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Forecasting Maximum Temperatures at Helena, Montana

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A western Indian symbol for rain. It also symbolizes man's dependence on weather and environment in the West.

U. S. DEPARTMENT OF COMMERCE
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FORECASTING MAXIMUM TEMPERATURES AT HELENA, MONTANA

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I. INTRODUCTION

Forecasting "today's" maximum afternoon temperature during nonadvective conditions depends primarily upon an accurate determination of net solar energy available for heating the lower atmosphere. In this study an approach similar to one developed by Williams [1], in which he converted net solar energy units to physical area units suitable for adiabatic chart WB D-11, is used. A maximum temperature formula based upon the average number of area units for each month is developed and a nomogram in the form of a plastic overlay for use on the pseudo-adiabatic chart is constructed. Corrections for cloudiness are included on the overlay.

II. DETERMINATION OF NET SOLAR ENERGY UNITS

With clear skies, net solar energy available for heating the lower atmosphere by convective processes is largely dependent on albedo of atmosphere and ground, energy absorption by soil, evaporation, and terrestrial radiation back into space. Using a procedure outlined by Dickey [2], net solar energy in calories cm^{-2} from sunrise to time of the maximum temperature is determined by the following steps:

1. The number of calories cm^{-2} day^{-1} incident at the top of the atmosphere on any desired date is taken from available insolation tables.

2. Since the amount of solar energy from sunrise to time of occurrence of maximum temperature (near 1500LST) is not a full day, the figure obtained in Step 1 is reduced by an estimated 20 percent.

3. The amount in Step 2 is reduced by 14 percent for sky albedo.

4. According to surface albedos compiled by Kung, Bryson and Lenchow [3], the average surface albedo during other than snow conditions is about 15 percent near Helena. Solar energy computed from Step 3 is reduced accordingly.

5. No figure is available for the amount of solar energy used in heating the soil; so a value of 20 percent given by Dickey is used. The value from Step 4 is reduced by 20 percent.

6. An estimate of the long-wave radiation back to space ($S_o\uparrow$) is based on an equation developed by Hewson and Longley [4]. That is,

$$S_o\uparrow = .285 \times 10^{-11}\ T^4 \text{ cal cm}^{-2} \text{ min}^{-1}$$

where T is the average daily temperature in degrees K.

By substituting mean daily temperature and multiplying by the number of minutes available from sunrise to 1500LST, the number of calories radiated back to space during this interval of time can be computed. Table I shows the number of calories lost to space on the 21st day of each month.

TABLE I

Number of calories per cm^{-2} reradiated to space between sunrise and 1500LST on the 21st of each month at Helena, Montana.

Month	Calories cm^{-2}
December	62
January	56
February	65
March	82
April	103
May	115
June	128
July	130
August	115
September	94
October	81
November	62

7. Evaporation at Helena is ignored because of general lack of moisture coupled with a porous soil.

Summarizing, the total available solar energy for heating the lower atmosphere can be written:

$$Q_T = Q_i\ (1-a)\ (1-b)\ (1-c)\ (1-d) - S_o\uparrow \quad (1)$$

where Q_T = number of calories cm^{-2} available to heat the lower atmosphere to maximum temperature, and

Q_i = Total calories cm^{-2} incident at top of atmosphere on any selected day.

a = Proportion of incident solar radiation beyond time of maximum temperature with a value of .20.

b = Albedo of atmosphere with a value of .14.

c = Albedo of surface with a value of .15.

d = Proportion of radiation absorbed by soil with a value of .20.

$S_O\uparrow$ = Loss of radiation to space between sunrise and 1500 LST.

Substituting these values into I yields:

$$Q_T = .47\ Q_i - S_O\uparrow \qquad (2)$$

Table 2 shows the amount of calories available for heating the lower atmosphere to a maximum temperature, and also the total calories incident at the top of the atmosphere. Values in the column labeled A will be discussed in the following section.

Date	Q_i cal cm^{-2}	Q_T cal cm^{-2}	A $(C^o)^2$
Dec. 21	230	46	24
Jan. 21	300	85	35
Feb. 21	470	157	65
Mar. 21	620	210	88
Apr. 21	820	282	122
May 21	930	322	142
Jun. 21	990	335	150
Jul. 21	930	309	141
Aug. 21	820	270	123
Sep. 21	620	198	89
Oct. 21	470	141	62
Nov. 21	300	73	31
Dec. 21	230	46	24

Table 2. Total calories incident at the top of the atmosphere (Q_i), calculated net calories available for heating the lower atmosphere to a maximum temperature (Q_T), and number of centigrade degree squares contained in area A, defined in Figure 1, on the 21st of each month.

III. DEVELOPMENT OF THE MAXIMUM TEMPERATURE FORMULA

A number of investigators [5], [6], [7], [8], have shown that the net amount of solar energy available to heat the lower atmosphere to its maximum temperature is proportional to an area on an adiabatic chart enclosed by an early morning temperature curve, the surface pressure line, and the dry adiabat extending from the surface maximum temperature up to and intersecting the temperature curve. Figure I shows a schematic of an estimate for a 1200Z summertime Helena temperature sounding on a pseudo-adiabatic chart. Area A is enclosed by the dry adiabat through the later observed maximum temperature T_M, the 880-mb pressure line (average surface pressure), and the 1200GMT temperature curve. T_O is a temperature determined by extending that portion of the temperature curve above any surface inversion downward with the same slope to the surface pressure. (Any surface inversion is ignored until later.) Z is the point of intersection between the temperature curve and the adiabat through the maximum temperature. Angle γ is the angle between the temperature curve and the surface pressure line, and angle β is the angle between the dry adiabat and the surface pressure line.

In Figure I, let S-Z be the distance above the surface pressure line measured in units of I degree centigrade (same units as the temperature coordinate). If angle γ, angle β, and area A are known, a maximum temperature formula can be derived:

$$S\text{-}Z = \frac{2A}{(T_M - T_O)} \quad \text{and} \quad S\text{-}Z = \frac{(T_M - T_O)}{(\text{Cot}\beta - \text{Cot}\gamma)}$$

hence:

$$T_M = T_O + [2A\ (\text{Cot}\beta - \text{Cot}\gamma)]^{1/2} \qquad (3)$$

[The dimensions of A are $(C^o)^2$]

Solar energy cannot be adapted to a pseudo-adiabatic chart directly; therefore, calories must be converted into the area units involved in A. The basic idea is to take a cross-section the size of one centigrade degree on the pseudo-adiabatic chart, find how many calories it takes to heat this one small square* one degree centigrade and divide this number into the total number of net calories available for raising the temperature from the early morning value to the afternoon maximum. This quotient is equivalent to the number of small squares contained in area A provided all the small squares start with the same temperature and pressure. However, each small square would take a different number

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*Each I centigrade degree square on the pseudo-adiabatic chart is equivalent to a square 8.8×10^3 centimeters on a side in the atmosphere. It should be noted in evaluating for heat capacity, the square is more precisely a slab one centimeter thick.

of calories to raise its temperature one centigrade degree since its density is dependent upon temperature and pressure. Because height Z has been selected to be variable, there is no precise method of evaluating either a total mass represented by area A, or even the average density. It is, therefore, necessary to employ a less accurate estimate.

An average density can be estimated with sufficient accuracy for this study by using the perfect gas law:

$$\rho = \frac{P}{RT}$$

Here P assumes Helena's average surface pressure, 880 mb.; T equals the average daily temperature for any given day (degrees K); R is the gas constant for dry air. The volume of the small slab of air is:

$$V = a^2(1 \text{ cm}) \qquad \text{where} \quad a = 8.8 \times 10^3 \text{ cm}.$$

and its mass is:

$$m = \rho V = \frac{P}{RT} a^2 (1 \text{ cm}).$$

Now, using the equation for computing heat capacity:

$$Q = m C_p dT$$

or

$$Q = \frac{P}{RT} a^2(1 \text{ cm}) C_p dT$$

and allowing the temperature change dT to equal one degree centigrade results in

$$\frac{P}{RT} a^2(1 \text{ cm}) C_p (1 \text{ deg})$$

for the number of calories necessary to heat one small slab one degree centigrade. The total number of calories available to heat the small slab to the maximum temperature is:

$$Q_T a (1 \text{ cm})$$

and the quotient

$$\frac{Q_T a (1 \text{ cm})}{\frac{P}{RT} a^2(1 \text{ cm}) C_p (1 \text{ deg})}$$

or

$$\frac{Q_T\ R\ T\ (1\ \text{cm})}{P\ C_p\ a\ (1\ \text{cm})\ (1\ \text{deg})}$$

gives a number which is equivalent to the number of centigrade degree squares contained in area A on the pseudo-adiabatic chart.

Hence $$A = \frac{Q_T\ R\ T}{P\ C_p\ a}\ \text{deg}^2.$$

By inserting this known number of squares into equation (3), it is easily seen how prediction of the maximum temperature is accomplished. By also holding the adiabatic slope constant, say at 45 degrees (a close approximation at the higher temperatures), equation (3) can be further simplified to:

$$T_M = T_o + \left[\frac{2Q_T\ R\ T}{P\ C_p\ a}(1 - \text{Cot}\gamma)\right]^{1/2}. \qquad (4)$$

IV. CONSTRUCTION OF AN OVERLAY

Instead of solving equation (4) each time, a nomogram on a transparent overlay was designed which is usable throughout the year. The construction of the nomogram is illustrated in Figure 2.

In panel (a) of Figure 2, two lines are drawn perpendicular to each other, the vertical line labeled Z and the horizontal line P_O. For this special case of a 90 degree angle between P_O and Z (an isothermal lapse rate) the contangent of γ in equation (4) is zero. The distance along P_O from the intersection point of P_O and Z is in centigrade degrees, equal to the square root of the number of small squares contained in area A. In computing the number of squares R, P_O , C_p , and a are constants. Q_T is the value on the 21st of each month, and T is the Helena mean temperature (degrees K) an the 21st of each month.

In panel (b) of Figure 2 is shown a scale of the number of squares in area A along the P_O line in convenient amounts. Also shown are the months corresponding to the number of squares computed from the Q_T and T for the month.

The line Z represents the slope of the temperature curve; so more P lines are drawn every 5 degrees of angle as shown in panel (c) of Figure 2.

The number of square units is plotted along each new P line by inserting the respective angle changes in equation (4). These points are connected by curved lines as shown in panel (c).

From the transmission percentages for various types of clouds given in Table 3, a reduction of the square units can be computed for each cloud type. The horizontal lines labeled by cloud type and the slanting lines in panel (d) of Figure 2 result from these computations. Panel (d) represents the finished device.

An example of the use of the device is illustrated in Figure 3. The dashed lines in this figure represent the transparent overlay. On a clear day in the middle of June 150 square units are available. Place Z along the temperature curve and follow the line labeled June on the overlay down to the P_O line. Follow the curved line to the surface pressure line (880 mb). At this point of intersection of the curved line and the surface pressure line read the forecast maximum temperature in degrees centigrade (20°C). This is then readily converted to °F from the temperature scales at the bottom of the chart (68°F).

Again, assume the same sounding, but assume that the day will be overcast with altocumulus clouds. From the line labeled June at the top of the device, follow the slanting line to its intersection with the Ac line, then down parallel to the 70 unit line to the P_O line, then along a curved line to its intersection with the surface pressure line. At this point read the estimated maximum temperature (15.5°C or 60°F). For days with a considerable inversion an estimate based on experience or direct information must be made in order to subtract a certain number of square units.

Overcast	Percentage Transmission
Cirrus	84
Cirrostratus	78
Altocumulus	50
Altostratus	41
Cumulus	25
Stratocumulus	20*

Table 3 - Transmission of solar energy through various types of clouds. [After Meyers (8).]

V. VERIFICATION

To test equation (4), 484 days were selected from all months during the period 1964-1967. Background information was obtained from Helena's

*Estimated

WBAN-10A and 10B forms, and Great Falls' temperature soundings (located 70 air miles to the north). Certain criteria had to be met before any individual day could be selected.

1. Cold or warm air advection, as indicated by the average temperature change in the Great Falls soundings from 1200Z to 0000Z, not to exceed 5 centigrade degrees.
2. No precipitation falling from sunrise to 1500 LST.
3. No snow on the ground.

Test data were divided into three categories: clear, partly cloudy, and cloudy. Clear days averaged cloudiness zero to three-tenths coverage during the period between sunrise and 1500 LST; partly cloudy days four-tenths to seven-tenths; and cloudy days eight-tenths to overcast. In addition, from inspection of WBAN 10 forms the predominate cloud type for the day was estimated.

Certain rules were followed in verifying each day: For cold and warm air advection a mean sounding was used. Since the plastic overlay was constructed for clear or overcast days, no adjustment was needed except for partly cloudy days when a mean between cloudy and clear values was used.* Estimates were made for inversions.

Table 4 shows the cumulative frequency distribution of errors for clear days. Table 5 and 6 are similar distributions of errors for partly cloudy and cloudy days. The average absolute error was 2.2 F° for clear days, 2.4 F° for partly cloudy, and 2.6 F° for cloudy days.

ABSOLUTE ERROR IN F°	≤1	≤2	≤3	≤4	≤5
CUMULATIVE FREQUENCY %	41	65	84	94	98

Table 4. Cumulative frequency distribution of errors for clear days.

ABSOLUTE ERROR IN F°	≤1	≤2	≤3	≤4	≤5
CUMULATIVE FREQUENCY %	33	62	80	88	94

Table 5. Cumulative frequency distribution of errors for partly cloudy days.

ABSOLUTE ERROR IN F°	≤1	≤2	≤3	≤4	≤5
CUMULATIVE FREQUENCY %	33	57	74	90	95

Table 6. Cumulative frequency distribution of errors for cloudy days.

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*Williams (1) used about 60% transmission for .5 cloud cover which has been adjusted to 50% for this paper.

Most days selected were from April 1 through November. Frequent snow cover during December through March prevented the method from being tested on many days during these months.

VI. CONCLUSION

The basic objective in developing equation (4) and constructing a plastic overlay was to provide a quick guide for forecasting maximum temperature year around. Its utility has been demonstrated through extensive use by Helena forecasters who use it about 85% of the forecasting time.

VII. ACKNOWLEDGMENT

Appreciation is expressed to the staff at Helena WBO and especially to Scientific Services Division, Western Region Headquarters, for their thorough evaluation.

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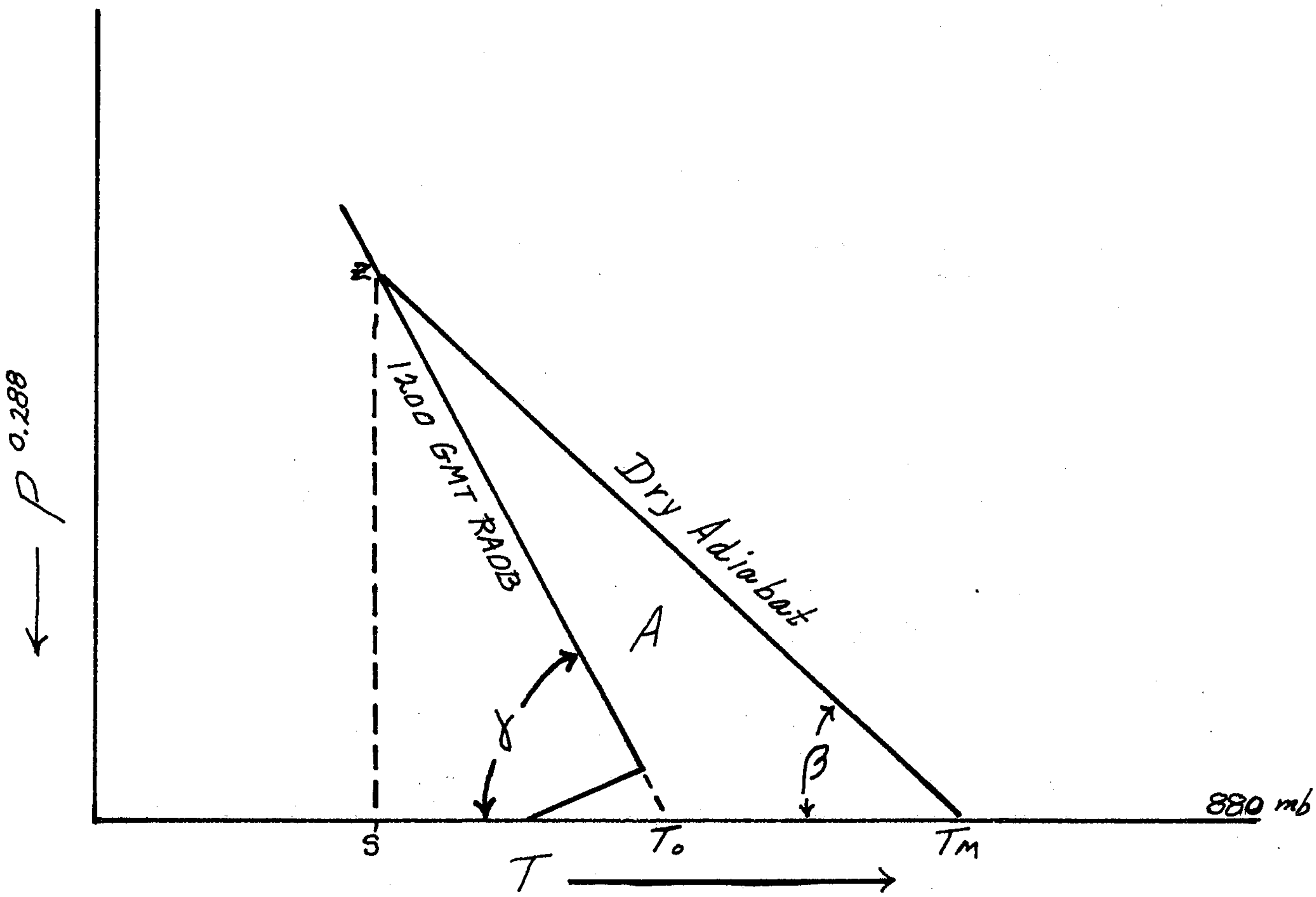

FIGURE 1. Schematic of a temperature sounding on a pseudo-adiabatic chart.

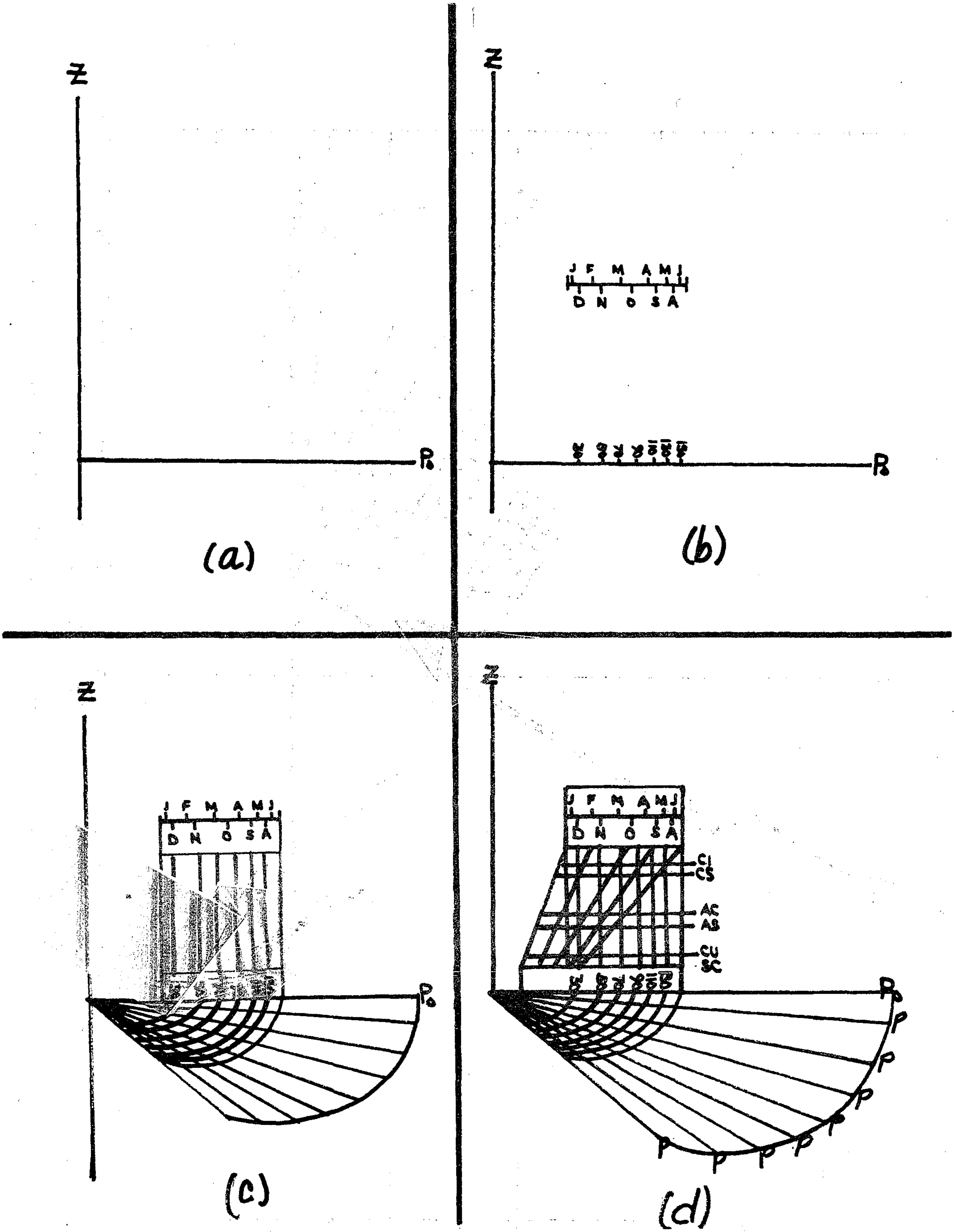

FIGURE 2. Stepwise illustration of construction of plastic overlay for computing maximum temperature from 1200Z Great Falls radiosonde. See text for detailed explanation. (Actual size to be used on Pseudo-Adiabatic Chart, D-11.)

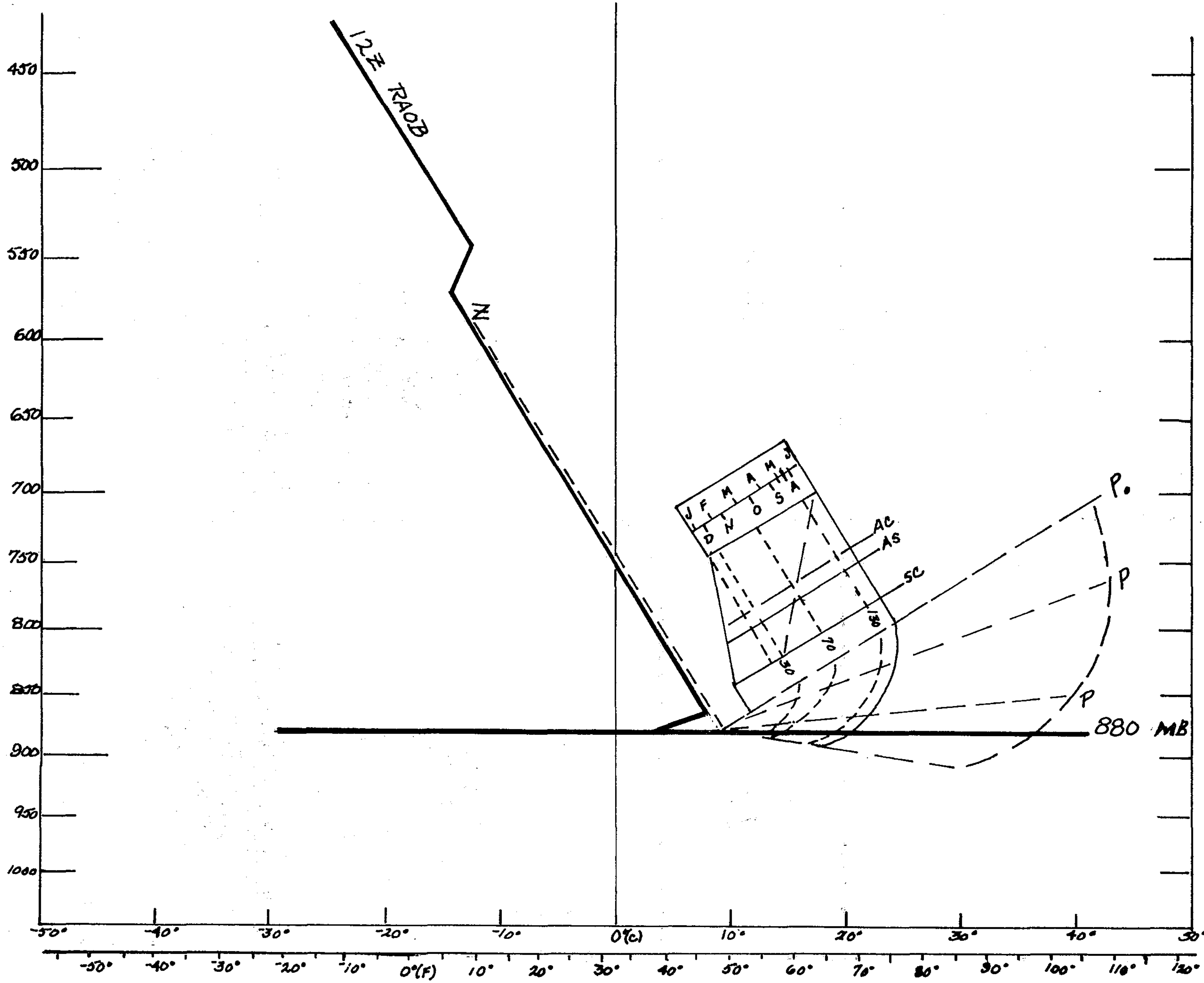

FIGURE 3. Illustration of use of transparent overlay. Dashed lines represent the overlay; 12Z Raob, Helena, Montana, solid lines.

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