ERRATA SHEET

Page	Line	
8	3	$\underline{Q}'$ = discharge at time t-Δt, in cfs;
8	4	$\underline{A}'$ = cross-sectional area at time t-Δt; in ft^2; and
8	27	the first two terms of the variable energy slope of Eq.(4).
12	10	associated with Q_m. The energy slope is determined from Eq.(14) in which

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Weather Service

NOAA Technical Memorandum NWS HYDRO-16

A DYNAMIC MODEL OF STAGE-DISCHARGE RELATIONS AFFECTED BY CHANGING DISCHARGE

D. L. Fread

Office of Hydrology

WASHINGTON, D.C.
November 1973

CONTENTS

ABSTRACT

A mathematical model is developed to simulate the dynamic relationship which exists between stage and discharge when the energy slope is variable due to the effects of changing discharge. The model enables either stage or discharge to be computed if the other is specified (measured or predicted) and the channel slope, cross-sectional area and roughness properties are known. The model is tested on several stage-discharge relations, which are influenced significantly by changing discharge, and found to provide excellent results. Also presented is a simple and easily-applied graphical procedure to estimate the magnitude of the changing discharge effect on stage-discharge ratings.

SECTION 1. INTRODUCTION

The hydrologist is frequently concerned with the conversion of flood discharges at a given location along a stream channel into corresponding stages or vice versa. This is accomplished via a relation between stage and discharge which applies to that particular location. Such a relation or "rating curve" is usually developed empirically from a number of previous streamflow measurements and the corresponding stages.

Unfortunately, the observed measurements of stage and discharge will not usually form a unique relation; i.e., a single value of stage does not correspond to a single value of discharge. This is illustrated in Figure 1. Deviations of the measurements from a single-value rating curve can be the result of such factors as:

1) Discharge measurement errors (Carter and Anderson, 1963);

2) Shifting control (Linsley, *et al*., 1949; Corbett, 1943) due to:

   a. scour and/or fill (Simons, *et al*., 1973);
   b. alluvial bed form changes (Simons and Richardson, 1961; Dawdy, 1961);
   c. seasonal changes in vegetative growth along the channel; and
   d. ice formation;

3) Variable energy slope (Linsley, *et al*., 1949; Corbett, 1943) due to:

   a. variable backwater from a downstream tributary, reservoir, or tidal estuary;
   b. conversion of discharge into or out of channel storage;
   c. return of overbank flow; and
   d. flow accelerations of unsteady, nonuniform flow (Henderson, 1966).

This study is concerned with only the effect of the last of the above factors; i.e., a variable energy slope caused by flow accelerations of unsteady, nonuniform flow (changing discharge).

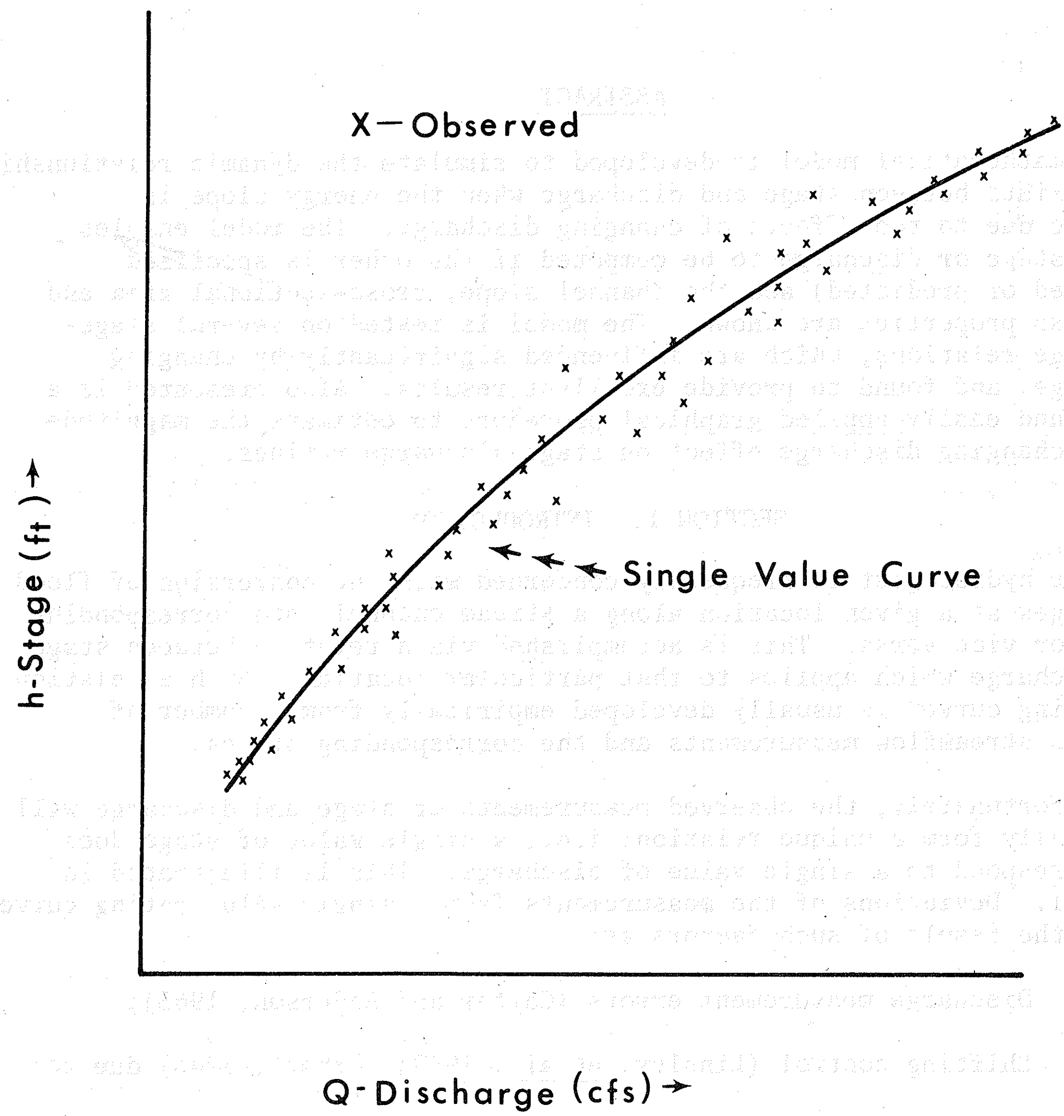

Figure 1. Stage-Discharge Relation.

Several authors; e.g., Linsley, et al. (1949), Corbett (1943), and Henderson (1966), have dealt with the stage-discharge relation influenced by a variable energy slope. It is well known that changing discharge can produce a so-called "hysteresis loop" in the stage-discharge rating curve such as the one shown in Fig. 2. For this type of rating curve, there exist two different stages for each discharge. The lesser of the two stage values is associated with the rising limb of the discharge hydrograph and the greater value occurs during the recession of the discharge. A hysteresis loop, which is caused by a variable energy slope due to changing discharge, is termed a "dynamic loop" herein since it occurs because of the changing or dynamic nature of the flood discharge.

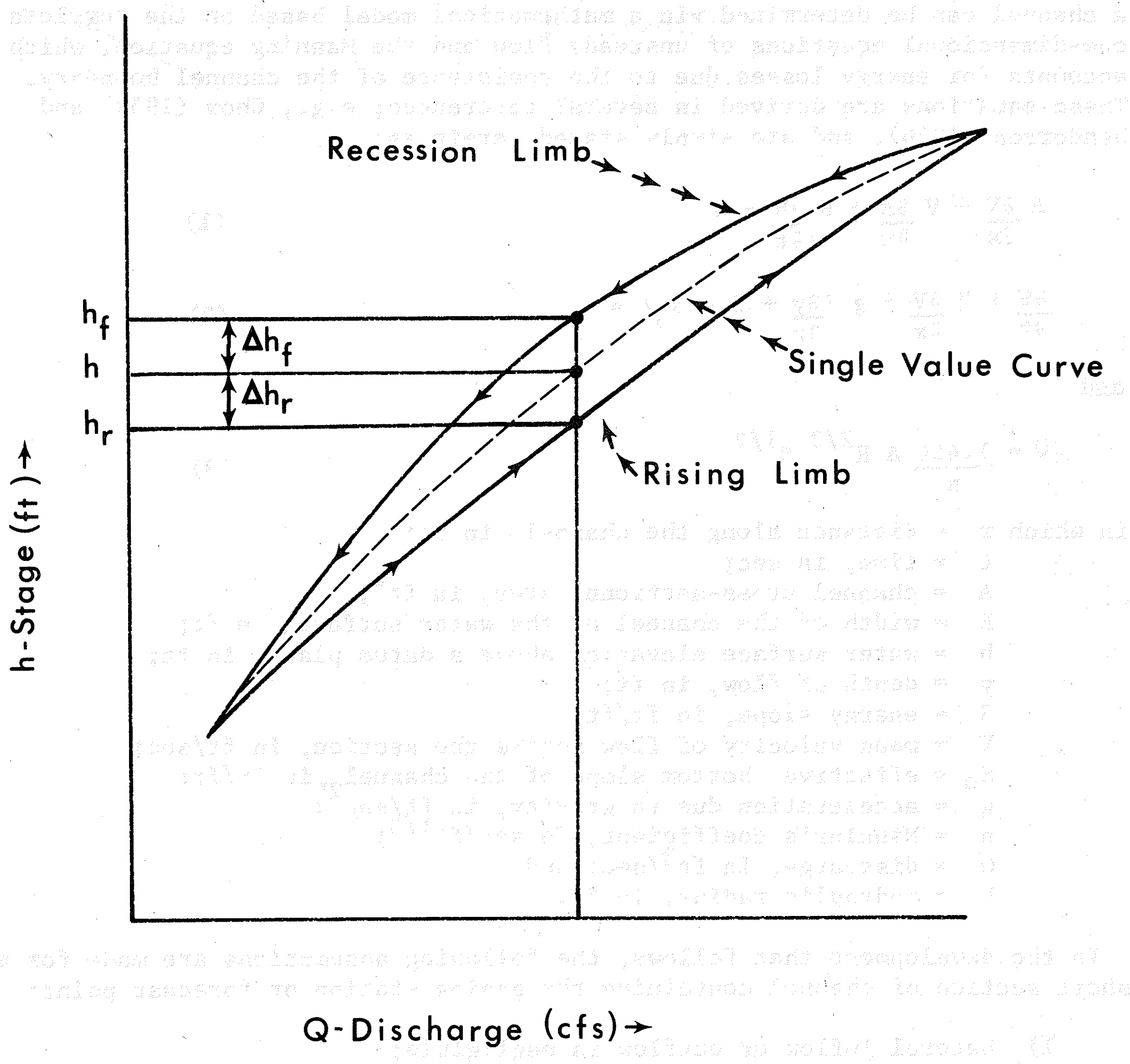

Figure 2. Stage – Discharge Relation with Hysteresis Loop Due to Changing Discharge.

SECTION 2. MODEL DEVELOPMENT

A unique, dynamic stage-discharge relation for a particular location along a channel can be determined via a mathematical model based on the complete one-dimensional equations of unsteady flow and the Manning equation, which accounts for energy losses due to the resistance of the channel boundary. These equations are derived in several references; e.g., Chow (1959) and Henderson (1966), and are simply stated herein as:

$$A \frac{\partial V}{\partial x} + V \frac{\partial A}{\partial x} + B \frac{\partial h}{\partial t} = 0 \qquad (1)$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial y}{\partial x} + S - S_o\right) = 0 \qquad (2)$$

and

$$Q = \frac{1.486}{n} A R^{2/3} S^{1/2} \qquad (3)$$

in which x = distance along the channel, in ft;
t = time, in sec;
A = channel cross-sectional area, in ft^2;
B = width of the channel at the water surface, in ft;
h = water surface elevation above a datum plane, in ft;
y = depth of flow, in ft;
S = energy slope, in ft/ft;
V = mean velocity of flow across the section, in ft/sec;
S_o = effective bottom slope of the channel, in ft/ft;
g = acceleration due to gravity, in ft/sec^2;
n = Manning's coefficient, in $sec/ft^{1/3}$;
Q = discharge, in ft^3/sec; and
R = hydraulic radius, in ft.

In the development that follows, the following assumptions are made for a short section of channel containing the gaging station or forecast point:

1) Lateral inflow or outflow is negligible;
2) The channel width is essentially constant; i.e., $\partial B/\partial x \approx 0$;
3) Energy losses from channel friction and turbulence are described by the Manning equation;
4) The geometry of the section is essentially permanent; i.e., any scour or fill is negligible;
5) The bulk of the flood wave is moving approximately as a kinematic wave which implies that the energy slope is approximately equal to the channel bottom slope; and
6) The flow at the section is controlled by the channel geometry, friction, and bottom slope and by the shape of the flood wave.

An expression for the energy slope is obtained by rearranging Eq (2) in the following form:

$$S = S_o - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \tag{4}$$

The four terms on the right side of Eq (4) represent the component slopes which produce the variable energy slope S due to changing discharge. From left to right respectively, the four slopes are attributed to: gravity force, pressure force, convective (spatial) acceleration, and local (temporal) acceleration.

Using assumption (2), Eq (1) may be expressed as:

$$\frac{\partial V}{\partial x} = -\frac{VB}{A}\frac{\partial y}{\partial x} - \frac{B}{A}\frac{\partial h}{\partial t} \tag{5}$$

Upon substituting Eq (5) and V=Q/A in Eq (4), the following expression for the variable energy slope S is obtained:

$$S = S_o + \left(\frac{BQ^2}{gA^3} - 1\right)\frac{\partial y}{\partial x} + \frac{BQ}{gA^2}\frac{\partial h}{\partial t} - \frac{1}{g}\frac{\partial (Q/A)}{\partial t} \tag{6}$$

Information concerning the characteristics of either the stage or discharge hydrograph generally is available only at the location for which the rating curve is required. Such lack of spatial resolution of the hydrograph requires that the derivative terms with respect to x in Eqs (4) and (5) be replaced by equivalent expressions which can be evaluated from the available information. Henderson (1966) shows that, if the bulk of the flood wave is moving approximately as a kinematic wave, the following expression may be used to eliminate the need for spatial resolution of the specified hydrograph:

$$\frac{\partial y}{\partial x} = -\frac{1}{c}\frac{\partial h}{\partial t} - \frac{2}{3}\frac{S_o}{r^2} \tag{7}$$

in which c is the kinematic wave velocity and r is the ratio of the channel bottom slope to an average wave slope.

The kinematic wave velocity c may be determined from observations of the time interval between equal rises in stage, h, at gaging stations along the channel. Also, c may be computed from a relationship given by Henderson (1966) and Chow (1959):

$$c = \frac{1}{B}\frac{dQ}{dh}, \qquad (8)$$

where dQ/dh is the slope of the single-value rating curve. If the channel is assumed to be prismatic, the kinematic wave velocity can be computed directly by substituting Eq (3) in Eq (8). After differentiation, the following is obtained:

$$c = KV = K\ Q/A, \qquad (9)$$

where

$$K = \frac{5}{3} - \frac{2A}{3B^2}\ dB/dh, \qquad (10)$$

and the hydraulic radius R is approximated by the hydraulic depth D; i.e.,

$$R \approx D = A/B. \qquad (11)$$

This is a good approximation of the hydraulic radius for large channels. If the hydraulic radius is used in lieu of the hydraulic depth, the term dB/dh in Eq(10) would be replaced by dP/dh, where P is the wetted perimeter of the channel cross-section; and, the term B^2 would be replaced by the product PB. From an inspection of Eq (10) it is evident that K has an upper limit of about 1.7 when dB/dh is negligible and a lower limit of about 1.3 for a triangular-shaped channel. It has been observed that K can be approximated as 1.3 for many natural channels (Corbett, 1943; Linsley, et al., 1949). Although there are a number of methods for determining the kinematic wave velocity, Eq (9) is used in this study.

It should be noted that the wave velocity is frequently greater than that computed by Eq (9); however, this may result from the fact that the wave is more nearly a dynamic wave than a kinematic wave. The dynamic wave velocity as given by Henderson (1966) is:

$$c_d = V \pm (g\ A/B)^{1/2}. \qquad (12)$$

A comparison of Eq (9) with Eq (12) indicates that the dynamic wave velocity can be considerably larger than the kinematic wave velocity, particularly for large A/B ratios. The dynamic wave predominates over the kinematic wave when the channel flow is pooled such as behind a dam or other constriction in the channel. Under this condition the flow is not controlled by channel geometry, friction, bottom slope, and the shape of the flood wave; therefore, flow in pooled areas where dynamic waves are formed is not treated herein. In some instances when pooling occurs only during the lower stages, the wave velocity changes from that of a dynamic wave to more nearly that of a kinematic wave as the stage increases. The portion of the rating curve, associated with the higher stages when the kinematic wave approximation is more applicable, could be determined approximately by the method developed herein.

The value of r in Eq (7) may be taken as a constant for a particular channel. Typical values of r range from 10 to 100. It is used in Eq (7) as part of a small correction which accounts for the fact that a typical flood wave is not exactly a kinematic wave. To arrive at a value for r, the wave slope is approximated from the characteristics of a typical flood event for a particular channel location. The wave slope is determined by dividing the height of the wave by its half-length, the latter obtained by assuming that the wave travels as a kinematic wave during the interval of time from the initiation of the wave to the occurrence of the wave peak at the location of concern. The half-length is determined from the product of the average kinematic wave velocity and the time to peak stage. Eq (9) is used to determine the average kinematic velocity, with Q and A taken as the average values during the flood event and K assumed equal to 1.3. Hence, the following expression is obtained for evaluating r:

$$r = \frac{56200\ (Q_p + Q_o)\ \tau\ S_o}{(h_p - h_o)\ \bar{A}} \qquad (13)$$

where:

Q_o = discharge at beginning of typical flood, in cfs;

Q_p = peak discharge for typical flood, in cfs;

h_o = stage at beginning of typical flood, in ft;

h_p = peak stage of typical flood, in ft;

$\bar{A}$ = cross-sectional area associated with the average stage, $(h_p + h_o)/2$, in ft^2; and

τ = interval of time from beginning of rise in stage until the occurrence of the peak stage, in days.

Since c and r are defined by Eqs (9) and (13), Eq (7) can be substituted in Eq (6), with the partial derivatives in the latter replaced by finite difference notation. After some rearrangement, the following equation is obtained:

$$S = S_o + \left[\frac{A}{KQ} + \left(1 - \frac{1}{K}\right) \frac{BQ}{gA^2} \right] \delta h_s + \frac{Q'/A' - Q/A}{g\Delta t} + \frac{2}{3}\frac{S_o}{r^2}\left(1 - \frac{BQ^2}{gA^3}\right) \qquad (14)$$

where:

Δt = small interval of time, in secs;

Q = discharge at time $t - \Delta t$, in cfs;

A = cross-sectional area at time $t - \Delta t$, in ft^2; and

δh_s = change in water surface elevation during the time interval Δt, in ft/sec ($h_s = \frac{h-h'}{\Delta t}$, where h′ is the stage at time $t - \Delta t$).

Eq (14) is the expression for the variable energy slope S which is caused by varying discharge. All the terms on the right side of the equation except S_o account for the effect of the dynamic characteristic of the flow. If the flow is steady (unchanging with time) the energy slope is constant and equivalent to the bottom slope, S_o. This is evident from Eq (14) since all terms on the right side of the equation except the first term vanish when the flow is steady; i.e., $Q' = Q$, $\delta h_s = 0$, and r is infinitely large since the wave slope vanishes for steady uniform flow.

An expression may be derived for the dynamic relation between stage and discharge when the energy slope is variable due to changing discharge. This can be obtained by substituting the hydraulic depth for the hydraulic radius in Eq (3) and using Eq (14) for the variable energy slope; i.e.,

$$Q - 1.486 \frac{AD^{2/3}}{n} \left[S_o + \left[\frac{A}{KQ} + \left(1 - \frac{1}{K} \right) \frac{BQ}{gA^2} \right] \delta h_s + \frac{Q'/A' - Q/A}{g\Delta t} + \frac{2S_o}{3r^2} \left(1 - \frac{BQ^2}{gA^3} \right) \right]^{1/2} = 0 \qquad (15)$$

This equation differs from others given in the literature; e.g., Linsley, et al. (1949), Corbett (1943), which include only terms equivalent to the first two terms of the variable energy slope of Eq (6).

Eq (15) forms the basis of a model that can be used to determine either discharge when the rate of change of stage is known (as in stream gaging) or stage when the rate of change of discharge is known (as in stream forecasting). These alternative conditions are denoted respectively as Case A and Case B. A brief description of each follows:

Case A ________ The discharge hydrograph is determined from a specified (observed) stage hydrograph. In this case, Q is unknown in Eq (15); the known quantities consist of constants (S_o, r, g, Δt), known functions (A, B, n, K) of the specified stage h, and known quantities (Q', A', h') associated with the time $t-\Delta t$.

Case B ———— The stage hydrograph is determined from a specified (predicted) discharge hydrograph. In this situation, h is the unknown in Eq (15) and the terms (A, B, n, K) are functions of h. The known quantities consist of constants (S_o, r, g, Δt), the specified discharge Q, and the quantities (Q', A', h') associated with the time $t-\Delta t$.

The unknowns Q and h in Case A and Case B respectively, are not expressed in an explicit manner; therefore, an iterative solution via a digital computer is required. A very efficient method for obtaining a solution is by Newton Iteration (Issacson and Keller, 1966). Details of the application of the Newton Iteration technique to the solution of Eq (15) for Case A and Case B are presented in Appendix A.

A Fortran IV computer program for modeling the dynamic relationship between stage and discharge for both Case A and Case B is presented in Appendix B. The program is written for execution on a CDC 6600 computer.

SECTION 3. APPLICATION OF MODEL

3.1 DATA REQUIREMENTS

In the application of Eq (15) to determine the dynamic relation of stage and discharge, the following information is required:

1) the effective bottom slope (S_0);

2) the cross-sectional area (A) and the surface width (B) as functions of the stage;

3) the Manning's coefficient (n) as a function of the stage; and

4) the specified (observed or predicted) stage or discharge hydrograph.

3.1.1 EFFECTIVE BOTTOM SLOPE

In natural channels, the bottom slope is often quite irregular due to the presence of deep bends, pools, crossings, shallow riffles and other irregularities that occur along the length of the channel. The bottom slope (S_0), used in Eq (15), is the slope that is effective in controlling the flow. A good measure of the effective bottom slope is the water surface profile associated with either the condition of low flow or peak flow. The low flow profile is obtained from the minimum recorded stages at gaging stations both up and downstream from the location for which the bottom slope is required. The peak flow profile is obtained from the maximum recorded stages. The low flow profile is preferred unless it is noticeably affected by the irregularity of the channel bottom, in which case the peak flow profile should be used. The effective channel bottom slope is computed by subtracting the downstream stage from the upstream stage, both referenced to the same datum, and dividing by the distance (in feet) separating the two gage locations.

3.1.2 CROSS-SECTIONAL AREA AND SURFACE WIDTH

These are geometric properties of the channel location and can be obtained from either hydrographic surveys or stream gaging records. Each is a function of the stage. By plotting values of A and B versus the stage, the extent of the variation can be determined. Approximately, three to ten values of A, B, and h will describe adequately the variation of area and width with stage, and linear interpolation will provide any intermediate values required during the computation of the stage-discharge relation.

It is assumed that the properties of the cross section are known throughout the duration of the specified stage or discharge hydrograph. Thus, if there is significant scour or fill at the cross section during this period, the computed stage-discharge relationship will be inaccurate because of the unknown variations of the cross-sectional properties. The primary effect of scour or fill on the stage-discharge relation is a change in the position of the mean single-value rating curve. The position is lowered when scour occurs and raised when fill occurs. Scour and fill have only a secondary effect on the shape and magnitude of the dynamic loop, since the two most dominant terms (S_o and δh_s) of the variable energy slope are not affected substantially by relatively small changes in the cross-sectional area brought about by scour and fill. Gradual long term changes in the cross-sectional properties which are monitored by periodic streamflow measurements should not prevent the use of the dynamic model. Significant scour and fill which produces rapid changes in the cross section will present difficulties in obtaining accurate stage-discharge relations; however, the dynamic model can provide a good approximation of the shape and magnitude of the dynamic loop.

3.1.3 MANNING'S COEFFICIENT, n

Stage-discharge measurements can be used to compute Manning's n via the following rearrangement of Eq (3); viz.,

$$n = 1.486\,\frac{AD^{2/3}\,S_o^{1/2}}{Q} \qquad (16)$$

where Q is the discharge associated with the mean single-value rating curve. It should be noted in Eq (16) that S_o, the effective channel bottom slope, is assumed to be equal to the energy slope. This curve is simply "sketched in" between (h,Q) points which have been obtained from discharge measurements. There should be points associated with both rising and falling discharges in order to obtain a representative mean rating curve.

Generally, n will vary with stage since the roughness properties of the channel change with stage due to (a) the variation of vegetation and channel bank irregularities with elevation and (b) the partial correlation of alluvial bed form changes with stage.

The following linear relation between n and stage is used herein:

$$n = n_{Lo} + \frac{(n_{L1} - n_{Lo})}{(h_{L1} - h_{Lo})}\,(h - h_{Lo}), \qquad (17)$$

where n_{L0} is the n value associated with the stage h_{L0} and n_{L1} is the value associated with the stage h_{L1}. Equation (17) applies throughout the range in stage, $h_{L0}<h<h_{L1}$. Sometimes the n,h relation varies sufficiently to require two linear relations; i.e., Eq (17) for a lower range of stage $h_{L0}<h<h_{L1}$ and a similar equation for an upper range in stage $h_{U0}<h<h_{U1}$.

If the n,h relation is subject to change due to seasonal changes in vegetation, man-made changes in the roughness properties of the channel bank or over-bank, or changes in alluvial bed forms, the relation may be updated using the most recent discharge measurements. The value of Q used to compute the updated value of n via Eq.(16) should be corrected to eliminate the dynamic effect. The corrected value of the discharge is determined from the following:

$$Q = Q_m - \frac{1.486}{n} A D^{2/3} \left(S^{1/2} - S_o^{1/2} \right) \tag{18}$$

where Q_m is the measured discharge. The values of A, D, and n are obtained from their currently known relationships with stage for the particular stage associated with Q_m. The energy slope S is determined from Eq.(13) in which Q is replaced by Q_m.

3.1.4 SPECIFIED HYDROGRAPH

The specified (known) stage hydrograph is obtained from a measured stage versus time record. The record may be daily, hourly, or have a smaller temporal resolution. The degree of resolution should be such as to provide an adequate definition of the variation of stage with time.

The specified discharge hydrograph is obtained from a record of the measured discharges or from a forecast discharge hydrograph. The latter may be the product of some flow routing technique which does not provide both discharge and stage; e.g., the Muskingham method, the Tatum method.

Δt in Eq.(15) should not be greater than the temporal resolution of the specified stage or discharge hydrograph. If the specified hydrograph has irregularities, a Δt which is less than the resolution of the hydrograph allows the resulting irregular stage-discharge rating curve to be defined more accurately. In this study, a Δt of 3 hours is used with specified hydrographs that have a 24-hour resolution.

3.2 TEST APPLICATIONS

The dynamic relation between stage and discharge is modeled herein for locations on the lower Mississippi, Atchafalaya, and Red Rivers. In each of these locations, the stage-discharge relation is influenced by a variable energy slope due to changing discharge.

The necessary information (bottom slope, cross-sectional area and surface width as functions of stage, and Manning's n) for each of the locations is given in Table 1. The cross-sectional properties represent time-average values. The departures of the cross-sectional properties for any 1 year from the average values shown in the table are less than approximately 3 percent. If the departures were considerably larger, cross-sectional proper ties applicable to each year would improve the accuracy of the computed stage-discharge relations. At each location, the floods, which were selected for testing the model, caused minimal scour and fill. The computed values

TABLE 1.--CROSS-SECTION AND HYDRAULIC DATA FOR LOCATIONS ON THE LOWER MISSISSIPPI, RED, AND ATCHAFALAYA RIVERS

Location	S_o	Year	n_{L0}	n_{L1}	h_{L0}	h_{L1}	n_{U0}	n_{U1}	h_{U0}	h_{U1}	h	A	B
Mississippi River		1966	0.0148	0.01354	5.	50.					16.0	72500.	3000.
Tarbert Landing, Louisiana	0.0000143	1967	0.01560	0.01542	5.	50.					34.0	134000.	3540.
		1969	0.0159	0.01392	5.	50.					41.2	164000.	3630.
											48.0	200000.	3690.
Mississippi River Red River Landing Louisiana	0.0000143	1963	0.0108	0.0113	15.	50.					15.0	55300.	3020.
											16.0	58000.	3060.
											18.0	64400.	3160.
											19.0	68000.	3360.
											30.0	104000.	3420.
											37.0	133500.	4310.
											44.0	163300.	4400.
Red River Alexandria, La.	0.0000233	1964	0.031	0.018	45.	55.	0.0180	0.0112	55.	75.	45.0	4800.	463.
		1966	0.031	0.018	45.	55.	0.0180	0.0130	55.	80.	52.0	8300.	528.
											60.0	13000.	576.
											80.0	26600.	696.
Atchafalaya River											1.0	20000.	960.
Simmisport, La.	0.0000211	1964	0.0395	0.0196	5.	16.1	0.0196	0.0143	16.1	40.	20.0	42100.	1250.
											40.0	66000.	1425.

of Manning's n vary somewhat for each flood. These variations are due apparently to the changing alluvial bed forms being not only a function of stage but also dependent on water temperature, discharge, and sediment load. Also, changes in bank vegetation due to seasonal effects may be responsible for some of the variability.

Using the information shown in Table 1 and observed stage-hydrographs, stage-discharge rating curves and discharge hydrographs were computed for selected floods at the locations discussed below.

3.2.1 MISSISSIPPI RIVER, TARBERT LANDING, LA.

Figures 3-8 illustrate computed stage-discharge relationships and discharge hydrographs for floods which occurred during the years 1966, 1967, and 1969.

In Fig. 3, the solid line representing the computed rating curve for the flood of 1966 was obtained by 1) using Eq (15) to compute the discharge hydrograph shown in Fig. 4 from the specified stage hydrograph shown in the insert of Fig. 3 and 2) plotting the observed stage against the computed discharge. Measured values of stage and discharge are shown in Fig. 3 also. The computed rating curve has a substantial dynamic loop; the discharge value of 550,000 cfs has two associated stages, one on the rising limb and one on the recession limb of the specified stage hydrograph, which differ by approximately 7 feet. The loop rating curve has irregularities which reflect each variation in the rate of change of the stage hydrograph. A comparison of the computed discharges shown in Fig. 4 with the observed discharges is quite good, the root mean square (rms) error being less than 4 percent.

The 1967 multiple-peaked stage hydrograph, shown in the insert of Fig. 5, yields the complex computed stage-discharge relation shown in the figure. Each peak of the specified stage hydrograph produces a different loop in the rating curve. As shown in Fig. 6 the computed discharge hydrograph agrees quite well with the discharge observations, the rms error being only about 3 percent.

The 1969 observed stage hydrograph shown in the insert of figure 7, yields the single-looped rating curve of the figure. The uniformity of the stage hydrograph produces the relatively smooth computed rating curve. A comparison of computed and observed discharges is shown in figure 8. The rms error is about 2 percent.

3.2.2 MISSISSIPPI RIVER, RED RIVER LANDING, LA.

The 1963 observed stage hydrograph, shown in the insert of Fig. 9, yields the computed stage-discharge rating curve of the figure. The dynamic loop is rather significant; for a discharge of 450,000 cfs, the maximum difference between stages of the rising and falling limbs of the hydrograph is about 9 feet. This is a result of the rather severe rate of change of stage which exists for both the rising and the recession limbs of the stage hydrograph. Also, noticeable variations in the rate of change of the stage hydrograph are reflected as irregularities superimposed on an otherwise

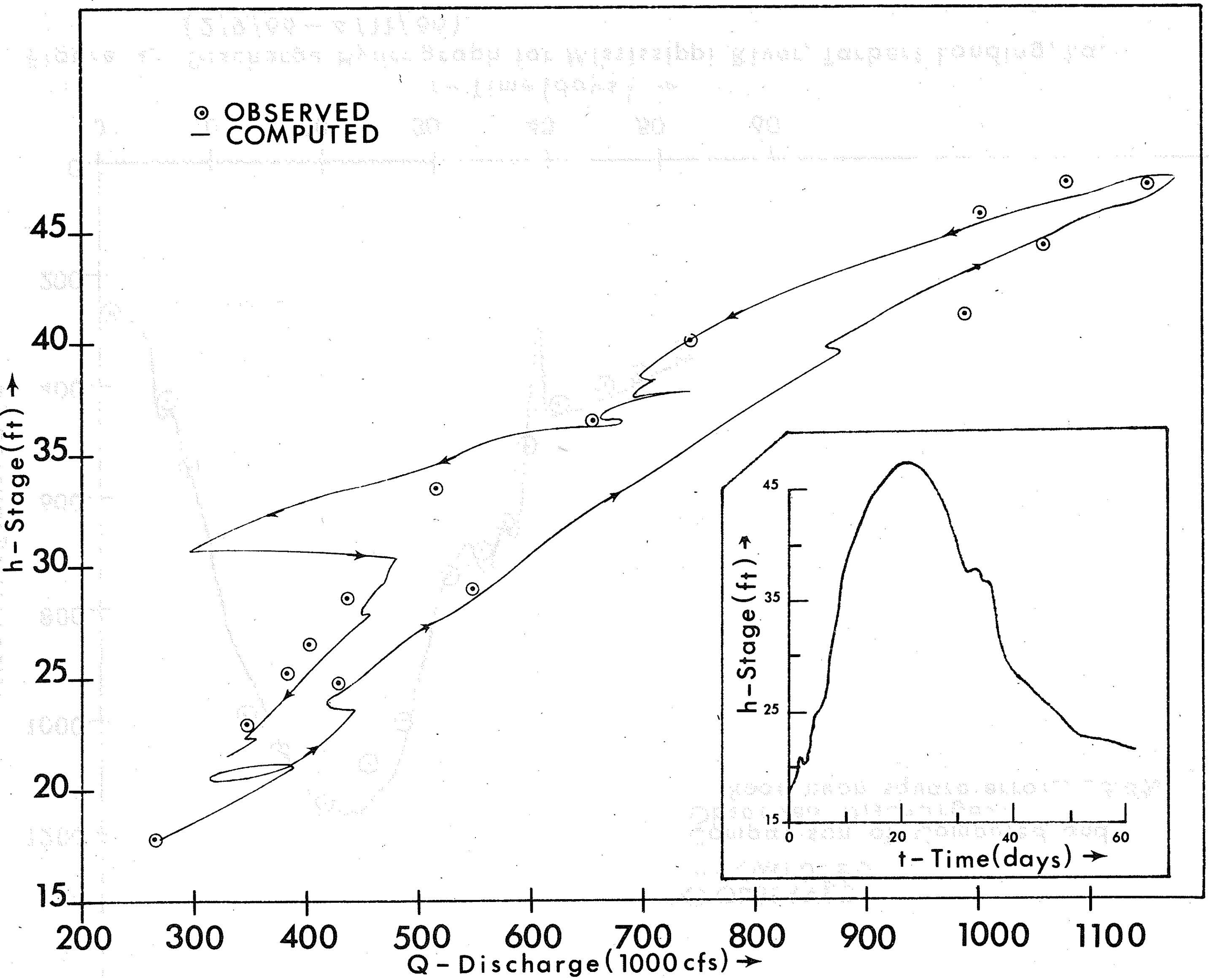

Figure 3. Stage-Discharge Relation for Mississippi River, Tarbert Landing, La. (2/9/66 – 4/11/66).

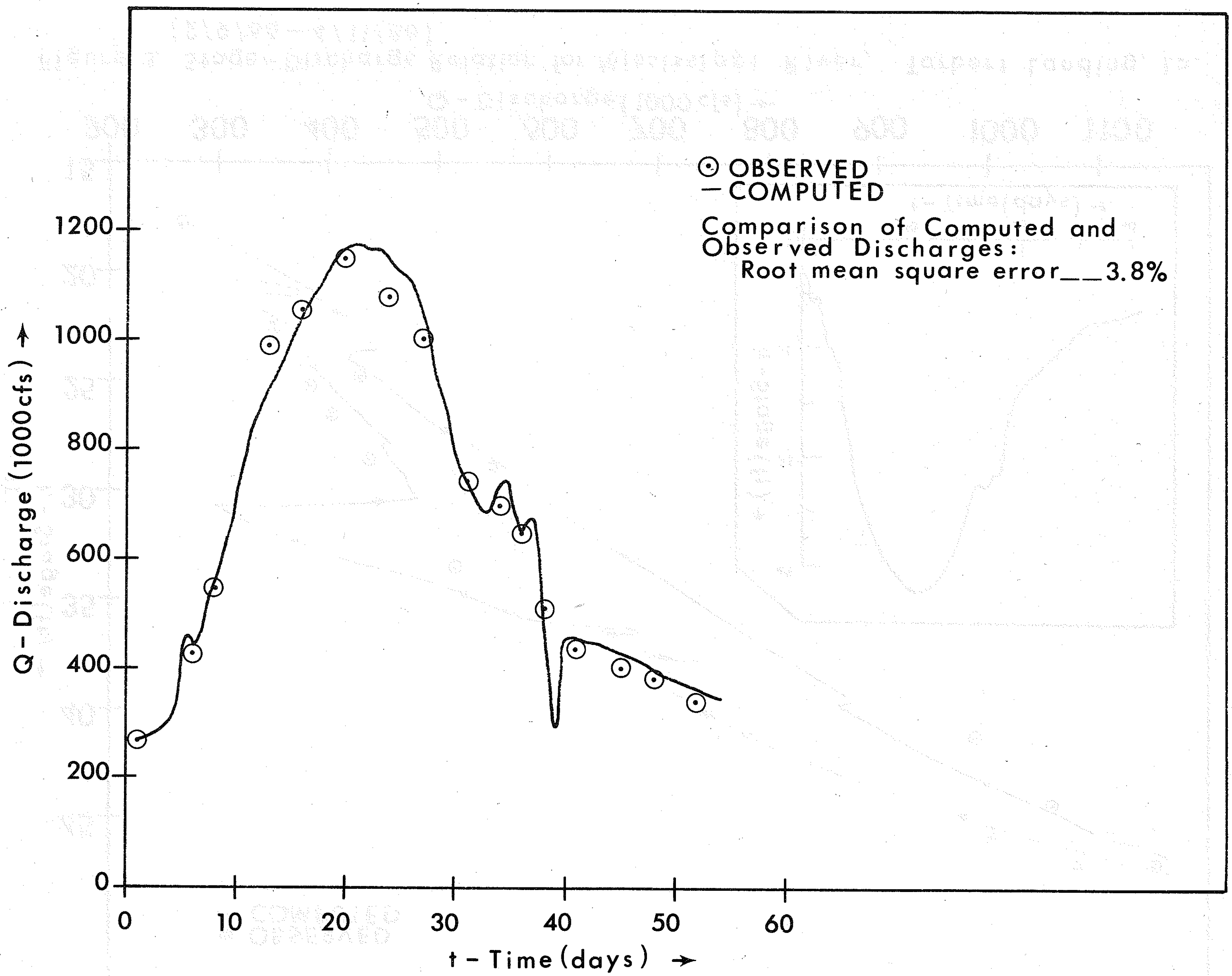

Figure 4. Discharge Hydrograph for Mississippi River, Tarbert Landing, La. (2/9/66 – 4/11/66).

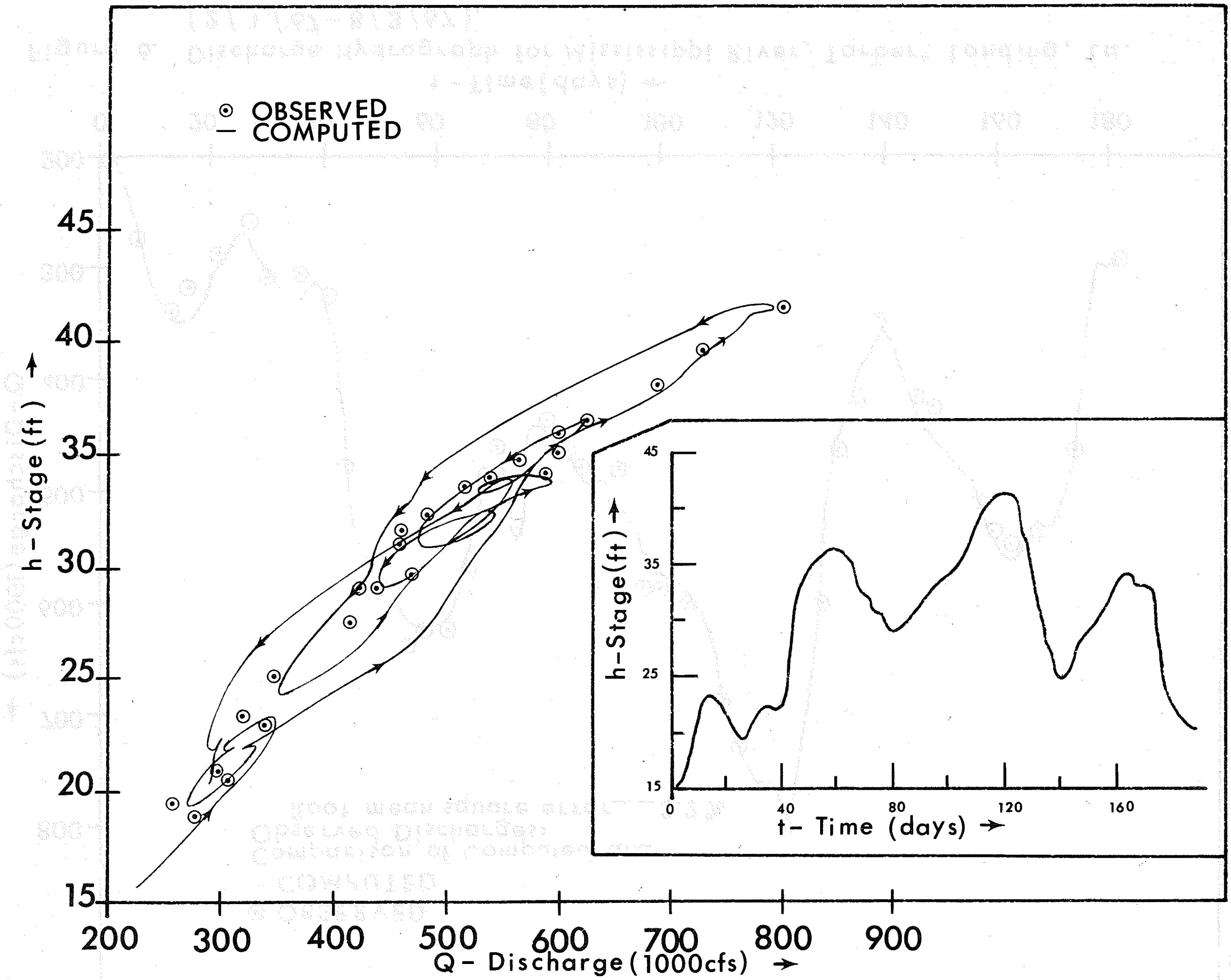

Figure 5. Stage - Discharge Relation for Mississippi River, Tarbert Landing, La. (2/1/67 — 8/3/67).

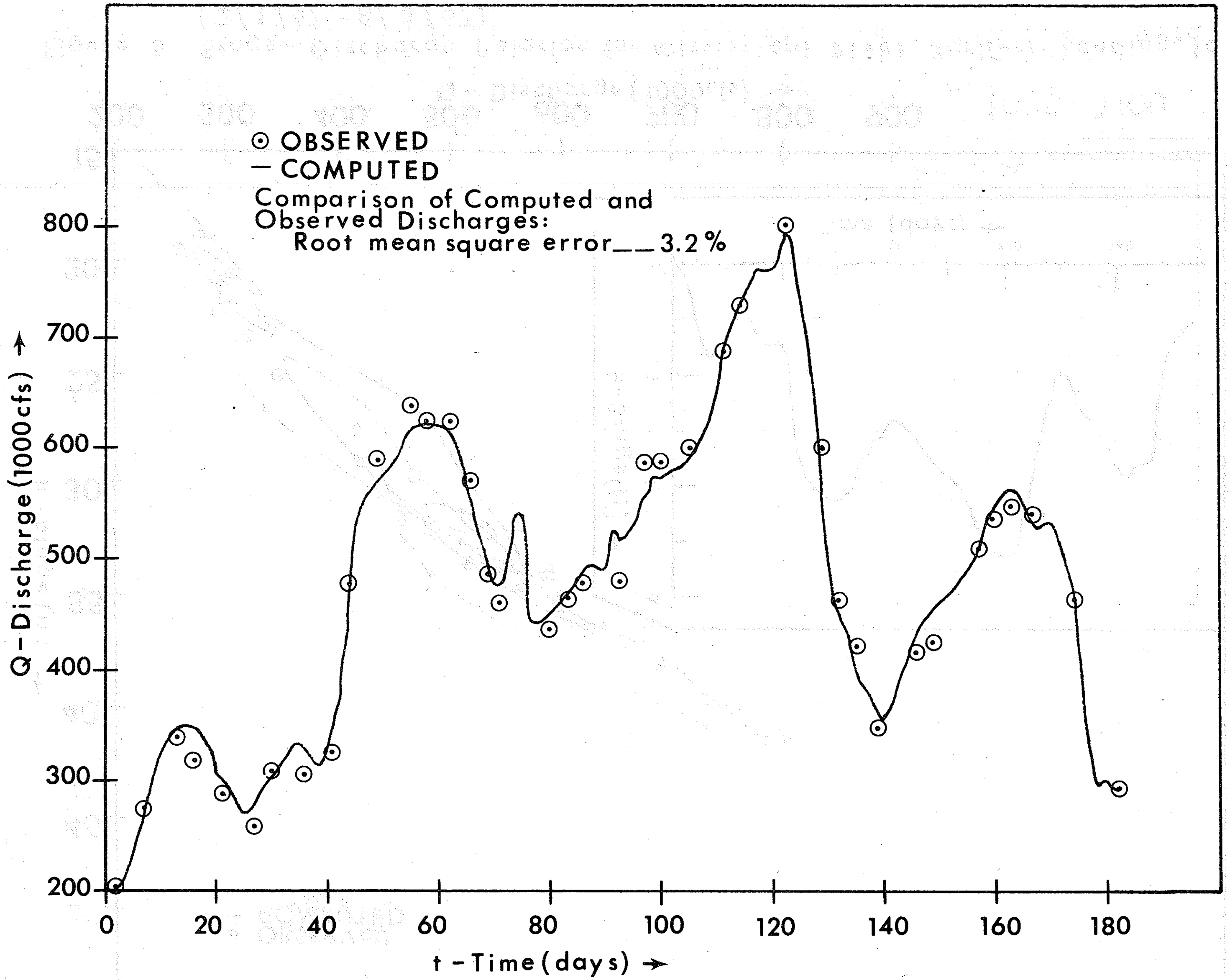

Figure 6. Discharge Hydrograph for Mississippi River, Tarbert Landing, La. (2/1/67 – 8/3/67).

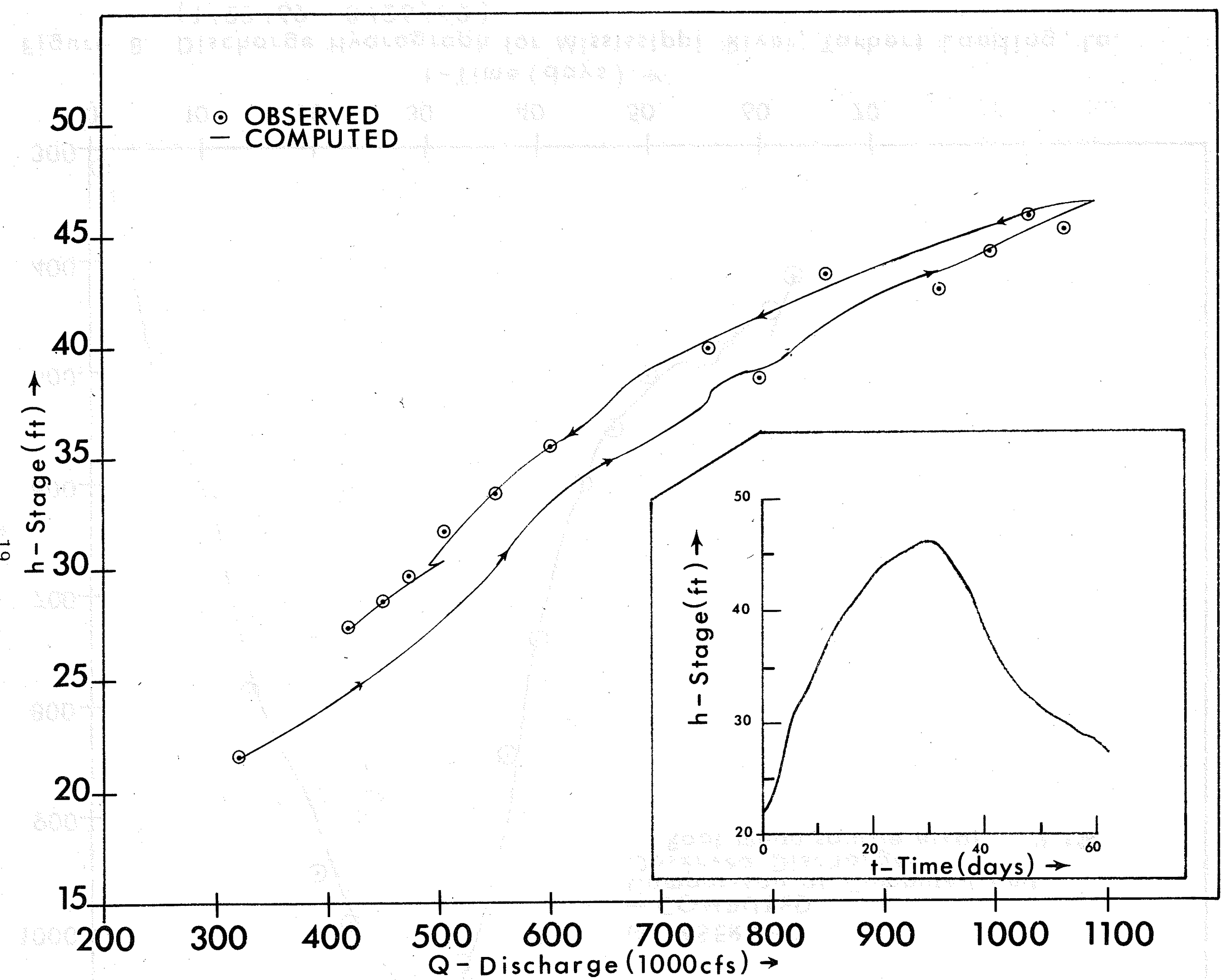

Figure 7. Stage - Discharge Relation for Mississippi River, Tarbert Landing, La. (1/23/69 — 3/26/69).

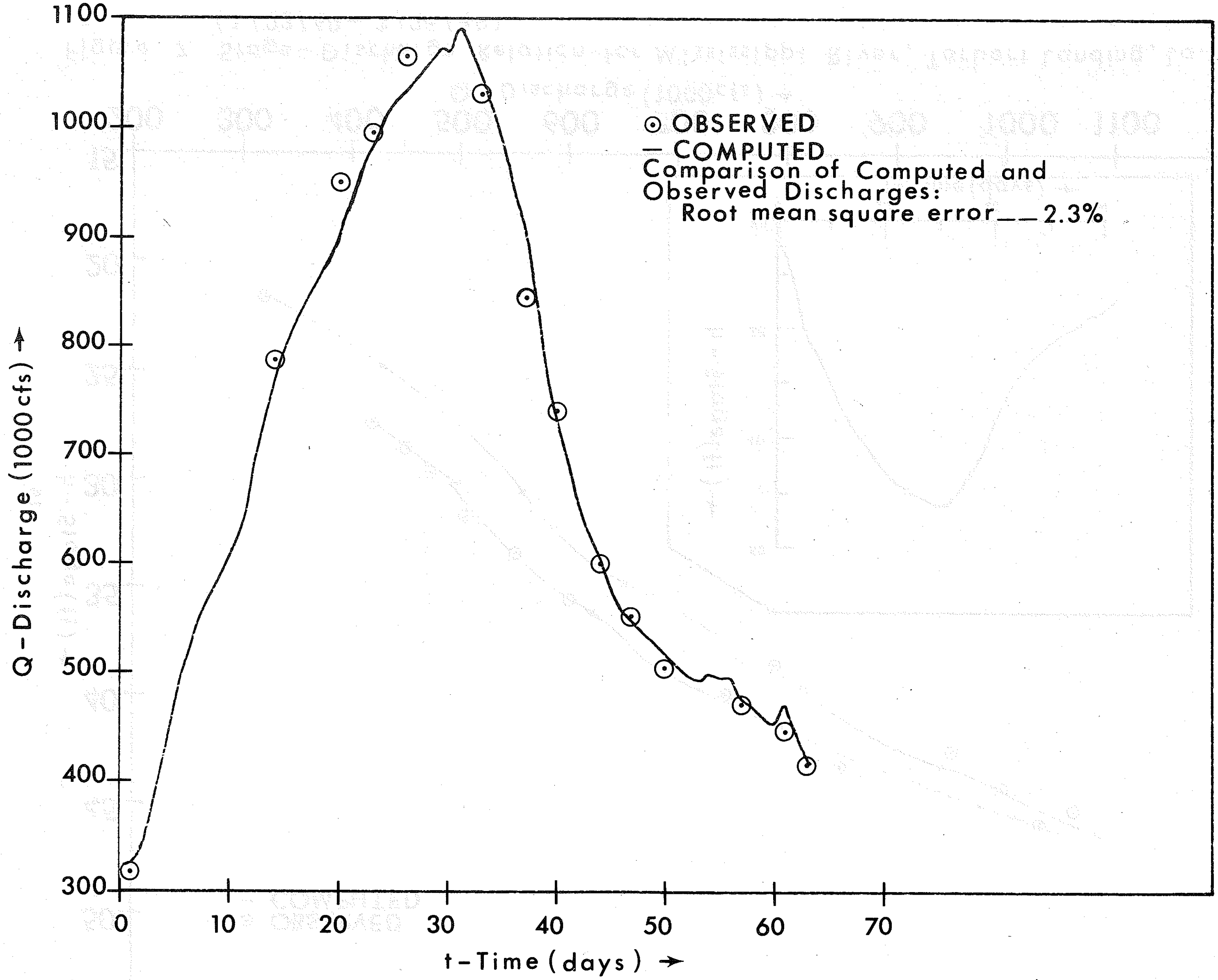

Figure 8. Discharge Hydrograph for Mississippi River, Tarbert Landing, La. (1/23/69 – 3/26/69).

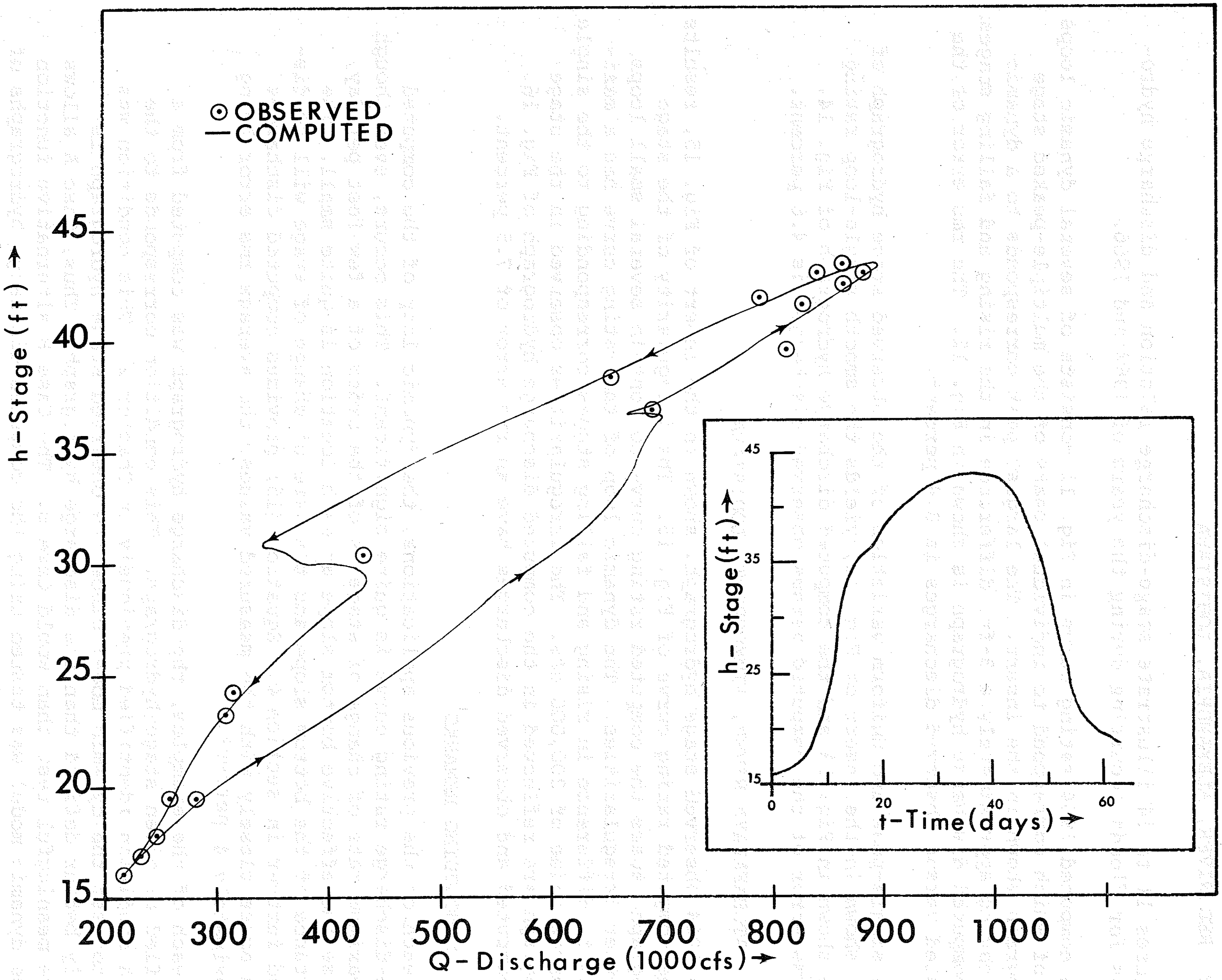

Figure 9. Stage – Discharge Relation for Mississippi River, Red River Landing, La. (3/11/63 – 5/13/63).

smooth single-looped rating curve. The computed discharge hydrograph, shown in Fig. 10, compares with measured discharges quite well, the rms error being 1.6 percent.

3.2.3 RED RIVER, ALEXANDRIA, LOUISIANA

Figures 11 to 14 illustrate stage-discharge relation and discharge hydrographs for floods occurring during the years of 1964 and 1966.

The computed 1964 rating curve in Fig. 11 consists of several dynamic loops each of which correspond to individual peaks of the multiple-peaked stage hydrograph shown in the insert. The largest peak corresponds to a dynamic loop having approximately a 3-ft. difference in the rising and falling stages. The computed discharge hydrograph is shown in Fig. 12. The rms error of the computed versus observed discharges is 8.4 percent.

The single-peak and uniform variation of the observed stage hydrograph of 1966, shown in the insert of Fig. 13, yields the smooth single-loop rating curve shown in Fig. 13 and the computed discharge hydrograph of Fig. 14. The rms error of the computed versus observed discharges is 4.6 percent.

3.2.4 ATCHAFALAYA RIVER, SIMMESPORT, LOUISIANA

The 1964 observed stage hydrograph, shown in the insert of Fig. 15, results in the computed rating curve of Fig. 15. The irregularity of the stage hydrograph causes the computed rating curve to contain several small loops and other irregularities. The dynamic loop of the rating curve has a maximum 8-ft. difference in rising and falling stages corresponding to the single discharge value of 200,000 cfs. The irregularities observed in the stage hydrograph are reflected in the computed discharge hydrograph of Fig. 16. The computed and observed discharges have an rms error of 7.5 percent.

3.2.5 CONCLUDING REMARKS

In each of the previous applications, the dynamic loop of the computed stage-discharge rating curve is quite significant. This occurs, even though the maximum rate of change of stage is of the order of a few feet per day, because the effective bottom slope at each location is quite small. The importance of the bottom slope and the rate of change of stage will be discussed further in section 4. Equation (15) provides computed discharges which agree closely with the measured values, the average rms error being approximately 4 percent.

In each of the examples, the discharge hydrograph was computed from a specified or given stage hydrograph. This condition corresponds to the Case A condition identified previously in Chapter 2. This condition was used to test the dynamic model since the observed stage hydrograph is usually better defined than the discharge hydrograph. Thus, Case A allows a more meaningful test than would Case B. The Case B alternative function of the dynamic model was tested using the computed discharge hydrographs of a few of the previous applications as the specified discharge hydrograph. None of the computed stage hydrographs are presented since each was found to be essentially identical with the corresponding observed stage hydrograph.

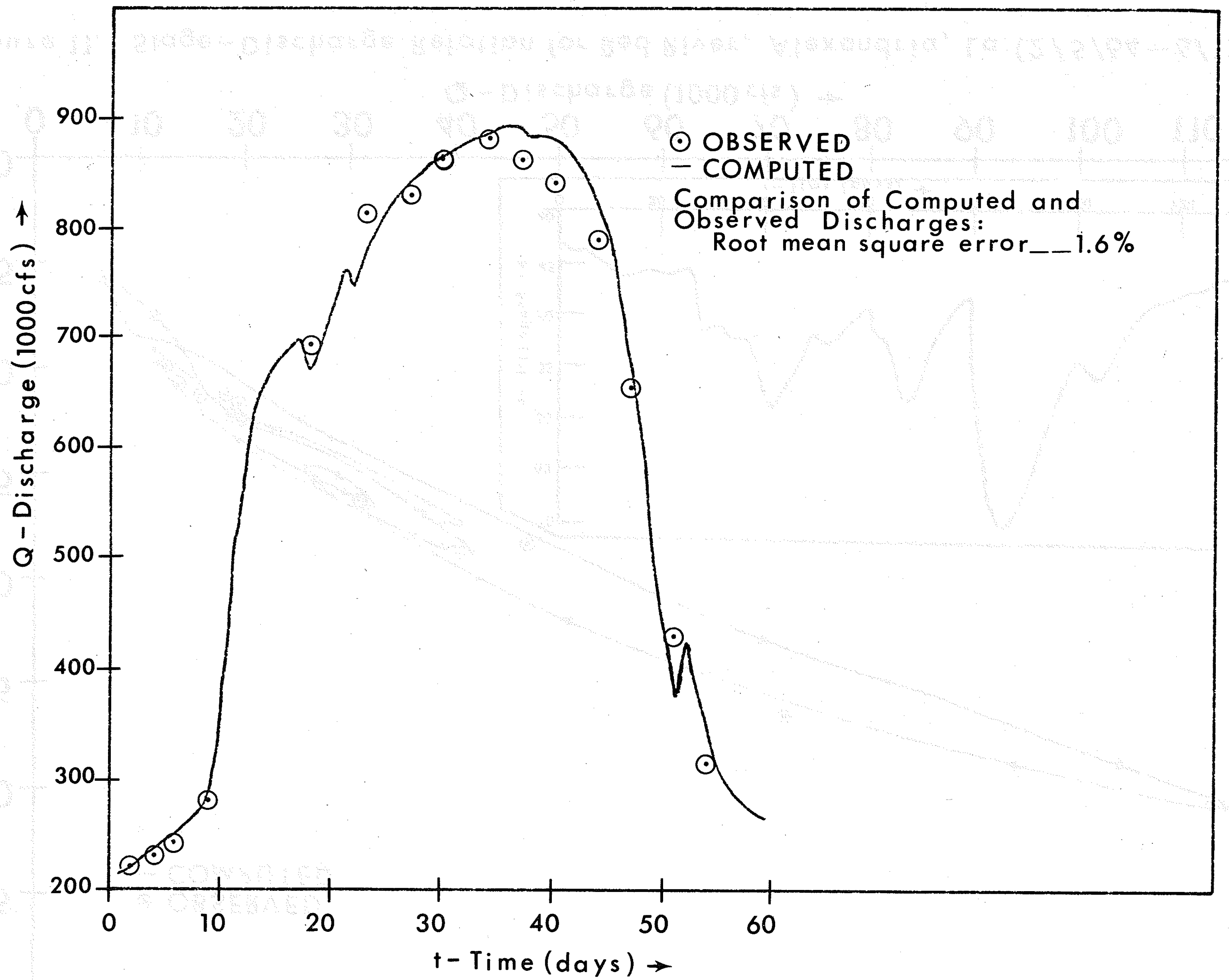

Figure 10. Discharge Hydrograph for Mississippi River, Red River Landing, La. (3/11/63 —5/13/63).

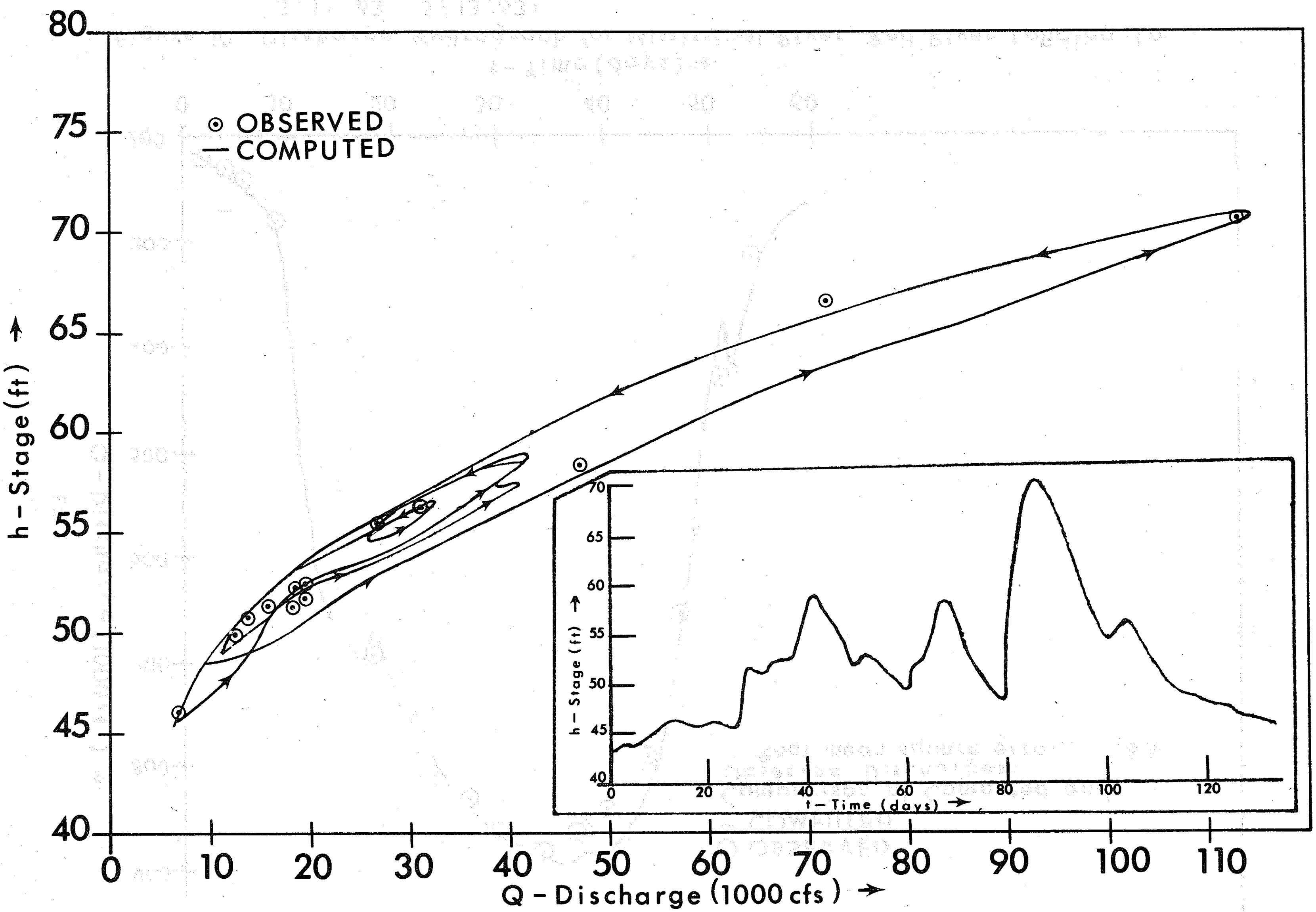

Figure 11. Stage-Discharge Relation for Red River, Alexandria, La. (2/5/64-6/17/64).

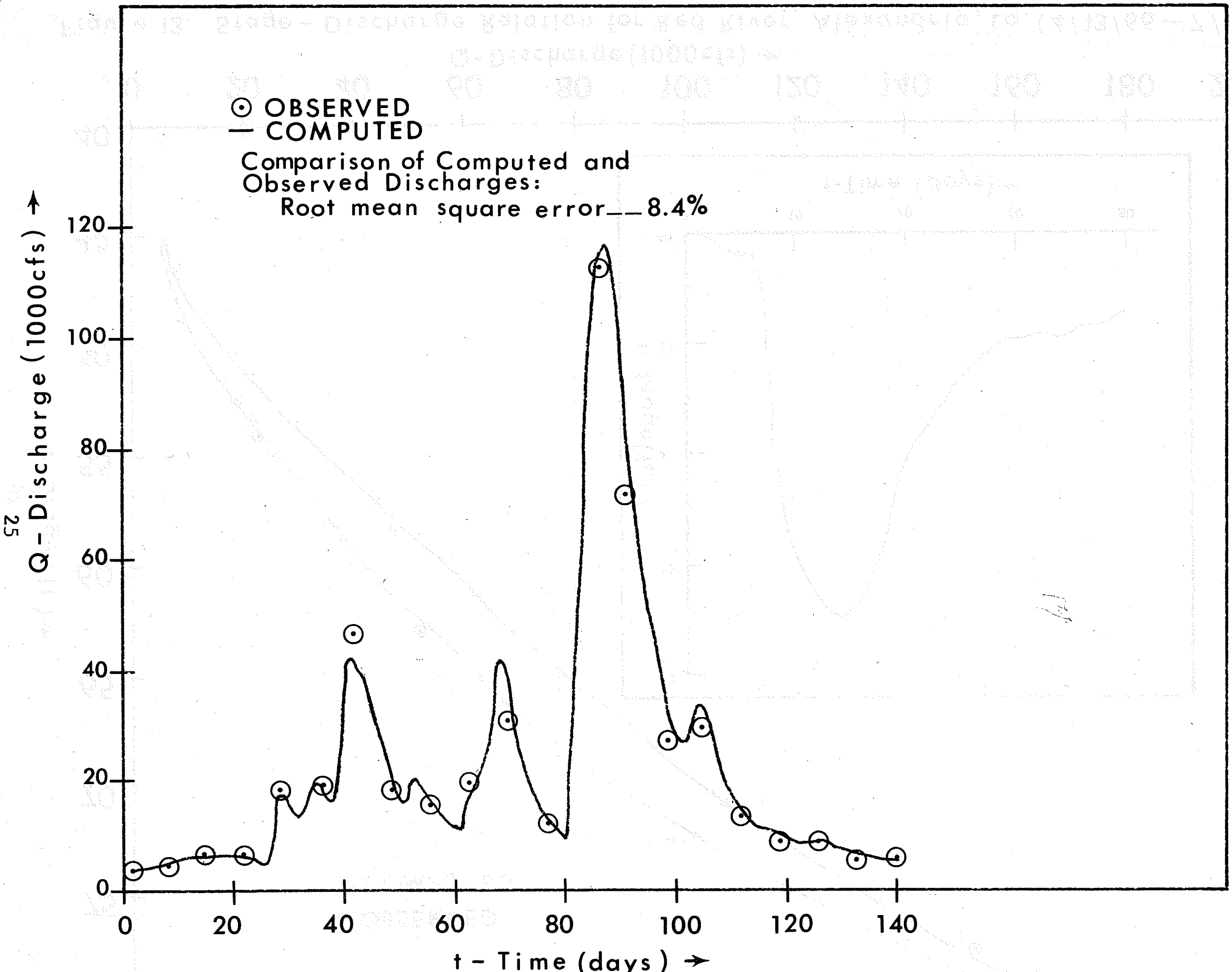

Figure 12. Discharge Hydrograph for Red River, Alexandria, La. (2/5/64— 6/17/64).

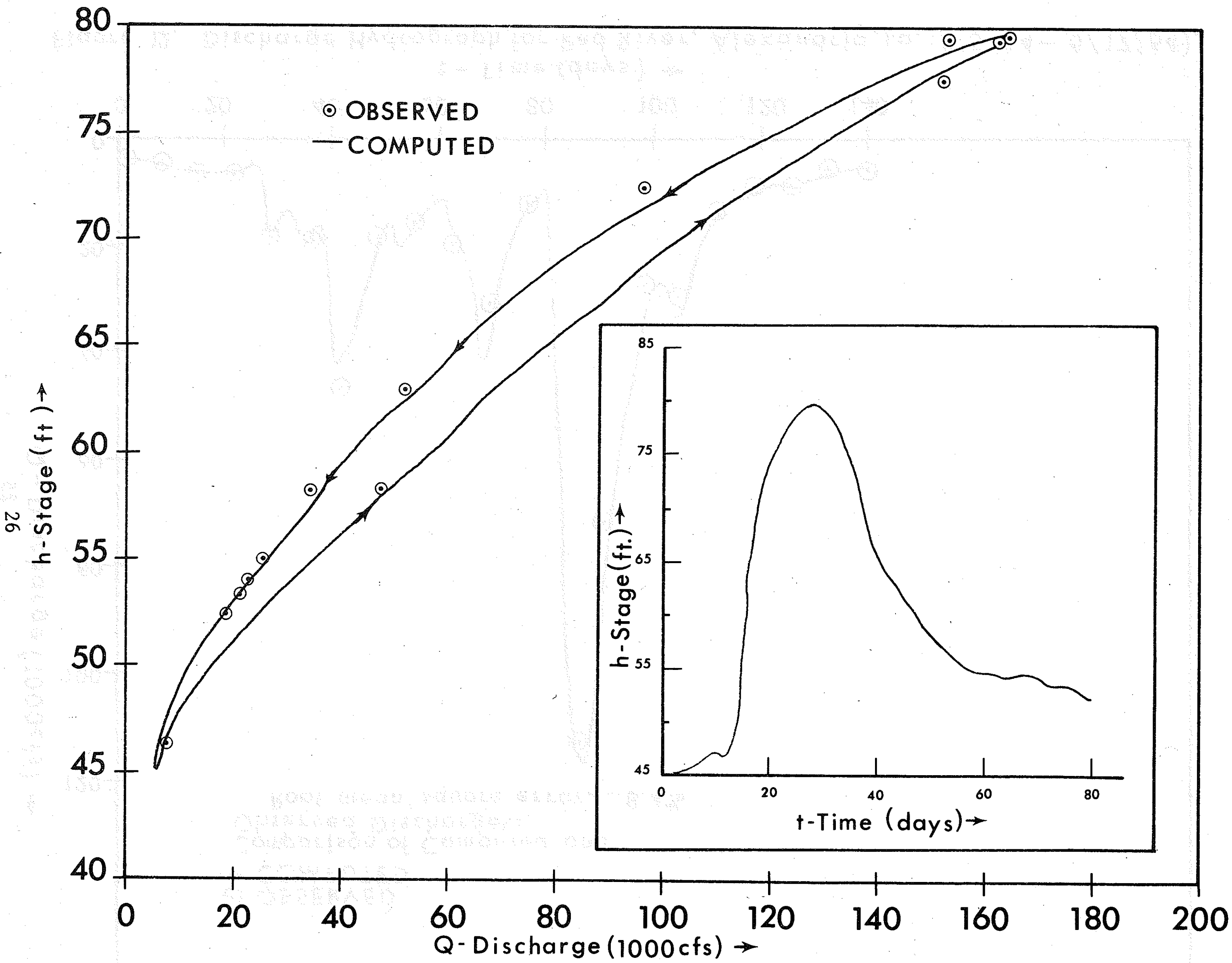

Figure 13. Stage – Discharge Relation for Red River, Alexandria, La. (4/13/66 – 7/1/66).

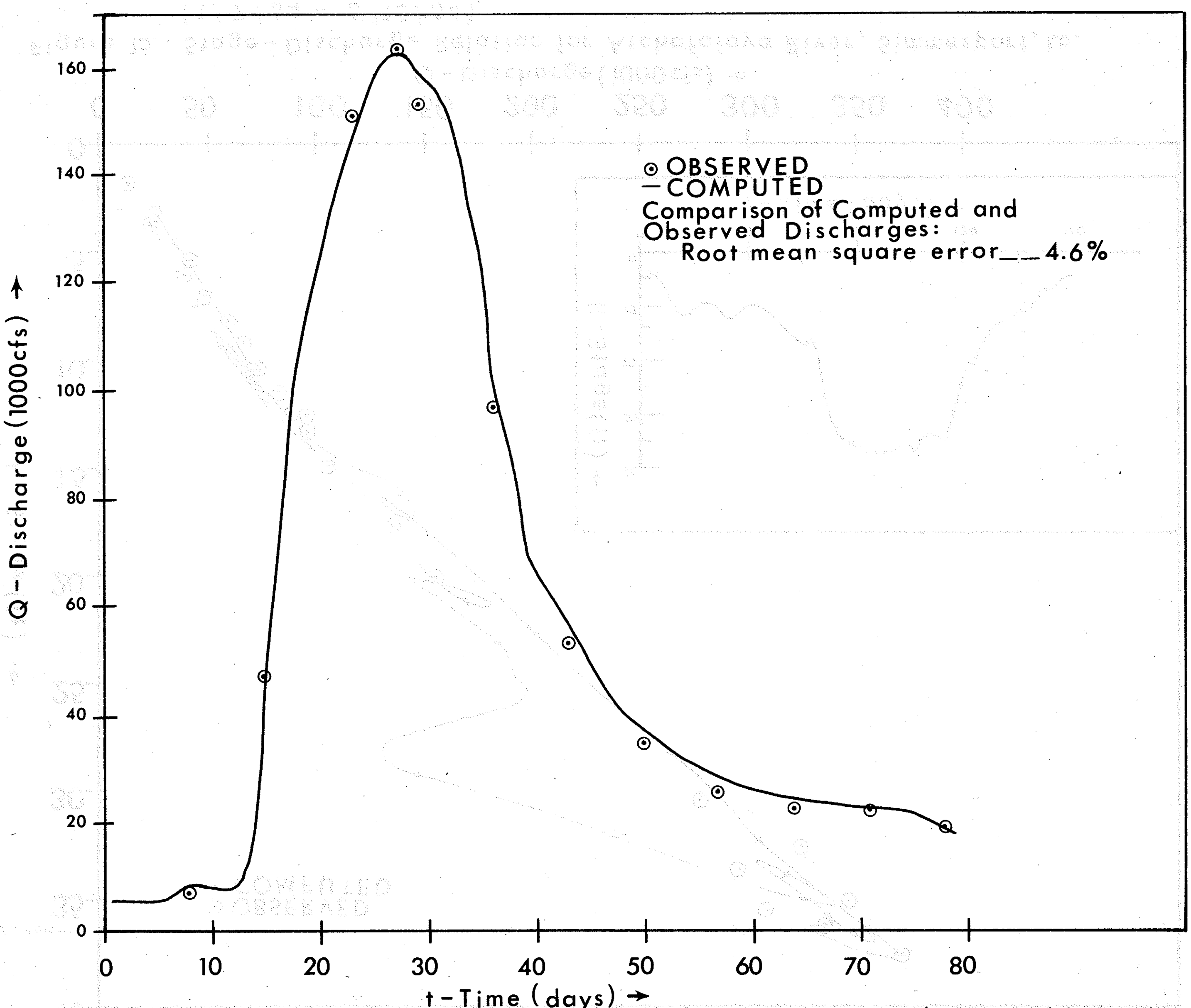

Figure 14. Discharge Hydrograph for Red River, Alexandria, La. (4/13/66 — 7/1/66).

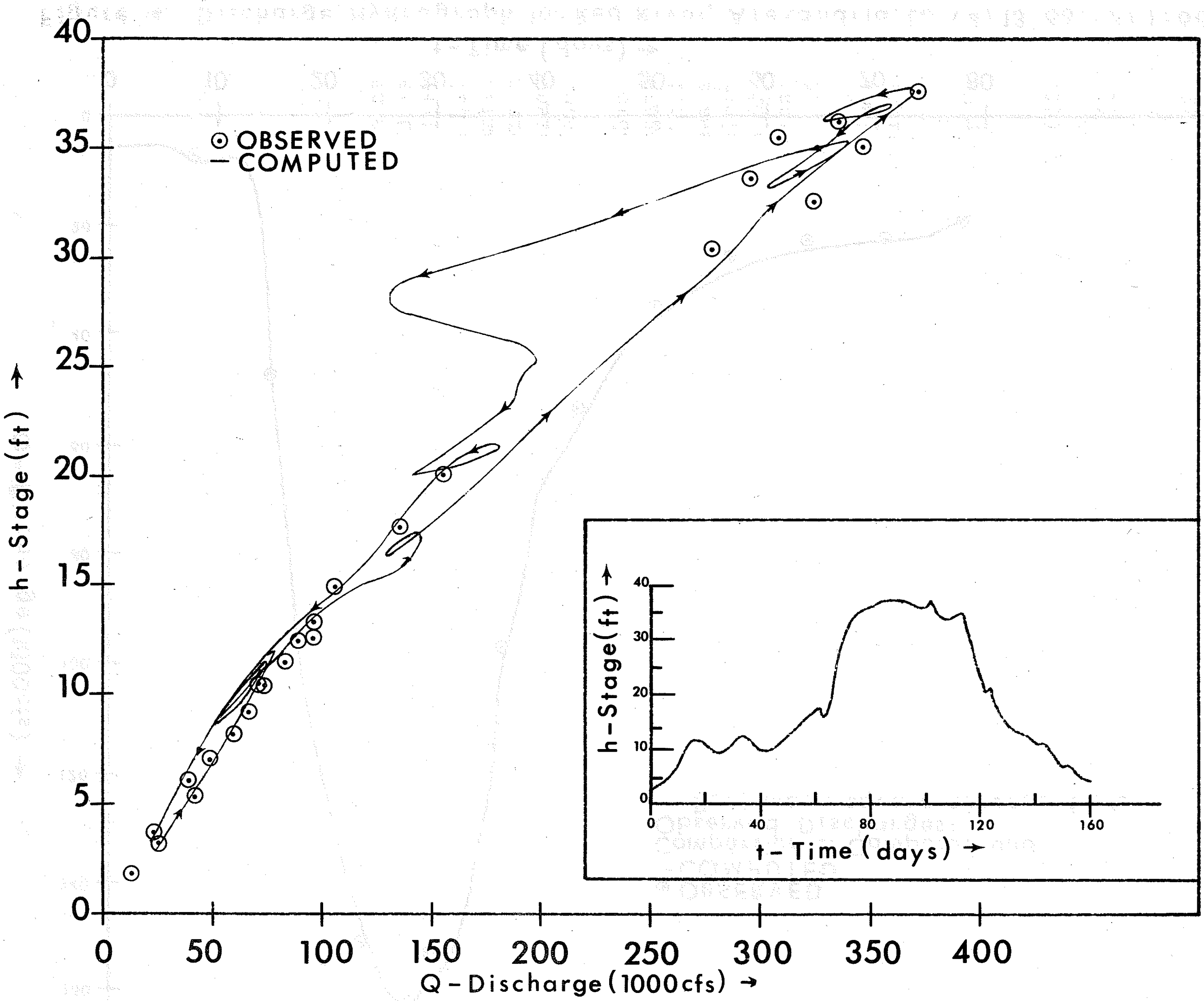

Figure 15. Stage-Discharge Relation for Atchafalaya River, Simmesport, La. (1/7/64 — 6/16/64).

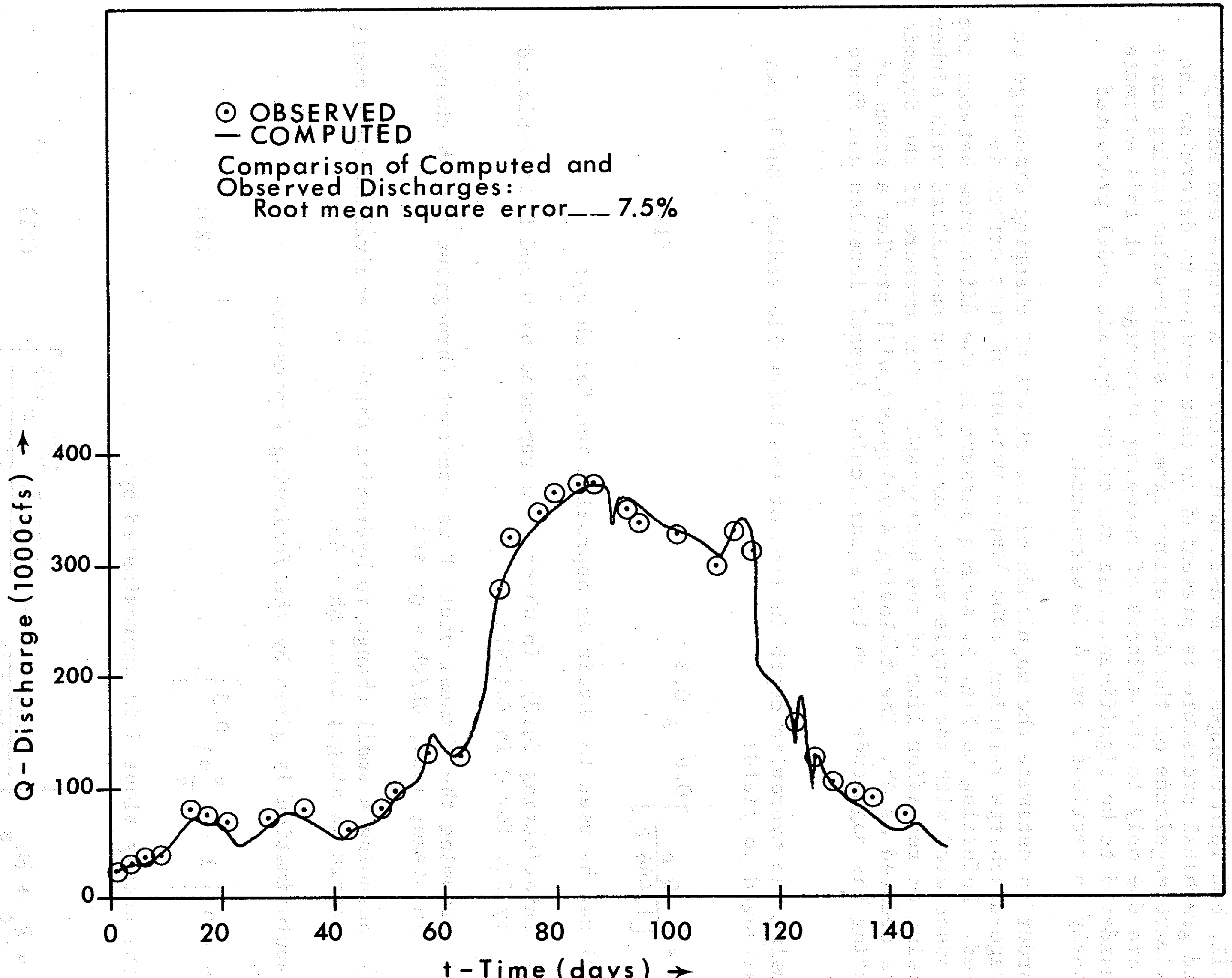

Figure 16. Discharge Hydrograph for Atchafalaya River, Simmesport, La. (1/7/64—6/16/64).

SECTION 4. GRAPHICAL ESTIMATION OF DYNAMIC LOOP

When stage-discharge measurements do not plot as a single-value rating curve, this is often attributed to some combination of the effects of scour and fill, bed form changes, or measurement errors. A simple and easily-applied graphical procedure is presented in this section to determine the approximate magnitude of the deviations from the single-value rating curve which are due only to the effects of changing discharge. If this estimate is considered to be significant, the use of the dynamic model presented previously in sections 3 and 4 is warranted.

In order to estimate the magnitude of the effect of changing discharge on the stage-discharge relation, some simple measure of this effect is required. Referring to Fig. 2, such a measure is the difference between the stage associated with the single-value curve and that associated with either the rising or recession limb of the hydrograph. This measure of the dynamic loop is denoted as Δh. The following development will provide a means of estimating the magnitude of Δh for a particular channel location and flood event.

By using the hydraulic depth in lieu of the hydraulic radius, Eq(3) can be rearranged to yield:

$$D = \left[\frac{Q\,n}{1.486\,B}\right]^{0.6} S^{-0.3} \tag{19}$$

Eq(19) can be used to obtain an approximation for Δh by:

1) substituting Eq(3), in which R is replaced by D and S is replaced by S_o, for Q in Eq(19);

2) assuming the channel width B is constant throughout the Δh change in stage; i.e., $dB/dh = 0$; and

3) assuming a small change in hydraulic depth is equivalent to a small change in stage; i.e., $\Delta h = \Delta D$.

The approximation is given by the following expression:

$$\Delta h \simeq D \left[1 - \left(\frac{S_o}{S}\right)^{0.3}\right], \tag{20}$$

where the energy slope S is approximated by:

$$S \simeq S_o + \delta h_s \left[\frac{0.52}{S_o^{1/2}\,\frac{D^{2/3}}{n}} + \frac{0.01\,S_o^{1/2}\,D^{2/3}}{D\;\;n}\right] \tag{21}$$

Eq(21) was obtained by:

1) using only the first two terms, which are the most significant, of the right side of Eq(14);

2) assuming K is constant and equal to 1.3; and

3) assuming Q in Eq(14) may be approximated by Eq(3) in which S_o is used in lieu of S.

An inspection of Eqs(20) and (21) indicates that the independent parameters necessary to approximate the magnitude of Δh in the dynamic loop are S_o, δh_s, D, and n. Thus, by allowing these parameters to assume values which encompass the practical range of each, Eqs(20) and (21) may be used to determine the Δh associated with various parameter values. The results of these computations are summarized by the family of graphical relationships shown in Fig. 17.

The following steps summarize the use of Fig. 17 to obtain an estimate of the Δh magnitude of the dynamic loop:

1) compute the value of $\frac{D^{2/3}}{n}$;

2) use Graph A and the values of $\frac{D^{2/3}}{n}$ and δh to obtain K_o;

3) use Graph B and the values of S_o and K_o to obtain S;

4) compute the value of S_o/S; and

5) use Graph C and the values of S_o/S and D to obtain Δh.

The following example illustrates the use of Fig. 17 to estimate Δh:

The approximate value of Δh is to be determined when $S_o = 0.00008$, D = 20.0 ft, n = 0.020, and δh = 1.0 ft/hr. First, the parameter $\frac{D^{2/3}}{n}$ is computed to be 368. With this value and the given value of δh, Graph A is used to obtain a value of 0.82 for K_o. Then, with the value of K_o and the given value of S_o, Graph B is used to obtain a value of 0.000137 for S. Finally, the ratio S_o/S is computed to be 0.584 and this value, along with the given value of D, is used in Graph C to obtain the Δh value of 2.9 ft.

The curves of Fig. 17 enable Δh to be determined for any combination of the relevant parameters (S_o, δh, D, and n). Of these, S_o and δh are the more dominant; therefore, by fixing the values of D and n at particular values, valuable insight concerning the effect of S_o and δh on the magnitude of Δh may be attained. Thus, if D and n are assumed constant at 10 ft and 0.015, respectively, the curves shown in Fig. 18 are obtained. If the values of D and n are assumed constant at 30 ft and 0.015, respectively, the curves shown in Fig. 19 are obtained. The following conclusions may be drawn from an examination of Figs. 18 and 19:

1) Δh increases as δh increases;

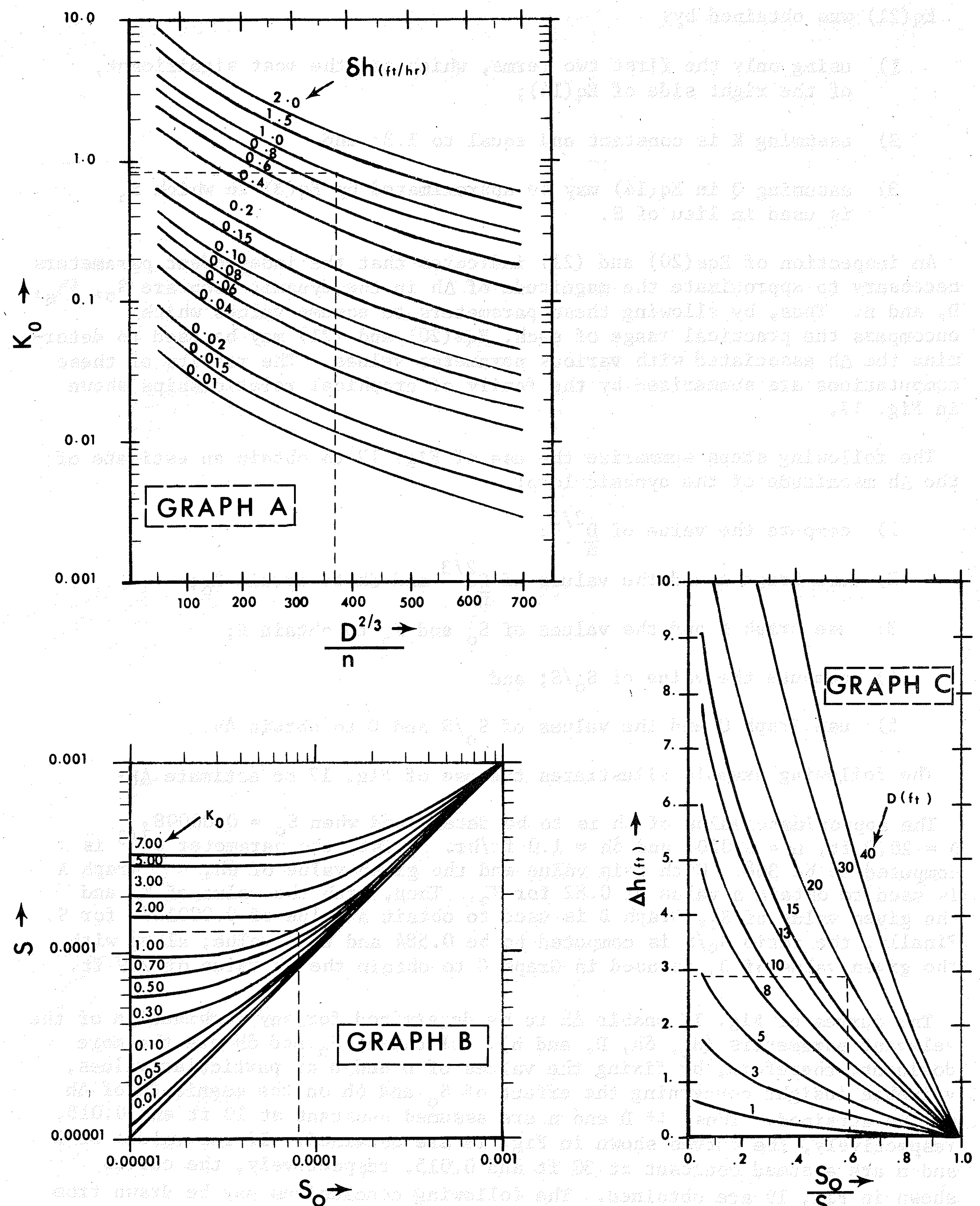

Figure 17. Generalized Graphical Estimation of Magnitude (Δh) of the Dynamic Loop.

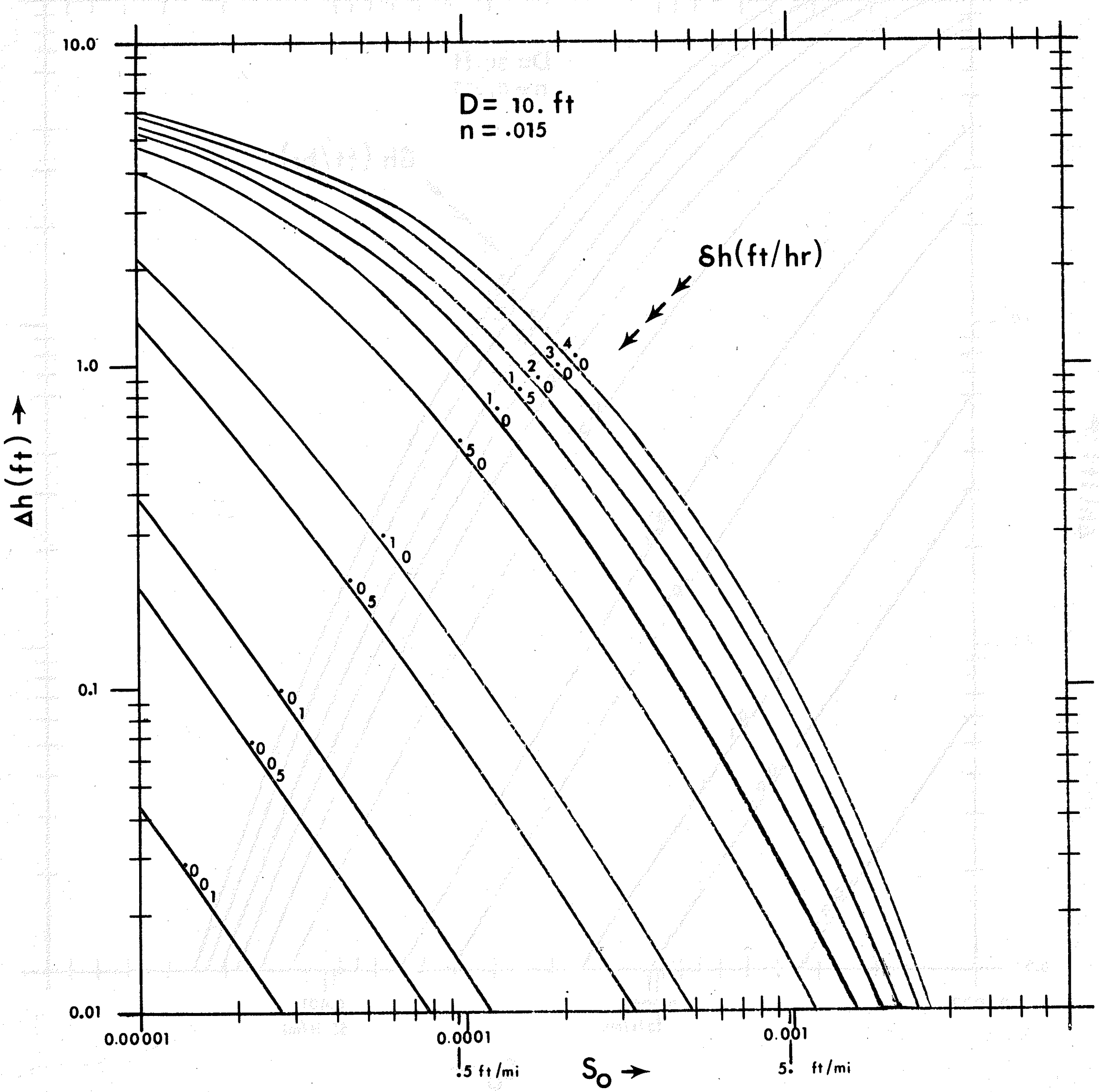

Figure 18. Graphical Estimation of Magnitude (Δh) of the Dynamic Loop for Constant D of 10 ft. and n of 0.015.

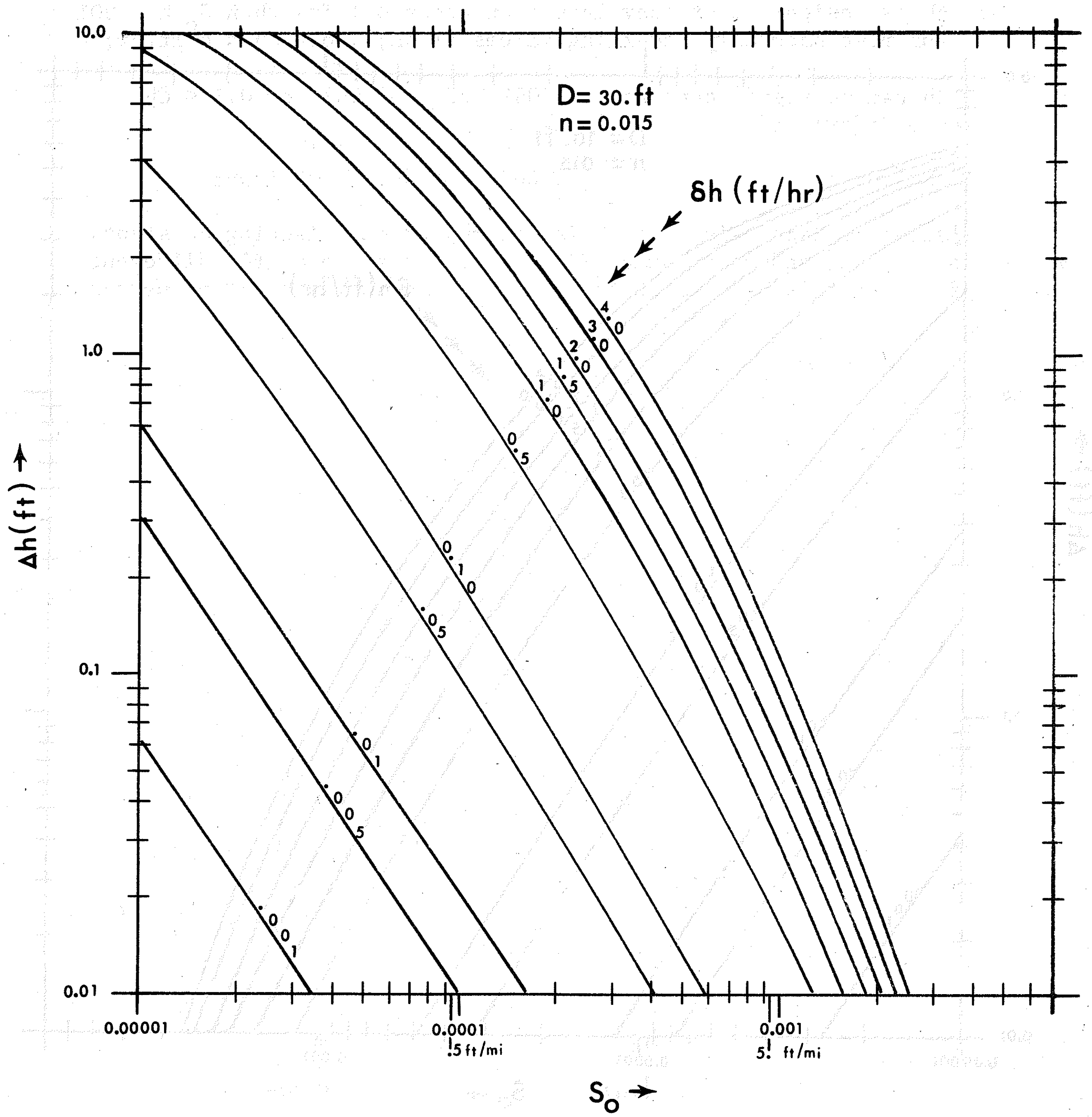

Figure 19. Graphical Estimation of Magnitude (Δh) of the Dynamic Loop for Constant D of 30. ft. and n of 0.015.

2) Δh decreases as S_o increases;

3) Δh increases as D increases;

4) Δh is insignificant (say less than about 0.1 ft) when $S_o > 0.001$ for most naturally occurring values of δh; i.e., $\delta h < 4$ ft/hr;

5) Δh can be significant when $0.0001 < S_o < 0.001$ for $0.1 < \delta h < 3$ ft/hr; and

6) Δh is significant when $S_o < 0.0001$ for $\delta h < 0.05$ ft/hr.

The relation of Manning's n to Δh is determined by selecting constant values for S_o, δh, and D and using Fig. 17 to determine Δh for different values of n. In this manner, it is found that Δh increases as n increases.

SECTION 5. SUMMARY

From the equations of unsteady flow, a mathematical model has been developed which computes either stage or discharge if the other is specified along with the channel slope, cross-sectional properties, and Manning's n. The model simulates the dynamic relation which exists between stage and discharge due to the effect of a variable energy slope caused by changing discharge. This effect, which is often observed as a loop in stage-discharge rating curves, was accurately modeled in several test applications; however, caution must be exercised when applying the model to locations where significant scour, fill and/or bed form changes occur since the model is only as accurate as the specified data.

The model can be used in forecasting to convert the forecast discharge hydrograph into a stage hydrograph which properly reflects the dynamic relation that exists between stage and discharge due to a variable energy slope. Also, the model can be used in stream gaging to convert an observed stage hydrograph into a discharge hydrograph when the effect of changing discharge is significant.

A convenient graphical procedure has been presented to estimate the magnitude of the changing discharge effect on stage-discharge ratings. This is useful in determining if the magnitude of the dynamic loop warrants the use of the mathematical model. The magnitude of the dynamic loop has been found to be related inversely to the channel bottom slope, and directly to the rate of change of stage, the hydraulic depth, and the Manning's n. As a general rule, the dynamic loop may be significant if the channel bottom slope is less than 0.001 ft/ft (about 5 ft/mile) and the rate of change of stage is greater than about 0.10 ft/hr. This rate of change of stage may decrease with decreasing values of the bottom slope.

ACKNOWLEDGMENTS

The author wishes to acknowledge the assistance given by Dr. E. L. Peck, Dr. R. A. Clark, and Mr. W. T. Sittner in reviewing the report and Mr. R. Tubella in drafting the figures.

REFERENCES

Carter, R. W., and I.E. Anderson, "Accuracy of Current Meter Measurements," Journal of the Hydraulics Division, ASCE, Vol. 89, No. HY4, Proc. Paper 3572, pp. 105-115, July, 1963.

Chow, V.T., Open-Channel Hydraulics, McGraw-Hill Book Company, New York, 1959.

Corbett, D.M., "Stream Gaging Procedure," Geological Survey Water Supply Paper 888, U.S. Government Printing Office, Washington, 1943.

Dawdy, D.R., "Depth-Discharge Relations of Alluvial Streams--Discontinuous Rating Curves," Geological Survey Water Supply Paper 1498-C, U.S. Government Printing Office, Washington, 1961.

Henderson, F.M., Open Channel Flow, MacMillan Company, New York, 1966.

Isaacson, E. and H.B. Keller, Analysis of Numerical Methods, John Wiley & Sons, Inc., New York, 1966.

Linsley, R.K., M.A. Kohler, and J.L.H. Paulhus, Applied Hydrology, McGraw-Hill Book Company, New York, 1949.

Simons, D.B., and E.V. Richardson, "The Effect of Bed Roughness on Depth-Discharge Relations in Alluvial Channels," Geological Survey Water Supply Paper 1498-E, U.S. Government Printing Office, Washington, 1961.

Simons, D.B., M.A. Stevens, and J.H. Duke, "Predicting Stages on Sand-Bed Rivers," Journal of the Waterways, Harbors and Coastal Engr. Division, ASCE, Vol. 99, No. WW2, Proc. Paper 9731, pp. 231-243, May, 1973.

APPENDIX A

SOLUTION BY NEWTON ITERATION

A nonlinear equation may be solved by a functional iterative technique such as Newton iteration. Consider the following equation expressed in functional form:

$$f(x) = 0 \tag{A-1}$$

The solution of Eq (A-1) is obtained in an iterative manner, proceeding from a first solution estimate x^k towards succeeding improved estimates x^{k+1}, which tend to converge toward an acceptable solution. The orderly procedure by which the improved solution estimate x^{k+1} is obtained so that it converges to an acceptable solution is known as Newton iteration and is described as follows.

A nonlinear equation such as Eq (A-1) may be linearized by using only the first two terms of its Taylor series expansion at x^k; i.e.,

$$f(x) = f(x^k) + \frac{df(x^k)}{dx^k}(x-x^k) \tag{A-2}$$

The right side of Eq (A-2) is the linear function of x^k that best approximates the nonlinear function f(x) which is evaluated at x^k. An iterative procedure, which will cause $f(x^k)$ to approach zero as the quantity $(x-x^k)$ approaches zero, can be obtained from Eq (A-2) by setting f(x) equal to zero and replacing x with x^{k+1}, which will be an improved solution estimate for x if the iterative procedure is convergent. Hence Eq (A-2) takes the form:

$$x^{k+1} = x^k - \frac{f(x^k)}{df(x^k)/dx^k} \tag{A-3}$$

where the k superscript denotes the number of iteration.

Eq (A-3), the general iteration algorithm of Newton, is repeated until the difference $(x^{k+1} - x^k)$ is less than ε which is a suitable error tolerance for the solution of Eq (A-1). When this occurs, the iteration process has converged; i.e., x^{k+1} has approached x to within the prescribed error tolerance ε.

The convergence of the iteration process depends on a good first solution estimate $x^{k=1}$. If the estimate is sufficiently close to x, convergence is attained; and it is at a quadratic rate; i.e. second order, since the iterative procedure involves the first derivative. The nonlinear equation which is solved by the Newton iterative algorithm in this report is a time dependent finite-difference equation. A first estimate of the solution is obtained by using the solution associated with the time t-Δt. In this study the iteration process always converged. The convergence process can be hastened when the first solution estimate $x^{k=1}$ is made closer to the acceptable solution. A simple linear extrapolation is used to provide better first solution estimates. Thus,

$$x_j^{k=1} = x_{j-1} + (x_{j-1} - x_{j-2})/2 \tag{A-4}$$

where the j subscript denotes the solution at time t and j-1 denotes the solution at time t-Δt, etc.

In the following, the Newton iteration algorithm is applied to Eq (15), which is presented here for convenient reference:

$$Q - 1.486 \frac{AD^{2/3}}{n} \left[S_o + [\frac{A}{KQ} + (1 - \frac{1}{K}) \frac{BQ}{gA^2}] \delta h_s + \frac{Q'/A' - Q/A}{g\Delta t} + \frac{2S_o}{3r^2}(1 - \frac{BQ^2}{gA^3}) \right]^{1/2} = 0 \tag{A-5}$$

First, the Case A condition is treated where the discharge Q is the unknown in Eq (A-5) and then the Case B condition is presented in which the stage h is the unknown.

<u>Case A</u>:

Eq (A-5) can be solved for Q at time t as follows:

$$Q^{k+1} = Q^k - \frac{f(Q^k)}{df(Q^k)/dQ^k} \tag{A-6}$$

where the superscript k denotes the number of iteration; $f(Q^k)$ is Eq (A-5) evaluated with the unknown Q replaced by the approximation Q^k; and the term $df(Q^k)/dQ^k$ is the derivative of $f(Q^k)$ with respect to Q^k. Thus,

$$f(Q^k) = Q^k - L_2 L_o^{1/2}, \tag{A-7}$$

where:

$$L_o = L_3 + \frac{L_4}{Q^k} + L_5 Q^k + L_6 (Q^k)^2 , \qquad \text{(A-8)}$$

$$L_2 = 1.486 \frac{AD^{2/3}}{n}, \qquad \text{(A-9)}$$

$$L_3 = S_o + \frac{2 S_o}{3 r^2} + \frac{Q'}{g A' \Delta t} , \qquad \text{(A-10)}$$

$$L_4 = \frac{A \delta h_s}{K} , \qquad \text{(A-11)}$$

$$L_5 = (1 - \frac{1}{K}) \frac{B \delta h_s}{g A^2} - \frac{1}{g A \Delta t} , \text{ and} \qquad \text{(A-12)}$$

$$L_6 = - \frac{2 S_o B}{3 r^2 g A^3} , \qquad \text{(A-13)}$$

in which

$$D = \frac{A}{B} , \qquad \text{(A-14)}$$

$$\delta h_s = \frac{(h - h')}{\Delta t} , \qquad \text{(A-15)}$$

$$K = \frac{5}{3} - \frac{2}{3} \frac{A \; dB/dh}{B^2} , \qquad \text{(A-16)}$$

$$dB/dh = \frac{(B-B')}{(h-h')} , \qquad \text{(A-17)}$$

$$n = n_{Lo} + \frac{(n_{L1} - n_{Lo})}{(h_{L1} - h_{Lo})} (h^k - h_{Lo}) . \qquad \text{(A-18)}$$

Also,

$$\frac{df(Q^k)}{dQ^k} = 1 - 0.5 \frac{L_2 L_1}{L_o^{1/2}} \qquad \text{(A-19)}$$

where:

$$L_1 = \frac{dL_o}{dQ^k} = \frac{-L_4}{(Q^k)^2} + L_5 + 2 L_6 Q^k . \qquad \text{(A-20)}$$

In the above equations, A and B are known functions of the stage and are evaluated at h; B', A', h' are known from the time period previous; i.e., t' or $t-\Delta t$; S_o, r, n_{Lo}, n_{L1}, h_{Lo}, h_{L1} are constants.

Case B:

Eq(A-5) can be solved for h at time t as follows:

$$h^{k+1} = h^k - \frac{f(h^k)}{df(h^k)/dh^k} \tag{A-21}$$

where the superscript k denotes the number of iteration; $f(h^k)$ is Eq(A-5) evaluated with the unknown h, which is also implicitly contained in the terms D, A, B, n, K, and δh_s, replaced by the approximation h^k; and the term $df(h^k)/dh^k$ is the derivative of $f(h^k)$ with respect to h^k. Thus,

$$f(h^k) = Q - 1.486 \frac{AD^{2/3}}{n} J_o^{1/2} \tag{A-22}$$

where:

$$J_o = J_2 + (J_3 A + J_4 \frac{B}{A^2})(h^k - h') + \frac{J_5}{A} + J_6 \frac{B}{A^3} \tag{A-23}$$

in which

$$J_2 = S_o + \frac{2 S_o}{3 r^2} + \frac{Q'}{g A' \Delta t}, \tag{A-24}$$

$$J_3 = \frac{1}{K Q \Delta t}, \tag{A-25}$$

$$J_4 = (1-\frac{1}{K}) \frac{Q}{g \Delta t}, \tag{A-26}$$

$$J_5 = -\frac{Q}{g \Delta t}, \text{ and} \tag{A-27}$$

$$J_6 = \frac{-2 S_o Q^2}{3 r^2 g}. \tag{A-28}$$

In the above,

$$K = \frac{5}{3} - \frac{2}{3} \frac{A}{B^2} dB/dh^k \quad \text{.....(evaluated at } h^k), \tag{A-29}$$

$$dB/dh^k = \frac{(B^k - B')}{(h^k - h')}, \tag{A-30}$$

and

$$\frac{df(h^k)}{dh^k} = -1.486\left[J_o^{1/2}\left[\frac{A}{n}\frac{d(D^{2/3})}{dh^k} + \frac{D^{2/3}\,B}{n} - \frac{A\,D^{2/3}\,dn/dh^k}{n^2}\right]\right.$$

$$\left. + \frac{0.5A\,D^{2/3}\,J_1}{n\,J_o^{1/2}}\right] , \qquad \text{(A-31)}$$

where:

$$J_1 = \frac{dJ_o}{dh^k} = J_3\,A + J_4\,\frac{B}{A^2} + (h^k - h') \left[J_3\,B + \frac{J_4}{A^2}\,(dB/dh^k\right.$$

$$\left. - 2\,\frac{B^2}{A})\right] - J_5\,\frac{B}{A^2} + \frac{J_6}{A^3}\,(dB/dh^k - 3\,\frac{B^2}{A}) , \qquad \text{(A-32)}$$

$$n = n_{Lo} + \frac{(n_{L1} - n_{Lo})}{(h_{L1} - h_{Lo})}\,(h^k - h_{Lo}) , \qquad \text{(A-33)}$$

$$dn/dh^k = \frac{(n_{L1} - n_{Lo})}{(h_{L1} - h_{Lo})} , \text{ and} \qquad \text{(A-34)}$$

$$\frac{d(D^{2/3})}{dh^k} = \frac{2}{3}\,D^{2/3}\,(\frac{B}{A} - \frac{dB/dh^k}{B}) . \qquad \text{(A-35)}$$

In the above equations A and B are specified functions of the stage and are evaluated at h^k; B', A', h' are known from the previous time $t-\Delta t$; and g, Δt, S_o, r, n_{Lo}, n_{L1}, h_{Lo}, h_{L1} are constants.

For either Case A or Case B, the solution of Eq(A-5) via Newton iteration requires only about two iterations when the following convergence criteria are used:

$$\left|Q^{k+1} - Q^k\right| < \varepsilon_Q \qquad \text{(A-36)}$$

and

$$\left|h^{k+1} - h^k\right| < \varepsilon_h , \qquad \text{(A-37)}$$

where:

$$\varepsilon_Q = 1.0 \text{ cfs} \qquad \text{(A-38)}$$

and

$$\varepsilon_h = 0.001 \text{ ft.} \qquad \text{(A-39)}$$

APPENDIX B

COMPUTER PROGRAM (DYNMOD)

The Fortran IV computer program presented in this section will compute either the Case A or Case B condition. An index labeled (IQH) functions as a decision variable so that the appropriate branches of the program are followed for either Case A or Case B. If IQH is assigned a value of one (integer), the Case A condition is performed in which discrete values of the discharge hydrograph are computed using an input of discrete values of the stage hydrograph. If IQH is assigned a value of two, the Case B condition is performed in which discrete values of the stage hydrograph are computed using an input of discrete values of the discharge hydrograph.

The program (DYNMOD) is designed with a (Main Program) and several subroutines for performing repetitive computations. Input/output information is handled by the Main Program. It also contains the basic program logic. Subroutine (QSOLVE) solves Eq (15) for the discharge Q using Newton Iteration. Subroutine (HSOLVE) solves Eq (15) for the stage (H) using Newton iteration. Subroutine (SECT) computes the geometrical properties of the cross-section for a particular elevation of the water surface. Subroutine (FRICT) computes Manning's n for a particular elevation of the water surface.

The output furnished by the program includes all the input information, as well as the following:

Time —— The time (in hours) associated with the computations being printed-out; the time corresponds to the temporal resolution of the specified (input) hydrograph.

Stage —— The actual stage (in ft above mean sea level or above a datum plane) which is read-in as the specified stage in the Case A condition or which is computed as in the Case B condition.

Discharge —— The actual discharge (in cfs) which is read-in as the specified discharge in the Case B condition or which is computed as in the Case A condition.

Normal discharge —— A fictitious discharge (in cfs) which would occur simultaneously with the actual stage if there were no dynamic effect due to a variable energy slope caused by changing discharge; this discharge would occur if the energy slope were equivalent to the effective channel bottom slope.

Dynamic effect —— A value (in cfs) which is the algebraic difference between Discharge and Normal Discharge; this represents the effect of the flow dynamics on the discharge.

Normal stage —— A fictitious stage (in ft above MSL) which would occur simultaneously with the actual discharge if there were no dynamic effect.

Dynamic effect —— Also used as a value (in ft) which is the algebraic difference between Stage and Normal Stage; this represents the effect of the flow dynamics on the stage.

Comment statements, inserted within the program, are provided as additional clarification.

The Fortran IV computer program (DYNMOD) and a typical INPUT/OUTPUT listing follows:

```
      PROGRAM DYNMOD (INPUT,OUTPUT)
      DIMENSION HU(200),T1(200),QU(200)
***      THIS PROGRAM IS A DYNAMIC MODEL OF THE STAGE-DISCHARGE RELATION.
***   THE MODEL CONSIDERS THE VARIABLE ENERGY SLOPE DUE TO CHANGING
***   DISCHARGE WHICH CAUSES A LOOP IN THE STAGE-DISCHARGE RATING CURVE.
      COMMON /A1/ CML1,CML2,CMU2,CMHL1,CMHL2,CMHU2
      COMMON /A2/ HS(10),BS(10),AS(10),NCS
***   GRAVITY ACCELERATION CONSTANT
      G=32.172
***   ITERATIVE CONVERGENCE CRITERION
      EPH=0.001
      EPQ=1.0
***   INPUT DATA
***      IF IQH=1,  THE STAGE HYDROGRAPH IS REQUIRED AS INPUT DATA AND THE
***    DISCHARGE HYDROGRAPH IS COMPUTED BY THE PROGRAM.
***      IF IQH=2,  THE DISCHARGE HYDROGRAPH IS REQUIRED AS INPUT DATA
***    AND THE STAGE HYDROGRAPH IS COMPUTED BY THE PROGRAM.
***   NU  IS THE NUMBER OF POINT VALUES OF THE SPECIFIED HYDROGRAPH.
***   NCS  IS THE NUMBER OF VALUES OF HS.
***   GZ  IS THE ELEVATION (FT) OF THE GAGE ZERO ABOVE MSL.
***   DT  IS THE DELTA TIME STEP (HRS) AT WHICH THE COMPUTATIONS PROCEED
***   DTHU  IS THE TIME INTERVAL (HRS) BETWEEN VALUES OF SPECIFIED
***                                                             HYDROGRAPH.
***   HS  IS THE WATER SURFACE ELEVATION (ABOVE MSL) FOR KNOWN VALUES OF
***   CROSS-SECTIONAL AREA AND WIDTH.
***   BS  IS THE KNOWN VALUE OF THE SURFACE WIDTH FOR THE ELEVATION HS.
***   AS  IS THE KNOWN VALUE OF THE CROSS-SECTIONAL AREA FOR ELEV. HS.
***   HU IS SPECIFIED STAGE HYDROGRAPH VALUE  FOR EACH DTHU (HRS)
***                                                   TIME INCREMENT.
***   QU  IS THE SPECIFIED DISCHARGE HYDROGRAPH VALUE FOR EACH DTHU
***                                            (HRS) TIME INCREMENT.
      READ 51, IQH
      PRINT 50, IQH
      READ 51,NU,NCS,GZ,DT,DTHU
      PRINT 55, NU,NCS,GZ,DT,DTHU
      READ 52,(HS(K),K=1,NCS)
      PRINT 60
      PRINT 52,(HS(K),K=1,NCS)
      READ 52,(BS(K),K=1,NCS)
      PRINT 61
      PRINT 52,(BS(K),K=1,NCS)
      READ 52,(AS(K),K=1,NCS)
      PRINT 62
      PRINT 52,(AS(K),K=1,NCS)
      IF(IQH-1) 8,8,9
    8 READ 52,(HU(K),K=1,NU)
      PRINT 63
      PRINT 52,(HU(K),K=1,NU)
***   CORRECT SPECIFIED STAGES TO MEAN SEA LEVEL DATUM
      DO 12 K=1,NU
   12 HU(K)=HU(K)+GZ
      GO TO 10
    9 READ 52, (QU(K),K=1,NU)
      PRINT 58
```

```
      PRINT 52, (QU(K),K=1,NU)
***   INPUT FRICTION COEFFICIENTS AND EFFECTIVE BOTTOM SLOPE
   10 READ 53, CML1,CML2,CMU2,CMHL1,CMHL2,CMHU2,SO
      PRINT 59,CML1,CML2,CMU2,CMHL1,CMHL2,CMHU2,SO
***   TYPICAL FLOOD DATA FOR COMPUTING CONSTANT (FR)
      READ 54, TP,QMAX,QMIN,HMAX,HMIN
      PRINT 67,TP,QMAX,QMIN,HMAX,HMIN
***   CONVERT DT TO SECONDS
      DTS=DT*3600.
***   COMPUTE HOUR ASSOCIATED WITH POINT VALUES OF SPECIFIED HYDROGRAPH.
      T1(1)=0.
      DO 11 K=2,NU
   11 T1(K)=T1(K-1)+DTHU
***   COMPUTE FR CONSTANT
      HMAX=HMAX+GZ
      HMIN=HMIN+GZ
      HA=0.5*(HMAX+HMIN)
      CALL SECT(HA,A,B,R,DB,DR)
      FR=56200.*(QMAX+QMIN)*TP*SO/A/(HMAX-HMIN)
      PRINT 56, FR
      CONR=2./3.*SO/FR/FR
      KT=1
      TT=0.
      IF(IQH-1) 30,30,31
***   COMPUTE INITIAL DISCHARGE
   30 HP=HU(1)
      CALL SECT(HP,A,B,R,DB,DR)
      CALL FRICT(HP,CM,DCM)
      QP= 1.486/CM*SQRT(SO)*A*R
      PRINT 57, QP
      GO TO 14
   31 Y1=(QU(1)*CML1/(1.486*SQRT(SO)*BS(1)))**(3./5.)
      DO 32 K=1,20
      CALL SECT(Y1,A,B,R,DB,DR)
      CALL FRICT(Y1,CM,DCM)
      F=QU(1)-1.486*SQRT(SO)/CM*A*R
      DF=-1.486*SQRT(SO)*(B*R/CM+A*DR/CM-A*R*DCM/CM/CM)
      Y=Y1-F/DF
      IF(ABS(Y-Y1)-EPH) 33,32,32
   32 Y1=Y
   33 HP=Y
      QP=QU(1)
   14 QF=QP
      HF=HP
      DHN=0.
      PRINT 68
      GO TO 43
   15 TT=TT+DT
***   INTERPOLATE AT TIME TT FROM SPECIFIED HYDROGRAPH
      DO 16 K=1,NU
      IF (TT-T1(K)) 17,16,16
   16 CONTINUE
      GO TO 26
   17 KK=K-1
```

```
      IF(IQH-1) 40,40,41
***   INTERPOLATION OF STAGE HYDROGRAPH
   40 HF=HU(KK)+(TT-T1(KK))/DTHU*(HU(K)-HU(KK))
      CALL QSOLVE (HF,HP,QP,EPQ,SO,CONR,DTS,DQ,G,QF)
      GO TO 43
***   INTERPOLATION OF DISCHARGE HYDROGRAPH
   41 QF=QU(KK)+(TT-T1(KK))/DTHU*(QU(K)-QU(KK))
      CALL HSOLVE (QF,QP,HP,EPH,SO,CONR,DTS,DH,G,HF)
   43 DQ=QF-QP
      DH=HF-HP
***   COMPUTE NORMAL DISCHARGE
      CALL SECT (HF,A,B,R,DB,DR)
      QN=1.486/CM*SQRT(SO)*A*R
      DQN=QF-QN
***   COMPUTE NORMAL STAGE
      HNK=HF+DHN/2.
      DO 22 K=1,20
      CALL SECT(HNK,A,B,R,DB,DR)
      CALL FRICT (HNK,CM,DCM)
      F=QF-1.486*SQRT(SO)*A*R/CM
      DF=-1.486*SQRT(SO)*(A*DR/CM+B*R/CM-A*R*DCM/CM/CM)
      HNKK=HNK-F/DF
      IF(ABS(HNKK-HNK)-EPH)23,22,22
   22 HNK=HNKK
   23 HN=HNKK
***   COMPUTE DIFFERENCES IN NORMAL AND DYNAMIC STAGES
      DHN=HF-HN
***   CHECK TO SEE IF COMPUTATIONS ARE TO BE PRINTED
      IF(TT-T1(KT)) 25,24,24
   24 KT=KT+1
***   PRINT COMPUTATIONS
      PRINT 69,TT,HF,QF,QN,DQN,HN,DHN
   25 QP=QF
      HP=HF
      GO TO 15
   26 CONTINUE
      STOP
   50 FORMAT(5X,*IQH=*I2)
   51 FORMAT(2I10,3F10.2)
   52 FORMAT(8F10.2)
   53 FORMAT(3F10.5,3F10.2,F10.6)
   54 FORMAT(5F10.2)
   55 FORMAT (5X,* NO. OF HYDROGRAPH PTS=*,I5,5X,
     1* NO. OF CROSS SECTION PTS=*,I3//5X,*GAGE DATUM ELEVATION(FT)=*,
     2F8.2//,5X,* DELTA TIME INCREMENT(HRS)=*,F6.1,5X,
     3* RESOLUTION OF SPECIFIED HYDROGRAPH(HRS)=*,F6.1)
   56 FORMAT(5X,*FR CONSTANT=*,F10.2)
   57 FORMAT(5X,*INITIAL STEADY DISCHARGE(CFS)=*,F10.0)
   59 FORMAT(2X*(CML1) IS N VALUE AT LOWEST STAGE OF LOWER RANGE OF STAG
     1E=*,F10.5,/2X,*(CML2) IS N VALUE AT HIGHEST STAGE OF LOWER RANGE
     2OF STAGE =*,F10.5,/2X,*(CMU2) IS N VALUE AT HIGHEST STAGE OF UPPER
     3 RANGE OF STAGE =*,F10.5,/2X,*(CMHL1) IS STAGE ASSOCIATED WITH CML
     41 =*,F10.2,/2X,*(CMHL2) IS STAGE ASSOCIATED WITH CML2 =*,F10.2,/2X
     5,*(CMHU2) IS STAGE ASSOCIATED WITH CMU2  =*,F10.2,/2X,*EFFECTIVE B
```

```
     6OTTOM SLOPE (SO) =*,F12.8)
  60 FORMAT(25X,* CROSS-SECTION FLEVATIONS(FT ABOVE MSL)*)
  61 FORMAT(25X,* CROSS-SECTION WIDTHS(FT)*)
  62 FORMAT(25X,* CROSS-SECTION AREAS (SQ. FT.)*)
  58 FORMAT(25X,*SPECIFIED DISCHARGE HYDROGRAPH(CFS)*)
  63 FORMAT(25X,* SPECIFIED STAGE HYDROGRAPH(FT)*)
  67 FORMAT(5X,* TIME TO PEAK(DAYS)=*,F6.1,5X,* MAX. ANNUAL DISCHARGE(C
    1FS)=*,F10.0,//,5X,*MIN. DISCHARGE(CFS)=*,F10.0,5X,* MAX. ANNUAL
    2STAGE(FT)=*,F8.2,5X,* MIN STAGE(FT)=*,F8.2)
  68 FORMAT(1X,*TIME(HRS)*,1X,*STAGE(FT MSL)*,1X,*DISCHARGE(CFS)*,1X,*N
    1ORMAL DISCH(CFS) *,1X,*DYNAMIC EFFECT(CFS)*,1X,*NORMAL STAGE(FT)*,
    21X,*DYNAMIC EFFECT(FT)*)
  69 FORMAT(F7.1,7X,F6.2,7X,F10.0,8X,F10.0,8X,F10.0,10X,F6.2,13X,F6.2)
     END
     SUBROUTINE QSOLVE (HF,HP,QP,EPQ,SO,CONR,DTS,DQ,G,QF)
***     THIS SUBROUTINE USES NEWTON ITERATION TO SOLVE EQ(15) FOR THE
***   UNKNOWN DISCHARGE WHEN THE STAGE HYDROGRAPH IS GIVEN.
***  COMPUTE CONSTANTS FOR EQ(15)
     CALL SECT(HP,A,B,R,DB,DR)
     FL3=SO+CONR+QP/G/A/DTS
     CALL SECT (HF,A,B,R,DB,DR)
     CALL FRICT (HF,CM,DCM)
     FK=5./3.-2./3.*A*DB/B/B
     FL2=1.486*A*R/CM
     DHS=(HF-HP)/DTS
     FL4=A*DHS/FK
     FL5=(1.-1./FK)*B*DHS/G/A/A-1./G/A/DTS
     FL6=-CONR/G*B/A/A/A
***  COMPUTE STARTING VALUE FOR ITERATIVE SOLUTION OF EQ(15)
     QK=QP+DQ/2.
***  SOLVE EQ(15) BY NEWTON ITERATION
     DO 20 K=1,20
     FL0=FL3+FL4/QK+FL5*QK+FL6*QK*QK
     FL1=-FL4/QK/QK+FL5+2.*FL6*QK
     F=QK-FL2*SQRT(FL0)
     DF=1.-0.5*FL2*FL1/SQRT(FL0)
     QKK=QK-F/DF
     IF(ABS(QKK-QK)-EPQ) 21,20,20
  20 QK=QKK
  21 QF=QK
     RETURN
     END
     SUBROUTINE HSOLVE (QF,QP,HP,EPH,SO,CONR,DTS,DH,G,HF)
***     THIS SUBROUTINE USES NEWTON ITERATION TO SOLVE EQ(15) FOR THE
***   UNKNOWN STAGE WHEN THE DISCHARGE HYDROGRAPH IS GIVEN.
***  COMPUTES CONSTANTS FOR EQ(15)
     CALL SECT(HP,A,B,R,DB,DR)
     FJ2=SO+CONR+QP/G/A/DTS
     FK=5./3.-2./3.*A/B/B*DB
***  COMPUTE STARTING VALUE FOR ITERATIVE SOLUTION OF EQ(15)
     HK=HP+DH/2.
***  SOLVF EQ(15) BY NEWTON ITERATION
     DO 20 K=1,20
     CALL SECT(HK,A,B,R,DB,DR)
```

```
      CALL FRICT(HK,CM,DCM)
      DH=HK-HP
      FJ3=1./QF/FK/DTS
      FJ4=(1.-1./FK)*QF/G/DTS
      FJ5=-QF/G/DTS
      FJ6=-CONR*QF*QF/G
      FJ0=FJ2+(FJ3*A+FJ4*B/A/A)*DH+FJ5/A+FJ6*B/A/A/A
      FJ1=FJ3*A+FJ4*B/A/A+DH*(FJ3*B+FJ4/A/A*(DB-2.*B*B/A))-FJ5*B/A/A+FJ6
     1/A/A/A*(DB-3.*B*B/A)
      F=QF-1.486*A*R/CM*SQRT(FJ0)
      DF=-1.486*(SQRT(FJ0)*(A/CM*DR+R*B/CM-A*R*DCM/CM/CM)+0.5*A*R/CM*FJ1
     1/SQRT(FJ0))
      HKK=HK-F/DF
      IF (ABS(HKK-HK)-EPH)21,20,20
   20 HK=HKK
   21 HF=HKK
      RETURN
      END
      SUBROUTINE SECT (Y,A,B,R,DB,DR)
      COMMON /A2/ HS(10),BS(10),AS(10),NCS
***      THIS SUBROUTINE COMPUTES THE GEOMETRICAL PROPERTIES OF THE
***   CROSS-SECTION AT A SPECIFIED WATER SURFACE ELEVATION.
      DO 10 K=2,NCS
      IF (Y-HS(K))5,5,10
    5 KT=K
      GO TO 15
   10 CONTINUE
   15 KL=KT-1
      DB=(BS(KT)-BS(KL))/(HS(KT)-HS(KL))
      B=BS(KL)+DB*(Y-HS(KL))
      A=AS(KL)+(AS(KT)-AS(KL))/(HS(KT)-HS(KL))*(Y-HS(KL))
      R=(A/B)**(2./3.)
      DR=2./3.*R*(B/A-DB/B)
      RETURN
      END
      SUBROUTINE FRICT(Y,CM,DCM)
      COMMON /A1/ CML1,CML2,CMU2,CMHL1,CMHL2,CMHU2
***    THIS SUBROUTINE COMPUTES MANNINGS  N  COEFFICIENT FOR A
***   SPECIFIED WATER SURFACE ELEVATION.
      CMU1=CML2
      CMHU1=CMHL2
      IF (Y-CMHL2)10,10,12
***   LOWER RANGE OF STAGE
   10 DCM=(CML2-CML1)/(CMHL2-CMHL1)
      CM=CML1+DCM*(Y-CMHL1)
      GO TO 20
***   UPPER RANGE OF STAGE
   12 DCM=(CMU2-CMU1)/(CMHU2-CMHU1)
      CM=CMU1+DCM*(Y-CMHU1)
   20 RETURN
      END
```

IOH= 1
NO. OF HYDROGRAPH PTS= 64 NO. OF CROSS SECTION PTS= 4

GAGE DATUM ELEVATION(FT)= 3.49

DELTA TIME INCREMENT(HRS)= 3.0 RESOLUTION OF SPECIFIED HYDROGRAPH(HRS)= 24.0

CROSS-SECTION ELEVATIONS(FT ABOVE MSL)

16.00 34.00 41.20 48.00

CROSS-SECTION WIDTHS(FT)

3000.00 3540.00 3630.00 3690.00

CROSS-SECTION AREAS (SQ. FT.)

72500.00 134000.00 164000.00 200000.00

SPECIFIED STAGE HYDROGRAPH(FT)

18.29	18.59	19.56	21.27	23.22	25.11	26.78	28.02
29.01	29.84	31.01	32.54	33.79	34.51	35.74	36.63
37.32	38.02	38.56	39.00	39.54	40.10	40.67	41.10
41.40	41.68	41.86	42.11	42.40	42.50	42.80	42.74
42.38	41.89	41.29	40.58	39.82	38.81	37.70	36.53
35.11	33.88	32.97	32.07	31.10	30.38	29.82	29.30
28.77	28.26	27.75	27.28	26.90	26.81	26.64	26.59
26.20	25.80	25.45	25.02	25.11	24.72	24.02	23.99

(CML1) IS N VALUE AT LOWEST STAGE OF LOWER RANGE OF STAGE= .01590
(CML2) IS N VALUE AT HIGHEST STAGE OF LOWER RANGE OF STAGE = .01392
(CMU2) IS N VALUE AT HIGHEST STAGE OF UPPER RANGE OF STAGE = .01392
(CMHL1) IS STAGE ASSOCIATED WITH CML1 = 5.00
(CMHL2) IS STAGE ASSOCIATED WITH CML2 = 50.00
(CMHU2) IS STAGE ASSOCIATED WITH CMU2 = 50.00
EFFECTIVE BOTTOM SLOPE (SO) = .00001430
TIME TO PEAK(DAYS)= 30.0 MAX. ANNUAL DISCHARGE(CFS)= 1064000

MIN. DISCHARGE(CFS)= 319000 MAX. ANNUAL STAGE(FT)= 42.74 MIN STAGE(FT)= 18.29
FR CONSTANT= 10.18
INITIAL STEADY DISCHARGE(CFS)= 323237

TIME(HRS)	STAGE(FT MSL)	DISCHARGE(CFS)	NORMAL DISCH(CFS)	DYNAMIC EFFECT(CFS)	NORMAL STAGE(FT)	DYNAMIC EFFECT(FT)
0.0	21.78	323237	323237	0	21.78	0.00
24.0	22.08	337255	329293	7962	22.52	-.44
48.0	23.05	371583	348613	22970	24.28	-1.23
72.0	24.76	423051	383157	39894	26.82	-2.06
96.0	26.71	471073	423371	47702	29.10	-2.39
120.0	28.60	512768	463363	49405	31.01	-2.41
144.0	30.27	546285	499490	46795	32.51	-2.24
168.0	31.51	563946	526434	37512	33.28	-1.77
192.0	32.50	580051	548379	31672	33.98	-1.48
216.0	33.33	594817	566860	27957	34.47	-1.14
240.0	34.50	634415	597768	36648	35.74	-1.24
264.0	36.03	695029	646434	48595	37.62	-1.59
288.0	37.28	728821	686503	42318	38.63	-1.35
312.0	38.00	735959	709323	26636	38.85	-.85
336.0	39.23	795864	751720	44144	40.59	-1.36
360.0	40.12	815691	781637	34054	41.16	-1.04
384.0	40.81	833019	805060	27959	41.55	-.74
408.0	41.51	861131	832588	28543	42.16	-.65
432.0	42.05	880282	857069	23213	42.58	-.53
456.0	42.49	897078	877289	19788	42.94	-.45
480.0	43.03	926800	902825	23975	43.56	-.53
504.0	43.59	954667	929480	25186	44.14	-.55
528.0	44.16	982978	956941	26038	44.73	-.57
552.0	44.59	998337	977562	20775	45.04	-.45
576.0	44.89	1007599	991974	15625	45.23	-.34
600.0	45.17	1020669	1005743	14926	45.49	-.32
624.0	45.35	1025197	1014426	10771	45.58	-.23
648.0	45.60	1040906	1027052	13854	45.89	-.29
672.0	45.89	1057379	1041684	15695	46.22	-.33
696.0	45.99	1053738	1046235	7503	46.15	-.16
720.0	46.29	1078225	1061904	16321	46.63	-.34
744.0	46.23	1058347	1057919	428	46.24	-.01
768.0	45.87	1025673	1038963	-13290	45.59	.28
792.0	45.38	994973	1014161	-19188	44.97	.41
816.0	44.78	960255	984291	-24036	44.26	.52
840.0	44.07	920788	949520	-28732	43.44	.63
864.0	43.31	882614	913100	-30486	42.63	.68
888.0	42.30	823985	865211	-41226	41.35	.95
912.0	41.19	769111	813793	-44681	39.82	1.37
936.0	40.02	725974	772823	-46849	38.55	1.47
960.0	38.60	666914	724001	-57087	36.75	1.85
984.0	37.37	637330	684064	-46734	35.83	1.54
1008.0	36.46	623426	655551	-32125	35.39	1.07
1032.0	35.56	596052	627045	-30993	34.51	1.05
1056.0	34.59	563779	596497	-32718	33.27	1.32
1080.0	33.87	551059	575686	-24627	32.72	1.15
1104.0	33.31	544904	563256	-18353	32.44	.87
1128.0	32.79	534895	551537	-16642	32.00	.79
1152.0	32.26	522738	539541	-16802	31.46	.80
1176.0	31.75	512287	528162	-15874	30.99	.76
1200.0	31.24	501137	516816	-15678	30.48	.76
1224.0	30.77	492438	506533	-14095	30.08	.69
1248.0	30.39	487519	498401	-10882	29.86	.53
1272.0	30.30	495700	497065	-1365	30.23	.07
1296.0	30.13	489268	493178	-3910	29.94	.19
1320.0	30.08	492234	492337	-103	30.08	.00
1344.0	29.69	472112	483136	-11024	29.15	.54
1368.0	29.29	463237	474479	-11242	28.73	.56
1392.0	28.94	457558	467071	-9513	28.47	.47
1416.0	28.51	445748	457736	-11988	27.91	.60
1440.0	28.60	464668	460653	4015	28.80	-.20
1464.0	28.21	440852	451458	-10606	27.68	.53
1488.0	27.51	415605	436120	-20515	26.46	1.05