

**Comments by National Crop Insurance Services on:
Actuarial Review for Price Volatility Factor Methodology**

Sumaria Systems, Inc., August 8, 2014

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Introduction

National Crop Insurance Services (NCIS) submits these comments on the “Actuarial Review for Price Volatility Factor Methodology” to the Risk Management Agency (RMA) on behalf of the approved insurance providers (AIPs). NCIS received comments from its members and the comments are incorporated into this response. In addition, individual AIPs may also be submitting separate comments.

NCIS appreciates the efforts of RMA and Sumaria Systems to conduct an actuarial review of the measure of implied volatility now being used to establish premium rates for the Revenue Protection (RP) and Revenue Protection with Harvest Price Exclusion (RP-HPE) plans of insurance. We also appreciate the opportunity to provide comment on the Sumaria review (denoted SR hereafter), and we encourage the continued practice of seeking public comment on all research studies and reviews conducted on crop insurance program features.

The primary SR recommendations are summarized on p. 4 as:

- Continue to use the Black-Scholes formula for price volatility estimation
- Continue to utilize a publicly available and external source of market price volatility
- Continue to use the underlying futures price as a forecast of future realized price
- Avoid using thinly traded options prices in computing the implied price volatility
- Review and update Price/Yield correlations used in rating
- Revise the formula for price variability in rate simulation to make it mathematically accurate

We first provide general comments on these recommendations and the role and use of volatility in premium determination, followed by editorial and more specific discussion on the review’s analysis and presentation.

Introduction

RMA currently uses implied volatility to characterize the distribution of futures prices during the growing season. RMA obtains implied volatility data from Barchart.com for each of the last five

days of the discovery month. The implied volatilities for these days are calculated using options premiums for at-the-money put and call options and the Black-Scholes Model (BSM). RMA time adjusts the volatilities for each of the last five days and then takes a simple average of the time adjusted volatilities. The resulting volatility factor is then used to determine the variance of the distribution of growing season futures prices.

RMA combines the variance estimate with the estimated expected price (monthly average futures price during the discovery month) in simulations to obtain the revenue add-on component for the premium rating for RP and RP-HPE plans of insurance. The simulation uses draws from the estimated price and yield distributions and imposes an historical price-yield correlation to obtain the joint yield and price distribution. Because the variance of the price distribution is determined by the volatility factor, the volatility factor affects price draws, which in turn affect the revenue add-ons and premium rates. The revenue add-on component is obtained as the difference between the simulated price risk and simulated yield risk. Adding the resulting revenue add-on rate to the Yield Protection (YP) rate provides the premium rates for revenue plans. The simulation accounts for the differences in the RP and RP-HPE plans of insurance and generates different rates.

This response is organized in the form of general comments, followed by technical considerations and corrections and summary and conclusions at the end.

General Comments

We address the details of the SR assessment of the BSM approach as well as broader issues on the efficacy of implied volatility in rate making. We also offer some suggestions to help improve public understanding of the role and impact of implied volatility values. We number our comments for the ease of reference.

1. Ineffectiveness of volatility in reflecting price risk and establishing premium rates.

The first two SR recommendations “Continue to use the Black-Scholes formula for price volatility estimation” and “Continue to utilize a publicly available and external source of market price volatility” essentially recommend the status quo in estimating a volatility value for simulation. We believe these recommendations derive from a restricted review of volatility and a limited assessment of alternatives—an assessment that did not address the nature of the effect that the use of measures of implied volatility have on premium rates. We are concerned that the current rate making approach is now, and is likely to continue, not meeting the objective of covering actual producer losses and providing for adequate private sector returns. This conclusion is supported by analyses conducted by several insurance and reinsurance companies and academics (e.g., Sherrick, Schnitkey and Woodard).

The premium rating system used for revenue plans is a logical statistical construction that relies on accurate characterization of yield and price risk facing an insured producer. A reason to use

implied volatility in estimating price risk is stated in the SR in many places. One example is found on page 8, “In other words, the volatility parameter implied by an option’s current market price in an efficient market should accurately reflect all relevant past and future information (i.e., which is why it is a “forward-looking” estimate). In that case, once implied volatility is known, any volatility estimate based on past prices alone should be redundant. This is the reason why implied volatility is generally considered by both academics and practitioners to be superior to alternative volatility forecasts.” Indeed, SR affirms this conclusion based on its literature review and empirical work and recommends the status quo with regard to estimating volatility for ratemaking. SR’s review is focused on how well the BSM approach now used by RMA does in estimating implied volatility, compared with selected alternative estimates of implied volatility.

The SR review, then, approaches the issue of volatility in a fairly narrow way. The role of volatility in generating valid premium rates should have been assessed. Because implied volatility can have a large impact on premium rates, it is critical that implied volatility generates premium rates that reflect the price risk that is actually realized. Premium rates must cover producers’ actual losses over time and generate sufficient competitive returns to the private sector delivery system; if these goals are not met, the rating system is deficient, regardless of how well BSM estimates implied volatility relative to other methods, measured against the yardstick of realized volatility. We next examine several indicators of the relationship of the implied volatility factor to valid premium rates.

Virtually no correlation of the volatility factor with actual price changes. One evaluation metric would be to determine whether the use of implied volatility results in higher premium rates when within-season prices actually change substantially and lower rates when such price changes are muted. Because an increase in implied volatility results in higher premium rates, other things equal, one can examine this metric by determining whether implied volatility is correlated with actual growing-season changes in prices that revenue plans of insurance are supposed to protect against.

Assuming a producer has a yield loss, rising prices during the growing season increase revenue to count and reduce the prospect of an indemnity under RP-HPE, compared with YP. For RP, rising prices raise the guarantee and contribute to a higher indemnity compared with both RP-HPE and YP. Declining prices during the growing season reduce revenue to count and increase the prospect of an indemnity under RP-HPE and RP, compared with YP.

Compared with YP, a given price increase under RP during the growing season may or may not result in a greater indemnity than would occur under a price decrease of the same amount, depending on the size of the yield shortfall. But, the greater the prospect of larger price changes, the greater the prospect of larger indemnities, compared with YP. To be effective in accounting for price risk, the volatility estimate used in ratemaking should be highly correlated with actual price changes.

Figures 1 and 2 below show the actual percentage change between the base prices and harvest prices used for major corn and soybean RP plans of insurance for 1968-2014. Corn and soybeans account for nearly 60% of total program premium. The futures prices are obtained from Barchart.com and the base prices and harvest prices for each year use RMA's method. The price changes are quite variable, and one might expect that the volatility factor which is based on implied volatility and exhibits considerable variation also would be correlated with the absolute value of the price changes. To examine that prospect, we obtained implied volatility estimates from Barchart.com for each of the last five days of February for 1990-2014. The average of the preceding values for each year is used as the volatility factor for each year (in line with RMA's method except the time-adjustment step, which was not material here).

Figure 1. Corn: Change in Futures Price from Base to Harvest, 1968-2014 1/

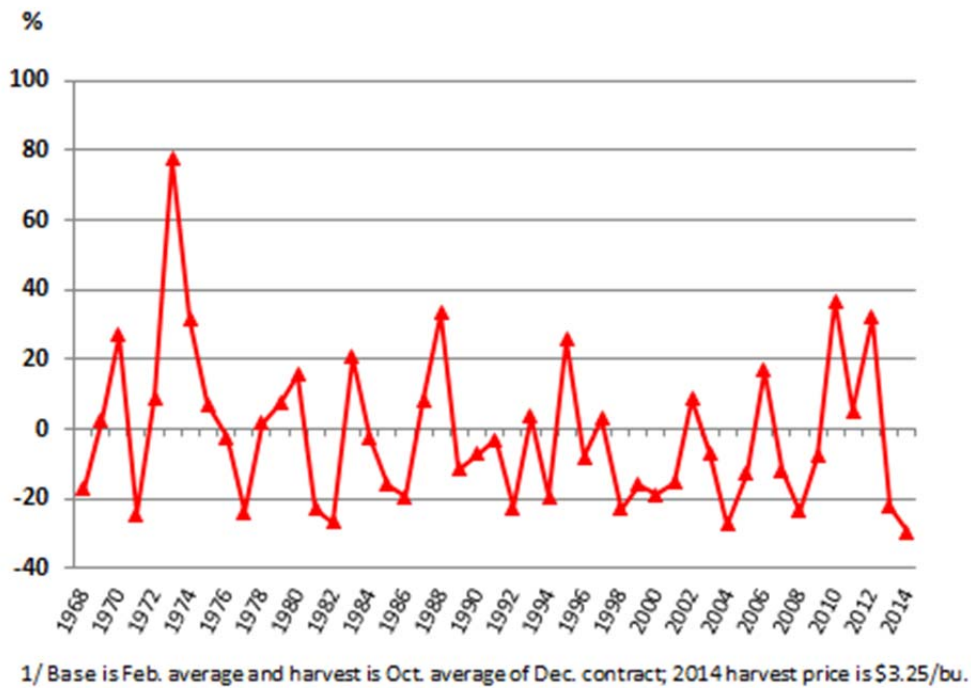
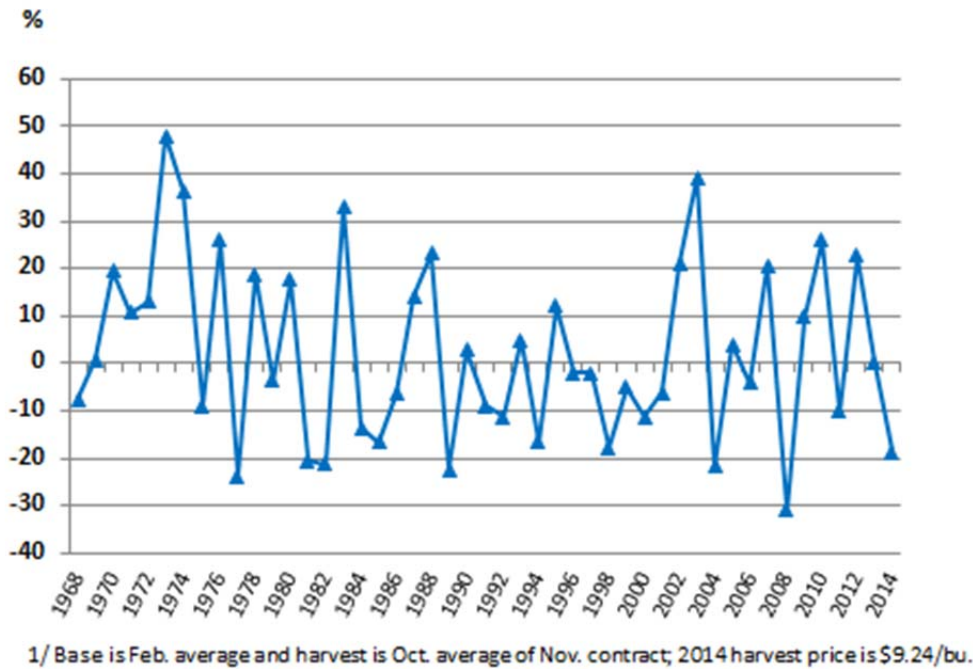


Figure 2. Soybeans: Change in Futures Price from Base to Harvest, 1968-2014 1/



Figures 3 and 4 show the estimates of the volatility factor and the percentage changes in base to harvest prices for corn and soybeans for 1990-2014. We used the absolute value of the price change as a percent of the base price as the measure of price risk actually experienced in each year. The harvest prices used for 2014 were the most recent daily prices shown on the RMA website for the price discovery month. The years are ranked by volatility, from smallest to largest. The charts show many years of low volatilities and large price changes and high volatilities and small price changes. There is little apparent association between implied volatility used in rate making and price risk actually experienced.

Figure 3. Corn: Absolute Value of Futures Price Change from Base to Harvest and Implied Volatility, Ranked from Smallest to Largest, 1990-2014 1/

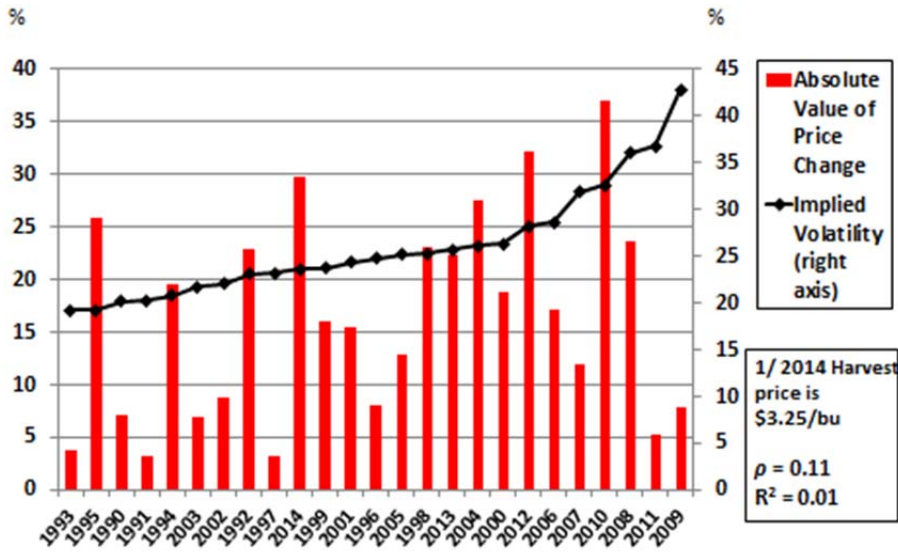


Figure 4. Soybeans: Absolute Value of Futures Price Change from Base to Harvest and Implied Volatility, Ranked from Smallest to Largest, 1990-2014 1/

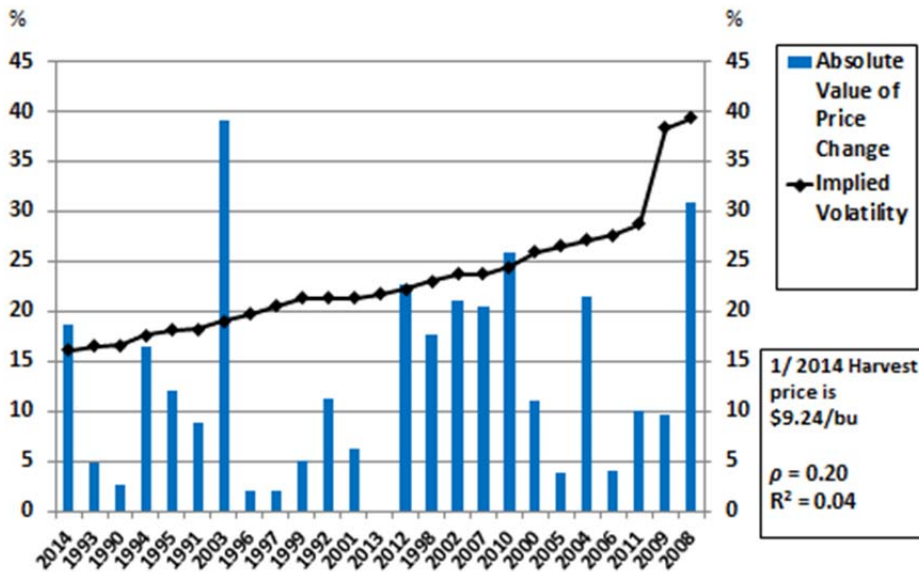


Table 1 shows that the Pearson correlation coefficients between the absolute value of the changes in prices as a percent of the base prices and the implied volatilities. The correlation coefficients and the coefficients of determination (R^2) are very small and not statistically significant. Thus, the correlation between actual price changes and the implied volatility factor used to establish price risk for premium ratings is not significantly different from zero. The current method of estimating implied volatility is not capturing the price risk that is actually being experienced.

Table 1. Relationship between actual corn and soybean price changes and implied volatility

	Pearson correlation coefficient	R^2
Corn		
Price change on implied volatility	0.06	0.004
Absolute value of price change on implied volatility	0.11	0.01
Soybeans		
Price change on implied volatility	0.19	0.04
Absolute value of price change on implied volatility	0.20	0.04

Frequent realization of low probability events. One could respond to the lack of correlation of volatility and actual prices by arguing that the actual price realization is but one, perhaps low probability, observation from a distribution that the implied volatility may be properly characterizing. However, there are now many years that show little relationship. These frequent realizations of “low probability” price changes, which are not reflected in premium rates, are contributing to underperformance of the crop insurance program. This conclusion is evident by examining the harvest price distribution that is generated from the volatility factor used in rate making. The actual observed changes in price have often occurred at the high and low ends of the estimated distributions, suggesting that the volatility factor is too low and premium rates are inadequate (see the highlighted years in table 2). From the point of view of the industry, companies could experience sustained cumulative losses waiting for the premium rating method to generate rates that appropriately cover the losses being incurred. It is difficult to imagine that a private property casualty insurance company would be reducing premium rates due to a lower implied volatility after incurring large losses and facing large price changes in the immediately prior years.

Table 2. The Implied Percentiles from the Corn Harvest Price Distribution /1

Year	Base Price (\$/bu)	Volatility Factor /2	Harvest Price (\$/bu)	Implied Probability (%)
2014 /3	4.62	19	3.25	3.8
2013	5.65	20	4.39	12.9
2012	5.68	22	7.50	91.2
2011	6.01	29	6.32	62.5
2010	3.99	26	5.46	90.9
2009	4.04	34	3.72	47.3
2008	5.40	29	4.13	21.5
2007	4.06	25	3.58	35.6
2006	2.59	23	3.03	78.9
2005	2.32	20	2.02	28.2
2004	2.83	21	2.05	7.5
2003	2.42	17	2.26	37.4
2002	2.32	18	2.52	71.4
2001	2.46	19	2.08	22.4
2000	2.51	21	2.04	18.9
1999	2.40	19	2.01	20.3
1998	2.84	20	2.19	11.6
1997	2.73	19	2.81	60.1
1996	3.08	20	2.84	37.3
1995	2.57	15	3.23	94.2
1994	2.68	17	2.16	11.1
1993	2.40	15	2.49	62.3
1992	2.70	18	2.09	9.5
1991	2.59	16	2.51	45.3
1990	2.47	16	2.30	35.2

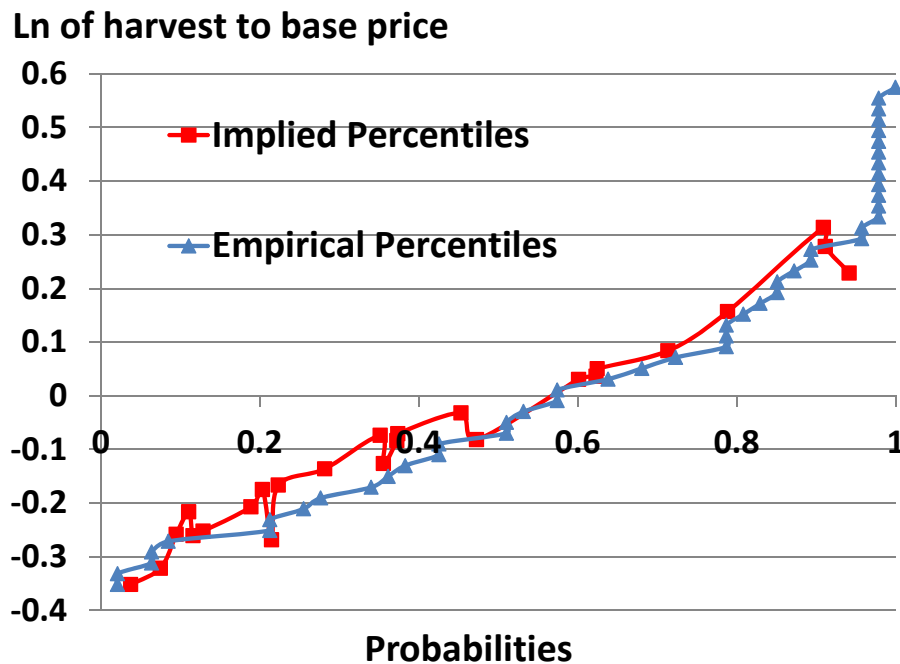
Notes. 1/ In line with RMA’s distribution assumption, the harvest price is lognormally distributed with a mean and a standard deviation. The standard deviation is the volatility factor per SR’s recommendation. The mean is then calculated in line with the formula given in equation 4 in Appendix 2, after setting the base price as the expected value of the harvest price. Because the mean and standard deviation parameters characterize the distribution for the harvest price, one can then calculate implied probabilities at the point of observed harvest price in each year.

2/ The implied volatility estimates are obtained from Barchart.com for each of the last five days of February for 1990-2014. The average of the preceding values for each year is obtained and time-adjusted by using the factor of 0.8 and rounded up to the two decimal points. Although the time-adjusting step somewhat differs from the RMA’s method, that should not create any material difference in the resulting volatility factor values.

3/ The estimate for the 2014 corn harvest price is preliminary and has not been finalized at the time of writing of this report.

Figure 5 makes a similar point to table 2 based on the distribution of the harvest price. The implied percentiles from the lognormal distribution of the ratio of harvest price to base price are constructed using RMA's approach and are compared with the percentiles from the empirical distribution constructed from the actual historical values of the natural logarithm of the ratio of harvest price to base price. Essentially, the use of the volatility factor to construct the distributions suggests a narrower price distribution than using the historical data; hence the former appears to consistently underestimate the risk in the tails of price distribution. Note that in the case of implied percentiles, the standard deviation of the distribution (volatility) is reset every year and exhibits considerable variation over the years; nevertheless, that does not appear to be helping against the yardstick of actual price realizations. Figure 5 also raises a question about of the appropriateness of the lognormal distribution assumption. Finally, the implied percentiles are based on the data over 1990-2014, while the empirical percentiles are based on the data running over 1968-2014. Nevertheless, there is considerable overlap between the two sample periods to rule out the possibility that the direction of bias is the artifact of chance.

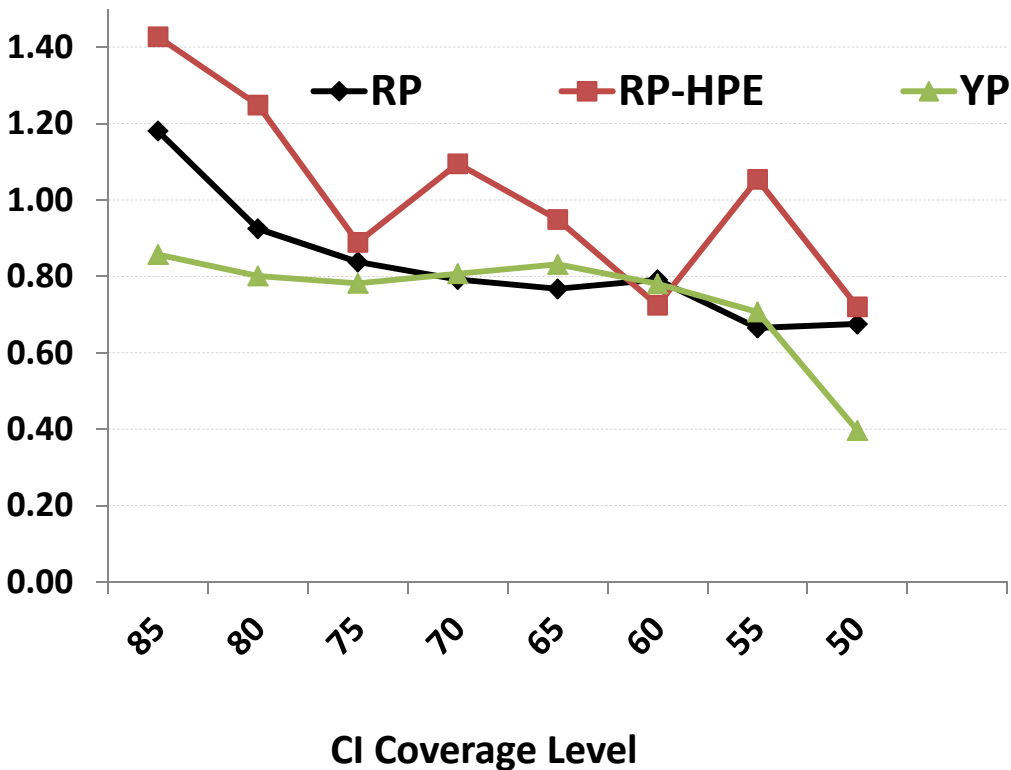
Figure 5. Implied Percentiles Versus Empirical Percentiles for the Distribution of the Ratio of Harvest Price to Base Price for Corn



Underperformance of revenue plan loss ratios, particularly at high coverage levels. Figure 6 illustrates the cumulative loss ratios for corn and soybeans combined during 2000-2013 for RP, RP-HPE and YP. During this period cumulative loss ratios for corn and soybeans across all coverage levels were 0.87 for RP, 1.12 for RP-HPE and 0.70 for YP. Much of the total premium was at coverage levels below 80%, where loss ratios were more similar across plans and lower than at the higher coverage levels. However, underperformance is most notable for 85%

coverage where the cumulative RP loss ratio is 1.18 and RP-HPE is 1.43, and at 80% coverage, the RP loss ratio is 0.92 and that for RP-HPE is 1.25. Further, if the most recent years are examined, 2008-2013, the cumulative RP loss ratio is 0.97, RP-HPE is 1.22 and YP is 0.74. The more recent data suggest the rating problems are getting worse over time.

Figure 6. Cumulative Loss Ratios for Corn and Soybeans Combined for Major Plans, 2000-2013



The SR study assessed implied volatility by comparing several formulas for computing implied volatility against the realized out-of-sample volatility. While this approach is appropriate as far as it goes, the ultimate question the rating system must address is whether the premium rates adequately cover losses and provide for a competitive rate of return to the delivery system. That appears doubtful, given the evidence just cited, trends in the program and recent program changes. In particular, future program performance at the higher coverage levels appears likely to be inadequate given:

- The lack of correlation between actual price changes and the measure of volatility used in ratemaking,
- The historical underperformance of revenue rates, especially at high coverage levels,
- The continuing trend toward higher coverage levels by producers,

- The Trend-Adjusted APH Yield Endorsement and the new Farm Bill’s APH exclusion which both effectively increase coverage to high levels, placing even more reliance on the adequacy of rates at high coverage levels,
- The new Farm Bill’s introduction of the Supplemental Coverage Option (SCO) and the Stacked Income Protection Plan (STAX) which will focus coverage at high levels, and
- The recent change in the rating methodology which has sharply lowered base rates.

In combination, these factors indicate RMA/SR need to address what is likely to be poor performance of revenue rates going forward.

2. Lack of analysis on premium rates and premiums.

Implied volatility is a key component in establishing revenue premium rates. SR notes on p. 5, “Thus, billions of dollars of premium are affected by a single parameter estimate.” Yet, other than for the issue of the transformation of implied volatility to variance for simulation, SR conducted no analysis of the effect of implied volatility on premium rates and premiums under the current method of estimation or under the set of alternatives examined. There is no assessment as to the validity of the rates generated by the current method or alternatives as noted in general comment #1. On a more micro level, there is no quantification of the differential impact on premium rates among the alternative methods studied.

Figure 7 illustrates the importance of volatility on premium rates and premium for one crop, wheat, using RMA’s cost estimator for the 2015 crop year. Total premium is an increasing (initially convex, after about volatility value of 0.25 concave) function of the volatility factor and appears to be quite sensitive to the volatility factor values: at the value of 0.2 for the volatility factor, increasing the volatility factor by 1-basis point increases the total premium by 5.04%:

$$\frac{(\$1,395 - \$1,114)/\$1,114}{(0.25 - 0.20)} = 5.04. \text{ Similarly, at the same value for the volatility factor, decreasing}$$

the volatility factor by 1-basis point decrease is the total premium by 4.72%:

$$\frac{(\$851 - \$1,114)/\$1,114}{(0.15 - 0.20)} = 4.72.$$

Figure 7. SCO Total Premium Related to Volatility: A Wheat Example

(100 acres Kansas Wheat farm; \$7.02/bu Base Price; APH Yield 48 bu/ac; RP Coverage = 75%)

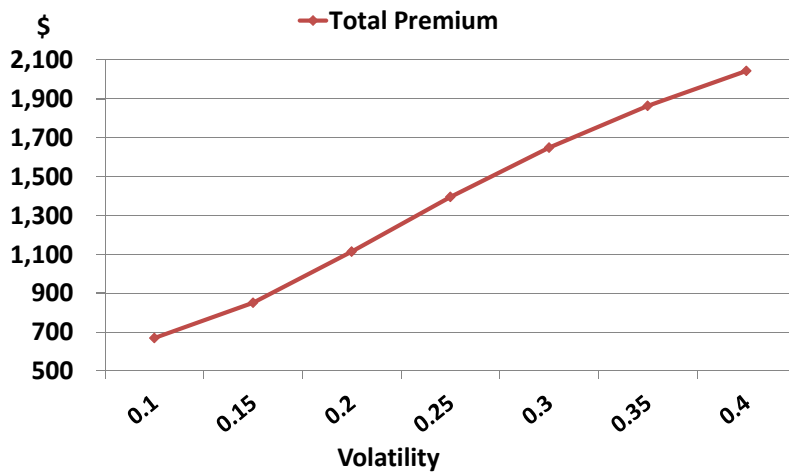
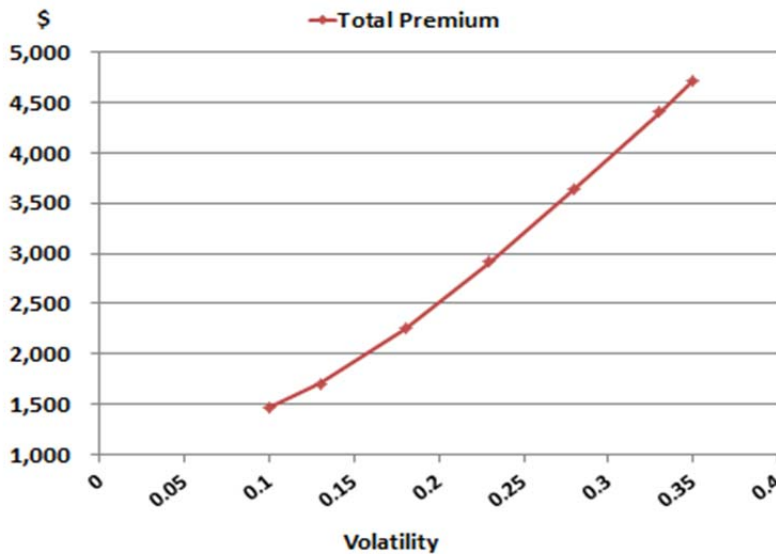


Figure 8 provides another example for Iowa soybeans, also for the 2015 crop year. The sensitivity of premium to volatility is slightly higher compared with the Kansas wheat example. At the volatility value of 0.18, increasing the volatility factor 1-basis point increases the premium 5.92%, while decreasing it 1-basis point decreases the premium by 4.85%. Also the total premium is a convex function of the volatility factor at a higher range of the volatility factor values compared with the wheat example above.

Figure 8. SCO Total Premium Related to Volatility: A Soybean Example

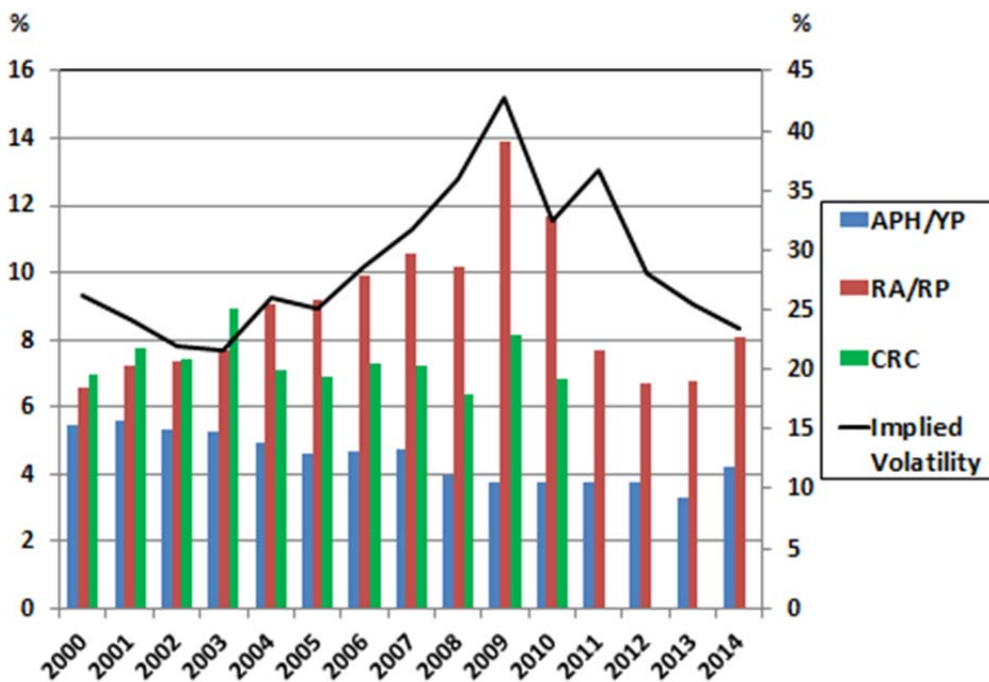
(100 acres Iowa, Kossuth County, Soybean farm; \$11.36/bu Base Price; APH Yield 50 bu/ac; RP at 85%)



In discussing BSM’s advantages of common acceptance and transparency, SR states, “The Black-Scholes volatility has these valuable attributes and we are unaware of any prominent alternative that would have similar advantages. Of course, such advantages must be weighed against any potential gains in accuracy that may be derived from adopting an alternative to the Black-Scholes model” (p. 39). One “gain” that should be weighed is the ultimate impact on premium rates and premium.

In addition, a change in implied volatility will have a different impact on premium by crop and region. The sensitivity of premium to volatility should be higher in areas where yield risk is low; hence the price risk forms the major component of risk, such as Illinois corn. Figure 9 shows average yield and revenue rates over time for Illinois corn for 75% coverage policies from 2000 to 2014. In particular, for 2013 and 2014, the figure indicates the price risk premium component accounts for about half the RP average premium. The figure also indicates how closely the average RP rate is associated with the changes in implied volatility. Some assessment of the crop and regional premium impacts of volatility changes, such as the effect of a 1-basis point change, would have been helpful to better understand the role of volatility across crops and regions.

Figure 9. Illinois Average Premium Rate for Corn at 75% Coverage Level and Implied Volatility



It would also have been useful to put the effect on premium of a volatility change in perspective by comparing it with a price change. In the example above, if instead the projected price

increased by 1%, the total premium would go up by 1% because the premium is the liability times the premium rate, and the liability is the price times the rate yield. That means, at least in the examples provided here, total premium is much more sensitive with respect to the one point increase in the volatility factor compared to the 1% increase in price. A decline in price from one year to the next, other things equal, reduces liability and premium. If volatility increases, other things equal, premium rates increase. Thus producers may find themselves facing lower insured value and having to pay more for the coverage.

3. Concerns with the Black-Sholes Model and alternatives considered.

If implied volatility is to remain in use as the method of characterizing the variance of the price distribution, is BSM the best approach to estimating implied volatility? The focus of the SR is on answering that question. In the SR literature review, several advantages and disadvantages to BSM are raised. Among the key identified strengths of BSM are: it is widely accepted, transparent, readily available through third party calculations, and alternative approaches to estimating implied volatility have mixed results when compared with BSM. Among the key concerns or limitations with BSM are: its assumption that prices are lognormally distributed, volatility is constant across all option maturities, options markets are liquid, markets are efficient with no arbitrage opportunities, there are no transaction costs, and the discount rate is constant. SR notes (p. 3) "... the literature also recognizes that implied volatility from the BSM has shortcomings and it is sometimes inconsistent with price/volatility behavior observed in the market".

The list of concerns is notable and raises the question as to whether the SR examined sufficient alternatives. All alternatives but one are versions of BSM—lognormal price distributions that assume efficiency. BSM assumes that volatility is constant. This assumption seems difficult to justify given the changes in price volatility over time. The remaining alternative considered is a VIX type model free method for which SR states "is not likely to have a great deal of relevance to our specific objectives" (p. 40) and results in three times more errors compared with other methods.

The SR literature review cites many other options that appeared to address BSM concerns and perform well for at least some crops. SR chose not to examine price distributions other than lognormal, model free approaches other than VIX type or composite approaches. SR also assumed away historical (time-series) approaches, arguing that historical information is embodied in options prices.

Markets may be generally efficient, particularly over time, but the diversity of results in the SR literature review indicated some alternative approaches fared better or were at least mixed compared with BSM for some commodities in some time periods. For example SR notes, "...empirical research on the relative performance of MFIV has been limited and the existing evidence as to whether MFIV provides better volatility forecasts than BSM (or time series

measures) has been mixed.” Further, Andersen and Bondarenko, cited by SR state “...there are indications that historical realized volatility contains additional information for future volatility. It is an intriguing research question to determine how best to combine the implied and historically observed volatility measures for forecast purposes.” (p. 7). Based on this and the literature cited in the study, estimation of time-series models (ARCH or GARCH type models) would constitute a fair benchmark to include in the SR analysis. Without examining such alternatives in the recent price environment, for us, the question remains unsettled as to whether alternative approaches, including historical approaches, may fare better in this time period for the exchanges and crops of interest: the Chicago Board of Trade (CBOT), the Kansas City Board of Trade (KCBOT), the Minneapolis Grain Exchange, Inc. (MGEX), and Intercontinental Exchange (ICE) for corn, cotton, grain sorghum, rice, soybeans, sunflowers, wheat, barley, and canola/rapeseed.

Because implied volatility is a market forecast of a parameter, implied volatility can be thought of as an estimate within a confidence interval. To illustrate the point, based on the lognormal distribution of the ratio of harvest price to base price, the 90% confidence interval estimate for the volatility (standard deviation parameter) could be constructed as (0.171, 0.244) for corn and (0.151, 0.216) for soybeans. (The calculations use the parameter estimates and their standard error estimates for the distribution for each crop as reported in Appendix 1, as well as the value of the t-statistic with degrees of freedom of 45). Notice that the RMA’s 2014 volatility factors are 0.19 and 0.13 for corn and soybeans, respectively, where the latter falls outside of its respective 90% confidence interval bounds. .

SR compares several BSM type models with a realized volatility measure. However, crop insurance policies are not written based on the realized volatility measure. From the AIPs’ and farmers’ perspectives, what matters is the price change from base to harvest. An alternative and more straightforward approach would have been to look at the distribution of price change from base to harvest (Vedenov and Power, 2008; Power, Vedenov and Hong, 2009; and Bulut and Collins, 2014). We used the price data illustrated in Figures 1 and 2 to find best fits among alternative distributions for the ratio of harvest price to base price. Figures 10 and 11 show the fitted distributions. Five distributions were considered: Normal, Lognormal, Burr, Weibull and Generalized Extreme Value. (This list is by no-means intended to be exhaustive.) We note Tejada and Goodwin studied the Burr distribution and recommended that it be used in crop insurance premium rating (Tejada and Goodwin, 2008). The details of this analysis is provided in Appendix 1. Note that two key findings are highlighted in the statistics presented in Appendix 1: “log likelihood statistics” and “variance”. The higher are the values of the log likelihood statistics, the better. The square root of the value of the variance is the standard deviation and that is the volatility estimate. Tables 3a and 3b below rank the distributions in terms of their fitting performance for corn and soybeans and report the resulting volatility estimates.

Table 3a. Fitted Price Distributions for Corn and Resulting Volatility Estimates

Distributions Considered	Fit Performance	Volatility Estimates
Generalized Extreme Value	11.9	25.3%
Burr	10.9	37.7%
Lognormal	9.5	20.6%
Normal	5.1	21.9%
Weibull	1.9	25.3%

Table 3b. Fitted Price Distributions for Soybeans and Resulting Volatility Estimates

Distributions Considered	Fit Performance	Volatility Estimates
Generalized Extreme Value	13.3	18.6%
Lognormal	13.1	19.0%
Normal	11.9	19.0%
Burr	11.5	20.5%
Weibull	10.2	20.5%

Generalized Extreme Value distribution (GEV) provides the highest fit for corn and soybeans and resulted in volatility estimates of 25.3% and 18.6%, respectively, which are both higher than the 2014 volatility factors used by RMA (19% and 13%, respectively) and at the same time are both lower than the 2009 counterparts (34% and 31%, respectively). Once the best fitting distribution is determined, which is subject to formal hypothesis testing and that could be periodically assessed, the method captures the inherent risk in base-to-harvest price changes and incrementally updates the volatility estimate with the new price information every year. The preceding methodology has the potential of delivering a more stable volatility estimate over the years. Finally, the distribution fitting exercise above, as well as the findings in Tejada and Goodwin (2008), raise questions about the appropriateness of the lognormality assumption made

in RMA's revenue premium rating as well as in the alternatives that SR analyzed. SR should consider evaluating this suggested methodology.

Figure 10. Fitted Distributions to the Ratio of Harvest Price to Base Price for Corn

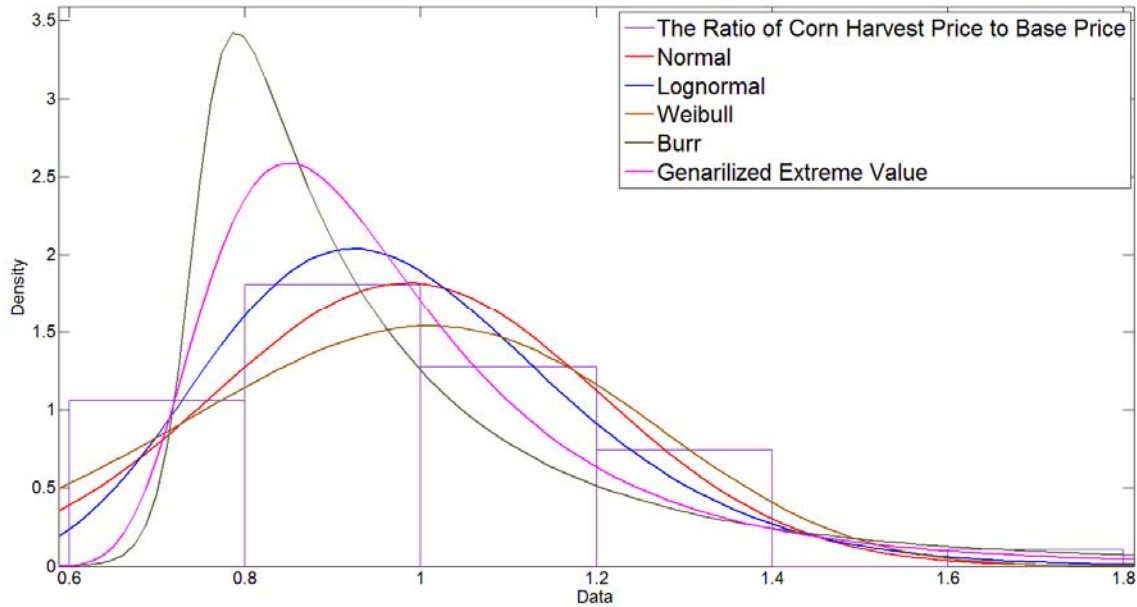
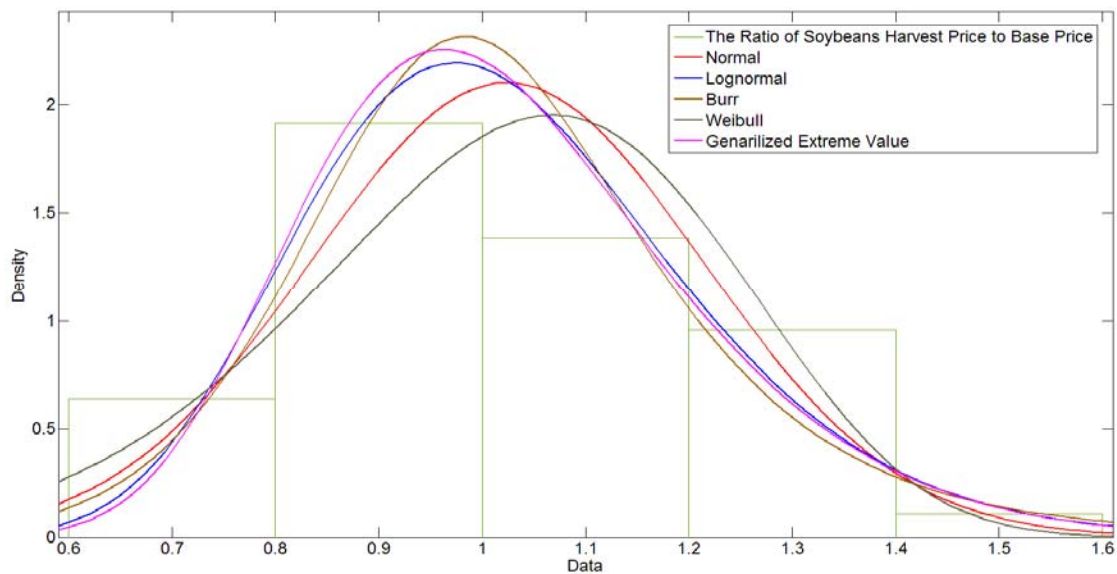


Figure 11. Fitted Distributions to the Ratio of Harvest Price to Base Price for Soybeans



4. Lack of analysis of the five-day price discovery period.

Another concern surrounds the use of implied volatilities from the last five days of the price discovery period. SR does no quantitative analysis of this issue. SR argues that since markets are

assumed to be efficient, all information available is embodied in the option prices on the last day of the five-day period. SR then argues, rather than using prices in one day, the use of a five-day average may help address any liquidity deficiency. A parallel argument, which SR does not make, would be to assert that a futures price on the last day of the discovery period embodies all known information and therefore is the single best estimate of the expected price. However, RMA uses the monthly average of futures prices, not a single day, and SR accepts that approach. SR should address the question of why one day or a few days of data is an appropriate method for a forecast of implied volatility but a monthly average is the appropriate forecast for the expected price.

It would also have been useful in the review to provide a better understanding of the sensitivity of the implied volatility calculation to option value changes during the last five days of the discovery month; e.g., what type of option value changes (or what type of futures price changes) in the last few days of the discovery period generate an implied volatility of 15% versus 30%? Finally, the review should address the issue of market efficiency, including the incentives for, and the capacity of, market participants to manipulate implied volatility during the last five days of the discovery period.

While we believe markets are generally efficient over time, we also believe competition is not perfect. It takes time for some market events to be properly evaluated, understood and priced into the market. In effect, markets may be open to manipulation during a short period of time, either for intentional or unintentional reasons. Inaccurate or misunderstood information or unusual market activity may generate a price on one day, but as the information and activity is better understood and gets built into the price, a different price prevails. This process may take time, and if inaccurate or misunderstood information or other non-representative events substantially affect the market within that five day window, then the price is distorted.

5. Adjustments for trade volume weighting and volatility transformation.

We appreciate SR's recommendation to "Avoid using thinly traded options prices in computing the implied price volatility." If BSM is to be used, we agree with the recommendation not to use data with zero trading volume and to consider trade volume weighting. We also agree with the recommendation to "Revise the formula for price variability in rate simulation to make it mathematically accurate." NCIS previously noted this issue through personal communications between Dr. Harun Bulut of NCIS and RMA and the study group (see Appendix 2). SR indicates that the adjustment is small (noting at a typical volatility of 25%, the difference in premium rates is slightly less than 0.1 percentage point for major crops).

6. Need for assessment of Price/Yield correlations.

SR states one purpose of the study is to do an "evaluation of the underlying price/yield correlations assumed and the interacting effect that price volatility and price/yield correlation have on revenue rates" (p. 6). However, that evaluation is limited to their comments.

Nevertheless, we agree with SR's recommendation to "Review and update Price/Yield correlations used in rating." The structure of global commodity markets has changed substantially over the past decade. Particularly, the volume of production from the Southern Hemisphere, Black Sea and other regions have influenced global trade and prices. This is likely influencing the U.S. production and price correlations. We note that SR makes this recommendation without any supporting analysis, such as the assessing the impact of the price/yield correlations currently used or potential changes to the correlations.

SR suggests RMA consider, in its ratemaking, alternative distributional formulations and use of copula methods that capture tail dependence (p. 68). SR also expresses its opinion regarding that "no practical applications of such methods that would offer important benefits to the Federal Crop Insurance Program are currently apparent. However, the issue should be revisited as the literature advances." We find the issue far too critical in terms of accounting for the tail risk that AIPs are undertaking. In fact, it has been observed for a corn farm in McLean County, Illinois that the best copula specification (chosen among a number of alternatives according to the standard model selection criteria) would produce premium rates for revenue coverage at the 75% coverage level that are as much as 40% higher than the premium rates based on the Gaussian copula which is currently used in the crop insurance revenue rating (Goodwin, 2014; see p. 19 and table 3 in p. 33). It has been also noted in the preceding reference (p. 10) that: "Assumptions made about the nature of dependencies among multiple sources of risk, such as yields and prices, in the empirical modeling of policy parameters have significant implications for the resulting values of the parameters and operation of the program. I believe this issue merits the attention that I am devoting to it here because of the increasingly prominent role that subsidized crop insurance plays in US agricultural policy as well as in the policy actions of legislators around the world." Given that there is no perfect model and the limitation of Gaussian copula in accounting for the tail risk is well-established, we recommend that the study group actually take a position and provide clear guidance on the use of copulas in modeling of dependency in crop insurance setting.

Technical Considerations and Corrections

1. RMA's current implied volatility estimation method:

--RMA's current method assumes constant volatility over the growing season within a year, however, the study does not test whether volatility factor is constant. At the same time, the volatility factor based on RMA's current method exhibits big swings between years (2009 versus 2014) as discussed above.

--In RMA's current method, are 500 draws enough to capture the risk in the tail of the joint distribution of price and yield? If, instead, 10,000 draws were used (as found in some of the literature cited above), how much difference would that make in resulting premium rates for revenue products?

2. SR's alternatives to the current method:

--Do all the models assessed in this study use the last five days of discovery period to estimate implied volatility?

3. SR's evaluation statistics:

--While the study group is referring to BSM in their report, they are using the formula given in Black (1976). The latter assumes non-randomness in the evolution of the commodity spot prices, including those for agricultural products. Non-randomness is thought to arise from the predicted pattern of spot prices of agricultural products typically rising prior to a harvest and falling following the harvest. This assumption can be questioned for three reasons: (i) for crop insurance purposes, all that matters is what happens to prices between planting and harvest; (ii) some close association between the futures and spot prices is expected during the month of harvest price discovery; and (iii) the futures prices data presented in figures 1 and 2 exhibits a rather random pattern in terms of base to harvest price changes. The formula under BSM is given in Hull (p. 291). Both formulas are provided in Appendix 3. We wrote a code in MATLAB to compare both formulas for a set of parameter values (see Appendix 3). Upon running the code, we verified a discrepancy. (The version in Hull results in 47.9409 cents, while the version in SR results in 46.4732 cents for the call option price in the example given.) Although the discrepancy appears to be minuscule, that is the artifact of a very small discount (interest) rate that is used in the example. The code also looks at the call options price obtained MATLAB's `blsprice(.)` routine for the same example and verifies that the latter results in the same call option price as the formula in Hull produces. We recommend that the study group clarify this issue.

-- In the study group's SAS code (p. 76; see the first data step), the commodity symbols are assigned to some of the commodities in a way that is not intuitive. For example, one would expect that 'CZO' would be assigned to 'CORN' not to 'SOYBEANS', while a symbol with 'S' would be assigned to 'SOYBEANS', instead the symbol 'PY' is assigned to the preceding commodity. Meanwhile, 'RRC' is assigned to 'RICE' and 'WZ' is assigned to 'WHEAT', as expected. We recommend that the study group review that part of the code to make sure the data is assigned to variables as intended.

--The study doesn't interpret the values of MAR or root MSE measures. For example, a value of 0.05 for these measures seems to indicate five basis points, is that correct?

--Reporting Root MSE and MSE together is not necessary, as the former is the square root of latter.

--This study doesn't specify the sample period being examined.

--The study does not test or check for autocorrelation structure in the realized volatility measure.

--The study doesn't test if the relationship between the realized volatility and forecasted volatility is actually statistically significant.

--The reported realized volatility formula is the sum of squared terms (indexed by time) which means the resulting magnitude will only go up over time (see figures 12 and 13 below). Notice that the resulting volatility in 2009 is much higher than that in 2014, which is consistent with the implied volatility values in those years. Note also that the realized volatility values in the figures are not time-adjusted as RMA does. The study should have written the formula for realized per day volatility and annualized it (see Hull, p. 282).

Figure 12. Realized Volatility per annum: 2014 December Corn Futures

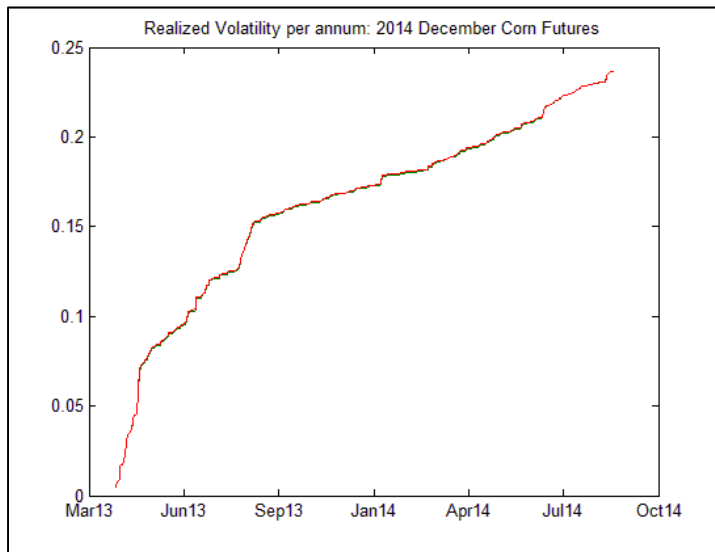
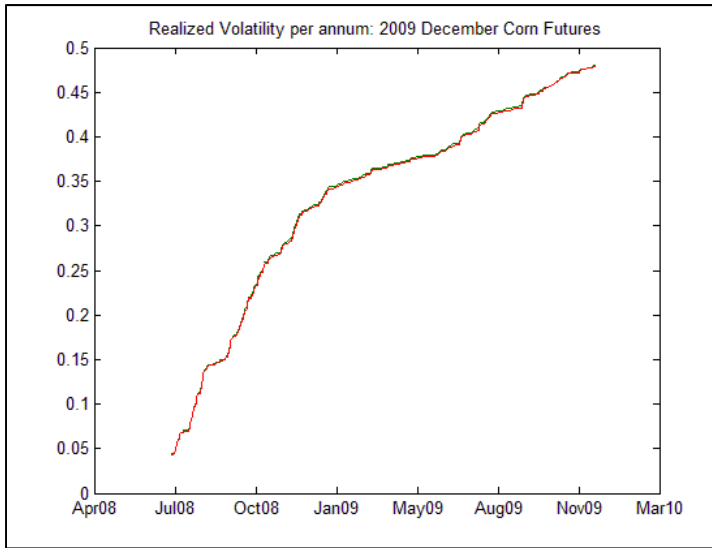


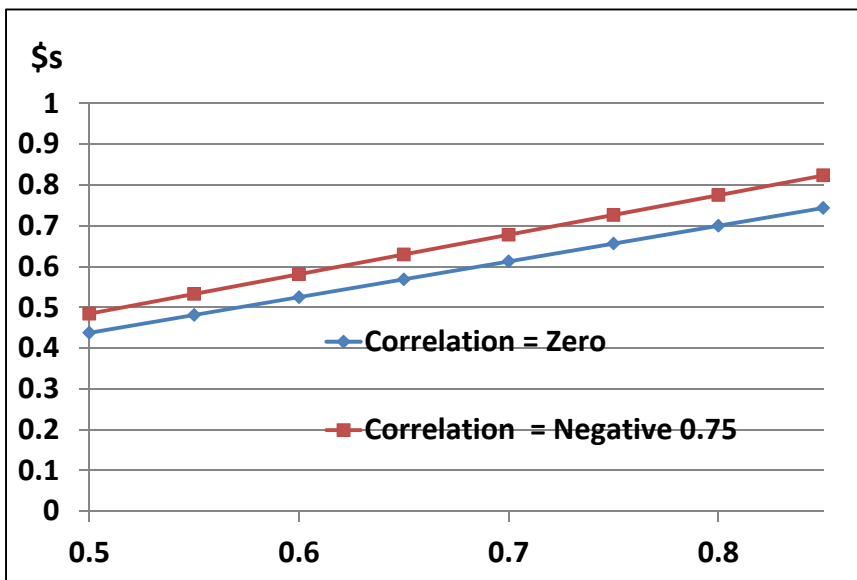
Figure 13. Realized Volatility per annum: 2009 December corn futures



4. SR's recommendation on price-yield correlation:

--The review states that as the price-yield correlation gets stronger, the rates for RP and RP-HPE would go down (p. 27). That is not correct for RP. Figure 14 presents an example that is based on simple four-point joint distribution given in Du, Hennessy and Feng (p. 240).

Figure 14. Expected indemnity for RP plan of insurance based on a four-point joint distribution



Summary and Conclusions

Our primary concern with the RMA/Sumaria volatility study is that the review of the volatility factor used to develop premium rates for revenue plans of insurance did not address the effects of implied volatility measures on premium rates themselves. Premium rates must cover producers' actual losses over time and generate sufficient competitive returns to the private sector delivery system. If these goals are not met, the rating system is deficient, regardless of how well BSM estimates implied volatility relative to other methods, measured against the yardstick of realized volatility. Our primary comments include:

1. Ineffectiveness of volatility in reflecting price risk and establishing premium rates.

Estimates of implied volatility were examined in relation to the percentage changes in base to harvest prices for corn and soybeans (which account for nearly 60% of total program premium) for 1990-2014. The data show virtually no association between the implied volatility factor used in rate making and price risk actually experienced. The correlation between actual price changes—the price risk actually experienced—and the measure of implied volatility used by RMA in ratemaking is not statistically different from zero.

If the volatility factor that is used in rate making is used to generate a price distribution for each past growing season, the actual observed changes in price are consistently occurring at the high and low ends of the estimated distributions, suggesting that the volatility factor is too low and premium rates are inadequate.

An examination of cumulative loss ratios (indemnities divided by total premium) for corn and soybeans combined shows notable underperformance of premium rates for 85% coverage, where the cumulative loss ratio for RP is 1.18 and for RP-HPE is 1.43. At 80% coverage, the cumulative RP loss ratio is 0.92 and that for RP-HPE is 1.25.

Future program performance, particularly at the higher coverage levels, appears likely to be inadequate given:

- The lack of correlation between actual price changes and the measure of volatility used in ratemaking,
- The historical underperformance of revenue rates, especially at high coverage levels,
- The continuing trend toward higher coverage levels by producers,
- The Trend-Adjusted APH Yield Endorsement and the new Farm Bill's APH exclusion which both effectively increase coverage to high levels, placing even more reliance on the adequacy of premium rates at high coverage levels,
- The new Farm Bill's introduction of the Supplemental Coverage Option (SCO) and the Stacked Income Protection Plan (STAX) which will provide coverage at high levels, and
- The recent change in the rating methodology which has sharply lowered base rates.

In combination, these factors indicate RMA/SR need to address what is likely to be poor performance going forward for premium rates of revenue plans of insurance. Regardless of whether BSM continues to be used, its output is adjusted or applied differently in some way, or an alternative approach to estimating volatility is employed, premium rates need to adequately reflect revenue risks, which does not now appear to be the case.

2. Lack of analysis on premium rates and premiums.

SR conducted no analysis of the effect of implied volatility on premium rates and premiums under the current method of estimation or under the set of alternatives examined. There is no assessment as to the validity of the rates generated by the current method or alternatives. Some assessment of the crop and regional premium impacts of volatility changes would have been helpful to better understand the role of volatility across crops, regions and time. We provide two examples based on Kansas wheat and Iowa soybeans for the 2015 crop year using RMA's cost estimator website and find that total premium is much more sensitive with respect to the one point increase in the volatility factor compared to the 1% increase in price.

3. Concerns with the Black-Sholes Model and alternatives considered.

Among the key identified strengths of BSM are: it is widely accepted, transparent, readily available through third party calculations, and alternative approaches to estimating implied volatility have mixed results when compared with BSM. Among the key concerns or limitations with BSM are: its assumption that prices are lognormally distributed, volatility is constant across all option maturities, options markets are liquid, markets are efficient with no arbitrage opportunities, there are no transaction costs, and the discount rate is constant.

Despite the identified concerns with BSM, the review did not to examine price distributions other than lognormal, model free approaches (other than VIX type) or composite approaches. Historical (time-series) approaches were not considered under the argument that historical information is embodied in options prices. We believe that time-series models (ARCH or GARCH type models) would constitute a fair benchmark to include in the review. Without examining such alternatives, the question remains unsettled as to whether alternative approaches, including historical approaches, may fare better in this time period for the exchanges and crops of interest.

SR should acknowledge the parameter uncertainty associated with the volatility factor estimate. Because implied volatility is a market forecast of a parameter, implied volatility can be thought of as an estimate within a confidence interval. We illustrate a case to that point. The consequences of extreme forecasts of this uncertain random variable may have significant costs for producers and the industry. We find that further problematic, having observed that these forecasts have not correlated with actual price changes.

As an alternative approach to BSM, we conducted a distribution fitting exercise for prices and found a better fitting alternative to the lognormal, which raises questions about the appropriateness of the lognormal assumption made in RMA's revenue premium rating method. Using such a best-fit distribution to historical prices has the potential of delivering a more stable volatility estimate over the years and should be considered by SR.

4. Lack of analysis of the five-day price discovery period.

The review does not assess the current method of using five days of option values to determine the volatility factor. The review should address why one day or a few days of data is appropriate for forecasting implied volatility but a monthly average is appropriate for forecasting the expected price. It would also have been useful in the review to provide a better understanding of the sensitivity of the implied volatility calculation to option value changes during the last five days of the discovery month. The review should address the issue of market efficiency, including the incentives for, and the capacity of, market participants to manipulate implied volatility during the last five days of the discovery period. While we believe markets are generally efficient over time, we also believe competition is not perfect. It takes time for some market events to be properly evaluated, understood and priced into the market. Inaccurate or misunderstood information or unusual market activity may generate a price on one day, but as the information and activity is better understood and gets built into the price, a different price prevails. This process may take time, and if inaccurate or misunderstood information or other non-representative events substantially affect the market within that five day window, then the price is distorted.

5. Adjustments for trade volume weighting and the issue regarding the volatility factor transformation.

If the current method of estimating implied volatility is to be used, we agree with the review's recommendation not to use data with zero trading volume and to consider trade volume weighting. We also agree with the recommendation to "Revise the formula for price variability in rate simulation to make it mathematically accurate."

6. Need for assessment of Price/Yield correlations.

While the review conducted no analysis of Price/Yield correlations, we agree with the review's recommendation to "Review and update Price/Yield correlations used in rating." There are many reasons why these correlations may change over time. We also believe the review should be more prescriptive in its recommendation on the use of copulas. We refer to the analysis and recommendation found in the recent literature in this regard, which suggest that the issue is far too critical in terms of accounting for the risk that AIPs are undertaking in the tail of the joint distribution of price and yield and it has practical implications for premium rate calculations. We

recommend that the review provide clear guidance to RMA on the use of copulas in modeling of dependency in crop insurance rating.

Our comment also addresses technical considerations and corrections.

Finally, in this response, our objective has been to provide alternative analytical considerations with the intent to improve SR's analysis. We recommend SR carefully review this response and address every point and concern that has been raised in it. The volatility factor methodology is critical for accurate and actuarially sound premium rating, which in turn determines the viability of the Crop Insurance program. Again, we thank RMA for examining the issue of volatility and the Sumaria team for undertaking the review.

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Appendix 1.

Best fit distribution analysis for the ratio of harvest price to base price, 1968-2014

CORN

1. Normal

Distribution: Normal
Log likelihood: 5.116
Domain: $-\text{Inf} < y < \text{Inf}$
Mean: 0.98514
Variance: 0.0481082

Parameter	Estimate	Std. Err.
mu	0.98514	0.0319934
sigma	0.219336	0.0229926

Estimated covariance of parameter estimates:

	mu	sigma
mu	0.00102358	-2.36271e-18
sigma	-2.36271e-18	0.000528661

2. Lognormal

Distribution: Lognormal
Log likelihood: 9.48001
Domain: $-\text{Inf} < y < \text{Inf}$
Mean: 0.984712
Variance: 0.0426212

Parameter	Estimate	Std. Err.
mu	-0.0369139	0.0302529
sigma	0.207404	0.0217418

Estimated covariance of parameter estimates:

	mu	sigma
mu	0.000915239	-4.91311e-20
sigma	-4.91311e-20	0.000472706

3. Burr

Distribution: Burr
Log likelihood: 10.8881
Domain: $0 < y < \text{Inf}$
Mean: 1.00463
Variance: 0.142063

Parameter	Estimate	Std. Err.
alpha	0.747675	0.02463
c	35.7776	22.3579
k	0.108204	0.0784048

Estimated covariance of parameter estimates:

	alpha	c	k
alpha	0.000606638	-0.43569	0.00160394
c	-0.43569	499.875	-1.70862
k	0.00160394	-1.70862	0.00614732

4. Weibull

Distribution: Weibull
 Log likelihood: **1.89567**
 Domain: $0 < y < \text{Inf}$
 Mean: 0.977968
 Variance: **0.0641492**

Parameter	Estimate	Std. Err.
A	1.0735	0.0381146
B	4.36911	0.436166

Estimated covariance of parameter estimates:

	A	B
A	0.00145272	0.00565795
B	0.00565795	0.190241

5. Generalized Extreme Value

Distribution: Generalized Extreme Value
 Log likelihood: **11.8692**
 Domain: $-\text{Inf} < y < \text{Inf}$
 Mean: 0.988611
 Variance: **0.0640637**

Parameter	Estimate	Std. Err.
k	0.181993	0.169435
sigma	0.144275	0.020851
mu	0.873983	0.0257379

Estimated covariance of parameter estimates:

	k	sigma	mu
k	0.0287083	-0.00126086	-0.00204122
sigma	-0.00126086	0.000434764	0.000335006
mu	-0.00204122	0.000335006	0.000662438

SOYBEANS

1. Normal

Distribution: Normal
Log likelihood: 11.8916
Domain: $-\text{Inf} < y < \text{Inf}$
Mean: 1.02403
Variance: 0.036058

Parameter	Estimate	Std. Err.
mu	1.02403	0.0276982
sigma	0.18989	0.0199058

Estimated covariance of parameter estimates:

	mu	sigma
mu	0.000767192	1.08575e-19
sigma	1.08575e-19	0.000396242

2. Lognormal

Distribution: Lognormal
Log likelihood: 13.1037
Domain: $-\text{Inf} < y < \text{Inf}$
Mean: 1.02433
Variance: 0.0360258

Parameter	Estimate	Std. Err.
mu	0.00715982	0.0268005
sigma	0.183735	0.0192606

Estimated covariance of parameter estimates:

	mu	sigma
mu	0.000718265	3.50521e-20
sigma	3.50521e-20	0.000370972

3. Burr

Distribution: Burr
Log likelihood: 11.5122
Domain: $0 < y < \text{Inf}$
Mean: 1.02345
Variance: 0.0419443

Parameter	Estimate	Std. Err.
alpha	1.01243	0.183269
c	9.07582	3.36689
k	1.04848	1.07259

Estimated covariance of parameter estimates:

	alpha	c	k
alpha	0.0335876	-0.578739	0.194191
c	-0.578739	11.3359	-3.42289
k	0.194191	-3.42289	1.15046

4. Weibull

Distribution: Weibull

Log likelihood: 10.202

Domain: $0 < y < \text{Inf}$

Mean: 1.02164

Variance: 0.0421894

Parameter Estimate Std. Err.

A 1.1037 0.0296177

B 5.7645 0.628316

Estimated covariance of parameter estimates:

	A	B
A	0.000877209	0.00619571
B	0.00619571	0.394781

5. Generalized Extreme Value

Distribution: Generalized Extreme Value

Log likelihood: 13.3116

Domain: $-\text{Inf} < y < \text{Inf}$

Mean: 1.02218

Variance: 0.0345515

Parameter Estimate Std. Err.

k -0.112214 0.14021

sigma 0.164326 0.0208666

mu 0.943889 0.0280784

Estimated covariance of parameter estimates:

	k	sigma	mu
k	0.0196587	-0.00157908	-0.00182115
sigma	-0.00157908	0.000435413	0.00019846
mu	-0.00182115	0.00019846	0.000788397

Appendix 2.

Note was previously prepared by Dr. Harun Bulut of NCIS and shared with RMA and the SR study group

On the Use of Volatility Factor in RMA's Revenue Rating

March 31, 2014

Issue: Based on Black-Sholes model, the volatility is defined as the standard deviation of $\ln(p_h/p_b)$, which is the same as the standard deviation of $\ln(p_h)$ because p_b is known at the planting time. Somewhat differently, RMA's revenue rating treats the volatility factor as the coefficient of variation (CV) measure for the harvest price (that is, the standard deviation of harvest price divided by the mean of harvest price, where the latter is assumed to be equal to base insurance price). We show that these two approaches result in different values and the difference is especially apparent at higher volatility values. The issue may be worth of investigating, as the sensitivity of premium rate calculation to the values of volatility factor is not well-known.

Analysis: Consider a log-normally distributed harvest price, p_h . The natural logarithmic transformation of p_h is normally distributed, that is

$$\ln(p_h) \sim N(M, s^2) \quad (1)$$

where M and s^2 denotes the mean and variance parameters. Note that M is referred as "*Ln Mean*" and s^2 is referred as "*Ln Variance*" in the RMA's Cost Estimator output (see Steps 5.5 and 5.4 in the attached, respectively). Denote the first two moments of original variable p_h as $\mu \equiv E(p_h)$ where $E(\cdot)$ is the expectation operator and $\sigma^2 \equiv Var(p_h)$ where $Var(\cdot)$ is the variance operator. One can write

$$E(p_h) = \mu = e^{M+s^2/2} \quad (2)$$

$$Var(p_h) = \sigma^2 = e^{s^2+2M} (e^{s^2} - 1). \quad (3)$$

Taking the natural logarithm of both sides of equation (2) and rearranging yields

$$M = \ln(\mu) - \frac{s^2}{2}. \quad (4)$$

The formulation in equation (4) corresponds to the formulation for "*Ln Mean*" in the Cost Estimator output. Thus, μ corresponds to the base insurance price. Similarly, taking the natural

logarithm of both sides of equation (3) and substituting the expression in equation (4) and rearranging the terms yields

$$s^2 = \ln \left(\left(\frac{\sigma}{\mu} \right)^2 + 1 \right). \quad (5)$$

Now, denote the volatility factor obtained from the futures markets (after being adjusted for the time difference) with v . RMA appears to assume the following

$$\sigma^2 = \mu^2 v^2. \quad (6)$$

Plugging the preceding expression in equation (5) yields

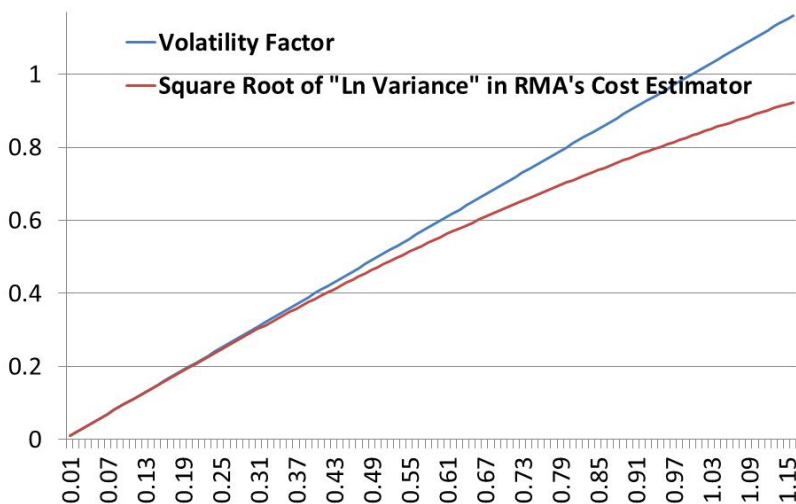
$$s^2 = \ln(v^2 + 1) \quad (7)$$

, which corresponds to the “Ln Variance” formulation in the Cost Estimator output (see Step 5.4 in the attached). From equation (7) the standard deviation of natural logarithm of harvest price is

$$s = \sqrt{\ln(v^2 + 1)}. \quad (8)$$

Figure 1 below presents some hypothetical values for the volatility factor and the resulting values for s from equation (8). The discrepancy between the two is especially apparent at higher volatility values.

Figure 1. Comparing a range of volatility factor values with the resulting values for the square root of “Ln Variance” in RMA’s Cost Estimator.



Appendix 3.

BSM call option pricing formulas (Hull, p. 291)

$$c = P_0 N(d_1) - Ke^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(P_0) + rT - \ln(K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

Sumaria, p. 20 (Black, 1976 call option pricing formula):

$$c = e^{-rT} (P_0 N(d_1) - KN(d_2))$$

$$d_1 = \frac{\ln(P_0) - \ln(K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

where c is the call option price, P_0 is the futures price, K is the strike price, $N(\cdot)$ is the standard normal CDF, T is the term of the option and r is the constant risk free interest rate.

MATLAB Code:

```
% Purpose: Compare the call option pricing formulas
% given in Hull (p. 291) and Sumaria (p. 20)

%%
% Data
call_price = cell(1,2);
c = call_price;

last_price = 763; % in cents
S0 = last_price;

strike = 765; % in cents
```

```

K = strike;

rate = 0.536; % percent
r = rate/100;

daystoexp= 248;
T = daystoexp/365;

impvol = 0.1898;
sig = impvol;

div_yield = 0;

%%
% Call option price formula in Hull, p. 291

d1 = (log(S0/K)+(r + sig^2/2)*T)...
    /(sig*sqrt(T));

d2 = d1 - sig*sqrt(T);

c{1} = S0*normcdf(d1,0,1)- K*(exp(-r*T))*normcdf(d2,0,1);

%%
% Call option price formula in Sumaria, p. 20

d1_alt = (log(S0/K)+( sig^2/2)*T)...
    /(sig*sqrt(T));

d2_alt = d1_alt - sig*sqrt(T);

c{2} = (S0*normcdf(d1_alt,0,1)- K*normcdf(d2_alt,0,1))*(exp(-r*T));

%%
% Check the resulting call option price from MATLAB's blsprice routine
[cCheck,pCheck] =
blsprice(last_price,strike,rate/100,daystoexp/365,impvol,div_yield);

%%
% Display the output to compare
display(c);
display(cCheck);

```