Actuarial Review for Price Volatility Factor Methodology

Responses to Review Comments

Sumaria Systems, Inc.

Sumaria

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Responses to Review Comments

We greatly appreciate the thoughtful comments and suggestions offered by the reviewers. We are also grateful to respond. In the discussions that follow, we attempt to respond to each issue raised in the reviews. In many cases, we believe that the specific questions and concerns relate to issues that fall outside of the scope of the Tasks and Work Requirements of this particular project. Many of these issues were addressed in an earlier report by Sumaria Systems.[1] More specifically, many of the concerns raised in the reviews pertain to how a measure of price volatility is used in the rating system rather than in how the price volatility factor is calculated. We refer to this earlier report in many cases.

Before proceeding to responses to individual items, it is helpful to briefly review the "Specific Tasks and Work Requirements" contained in the original solicitation (Number D13PS00570).

2.3 Specific Tasks and Work Requirements: Task Order against the IDIQ

2.3.1 Review of price volatility factor calculation methodology Draft Report (FFP)

Price volatility factors are necessary to the development of premium rates for crop insurance programs offering revenue coverage based on commodity futures markets. Price volatility factors are currently based on the average implied volatilities for close-to-the-money puts and calls during the last five trading days of the projected price-monitoring period for the given commodity (as determined by Barchart.com). The market estimates reflect an annualized value that is further adjusted by the square root of the percent of the year under which the insurance contract is in place (i.e. the square root of the percent of a calendar year between the projected price and harvest price discovery). However, imbedded within the option premium, and therefore implied volatility valuation, are certain assumptions or costs that may not be appropriate within the crop insurance setting.

The contractor shall perform the following tasks:

1. *Analyze the data and assumptions and assess the adequacy of RMA's existing method for establishing price volatility factors for COMBO (see brief outline at: www.rma.usda.gov/pubs/2011/volatilitymethodology.pdf).*
2. *Review various comments/suggestions RMA has received regarding the determination and use of implied volatilities in establishing premium rates.*
3. *Review existing literature regarding implied volatility determination.*
4. *Compare and contrast two or more alternative methods for calculating implied volatility to the method used by RMA.*

[1] See "A Comprehensive Review of the RMA APH and COMBO Rating Methodology: Final Report," (April 2010), which is available online at http://www.rma.usda.gov/pubs/2009/comprehensivereview.pdf.

5. *Review the use of the price volatility factor within the revenue rate simulation model underlying COMBO, including an evaluation of the underlying price/yield correlations assumed and the interacting effect price volatility and price/yield correlation have on revenue rates.*
6. *Provide a draft technical report outlining all findings and recommendations.*

The Contractor shall produce a draft report that provides the analysis, results, and recommendations.

Responses to NCIS Review Comments

General Comments

Many of the NCIS review comments argue in favor of an alternative retrospective approach to measuring price volatility. An argument is made that the Black-Scholes (BS) volatility has not tended to provide an accurate representation of the market's estimate of the volatility associated with future prices. The fact that the market's "best estimate," as it is reflected in the BS volatility, appears to lack precision when compared to ex-post realized prices is taken to suggest that the approach is conceptually flawed. Analyses of the relationships between price movements between planting and harvest and the BS implied volatilities are used to argue that there is very little correlation between the ex-ante forecast of the BS volatility and the ex-post realized price differences. An analysis of alternative distributional assumptions is applied to bolster this argument.

We believe these assertions are flawed on several grounds. First, as the original report has maintained, the relevant issues pertain not only to the accuracy of the predicted variability but also on suggestions for alternative approaches that are superior on intuitive, theoretical, or empirical grounds. Because options are a fairly recent phenomenon and in light of the annual nature of crop production (one observation per year), the amount of data available for an empirical analysis is necessarily limited. In particular, we have a total of 25 observations that allow a direct comparison between the options implied BS volatility and the actual price changes. Thus, the lack of precision regarding how well the BS volatility reflects actual price changes may have much to do with the relatively limited data available. On conceptual grounds, the BS is bolstered by a fundamental economic assumption of the rationality of agents and the efficiency of markets.

1. Ineffectiveness of BS Volatility in Reflecting Price Risk

Response to Correlation Comment. A Pearson (linear) correlation of the BS volatilities and realized price changes is presented in the review. It is noted that the linear correlation coefficients are generally close to zero. Of course, linear correlation is a narrow approach to measuring dependence. Monotonic transformations of the relevant variables, which by definition do not change ordering, can have drastic effects on Pearson correlation coefficients. An approach that may offer advantages involves rank correlation. We repeated the correlation analysis of squared realized returns (represented by the log of the ratio of harvest to planting prices) and the Bridge/CRB monthly average implied volatilities. Note that use of rank correlation lessens the reliance of the results on the particular metric used to represent price differences. For example, using the absolute value of returns yields the same correlation structure as squared returns.

As we have noted, the BS volatility reflects the market's collective judgment regarding the uncertainty of future prices and it is difficult to draw strong conclusions from its actual ex-post performance in such a small sample. However, the Spearman correlation coefficients do indeed demonstrate a correspondence between the BS implied volatilities and the squared returns (the squared logarithm of the ratio of harvest to planting prices). Table 1 presents the Spearman rank correlation matrix for corn and soybeans, the same commodities evaluated in the NCIS review. The relationship is statistically significant at the $\alpha = .10$ level with a rank correlation of 0.36 for corn. In the case of soybeans, the correlation coefficient is 0.30 and narrowly misses being statistically different from zero (with a p-value of 0.14). While we would caution against making definite conclusions about this relationship with such a small sample, it certainly is not the case that there is "virtually no correlation."

Response to comment on frequent realization of low probability events. Probabilities listed in the column titled "Implied Probability %" in Table 2 of the NCIS document are derived using the normal probability density function. An example of the normal density for a random variable X with mean μ and variance σ^2 is depicted in the Figure 1 below.

Probabilities calculated for different years can be compared with each-other even though the mean μ and variance σ^2 change every year as one could easily transform the normal random variable X with mean μ and variance σ^2 to a standard normal random variable Z with mean 0 and variance 1.

As depicted in Figure 1 below, actual values x of the random variable X that are two standard deviations away from the mean, $\mu - 2\sigma \geq x \geq \mu + 2\sigma$, have a 5 percent probability of occurrence. Said differently, there is a 5 percent probability of x values being two standard deviations away from the mean or 2.5 percent probability of $x \leq \mu - 2\sigma$ and 2.5 percent probability of $x \geq \mu + 2\sigma$.

We updated the harvest price in Table 2 of the NCIS comments, and its corresponding probability, for 2014. The harvest price is \$3.49 and the implied probability is 8.5%. The original results of Table 2 in the NCIS document with the updated values for 2014 are presented in Table 2 below. The implied probability values in the NCIS Table 2 and Table 2 below give the probability of $P \leq P_{actual}$ where P is corn price and P_{actual} is the observed corn harvest price. For example in 2014, $P_{actual} = 3.49\$ / \text{bu.}$ and the probability that the price of corn would be less than or equal to \$3.49/bu. is 8.5 percent. For 2012, $P_{actual} = 7.50\$ / \text{bu.}$ and the probability that the price of corn would be less than or equal to \$7.50/bu. is 91.2 percent, or said differently, the probability that the price of corn would be greater than or equal to \$7.50/bu. is 8.8 percent.

The connection between the interpretation of the implied probability values in the NCIS Table 2 and Table 2 below, as well as the graphical presentation in Figure 1, is illustrated by the following two examples. If $P_{actual} = \mu - 2\sigma$ then the probability that the price of corn would be less than or equal to P_{actual} is 2.5 percent. Similarly, if $P_{actual} = \mu + 2\sigma$ then the probability that the price of corn would be less than or equal to P_{actual} is 97.5 percent, or said differently the probability that the price of corn would be greater than or equal to P_{actual} is 2.5 percent.

Looking at the values of the implied probability in the NCIS Table 2 and Table 2 below, it is obvious that none of the probability values is less than 2.5 percent or greater than 97.5 percent. This means that none of the observed harvest prices is two or more standard deviations away from the mean – an occurrence with a 5 percent probability. Similarly, none of the probability values is less than 5 percent or greater than 95 percent. This means that none of the observed harvest prices is 1.645 or more standard deviations away from the mean – an occurrence with a 10 percent probability.

Another way to look at the historical price values presented in the NCIS Table 2 and Table 2 below is as follows. Under the assumptions that price follows a lognormal distribution (and the natural logarithm of the price follows a normal distribution) one would expect that in 5 out of 100 years the natural logarithm of the observed price would be either two standard deviations below its mean or two standard deviations above the mean. Similarly, one would expect that in 10 out of 100 years the natural logarithm of the observed price would be either 1.645 standard deviations below its mean or 1.645 standard deviations above the mean.

Given that there are only 25 years presented in the NCIS Table 2 and Table 2 below, one would expect that in at least one of the 25 years the natural logarithm of the observed price would be either two standard deviations below its mean or two standard deviations above the mean. Similarly, one would expect that in at least 2 of the 25 years the natural logarithm of the observed price would be either 1.645 standard deviations below its mean or 1.645 standard deviations above the mean. As noted above there are no such occurrences. This means that the volatility factor used in deriving premium rates has been too high. As an exercise, we lowered the volatility factor for all years to the level required to produce at least one occurrence with a 5 percent probability and at least two occurrences with a 10 percent probability. Our results show that the volatility factor should be reduced by 15 percent to produce at least two occurrences with a 10 percent probability and by 20 percent to produce at least one occurrence with a 5 percent probability.

One important conclusion resulting from the analysis presented above is that if there are any actuarial issues related to the methodology used to derive premium rates which cause premium rates to be lower than they should be, the issue is not the volatility factor used to derive the premium rates.

Summary Response. The NCIS review criticizes the BS volatility on a number of grounds having to do with a retrospective, ex-post evaluation of the relationship between the implied volatility and price changes. We demonstrate that alternative considerations of this relationship, made using rank correlation, do reflect the expected relationship, albeit weakly in some cases. We believe the sample is too short to place a heavy emphasis on these results. We would also note that any method for measuring price volatility that is retrospective necessarily makes unreasonable assumptions regarding the stationarity of the price distribution. This is discussed in detail below. Finally, the analysis presented in the review stops short of providing a specific recommendation regarding an alternative to the BS implied volatility.

2. Lack of Analysis on Premium Rates and Premiums

The analysis presented in the NCIS document argues that the volatility review stops short of providing an effective analysis of how the volatility shapes revenue premium rates. The review demonstrates the sensitivity of revenue protection premium rates to changes in volatility. We most certainly agree that the volatility is a very important factor in determining the price of revenue coverage. Item 5 of the Task Work Order requested that the contractor "*Review the use of the price volatility factor within the revenue rate simulation model underlying COMBO, including an evaluation of the underlying price/yield correlations assumed and the interacting effect price volatility and price/yield correlation have on revenue rates.*" This review is contained on pages 23-36 of the volatility report. Likewise, much of this material was extensively reviewed in the 2010 Combo rating review.

Our review identified a minor error in the manner in which the volatility is used in the "Cost Estimator" to determine premiums. This issue was pointed out in previous correspondence with NCIS and we agree that a minor change is needed to address this issue.

The 2010 report made a number of recommendations for future study that we believe remain important, though outside of the scope of this particular review. We noted there (pages 106-110) that a minor change in the manner in which rate relativities is applied has conceptual advantages. We also recommended the use of a direct analytical calculation of rates rather than dependence on a relatively small number of "pseudo-random" draws. We also suggested further study of the price-yield correlations. We agree with the NCIS document that this remains an important topic for additional evaluation. Our focus in this review was on the determination and measurement of expected price volatility and thus a detailed assessment of the price and yield correlations was beyond the scope of the study.

3. Concerns with the Black-Scholes Model and Alternatives Considered

Our review considered a number of alternatives to the standard BS implied volatility that have been proposed in the academic literature and in commentary on RMA's approach to measuring volatility. Our conclusion was that the BS tends to perform comparably to a number of these proposed alternatives and further that it has important advantages in terms of transparency and acceptance by the financial community. Of course, it is impossible to evaluate the universe of all possible alternatives. The review argues that we confined our analysis only to the case of log-normality. This is not correct. We considered the volatility model of Egelkraut, Garcia, and Sherrick (2007).[2] This methodology calibrates a log-normal distribution across the entire range of strikes. This is different from assuming that the distribution of prices is necessarily log-normal across all strikes. The distinction is subtle, but rather than calibrating a log-normal distribution to each strike, we calibrate it to the entire range of positively traded strikes. As Egelkraut, Garcia, and Sherrick (2007) note, this is equivalent to approximating the risk neutral density of prices using a log-normal distribution. The resulting volatility measure may differ from a standard BS estimate across the entire range of strikes.

Before proceeding to a wider evaluation of alternatives to the log-normal, it is important to address the evaluation of alternative distributions estimated using historical price changes that is presented in the NCIS review. Table 3 of the NCIS document presents a number of alternative distribution functions that have been fit to historical price differences spanning 1968-2014. We believe that this analysis is invalid because of the fundamental assumption that is implicit in the approach—that the price density is stationary across this entire period. Put differently, to our understanding of the analysis, the volatility estimates that are presented in Table 3 of the NCIS document are assumed to be constant across this entire span (or, at the very least, the parameters that characterize the distribution of prices is assumed to be constant across the entire period). A valid density can only be estimated when the data are identically and independently distributed (iid). Changing market conditions necessarily suggest that this pricing density is different each year.

This goes to a very important distinction between ex-ante and ex-post measures of price volatility. Using historical data to fit a distribution or otherwise measure volatility is going to result in a static (or at least slowly adjusting) measure of volatility. One would wonder how the program would have performed between 2003 and 2008, when volatilities doubled. Had the volatility been fixed at a level determined by the standard deviation of annual price returns between 1960 and 2007, insurers and reinsurers would have had rates based upon volatility measures of 0.18 and 0.16 for corn and soybeans, respectively, heading into the 2008 crop year. Implied volatilities exceeded 0.35 in 2008, suggesting a significant potential for large payouts that would not be covered by premiums. Large price swings did indeed arise in the years following 2007 and the industry would have realized significant financial losses had revenue rates been based upon a static volatility measure.

Although we disagree with the implications of the analysis presented in the NCIS report, we did undertake some additional analysis in the same vein. It was unclear to us exactly how the price differences were expressed, but we chose to use the return between planting and harvest, measured as the log of the ratio of prices. Without loss of generality, we shift the density to the

[2] See Egelkraut, T.M., P. Garcia, and B.J. Sherrick. 2007. "The Term Structure of Implied Forward Volatility: Recovery and Informational Content in the Corn Options Market." *American J. of Agricultural Economics* 89(1): 1-11.

right by adding one (that is, we use $1+\ln(P_H/P_P)$). We fit a range of relevant distributions to the price differences, including the Burr and log-normal. Goodness of fit statistics, including Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von-Mises tests are presented in Table 3 below. In the case of corn, every statistic favors a log-normal distribution while for soybeans the specification tests are split between the Burr and log-normal distributions (with the test statistics being very close to one-another). In light of the concerns regarding the assumed static nature of distributions, we do not place relevance on these results. However, within the context of the evaluation presented in the review, these results demonstrate that the differences in goodness of fit are modest and that the log-normal distribution does receive strong support. Figure 2 below presents the fitted cumulative distribution functions for the alternatives.

The NCIS review also recommends consideration of retrospective GARCH or ARCH type models as a means for representing time-varying volatility. We do not believe such models would be appropriate for RMA's needs for several reasons. First, such models are typically applied to high frequency data. To apply the models to the case of representing price volatility between planting and harvest time, one would need to simulate the variance for a very long out-of-sample period. For a stationary GARCH model, these long-term volatility projections typically revert to the unconditional mean.[3]

The NCIS review also recommended consideration of alternative distributions for the risk free pricing density. The review specifically recommended consideration of a Burr distribution, which has been shown to offer considerable flexibility in terms of representing skewness and kurtosis. We appreciate this suggestion and agree that considering the Burr broadens the scope of our conclusions. Table 4 below repeats the analysis presented in Table 6.1 of the original report with the addition of volatilities estimated using the Burr Type 3 and Burr Type 12 distributions. The results are again very comparable though in no case does either version of the Burr provides a better fit to the realized volatilities than the log-normal. Again, it is reassuring that the results are robust with respect to alternative distributions.

Finally, the NCIS review raises some valid questions regarding a potential inconsistency between the use of monthly averages to calculate expected prices and the use of only five days in calculating volatilities. We agree that this issue merits additional consideration on the part of RMA. The use of such averages always attempts to balance the effects of random variation (which is diminished by averaging) and the accuracy of projections.

4. Adjustments for Trade Volume Weighting and Volatility Transformation

In the volatility report, we argued that care must be exercised when dealing with thinly traded options. We recommended dropping option quotes with zero trades and a consideration of

[3] See, for example, Christoffersen, P.F., and F.X. Diebold. 2000. "How Relevant Is Volatility Forecasting for Financial Risk Management." *Review of Economics and Statistics* 82:12–22.

volume-weighting in the calculation of volatilities. In that the Barchart methods use near-the-money option quotes, which are the most fluidly traded instruments, we believe that the issue of trade volume weighting is not critical for the current methods. However, any alternative analytical method that might be adopted for estimating volatilities should give careful consideration to volume. It appears we are largely in agreement with NCIS on this issue.

5. Need for Assessment of Price/Yield Correlations

The NCIS review argues that the issue of parameterizing price/yield correlation deserves greater attention. In accordance with the Task Order Statement of Work, we did review the price/yield correlation issue. On page 28 of our report, we concluded that "Our suggestion is that RMA consider updating price-yield correlations both spatially and using newer data. Further we suggest RMA follow the recent developments related to applying copulas for modeling revenue. However, we make no stronger recommendation at this time." On page 109 of the 2010 "Combo Report," we also noted that a number of questions regarding assumed price/yield correlations remain relevant. We recommended that "RMA evaluate estimating price-yield correlations at a level below the state level as there may be clear reason to allow correlation to vary across production regions in a state." We believe these recommendations remain relevant. A more detailed study of the proper parameterization of price/yield dependencies is clearly outside of the scope of the volatility study, but we agree it merits additional investigation.

Technical Considerations and Corrections

1. RMA's current implied volatility estimation method.

The review argues that RMA's methods assume constant volatility across the growing season. We disagree. As the review correctly notes, the volatility is a *forecast* made at a point in time. This forecast will of course be updated by the market, but the important point is that rates must be determined prior to the growing season.

Questions are also raised regarding the suitability of 500 random draws. As we noted above, we have recommended further evaluation of this issue in the 2010 review. We agree that 500 random draws is a small number and further that direct analytical solutions to the determination of rates may be preferred to simulations run within the ADM software.

2. SR's alternatives to the current method.

All of the evaluations undertaken in this study used daily closing settlement quotes for the relevant options.

3. SR's evaluation statistics.

The NICS report argues that modest differences may arise due to the use of the Black (1976) implied volatility formula and a slight variant presented by Hull. We provide below an explanation for the discrepancy. It is not correct to state that Black's formula assumes non-randomness in the evolution of the commodity spot prices as Black's models uses futures rather than spot prices as the underlying price. The fact that futures prices in a risk-neutral world have zero drift accounts for the discrepancy between the two formulas, BSM and Black's. A more technical explanation follows.

In the formulas below, c is the call option price, S_0 is the price of an asset (other than a futures), F_0 is the futures price, K is the strike price, $N(.)$ is the standard normal CDF, T is the term of the option and r is the constant risk free interest rate.

BSM's call option formula:

$$c = S_0\ N(d_1) - K\ e^{-rT} N(d_2)$$

$$\text{where}\quad d_1 = \frac{\ln(S_0) + rT - \ln(K) + \sigma^2\,\mathrm{T}/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Black's call option formula:

$$c = e^{-rT} F_0\ N(d_1) - e^{-rT} K\ N(d_2)$$

$$\text{where}\quad d_1 = \frac{\ln(F_0) - \ln(K) + \sigma^2\,\mathrm{T}/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

To highlight the well-known difference between the two call pricing formulas we will display the first part of each formula side-by-side:

BSM's formula: $c = S_0\ N(d_1) - K\ e^{-rT} N(d_2)$

Black's formula: $c = e^{-rT} F_0\ N(d_1) - e^{-rT} K\ N(d_2)$.

As noted, the difference between the two formulas is that the asset price in the first term of the BSM's formula, S_0 is not discounted, while the futures price in the first term in the Black's formula, F_0 is discounted at the risk free rate.

The reason for S_0 not being discounted is that different risky assets (stocks, stock indexes, exchange rates, etc.) in a risk-neutral world would return the risk free rate, r. Thus, their value at time T in a risk-neutral world would be equal to $S_T = e^{rT} S_0$. Discount this future value to the present and you get: $S_0 = e^{-rT} S_T = e^{-rT}(e^{rT} S_0) = S_0$.

The reason for F_0 being discounted is the well-known fact that futures in a risk-neutral world would return zero. In other words, the expected value of futures at time T in a risk-neutral world is $E(F_T) = F_0$. Discount this future value to the present and you get: $e^{-rT}F_0$.

Even in light of the discussion above, note that we do not use Black's formula directly in our calculation of the BS volatility[4] but instead use embedded procedures in SAS. In particular, the implied volatility is given by

```
proc fcmp outlib=sasuser.funcs.options;
  function blkschc(strike_price, term, p_, rate, volty);
    return(blkshclprc(strike_price, term, p_, rate, volty));
  endsub;
    function bsvoltyc(settlement, strike_price,term, p_, rate);
          array opts[5] initial abconv relconv maxiter status
            (.3 .001 1.0e-6 500 -1);
  iv = solve("blkschc", opts, settlement, strike_price, term, p_, rate, .);
  return(iv);
  endsub;
run;
```

This is technically correct and consistent with Hull.

The NCIS review does point out that the calibration of a log-normal distribution across the entire range of relevant strikes uses Black's original formula. However, as we have noted above, this does not directly correspond to the BS volatility but rather is the case of a log-normal approximation to an unknown pricing density.

The NCIS report notes that the commodity symbols appearing in SAS seem confusing. We agree but would note that these are the correct symbols used by CRB/Bridge. We verified this with Bridge. The specific codes are: CZO = soybeans, LO = crude oil, PY = corn, RRC = rough rice, WZ = Chicago wheat.

The mean absolute and squared error statistics presented are defined as given in the table captions. In particular, they are the average absolute and squared differences of the estimated volatility and the realized volatility. Likewise, when comparing price differences, these are again given in terms of differences in absolute prices (cents per bushel).

The sample period encompasses all available options for the relevant options. The data generally begin in the 1985-1990 period, depending on the contract.

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[4] We updated our results of table 6.1 in the original report (presented in Table 4) to include the results derived using the Black formula. There are only trivial changes between the results based on the Black and BSM formula.

The NCIS document also argues that the volatility study does not test for autocorrelation in the realized volatility. We agree, but see no relevance to our conclusions. There is but a single realized volatility for each year and the comparisons are all made to this realized volatility.

The NCIS review further notes that the volatility study does not test the significance of the differences in alternative volatility measures. Table 5 below has added t-tests of the significance of the differences for each alternative volatility measure. Given the very large sample size, the differences are nearly always significant, but the results reinforce conclusions favoring the standard Black Scholes and log-normal volatility measures.

The NCIS report argues that realized volatilities should have been adjusted for time differences. We disagree in that all volatilities apply to roughly the same time basis—planting to harvest. These could all be annualized but it would not change the conclusions.

Table 1. Spearman Correlation Coefficients (BS Implied Volatility and ln(Harvest/Planting)^2).

Spearman Correlation Coefficients Prob > |r| under H0: Rho=0 Number of Observations	sretc	ivc	srets	ivs
sretc Squared Returns Corn	1.00000 54	0.36348 0.0808 24	0.49015 0.0002 54	0.29154 0.1574 25
ivc BS IV Corn	0.36348 0.0808 24	1.00000 24	0.29304 0.1646 24	0.89304 <.0001 24
srets Squared Returns Soybeans	0.49015 0.0002 54	0.29304 0.1646 24	1.00000 54	0.30462 0.1387 25
ivs BS IV Soybeans	0.29154 0.1574 25	0.89304 <.0001 24	0.30462 0.1387 25	1.00000 25

Table 2. The Implied Percentiles from the Corn Harvest Price Distribution

Year	Base Price ($/bu.)	Volatility Factor	Harvest Price ($/bu.)	Implied Probability (%)
2014	4.62	19	3.25 (3.49)*	3.8 (8.5)*
2013	5.65	20	4.39	12.9
2012	5.68	22	7.5	91.2
2011	6.01	29	6.32	62.5
2010	3.99	26	5.46	90.9
2009	4.04	34	3.72	47.3
2008	5.4	29	4.13	21.5
2007	4.06	25	3.58	35.6
2006	2.59	23	3.03	78.9
2005	2.32	20	2.02	28.2
2004	2.83	21	2.05	7.5
2003	2.42	17	2.26	37.4
2002	2.32	18	2.52	71.4
2001	2.46	19	2.08	22.4
2000	2.51	21	2.04	18.9
1999	2.4	19	2.01	20.3
1998	2.84	20	2.19	11.6
1997	2.73	19	2.81	60.1
1996	3.08	20	2.84	37.3
1995	2.57	15	3.23	94.2
1994	2.68	17	2.16	11.1
1993	2.4	15	2.49	62.3
1992	2.7	18	2.09	9.5
1991	2.59	16	2.51	45.3
1990	2.47	16	2.3	35.2

Note: * Numbers in parentheses are the updated values.

Table 3. Evaluation of Alternative Distributions for $[1+\ln(P_{harvest}/P_{planting})]$

A. Corn

All Fit Statistics														
Distribution	-2 Log Likelihood		AIC		AICC		BIC		KS		AD		CvM	
Logn	-31.30874	*	-27.30874	*	-27.07345	*	-23.33077	*	0.50356	*	0.28839	*	0.03458	*
Exp	104.68363		106.68363		106.76056		108.67262		3.72487		16.39553		3.48306	
Burr	-28.64357		-22.64357		-22.16357		-16.67662		0.56376		0.35560		0.04496	
Weibull	-20.73609		-16.73609		-16.50080		-12.75812		0.76120		0.82758		0.10276	
Note: The asterisk (*) marks the best model according to each column's criterion.														

B. Soybeans

All Fit Statistics														
Distribution	-2 Log Likelihood		AIC		AICC		BIC		KS		AD		CvM	
Logn	-37.86757		-33.86757	*	-33.63227	*	-29.88960	*	0.45939	*	0.23761		0.03079	
Exp	109.50650		111.50650		111.58342		113.49548		3.65722		17.30684		3.68605	
Burr	-38.25727	*	-32.25727		-31.77727		-26.29032		0.48940		0.18357	*	0.02808	*
Weibull	-37.53347		-33.53347		-33.29818		-29.55550		0.60926		0.27648		0.04386	
Note: The asterisk (*) marks the best model according to each column's criterion.														

Table 4. Volatility Comparisons Including Burr 3 and Burr 12

(Refers to Table 6.1 of Volatility Report)

	All	Corn	Soybeans	Wheat
	(n=1,251)	(n=539)	(n=554)	(n=158)
Method	**Mean Absolute Error**			
Standard Black Model	0.0478	0.0409	0.0402	0.1007
Standard Black Scholes (Barcharts)	0.0479	0.0418	0.0389	0.1031
Log-Normal (Restricted Mean)	0.0487	0.0417	0.0410	0.1021
Weighted Log-Normal (Restricted Mean)	0.0496	0.0417	0.0429	0.1028
Egelkraut, Garcia, and Sherrick (Unweighted)	0.0528	0.0467	0.0408	0.1191
Egelkraut, Garcia, and Sherrick (Weighted)	0.0532	0.0475	0.0410	0.1191
Burr Type 3	0.0539	0.0477	0.0420	0.1200
Burr Type 12	0.0523	0.0462	0.0404	0.1186
Model-Free Volatility	0.1488	0.0456	0.2366	0.2012
	Mean Squared Error			
Standard Black Model	0.0045	0.0032	0.0028	0.0157
Standard Black Scholes (Barcharts)	0.0046	0.0033	0.0028	0.0162
Log-Normal (Restricted Mean)	0.0046	0.0033	0.0029	0.0159
Weighted Log-Normal (Restricted Mean)	0.0047	0.0033	0.0030	0.0159
Egelkraut, Garcia, and Sherrick (Unweighted)	0.0059	0.0045	0.0035	0.0202
Egelkraut, Garcia, and Sherrick (Weighted)	0.0060	0.0045	0.0034	0.0204
Burr Type 3	0.0060	0.0045	0.0036	0.0201
Burr Type 12	0.0058	0.0043	0.0034	0.0198
Model-Free Volatility	0.1656	0.0040	0.3514	0.0585

Table 5. T-Tests of Mean Errors

(Refers to Table 6.1 of Volatility Report)

	All	Corn	Soybeans	Wheat
	(n=1,251)	(n=539)	(n=554)	(n=158)
Method	**Mean Absolute Error**			
Standard Black Scholes (Barcharts)	0.0479	0.0418	0.0389	0.1031
Log-Normal (Restricted Mean)	0.0487	0.0417	0.0410	0.1021
Weighted Log-Normal (Restricted Mean)	0.0496	0.0417	0.0429	0.1028
Egelkraut, Garcia, and Sherrick (Unweighted)	0.0528	0.0467	0.0408	0.1191
Egelkraut, Garcia, and Sherrick (Weighted)	0.0532	0.0475	0.0410	0.1191
Model-Free Volatility	0.1488	0.0456	0.2366	0.2012
	t-test of Mean Error = 0			
Standard Black Scholes (Barcharts)	-13.1420	-9.5873	-1.8972	-16.5655
Log-Normal (Restricted Mean)	-9.5978	-7.4063	1.7687	-16.2608
Weighted Log-Normal (Restricted Mean)	-9.2277	-7.7695	2.4561	-15.7173
Egelkraut, Garcia, and Sherrick (Unweighted)	-23.8621	-16.9253	-11.7365	-18.6134
Egelkraut, Garcia, and Sherrick (Weighted)	-23.5253	-17.5388	-10.7949	-18.3290
Model-Free Volatility	3.9894	-11.5941	7.4533	-15.3835

Figure 1. Graphical presentation of the normal probability density function.

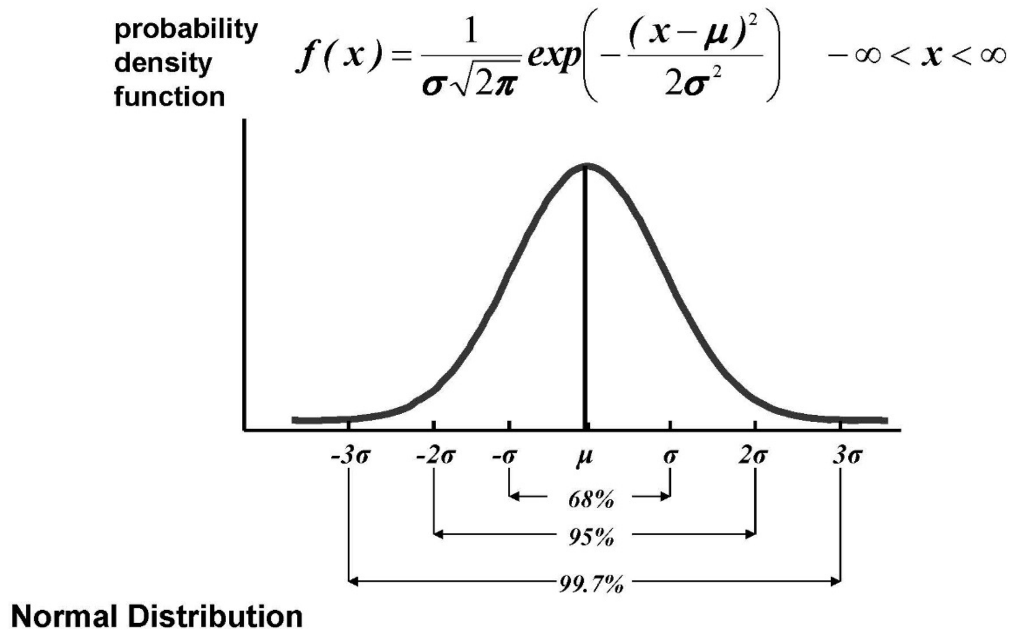
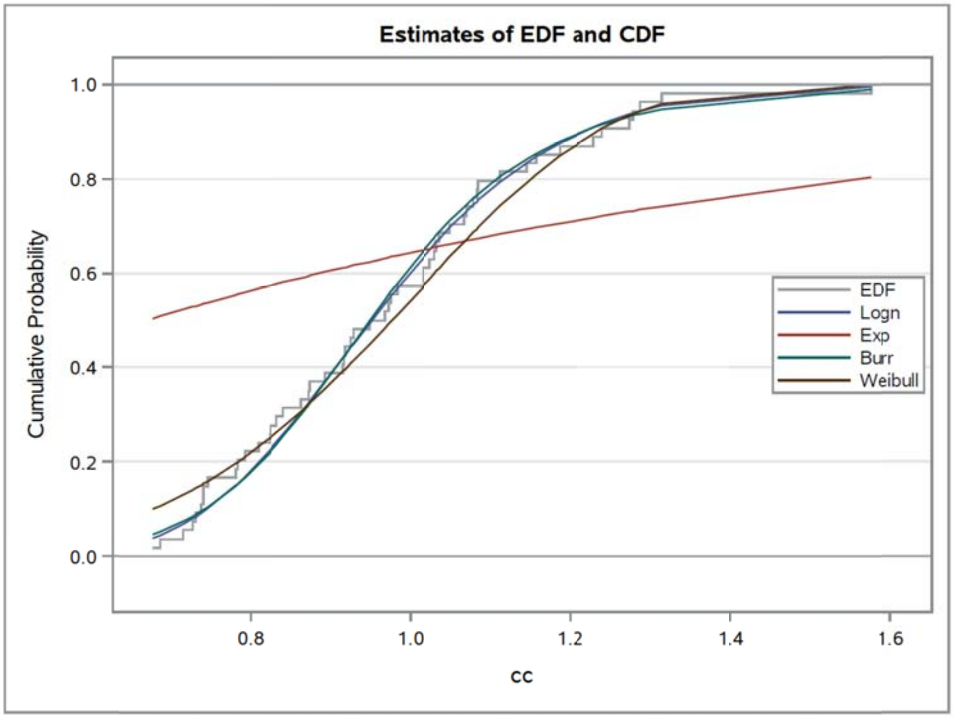
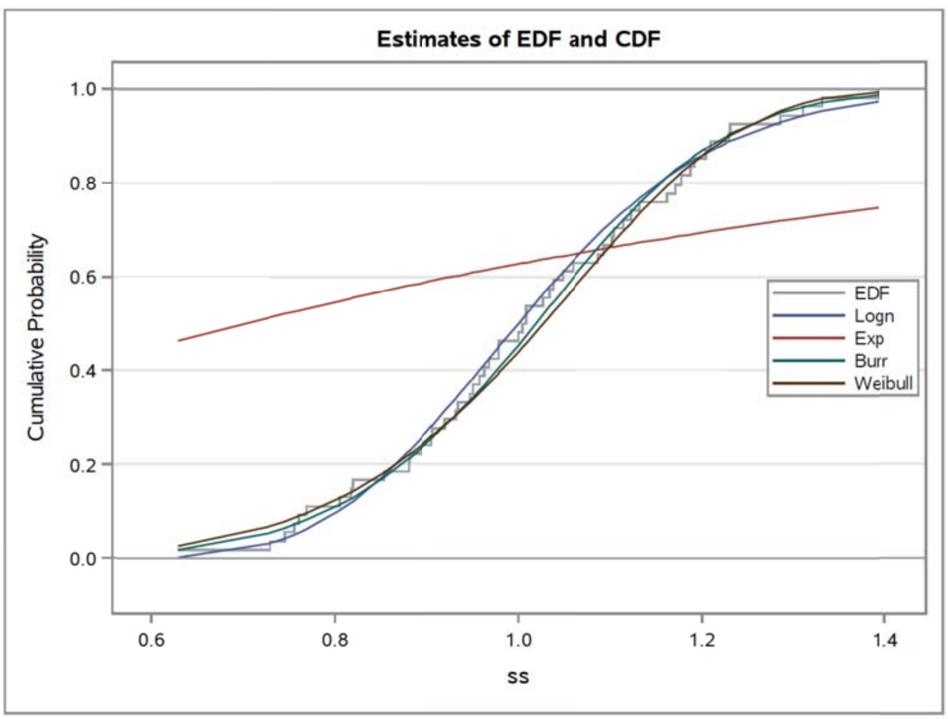


Figure 2. Estimated Cumulative Distributions for [1+ ln($P_{harvest}$/$P_{planting}$)]

A. Corn



B. Soybeans



Responses to Munich-RE Review Comments

1. Adequacy of the BSM as the Core of RMA's Revenue Protection Rate Making

The review comments of Munich-Re argue that reliance on financial asset markets (options) to derive measures of crop price volatility may induce biases not consistent with market fundamentals. We would argue that, if one accepts the efficiency of such markets, then market-based measures of volatility should correspond to market fundamentals. We have considered a range of alternative volatility measures and have not been able to derive an alternative that would offer advantages over the current method.

A number of arguments regarding the superiority of retrospective ex-post measures of price volatility have been advanced. As we have noted in our responses to the NCIS review, we disagree with these assertions and believe that reliance on static or slowly-adjusting volatility measures could result in substantial risks to the industry. We have also pointed out that assertions that the implied volatilities do not necessarily correspond to actual price changes are questionable. This is complicated by a relatively small sample but we do demonstrate statistically significant rank correlations among these alternatives.

2. As a Public-Private Partnership, Alternatives to the BSM Should Be Considered

The Munich-RE review argues that the risk exposure of insurers and reinsurers necessarily justifies recognition of longer-term risk factors. We do not feel that we are in a position to offer an opinion on these issues as the scope of our analysis was necessarily confined to the specific issue of measuring price volatility in rating annual revenue insurance contracts.

Without making inappropriate inferences, it seems that a subtle argument regarding the necessity for loading of price risk much as is done for yield risk is being made. We do not have an opinion on this issue as it is clearly outside of the scope of our assignment.

3. Proposed Adjustments to the BSM volatility

An argument is made regarding the fact that prices and volatilities necessarily adjust to reflect new information during the growing season. This is true but we do not believe it is necessarily relevant to the manner in which the BSM is used in rating. As we have noted in the responses to NCIS, the volatility is a *forecast* made at a point in time. This forecast will of course be updated by the market, but the important point is that rates must be determined prior to the growing season.

The Munich-RE review also argues that the BSM omits potentially relevant information contained in the tails of the price distribution. An argument is made that, despite the very thinly traded nature of strikes in the tails, information regarding tail behavior could be extracted. Much of our evaluation of alternatives was directed toward considering alternatives that use such information—as long as the assets were traded. For example, calibration of a log-normal across the entire range of strikes results in a volatility that differs from the BSM volatility for near the money strikes. We do not find that such alternatives would necessarily improve the precision of volatility estimates.

Responses to Rain and Hail Review Comments

Rain and Hail provides a very thoughtful review that mainly emphasizes a need for further evaluation. The review notes that there is a perceived lack of correspondence between the BS implied volatilities and actual price changes. As we have noted in our responses to NCIS, we do not agree with the conclusion and further do not believe a preferred alternative has been identified. As we note, rank correlations between price changes and volatilities do demonstrate the expected relationships, albeit somewhat weakly. This is largely due to the relatively small sample and the fact that prediction of volatility is a challenging task.

Rain and Hail argue that RMA should consider floors in the volatility or other changes in the SRA that would mitigate the exposure of the industry to price risk. We do not offer an opinion on these suggestions as they are issues outside of the scope of our assignment. Our evaluation was limited to the suitability of the BS volatility in representing price volatility and the consideration of potential alternatives.