

NOAA Technical Memorandum NWS WR-122



A METHOD FOR TRANSFORMING TEMPERATURE DISTRIBUTIONS TO NORMALITY

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Salt Lake City, Utah
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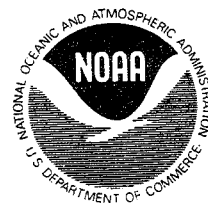
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LIST OF SYMBOLS

<u>Symbols Used In Text</u>	<u>Equivalent Symbols Used In Appendix</u>	
A	A	A term equal to $\ln(\bar{T}') - \frac{\sum \ln(T_i')}{N}$.
\hat{a}	\hat{a}	A term in the gamma distribution equal to $\frac{1 + \sqrt{1 + (4/3)A}}{4A}$.
\hat{B}	\hat{B}	A term in the gamma distribution equal to $\frac{T'}{\hat{a}}$.
N	N	Number of elements in the distribution.
P(T)	P(x)	The cumulative frequency of T.
Q(T)	Q(x)	A term equal to $1 - P(T)$.
S	S	Standard deviation.
T	x	Temperature.
\bar{T}	\bar{x}	Mean temperature.
T'	x'	The absolute value of $T - T_{BASE}$.
T_{BASE}	x_{BASE}	Base temperature.
T_i	x_i	A sample temperature of the distribution where $i = 1, 2, \dots, N$.
T_i'	x_i'	The absolute value of $T_i - T_{BASE}$.
\bar{T}'	\bar{x}'	The mean of the T' distribution.
T_M	x_{MEDIAN}	Median temperature.
T_{MODE}	--	Mode.
T_X	$x_{MAX/MIN}$	The distribution's highest or lowest temperature, depending on the sign of the skew.
$Z_{N/(N+1)}$	$Z_{N/(N+1)}$	A point where the area under the standard normal curve from $-\infty$ to z is $\frac{N}{N+1} \cdot 100$ percent of the curve's total area.
3.891	$Z_{0.99995}$	The value of $Z_{0.99995}$
$\Gamma(\hat{a})$	$\Gamma(\hat{a})$	The gamma function of a.
$\gamma(\hat{a}, T'/\hat{B})$	$\gamma(\hat{a}, x'/\hat{B})$	The incomplete gamma function of $(\hat{a}, T'/\hat{B})$.

A METHOD FOR TRANSFORMING TEMPERATURE DISTRIBUTIONS TO NORMALITY

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ABSTRACT. A method of transforming and fitting temperature distributions to the normal curve using the gamma distribution is presented. At the same time, the worth of the programmable calculator is once again demonstrated. The concept of "base temperature" is introduced in the presentation: a concept which permits the gamma distribution to be used with non-positive as well as positive values. An actual temperature distribution is transformed to normality using steps contained in a program designed to normalize meteorological variates. The resultant normal curve is then used to determine cumulative temperature probabilities.

A METHOD FOR TRANSFORMING TEMPERATURE DISTRIBUTIONS TO NORMALITY

Most meteorological variates are not normally distributed. This is true in the case of temperature, whose distributions are often skewed.

It is desirable to transform and fit skewed distributions, such as temperature, to the normal curve. This is because many useful statistical tests such as analysis of variance, hypothesis testing about the mean, etc., are valid only for normal distributions. In addition, the temperature distribution once transformed can be represented as a smooth curve on probability graph paper, thus allowing easy determination of temperature frequency.

How can temperature distributions be transformed to normality? One way is to use the gamma distribution to fit and transform the temperature distribution (Panofsky and Brier 1958). Unfortunately, the transforming process is usually tedious, involves complicated equations, and hard-to-read tables.

Enter the programmable calculator! These electronic marvels make the process of "normalizing" temperatures virtually painless, especially if the calculator is designed to accept magnetic program cards.

However, the gamma distribution can be used only if the distribution to be transformed consists of positive numbers. Obviously, temperatures expressed in (°F) or (°C) can assume values of zero or below.

One way of solving this problem is to select a "base temperature" which, when added to each temperature in the distribution, will make all the temperatures positive. The base temperature should be a few degrees lower than the distribution's lowest temperature if the distribution has positive skew, or a few degrees higher than the distribution's highest temperature if the distribution has negative skew 1/.

The author's formula for objectively determining the base temperature is as follows:

$$T_{BASE} = \sqrt{\frac{3.891}{Z \left[\frac{N}{N+1} \right]}} \cdot (T_X - T_M) + T_M$$

Values of $Z \left[\frac{N}{N+1} \right]$ can be obtained from Figure 1.

The program to normalize meteorological distributions consists of 435 steps 2/. It is designed to run on a Texas Instruments SR-52 calculator. The program flowchart is shown in Figure 2.

For purposes of illustration, the author will show how January minimum temperatures at Laramie, Wyoming, are transformed to normality using steps outlined in the program 3/. Laramie's winter temperature distributions show large skew and wide extremes--in other words, a good test for the program.

The sample temperatures are selected and the resultant distribution checked for the sign of its skew. In this case, a random sample of 100 elements is selected from a population of 620 January minimum temperatures at Laramie. The sample temperatures are then arrayed, as in Table 1. Since $T_M = 12^\circ\text{F}$, $T_{MODE} = 14^\circ\text{F}$, and $T_X = 33^\circ\text{F}$ (maximum T_i) and -46°F (minimum T_i), the sample distribution is assumed to have negative skew.

T_{BASE} needs to be determined, because the distribution has negative skew and contains temperatures $\leq 0^\circ\text{F}$. Since $N = 100$, $N/(N+1) = 100/101 = 0.9900990099$. Using Figure 1, $Z_{100/101}$ is found to be equal to

1/ $\sum |T_i - T_{BASE}|$ should be kept as small as possible. Otherwise $\Gamma(\hat{a})$, a term influenced by the size of $\sum |T_i - T_{BASE}|$, will become too large for the calculator to handle.

2/ See appendix.

3/ The author had previously normalized Laramie's temperatures, while stationed at WSFO Cheyenne.

2.33. Substituting 12°F for T_M , 33°F for T_X , and 2.33 for $Z_N/(N+1)$ yields a T_{BASE} of 39.13763204°F. Pressing the keys FIX, 0, and RUN truncates the fractional part of the number and places the integer 39 in the data register reserved for T_{BASE} .

The next steps involve the computation of sums which are used to solve later equations. Each T_i is entered into the program by successive keystrokes, with sums for T_i , T_i^2 , $T_i' = |T_i - T_{BASE}|$, $\ln(T_i')$, and i accumulated in appropriate data registers. After all 100 temperatures are entered, the following sums are obtained:

$$\begin{aligned}\sum T_i &= 997 \\ \sum T_i^2 &= 28,953 \\ \sum T_i' &= 2,903 \\ \sum \ln(T_i') &= 325.5576843\cdot\cdot^4\end{aligned}$$

The above sums are used to solve equations for the distribution's mean, standard deviation and skew, and the terms A, \hat{a} , and \hat{B} which are used to compute $\Gamma(\hat{a})$ and values of $\gamma(\hat{a}, T'/\hat{B})$. Substituting the appropriate data register contents into the equations yield the following:

$$\bar{T} = \sum T_i / N = 997/100 = \underline{9.97^\circ\text{F}}$$

$$\begin{aligned}S &= \sqrt{(\sum T_i^2 - NT^2)/(N-1)} = \sqrt{(28,953 - 100(9.97)^2)/(100-1)} \\ &= \underline{13.86^\circ\text{F}}\end{aligned}$$

$$\text{SKEW} = 3(\bar{T} - T_M)/S = 3(9.97 - 12)/13.86 = \underline{-0.439}$$

$$\begin{aligned}A &= \ln(\sum T_i' / N) - \sum \ln(T_i') / N = \ln(2,903/100) - 325.5576843\cdot\cdot / 100 \\ &= \underline{0.1127529346\cdot\cdot}\end{aligned}$$

$$\begin{aligned}\hat{a} &= (1 + \sqrt{1 + (4/3)A}) / 4A = \frac{(1 + \sqrt{1 + (4/3) \cdot (0.1127529346\cdot\cdot)})}{4(0.1127529346\cdot\cdot)} \\ &= \underline{4.595307802\cdot\cdot}\end{aligned}$$

$$\hat{B} = (\sum T_i' / N) / \hat{a} = (2,903/100) / 4.595307802\cdot\cdot = \underline{6.317313496\cdot\cdot}$$

$$\Gamma(\hat{a}) = \int_0^{\hat{a}} t^{\hat{a}-1} e^{-t} dt$$

$$\approx \frac{\sqrt{2\pi} (\hat{a}+5)^{(\hat{a}+5)} e^{-[(\hat{a}+5) - \frac{1}{12(\hat{a}+5)} + \frac{1}{360(\hat{a}+5)^3}]}{(\hat{a}+4) \cdot (\hat{a}+3) \cdot (\hat{a}+2) \cdot (\hat{a}+1) \cdot \hat{a}}$$

$$= \underline{13.29286832\cdot\cdot}, \text{ with error}/\Gamma(\hat{a}) < 2(10^{-7})$$

$$\text{Thus, error} < \underline{2.7(10^{-6})}$$

4/ The two dots preceding the superscript indicate the number is accurate to 12 significant digits in the data register. However, the display register in the SR-52 is capable of showing only 10 of these digits.

The solutions for \hat{a} , \hat{B} , and $\Gamma(\hat{a})$ are stored in data registers, so they can be recalled for subsequent computations.

By entering a temperature T into the program at this point, a value of the incomplete gamma function corresponding to T can be obtained. The formula for the incomplete gamma function is:

$$\begin{aligned} \gamma(\hat{a}, T'/\hat{B}) &= \int_0^{T'/\hat{B}} t^{\hat{a}-1} e^{-t} dt \\ &= (T'/\hat{B})^{\hat{a}} \cdot e^{-(T'/\hat{B})} \sum_{n=0}^{\infty} \frac{(T'/\hat{B})^n}{\hat{a}(\hat{a}+1)\cdots(\hat{a}+n)} . \end{aligned}$$

An iterative technique is used to solve the above formula. The iterative process continues until the series portion of the formula achieves a predetermined accuracy--in this case, accuracy to seven significant figures. The series is then multiplied by the terms preceding it, yielding a value of the incomplete gamma function.

Dividing $\gamma(\hat{a}, T'/\hat{B})$ by $\Gamma(\hat{a})$ yields the cumulative frequencies $P(T)$ and $Q(T) = [1 - P(T)]$ for a temperature T . Values of $\gamma(\hat{a}, T'/\hat{B})$, $P(T)$, and $Q(T)$ for selected January minimum temperatures at Laramie are listed in Table 2.

Information concerning temperature which was previously unknown or hard to determine can be easily interpolated from fitted gamma distributions such as the one in Figure 3. The January minimum temperature at Laramie should be $>28^{\circ}\text{F}$ in only about 5 percent of the observations; 21 percent of the observed minimums should be $<0^{\circ}\text{F}$; 80 percent of the observations should lie between -8°F and 25°F , with 36 percent of the observations being between 16°F and 32°F . In other words, the gamma curve makes it easy to express temperature data in probabilistic terms.

Thus, temperatures as well as other meteorological distributions can be transformed to normality by using the gamma distribution as a transforming agent. The process can be hastened considerably and actually made easy by letting a programmable calculator do the dirty work. Hopefully, this presentation will prove useful to those who wish to try the method out at their own station, or on other meteorological variates.

ACKNOWLEDGEMENTS. The author wishes to express his sincere thanks to Robert G. Beebe, Meteorologist in Charge of WSFO, Cheyenne, Wyoming. Bob's leadership, which encouraged on-station research while the author was stationed at Cheyenne, was the main inspiration for this paper. I also wish to thank my wife, Mary, who faithfully served as a proofreader and favorite critic as the paper evolved.

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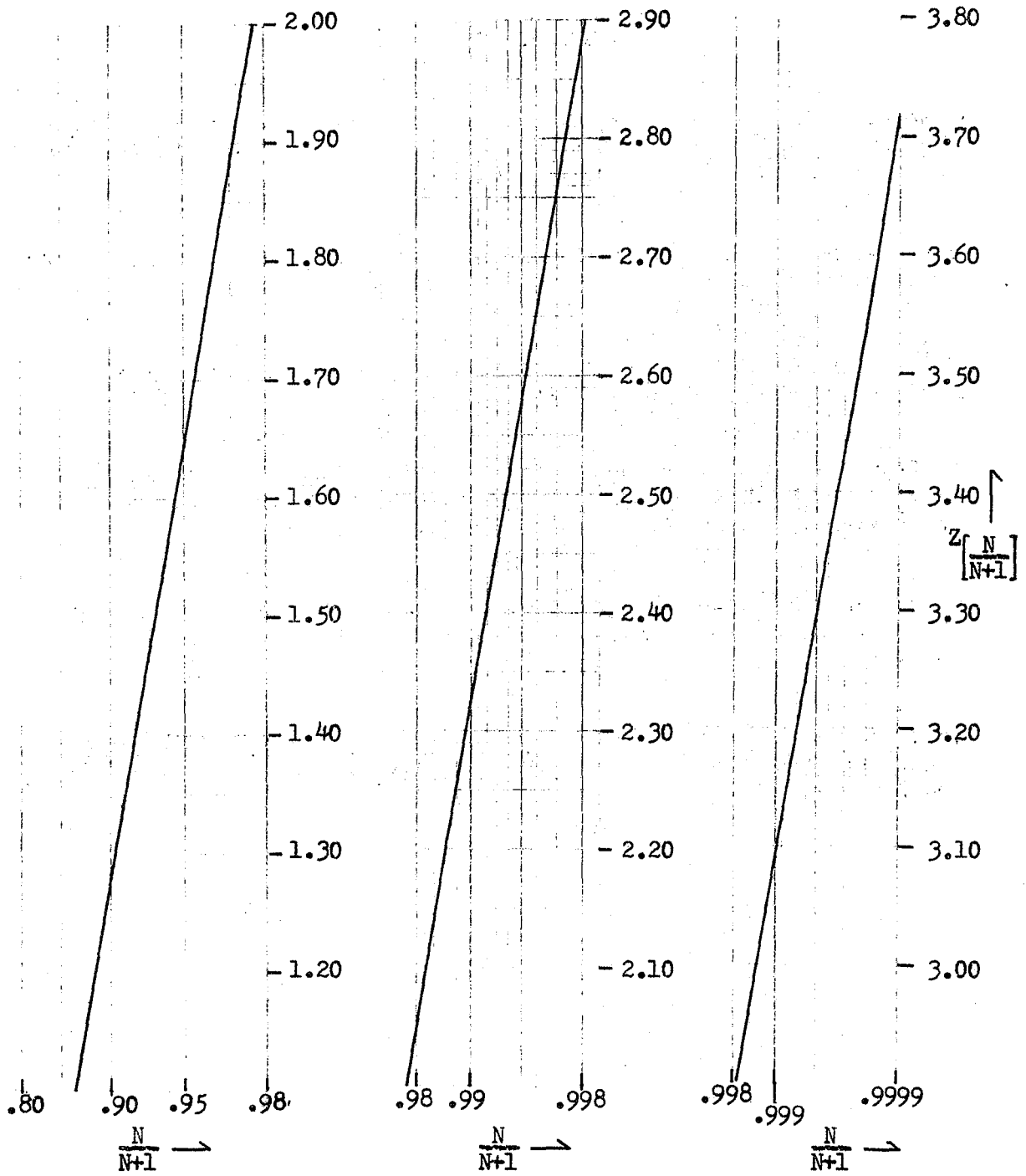


Figure 1. Graph Depicting $N/(N+1)$ vs. $Z_{N/(N+1)}$.

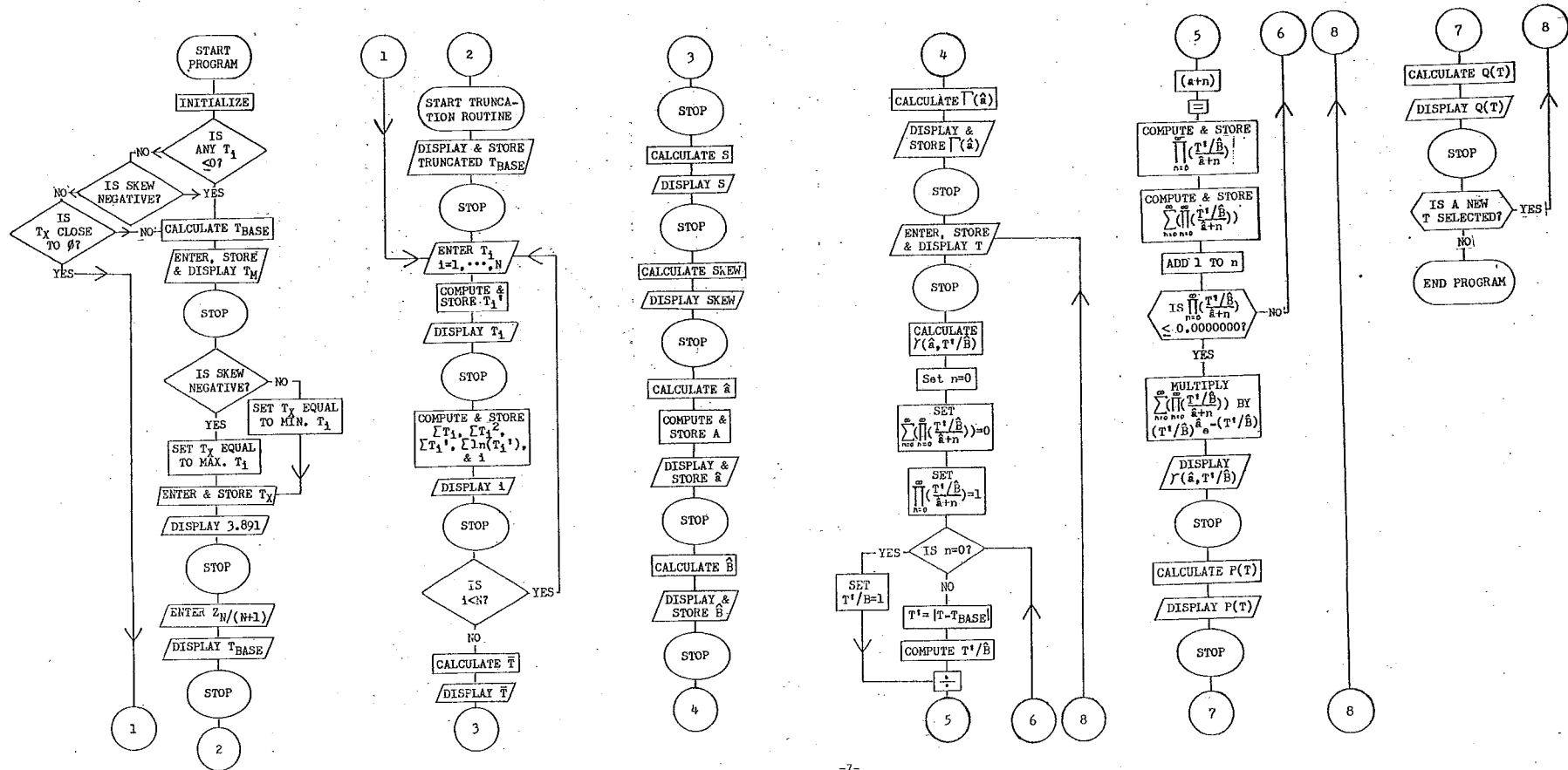


Figure 2. Flowchart of the Program Which Transforms Temperature Distributions to Normality

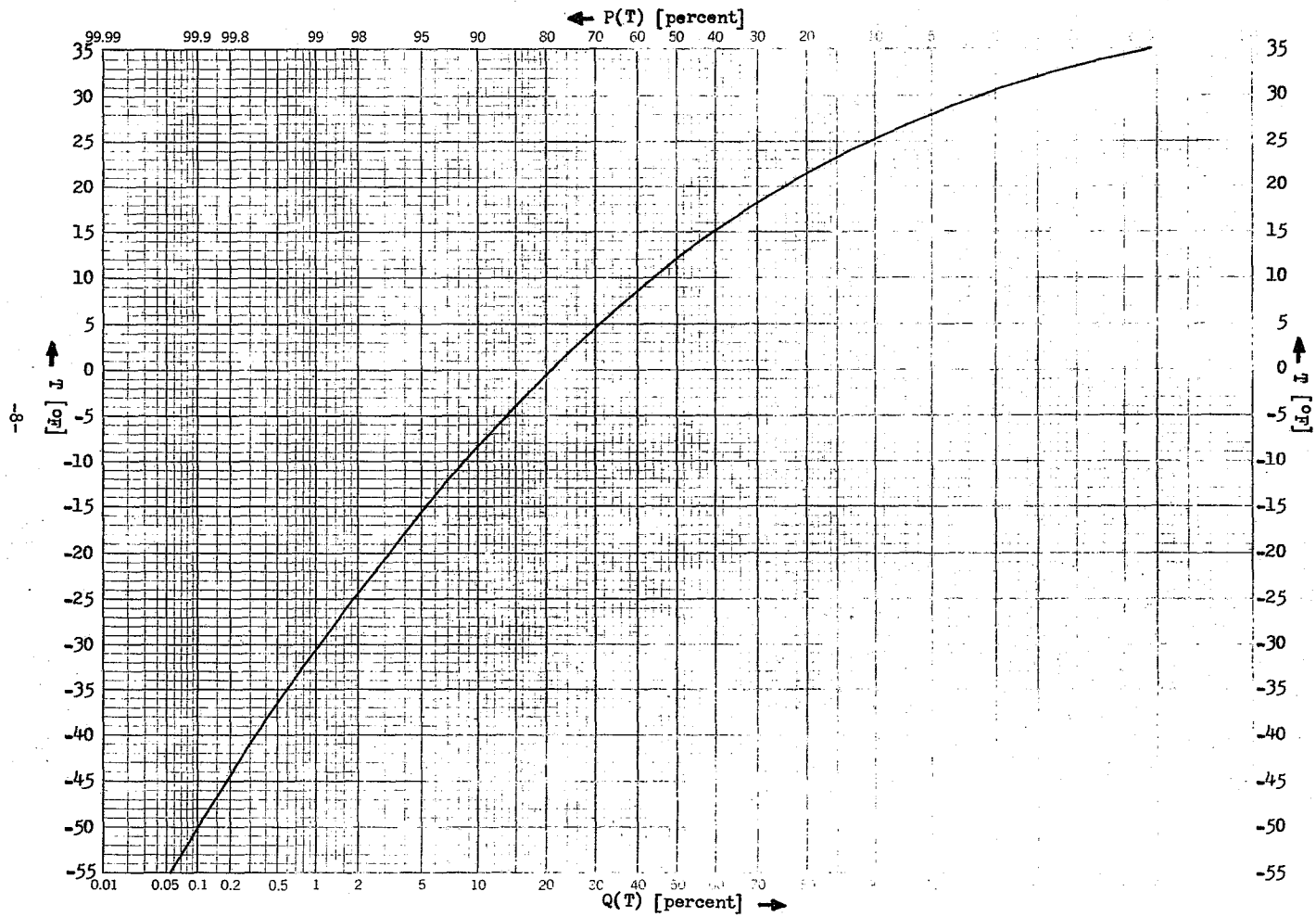


Figure 3. Graph Depicting the January Minimum Temperature at Laramie, Wyoming, (T) vs. the Cumulative Frequency of Temperature [$P(T)$ and $Q(T)$].

TABLE 1

SAMPLE OF 100 JANUARY MINIMUM TEMPERATURES AT LARAMIE, WYOMING
USED TO TEST THE PROGRAM
(TEMPERATURES ARRANGED FROM HIGHEST TO LOWEST VALUE)

i	T ₁ (°F)	i	T ₁ (°F)	i	T ₁ (°F)
1	33	41	14	81	1
2	33	42	14	82	0
3	31	43	14	83	0
4	30	44	14	84	0
5	29	45	14	85	-1
6	27	46	14	86	-1
7	27	47	14	87	-2
8	26	48	13	88	-4
9	25	49	13	89	-5
10	25	50	12	90	-7
11	25	51	12	91	-8
12	24	52	11	92	-13
13	24	53	11	93	-13
14	23	54	10	94	-15
15	23	55	10	95	-16
16	23	56	10	96	-17
17	22	57	10	97	-18
18	22	58	9	98	-22
19	22	59	9	99	-33
20	22	60	9	100	-46
21	21	61	9		
22	21	62	8		
23	20	63	8		
24	20	64	8		
25	20	65	7		
26	19	66	7		
27	19	67	6		
28	18	68	6		
29	18	69	5		
30	18	70	5		
31	18	71	5		
32	18	72	5		
33	17	73	5		
34	17	74	4		
35	17	75	3		
36	17	76	3		
37	17	77	2		
38	16	78	2		
39	16	79	2		
40	15	80	2		

TABLE 2

VALUES OF $\gamma(a, T'/B)$, P(T), AND Q(T) FOR SELECTED
JANUARY MINIMUM TEMPERATURES AT LARAMIE, WYOMING

T(°F)	$\gamma(a, T'/B)$	P(T)	Q(T)
37	0.00085	0.00006	0.99994
35	0.01592	0.00120	0.99880
30	0.35198	0.02648	0.97352
25	1.45297	0.10930	0.89070
20	3.26784	0.24583	0.75417
15	5.40220	0.40640	0.59360
12	6.66542	0.50143	0.49857
10	7.46000	0.56120	0.43880
5	9.20132	0.69220	0.30780
0	10.54511	0.79329	0.20671
-5	11.51347	0.86614	0.13386
-10	12.17533	0.91593	0.08407
-15	12.60905	0.94856	0.05144
-20	12.88370	0.96922	0.03078
-25	13.05274	0.98194	0.01806
-30	13.15430	0.98958	0.01042
-35	13.21409	0.99407	0.00593
-40	13.24867	0.99668	0.00332
-45	13.26836	0.99816	0.00184
-50	13.27942	0.99899	0.00101
-55	13.28556	0.99956	0.00055
-60	13.28893	0.99970	0.00030

APPENDIX

ACTUAL SR-52 PROGRAM

USED TO NORMALIZE METEOROLOGICAL VARIATES

(X replaces T in most of the symbology, since the program is used to normalize meteorological variates other than temperature).

SR-52 User Instructions

MEAN, STANDARD DEVIATION,
SKEW, $\hat{\alpha}$, AND $\hat{\beta}$

PAGE 1 OF 3

A			
X _{BASE}	X̄	S	SKEW
X _i	i	α	β
			INIT. MEDIAN

B			

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	LOAD PROGRAM CARD			
2	CLEAR REGISTERS		*E	∅
3	ENTER MEDIAN	X _{MEDIAN}	E	X _{MEDIAN}
4	CALCULATE X _{BASE}	X _{MAX/MIN}	*A	3.891
	$\sqrt{\frac{2 \ln 2}{\pi(N+1)} [X_{MAX/MIN} - X_{MEDIAN}]}$	Z[N/(N+1)]	RUN	X _{BASE}
	+ X _{MEDIAN}	*FIX M, M=0,1,9	RUN	X _{BASE} (TRUNCATED TO 10 DECIMAL PLACES)
	NOTE: USE STEPS 3 AND 4 WHEN ANY OF THE DISTRIBUTION HAS A NEGATIVE SKEW.	X _i 'S ARE ≤ ∅, AND/OR		WHEN THE
5	INDEPENDENT VARIABLE X _i	X _i	A	X _i
			B	i, i=1, ..., N
	REPEAT STEP 5 FOR ALL X _i 'S IN THE DISTRIBUTION			
6	COMPUTE THE MEAN		*B	X̄
7	COMPUTE THE STANDARD DEVIATION		*C	S
8	COMPUTE THE SKEW		*D	SKEW
9	COMPUTE α̂		C	α̂
10	COMPUTE β̂		D	β̂
	$\bar{X} = \sum_{i=1}^N X_i / N$			
	$S = \sqrt{\frac{1}{N-1} (\sum_{i=1}^N X_i^2 - N\bar{X}^2)}$			
	$SKEW = \frac{3(X - X_{MEDIAN})}{S}$			
	$\hat{\alpha} = [1 + \sqrt{1 + 4A/3}] / 4A$ WHERE $A = \ln \bar{X} - \frac{1}{N} \sum_{i=1}^N \ln X_i$, $X_i' = X_i - X_{BASE} $			
	$\hat{\beta} = \bar{X}' / \hat{\alpha}$ WHERE $\bar{X}' = [\sum_{i=1}^N X_i - X_{BASE}] / N$			

TITLE \bar{X} , S, SKEW, $\hat{\alpha}$, AND $\hat{\beta}$ PAGE 2 OF 3
PROGRAMMER MORRIS S. WEBB, JR. DATE 4/16/77

SR-52 Coding Form

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LABELS
000	46	*LBL		54)			00	∅	∅	∑ X _i '	A X _i
	17	*B		56	*RTN			08	8		i ⁱ⁼¹	B i
	53	(040	46	*LBL		23	LNx			C α̂
	43	RCL		11	A			44	SUM			D β̂
	∅∅	∅	COMPUTE	53	(∅∅	∅			E X _{MEDIAN}
005	∅6	6	X̄	42	STO			∅9	9			A' X _{BASE}
	55	÷		∅∅	∅			∅1	1			B' X̄
	43	RCL		045	∅4	4		44	SUM			C' S
	∅1	1		75	-			∅1	1			D' SKEW
	∅∅	∅		43	RCL	COMPUTE		∅∅	∅			E' INIT.
010	54)		∅∅	∅	X _i '		43	RCL			REGISTERS
	56	*RTN		∅3	3	= X _i -X _{BASE}		∅1	1		DISPLAY	∅0
	46	*LBL		050	54)		∅∅	∅			∅1 MEDIAN
	18	*C		4∅	*X ²	PLAY X _i		81	HLT			∅2 X _{MAX/MIN}
	53	(3∅	*√			090	46	*LBL		∅3 X _{BASE}
015	53	(42	STO			13	C	COMPUTE		∅4 X _i
	53	(∅∅	∅			53	(α̂	∅5 X _i '
	43	RCL		055	∅5	5		53	(AND STORE		∅6 ∑ X
	∅∅	0		43	RCL			43	RCL	IN R ₁₂		∅7 ∑ X ²
	∅7	7		∅∅	∅			095	∅∅	∅		∅8 ∑ X ^r
020	75	-		∅4	4			∅8	8			∅9 ∑ ln X ^r
	43	RCL		81	HLT			55	÷			∅10 N
	∅1	1		060	46	*LBL	COMPUTE	43	RCL			∅11 A
	∅∅	∅		12	B	SUMS		∅1	1			∅12 α̂
	65	X		43	RCL			100	∅∅	∅		∅13 β̂
025	17	*B	COMPUTE	∅∅	∅			54)	(COMPUTE		∅14
	4∅	*X ²	S	∅4	4			23	LNx	A AND		∅15
	54)		065	44	SUM		75	-	STORE		∅16
	55	÷		∅∅	∅	∑ X _i		43	RCL	IN R ₁₁		∅17
	53	(∅6	6	i ⁱ⁼¹		105	∅∅	∅		∅18
030	43	RCL		4∅	*X ²			∅9	9			∅19
	∅1	1		44	SUM			55	÷			FLAGS
	∅∅	∅		070	∅∅	∅	∑ X _i ²	43	RCL			∅
	75	-		∅7	7	i ⁱ⁼¹		∅1	1			1
	∅1	1		43	RCL			110	∅∅	∅		2
035	54)		∅∅	∅			54)			3
	54)		∅5	5							4
	3∅	*√		075	44	SUM						

SR-52
Coding Form

SR-52
Coding Form

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LABELS
000	46	*LBL		04	4	STEP 45)		52	EE	IF NOT,	A) $\Gamma(a)$	
	17	*B'		05	5			22	INV	GO TO	B) X	
	53	(040	17	*B'	(IF n=0,	90	*IFZRO	STEP 082)	C) $\gamma(a, x/\beta)$	
	53	(41	GTO	CALCULATE		00	0		D) P(x)	
	43	RCL		00	0	X/\beta AND	060	03	3		E) Q(x)	
005	00	0		04	4	GO TO STEP		03	3		A' (a+5)	
	04	4		06	6	46)		43	RCL		B' X/\beta	
	75	-	COMPUTE	045	01	1		01	1		C'	
	43	RCL	AND	55	÷			07	7		D'	
	00	0	DISPLAY	53	(085	65	X		E'	
010	03	3	X/\beta	43	RCL			17	*B'		REGISTERS	
	54)		01	1	[X/\beta]		45	Y^X		00	
	40	*X^2		050	02	2	[a+n]	43	RCL		01) MEDIAN	
	30	*√		85	+			01	1		02) X _{MAX/MIN}	
	55	÷		43	RCL		090	02	2		03) X _{BASE}	
015	43	RCL		01	1			65	X		04) X _{OR} X _i	
	01	1		05	5			17	*B'		05) X _i '	
	03	3		055	95	=		94	+/-		06) Σ X	
	54)		49	*PROD	$\prod_{n=0}^{\infty} \frac{x/\beta}{a+n}$		22	INV		07) Σ X^2	
	56	*RTN		01	1	$= R_{16}$	095	23	LN X		08) Σ X^P	
020	46	*LBL	COMPUTE	06	6			95	=		09) Σ ln X^P	
	13	C	$\gamma(a, x/\beta)$	43	RCL	(RECALL		22	INV		10) N	
	00	0		060	01	1	R ₁₆)	57	*FIX		11) A	
	42	STO	(STORE	06	6			42	STO	(STORE	12) a	
	01	1	ZERO IN	44	SUM	$\sum_{n=0}^{\infty} R_{16}$		01	1	$\gamma(a, x/\beta)$	13) B	
025	05	5	R ₁₅ AND	01	1	$= R_{17}$		08	8	IN R ₁₆)	14) $\Gamma(a)$	
	42	STO	R ₁₇)	07	7			81	HLT		15) n	
	01	1		065	01	1		46	*LBL		16) $\prod_{n=0}^{\infty} \frac{x/\beta}{a+n}$	
	07	7		44	SUM	(LOOP		16	*A'		17) $\sum_{n=0}^{\infty} R_{16}$	
	01	1		01	1	COUNTER)		53	(COMPUTE	18) $\gamma(a, x/\beta)$	
030	42	STO	(STORE 1	05	5			43	RCL	AND	19	
	01	1	IN R ₁₆)	43	RCL			01	1	DISPLAY	FLAGS	
	06	6		070	01	1		02	2	(a+5)	0	
	43	RCL	(RECALL	06	6			85	+		1	
	01	1	n)	57	*FIX	(IF R ₁₆ >		05	5		2	
035	05	5		07	7	0.0000000		54)		3	
	90	*IFZRO	(IF n=0,	52	EE	GO TO					4	
	00	0	GO TO	075	22	INV	STEP 033;					

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LABELS
112	56	*RTN		75	-			54)			A) $\Gamma(a)$
	46	*LBL		53	(55	÷			B) X
	14	D		152	01	1		53	(C) $\gamma(a, x/\beta)$
	53	(02	2			16	*A'			D) P(x)
	43	RCL		65	X			192	75	-		E) Q(x)
117	01	1		16	*A'			02	2			A' (a+5)
	08	8	COMPUTE	54)			54)			B' X/\beta
	55	÷	P(x)	157	20	*1/x		55	-			C'
	43	RCL		85	+			53	(D'
	01	1		53	(197	16	*A'		E'
122	04	4		03	3			75	-			REGISTERS
	54)		06	6			01	1			00
	57	*FIX		162	00	0		95	=			01) MEDIAN
	05	5		65	X			42	STO			02) X _{MAX/MIN}
	56	*RTN		16	*A'			202	01	1		03) X _{BASE}
127	43	*LBL		45	Y^X			04	4			04) X _{OR} X _i
	15	E		03	3			81	HLT			05) X _i '
	01	1	COMPUTE	167	54)		46	*LBL			06) Σ X
	75	-	Q(x)	20	*1/x			12	B			07) Σ X^2
	14	D		54)			22	INV	ENTER		08) Σ X^P
	95	=		94	+/-			57	*FIX	AND DIS-		09) Σ ln X^P
132	81	HLT		22	INV			42	STO	PLAY X		10) N
	46	*LBL		172	23	LN X		00	0			11) A
	11	A		55	÷			04	4			12) a
	02	2		43	RCL			212	81	HLT		13) B
137	65	X		01	1							14) $\Gamma(a)$
	59	*√		02	2							15) n
	55	÷		177	55	÷						16) $\prod_{n=0}^{\infty} \frac{x/\beta}{a+n}$
	16	*A'	COMPUTE	53	(17) $\sum_{n=0}^{\infty} R_{16}$
	95	=	$\Gamma(a)$	16	*A'			217				18) $\gamma(a, x/\beta)$
142	30	*√	AND STORE	75	-							19
	65	X	IN R ₁₄	04	4							FLAGS
	16	*A'		182	54)						0
	45	Y^X		55	÷							1
	16	*A'		53	(2
147	65	X		16	*A'							3
	53	(75	-							4
	16	*A'		187	03	3						

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- 92 Smoke Management in the Willamette Valley. Earl M. Bates, May 1974. (COM-74-11277/AS)
- 93 An Operational Evaluation of 500-mb Type Stratified Regression Equations. Alexander E. MacDonald, June 1974. (COM-74-11407/AS)
- 94 Conditional Probability of Visibility Less than One-Half Mile in Radiation Fog at Fresno, California. John D. Thomas, August 1974. (COM-74-11555/AS)
- 95 Climate of Flagstaff, Arizona. Paul W. Sorenson, August 1974. (COM-74-11678/AS)
- 96 Map Type Precipitation Probabilities for the Western Region. Glenn E. Rasch and Alexander E. MacDonald, February 1975. (COM-75-10428/AS)
- 97 Eastern Pacific Cut-off Low of April 21-28, 1974. William J. Alder and George R. Miller, January 1976. (PB-250-711/AS)
- 98 Study on a Significant Precipitation Episode in the Western United States. Ira S. Brenner, April 1975. (COM-75-10719/AS)
- 99 A Study of Flash Flood Susceptibility--A Basin in Southern Arizona. Gerald Williams, August 1975. (COM-75-11360/AS)
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- 104 Objective Aids for Forecasting Minimum Temperatures at Reno, Nevada, During the Summer Months. Christopher D. Hill, January 1976. (PB-252-866/AS)
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- 120 Some Meteorological Aspects of Air Pollution in Utah with Emphasis on the Salt Lake Valley. Dean N. Jackman and William T. Chapman, June 1977.
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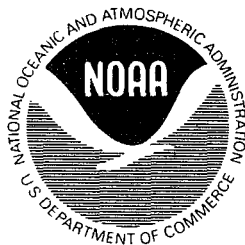
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