of interpolation error is less than 2%. And 2% of a very low rate is a quite low number. At a 65% premium rate of 10%, the amount of error drops to less than 0.2%. The reason why the error is so low is that even with a highly curved relationship, the interpolation points are so close together that a linear approximation works well. Thus I conclude that linear approximation of premium rates for effective coverage levels between 65% and 85% is appropriate.

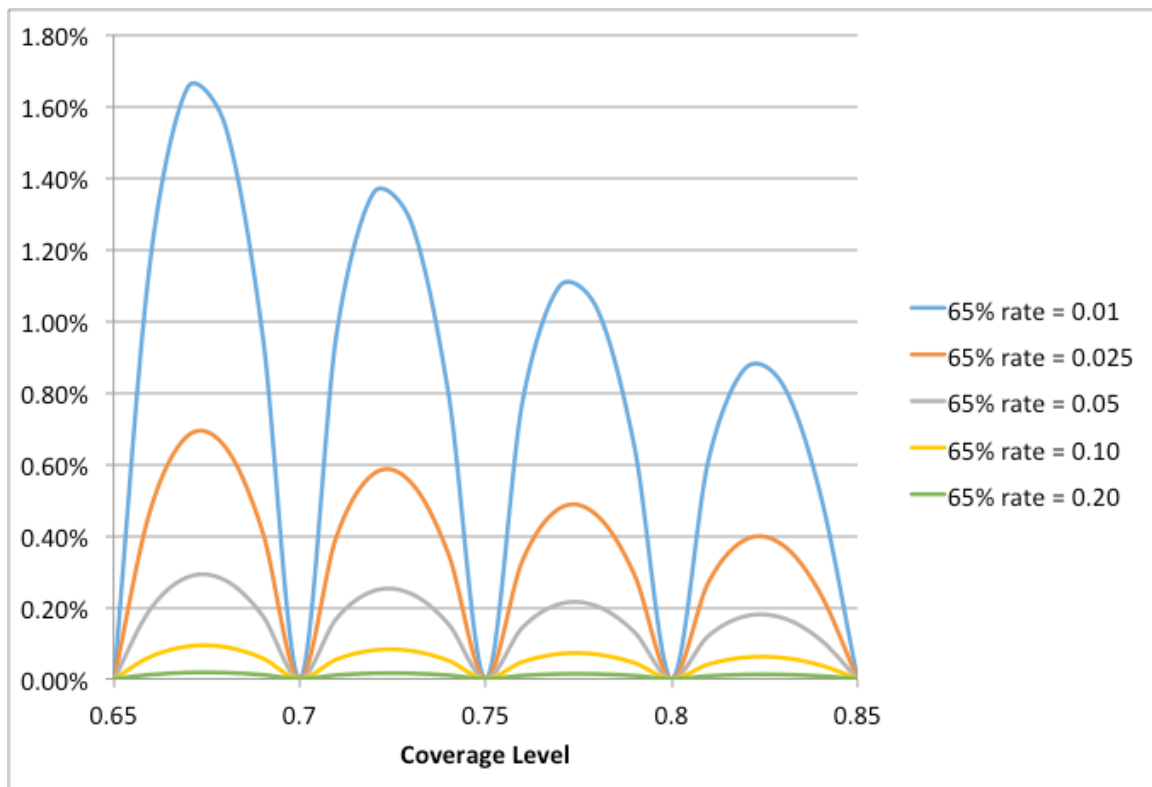


Figure 7. Interpolation Errors Using the Truncate Normal Distribution

The method I use to determine extrapolation errors made in estimating premium rates for greater than 85% coverage levels is to first assume that yields follow either the beta distribution or the truncated normal distribution. The increase in premium rate between 80% and 85% coverage levels is then calculated for each yield distribution, which is uniquely identified by the chosen parametric form and the target 65% premium rates. The change in premium rate between the 80% and 85% coverage levels for each distribution is then used to determine the premium rates for effective coverage levels greater than 85%. That is, the assumed known actuarially fair premium rates between 80% and 85% coverage levels are used to determine the slope of a line relating premium rate to coverage level. This slope is multiplied by the difference in coverage between the effective coverage level in question and the 85% coverage level. The resulting product is then added to the 85% premium rate to determine the extrapolated premium rate. The resulting error for each effective coverage level is simply the difference between the extrapolated premium rate and the actuarially fair premium rate for each effective coverage level, which is determined by Monte Carlo methods for each distribution. All calculations were done using premium rates rather than CLDs. This is equivalent to using CDLs because CLDs are simply the ratio of a premium rate to the 65% premium rates. Actuarially fair premium rates are shown in Figures 8 and 9 for the truncate normal and beta yield distributions respectively.

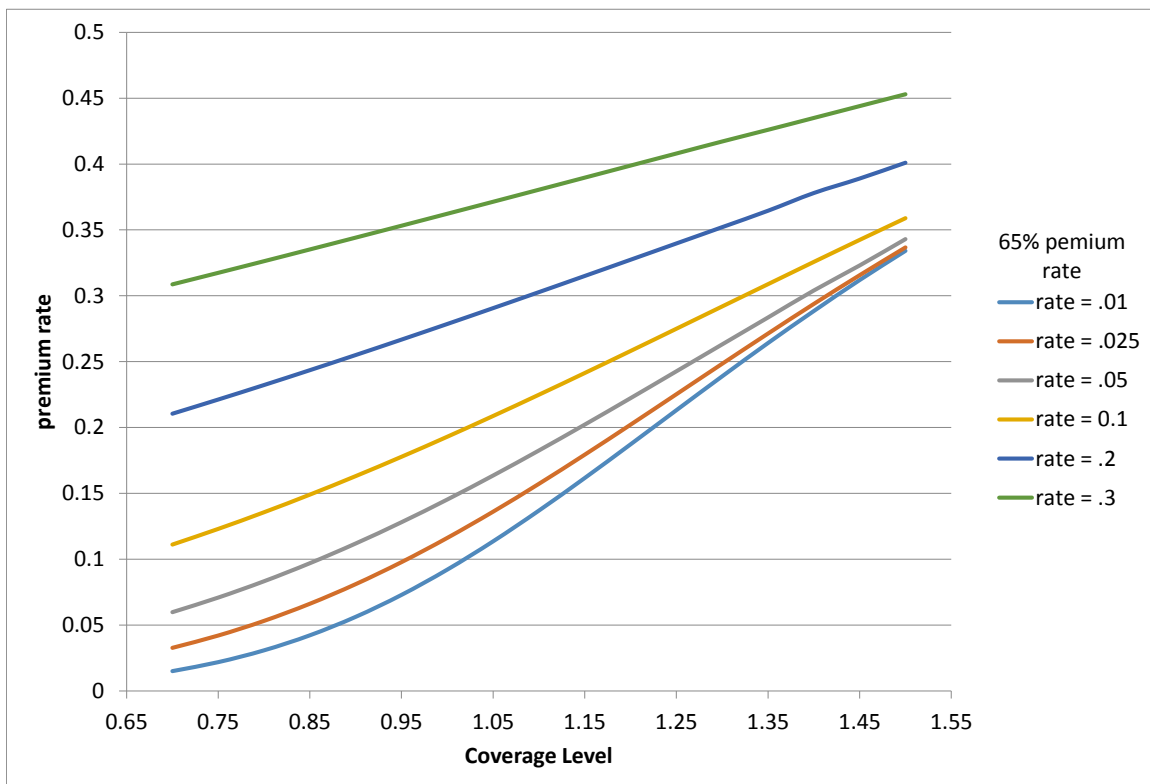


Figure 8. Actuarially Fair Premium Rates for the Truncated Normal Yield Distribution

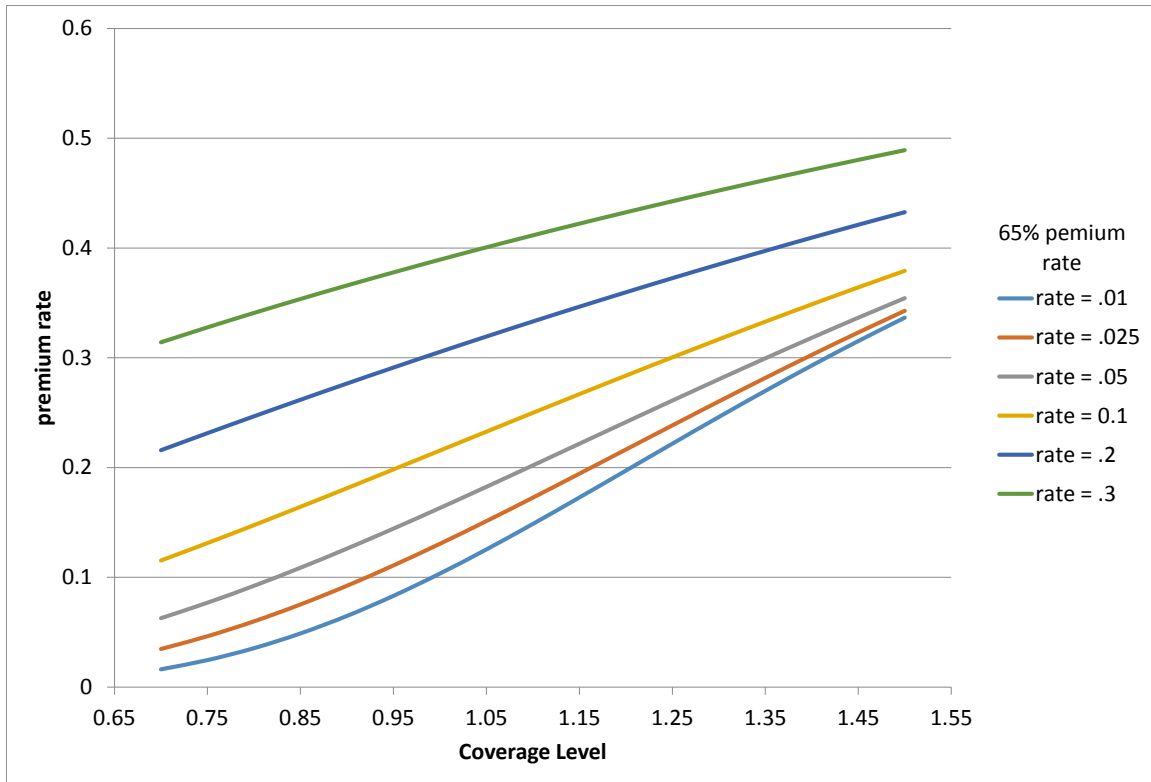


Figure 9. Actuarially Fair Premium Rates for the Beta Yield Distribution

As can be seen, there are some significant differences between the two yield distributions in how actuarially fair premiums increase with coverage level. At very high coverage levels, the truncated normal distribution implies an approximately linear relationship whereas the beta distribution implies a concave relationship. An approximately linear relationship implies that extrapolation errors will be quite small. A concave relationship implies that extrapolation errors will be positive (extrapolation overestimates premium rates). Both distributions show a convex relationship between coverage level and premium rate at 65% premium rates of 0.01 and 0.025. This suggests that extrapolation errors for low risk farmers will be negative. For intermediate rates the truncated normal appears to maintain a convex relationship for 65% premium rates of 0.05 and 0.10 whereas the beta is convex at the 0.05 rate and a bit concave at the 0.10 rate.

Figures 10 and 11 present the extrapolation errors that correspond to Figures 8 and 9 respectively. As expected, both distributions imply large extrapolation errors for yield distributions calibrated to low 65% premium rates. The reason is that the relationship between coverage level and premium rate is convex at the 85% coverage level when risk is low so linear extrapolation leads to underestimates of what premium rates should be. The truncated normal maintains this convex relationship even for higher risk yield distributions. Thus, as shown in Figure 10, a truncated normal distribution implies that extrapolation errors will only be negative and they will be large for effective coverage levels above 90% unless the 65% premium rate is quite high. In contrast a beta yield distribution implies that at high

premium rates extrapolation errors will be positive. With the yield distribution calibrated to a 65% premium of 0.10, the beta distribution implies nearly a linear relationship so at that rate the extrapolation errors will be low.

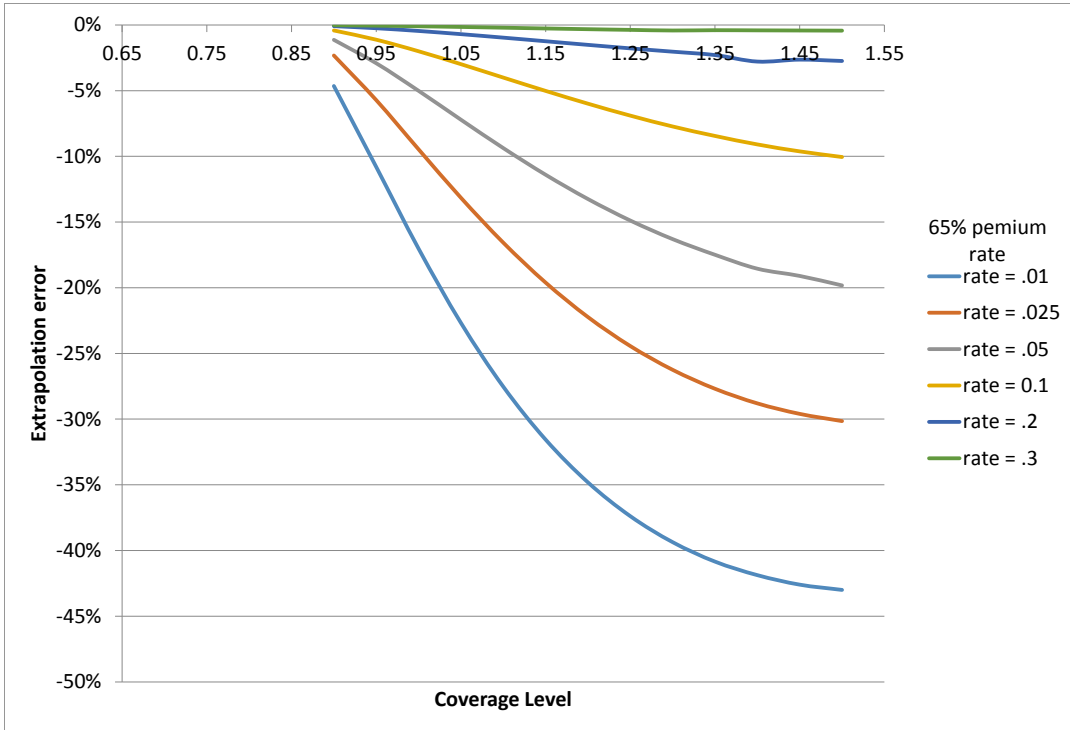


Figure 10. Extrapolation Errors for the Truncated Normal Yield Distribution

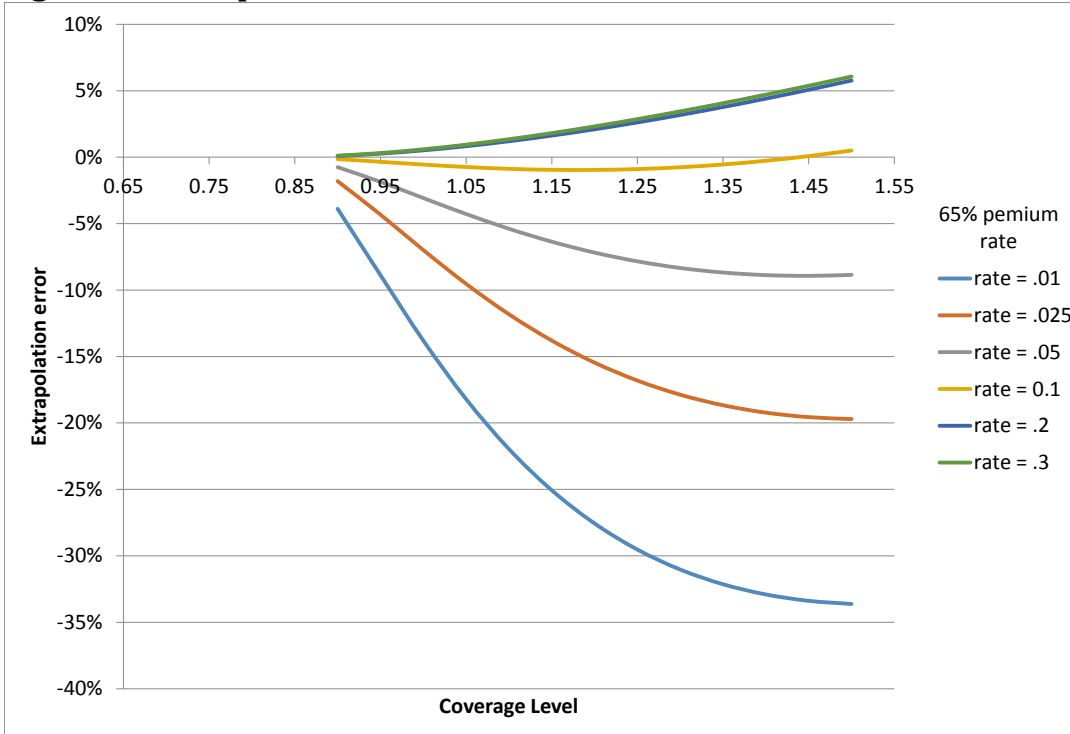


Figure 11. Extrapolation Errors for the Beta Distribution

Discussion

The analysis presented here demonstrates that substantial extrapolation errors are likely if the relationship between actuarially fair premium rates and coverage levels above 85% is either convex or concave. If the relationship is convex, as it will be for low-risk farmers, then the errors will be negative, leading to underestimation of premium rates using the proposed extrapolation method. When the relationship is concave, as it could be at very high coverage levels or if the probability density function is skewed to the right, then the errors will be positive, leading to overestimation of rates.

The foregoing conclusions assumes that the current coverage level differentials used by RMA to determine 80% and 85% premium rates are consistent with the actuarially fair premiums that were used in this analysis. This assumption is, at best, only approximately true because the equations used to estimate the 80% and 85% coverage level differentials are not estimated using Monte Carlo methods from a well-specified probability distribution of yields. Rather they are estimated econometrically from loss cost data. Thus it is not really possible to determine exactly how large an error will be made using the proposed extrapolation methods.

However, we know that the actuarially fair value of a yield insurance contract is simply the average indemnity payoff from the contract, conditional on the indemnity being positive, multiplied by the probability that an indemnity will be received. Whether this actuarially fair value is convex, concave, or linear with respect to coverage level depends on the change in the probability of collecting an indemnity and the change in the average size of an indemnity if one is received. Consider two yield distributions with the same mean but one is high risk and one low risk. As coverage increases from 80% to 85% to 90% coverage the change in the probability of receiving an indemnity increases faster for the low risk distribution because more of the mass of the probability distribution is near the mean than for the high risk distribution. The change in the average size of a payment is more similar for the two distributions. This implies that the degree of convexity for the lower risk distribution will be greater for coverage levels above 85% than for higher risk distributions. This result implies that unless the RMA coverage level differential equations greatly overestimate the actual slope in premium rates between 80% and 85%, then for low-risk farmers, the proposed RMA extrapolation will lead to farmers being charged too little—perhaps significantly so—for effective coverage levels above 85%. For high risk farmers, the results presented here suggest that unless actuarially fair premium rates become concave in effective coverage levels, then extrapolation errors will be much smaller than for low risk farmers.

For low-risk farmers, if moral hazard issues arise (as seems likely) from offering effective coverage levels greater than 85% then the underestimation bias of the extrapolation method will be exacerbated because the actual indemnities paid out will exceed those implied by the Monte Carlo methods used here. Thus it seems

advisable to adopt methods that at a minimum, will not lead to systematic underestimation of premium rates for effective coverage levels greater than 85%.

One option that RMA could use as an alternative would be to adopt the proposed methods to interpolate premium rates for effective coverage levels that are less than 85% and to then use the truncated normal draws for determine coverage level differentials for coverage levels greater than 85%. The yield draws are already available for rate-making purposes for revenue insurance plans of insurance. It would be feasible to simply use them instead of adopting a method that does not take into account the fact that premium rates are likely to be convex in effective coverage levels for the majority of farmers in the program.

Answers to Requested Items

(A) Are adequate, credible, and reliable rate-making data available? Is it likely that the data will continue to be available? Is the data vulnerable to tampering if the proposed ate methodology is approved?

Loss cost data above 85% coverage levels are not readily available hence the need to develop a method to determine how premium rates should increase for above-85% coverage levels. Simulation “data” based on assumptions about the functional form of yield distributions could be used instead of loss cost data. Data for losses less than 85% coverage level are readily available.

(B) Are the explicit and implicit assumptions used in the rating process reasonable?

Yes for interpolation of rates for effective coverage levels less than 85%. The linear assumption is appropriate as explained in my report above.

But the linear assumption is not reasonable for extrapolating premium rates above 85% because the relationship between premium rates and effective coverage levels is likely nonlinear.

(C) Are the technical analyses (e.g., stochastic and other simulations) technically correct? Do they provide credible, relevant results?

The algebra provided was correctly done. No simulations were provided.

(D) Is the data used for the analysis appropriate, reliable, and the best available?

No data were used in the analysis. Monte Carlo simulations could have revealed the limitations of the proposed methods.

(F) Are the proposed premium rates likely to cover anticipated losses and a reasonable reserve?

Not for low risk producers who drop yields from their APH. Perhaps for higher risk producers who drop yields. Because it is more likely that high risk producer will drop yields because most of them live in or adjacent to high risk counties, the probability of a high risk producer dropping yields is greater than the probability of a low risk producer who drops yields. Thus the aggregate amount that premiums fall short of anticipated losses is difficult to determine.

(G) Is the actuarial methodology appropriate for the insured risks?

Not for determining premium rates above 85%. The relationship between rates and coverage levels is most likely nonlinear. The method used assumes a linear relationship, which is only appropriate for specific cases.

Short Bio of Bruce Babcock.

Bruce Babcock is a partner in AgRisk Management. He has developed rating methods for crop insurance products that are widely in use today. He holds a PhD in Agricultural and Resource Economics from the University of California, Berkeley and MS and BS degrees from UC Davis. He currently holds the Cargill Chair of Energy Economics at Iowa State University.