



# **Determination of the Earth's mathematical surface in Africa towards the realization of the International Height Reference System (IHR)**

## **Final Report**

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### **1. Overview**

This report summarizes the main activities and achievements of the IUGG grant entitled “Determination of the Earth’s mathematical surface in Africa towards the realization of the International Height Reference System (IHR)”. The grant ran in the period 2016–2018. The lead applicant is IAG, represented by the IAG Secretary General, Hermann Drewes, DGFI, Munich, Germany. The Project Principal Participant of the IAG lead applicant is Hussein Abd-Elmotaal, Minia University, Egypt. The supporting applicant is IASPEI represented by the IASPEI Secretary General, Johannes Schweitzer, NOR SAR, Kjeller, Norway. The Project Principal Participant of the IASPEI supporting applicant is Rashad Kebeasy, National Research Institute of Astronomy and Geophysics, Egypt.

### **2. Activities and Achievements**

A number of very important activities and tasks of the project have been achieved during the period of the IUGG grant. They are summarized in the following sections.

#### **2.1 Collecting Gravity Data**

This task represents the core of the project. It is a very hard task, since most institutions don't like to release the gravity data they have. A very laborious work has been done by the IAG PPP, Hussein Abd-Elmotaal, to collect the gravity data along the life time of the project (even long before starting the IUGG grant).



Figure 1 shows the gravity data available as of 2014, from which the gravity database AFRGDB\_V1.0 (Abd-Elmotaal et al., 2015) has been created. It consists of **land**, **shipborne** and **altimetry** derived gravity anomalies data (cf. Fig. 1).

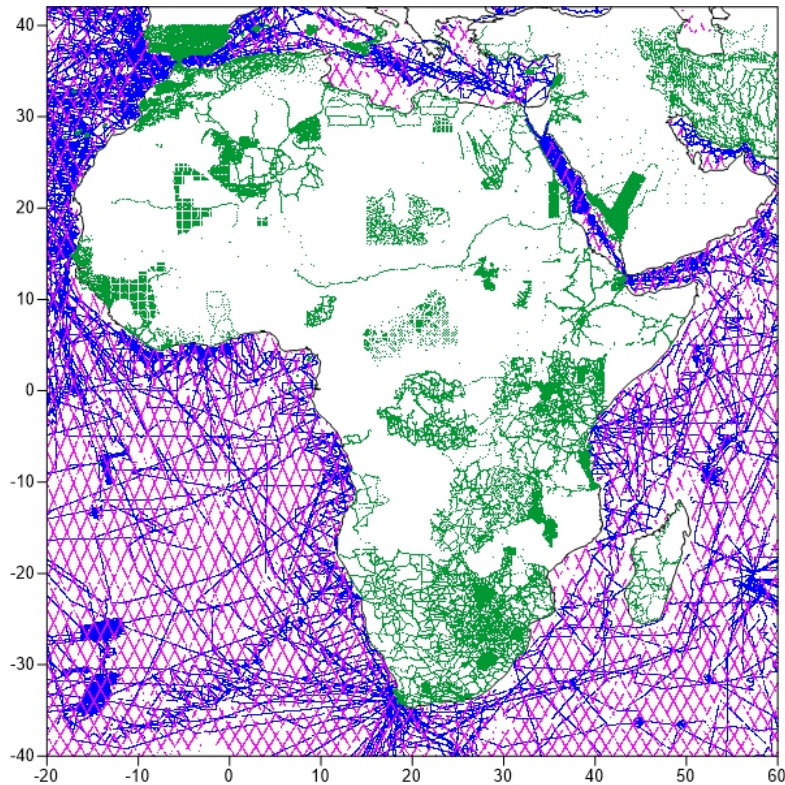


Fig. 1: The available gravity data for Africa as of 2014, used to create the AFRGDB\_V1.0 gravity database (after Abd-Elmotaal et al., 2015).

In 2016, a new data set on land has become available by the Bureau Gravimétrique International (BGI) (thanks to Sylvain Bonvalot). This new data set is illustrated in Fig. 2. The new gravity set is located mainly at the very large data gaps of the previously available data set used to generate the AFRGDB\_V1.0 gravity database (compare Figs. 1 and 2). Accordingly, this new data set has been employed to externally validate the AFRGDB\_V1.0 gravity database.

It should be mentioned that collecting the gravity data is a dynamic process, and additional data are looked for and hopefully it will become available in the future.

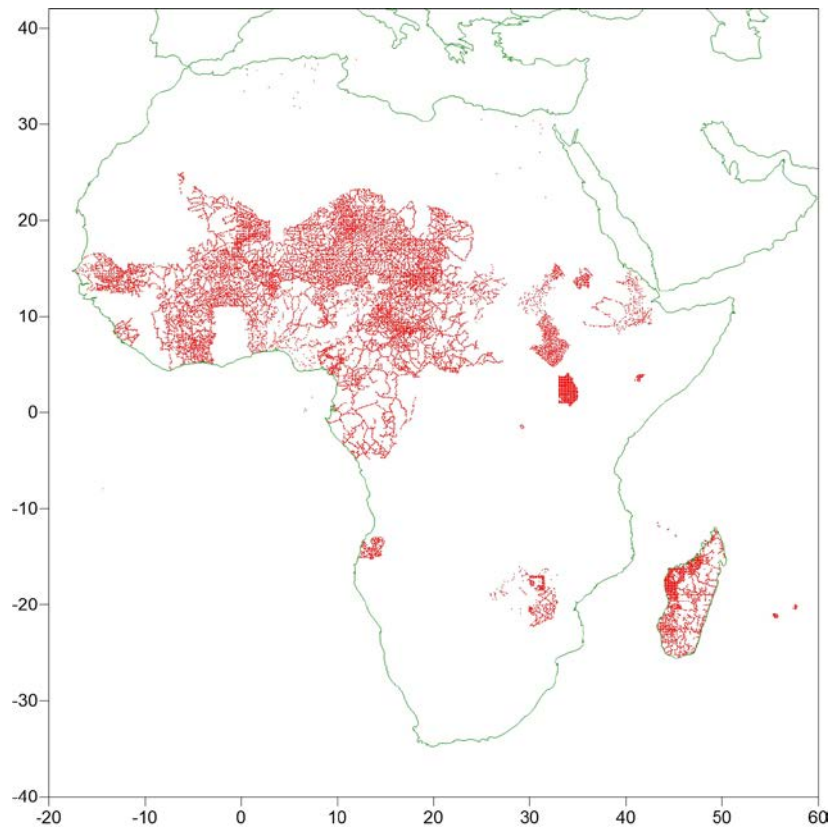


Fig. 2: The new land gravity data for Africa as of 2016 (after Abd-Elmotaal et al., 2017b).

## 2.2 Create and Maintain a Webpage of the Project

A webpage of the project ([mn.minia.edu.eg/Geodesy/AFRgeo/index.htm](http://mn.minia.edu.eg/Geodesy/AFRgeo/index.htm)) has been created and maintained. It is hosted at Minia University (where Hussein Abd-Elmotaal comes from). It contains a lot of useful information about the project, its data and obtained results and achievement. It also hosts a number of important publications and reports related to the project.

## 2.3 Gross-Error Detection and Validation

The gross-error detection for the shipborne and altimetry data has been carried out in a previous stage (cf. Abd-Elmotaal and Makhloof, 2013; Abd-Elmotaal and Makhloof, 2014). A gross-error detection has been implemented on the land data set using a smart gross-error detection technique, performing the algorithm developed by Abd-Elmotaal and Kühtreiber (2014). This gross-error approach is based on the least-squares prediction algorithm. The technique works first to estimate the gravity anomaly value at the data station using other values than the current data point. It thus compares the estimated value to the data value for possible blunder detection. Hence the technique measures the influence of removing the data



value of a current point on the neighbourhood stations. Only if the value of a certain station proves to be blunder, it is then removed from the data base. This approach saves as much data as possible. Using that approach, only 0.5% of the land data points are found to be blunders. More details are found in (Abd-Elmotaal and Kühtreiber, 2014).

## 2.4 Establishing Topo/Bathymetry Detailed DTM for Africa

A fine digital terrain model (DTM) for Africa and the surrounding region covering the window ( $-42^\circ \leq \phi \leq 44^\circ; -22^\circ \leq \lambda \leq 62^\circ$ ) using the Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) Global Digital Elevation Model (GDEM) at a  $3'' \times 3''$  resolution (which corresponds to roughly 90 m resolution on the earth's surface) has been created (Abd-Elmotaal et al., 2017a). The ASTER-GDEM model, which is available only on land, has been smoothed from its original  $1'' \times 1''$  resolution to the used  $3'' \times 3''$  resolution using the block average operator technique employing special characteristics at the coastal borders. The  $30'' \times 30''$  SRTM30+ has been used, after being interpolated to  $3'' \times 3''$  grid size, to fill-in the missing sea regions of the ASTER-GDEM model. The created  $3'' \times 3''$  DTM has been compared with the available point data both on land and on sea areas for a set of more than one million points. Residuals follow perfectly the Gaussian normal distribution. The fine  $3'' \times 3''$  DTM for Africa is illustrated in Fig. 3.

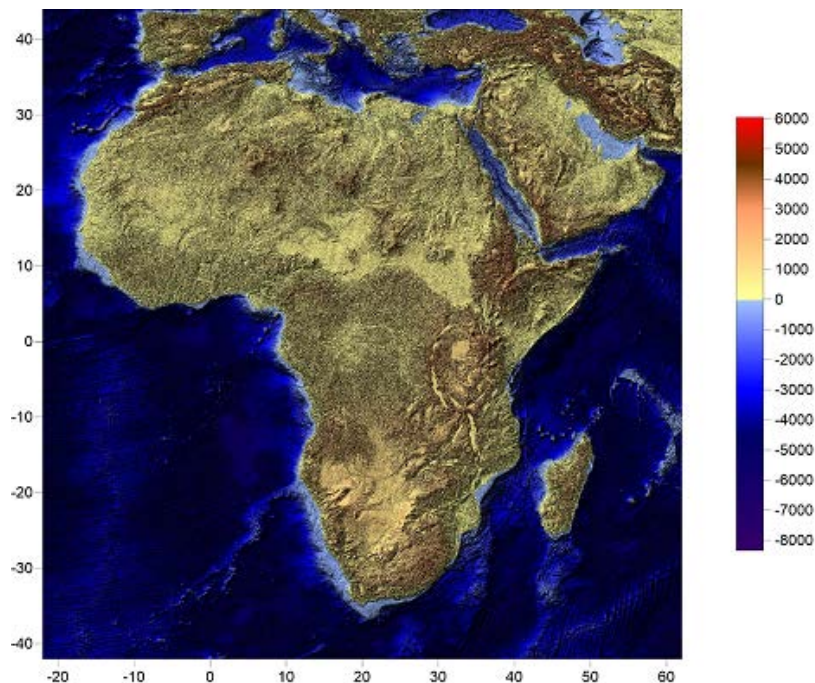


Fig. 3: The fine  $3'' \times 3''$  DTM for Africa (after Abd-Elmotaal et al., 2017a). Units in [m].



It is worth mentioning that a number of DTMs for Africa have been created from the established most fine  $3'' \times 3''$  DTM employing the block average operator technique. The following DTM resolutions are available:  $6'' \times 6''$ ,  $15'' \times 15''$ ,  $30'' \times 30''$  and  $3' \times 3'$ . Such a set of different resolution DTMs are needed for the terrain reduction computation.

## **2.5 Evaluation of the African Gravity Database AFRGDB\_V1.0**

As stated above, recently a new data set on land has been made available by the Bureau Gravimétrique International (BGI). The new data set consists of 33,971 gravity data points. There is no overlap between the old and the new data sets on land. The free-air gravity anomalies for the new data set range between  $-148.4$  mgal and  $453.6$  mgal with an average of about  $4.9$  mgal and a standard deviation of  $28.3$  mgal. Figure 2 shows the distribution of the new gravity data set on land for Africa. Comparing Figs. 1 and 2 shows that most of the new data points are located in the very large data gaps (especially at West Africa) of the old data set used for generating the AFRGDB\_V1.0 gravity database. This enables the comparison at those data points between the new data and the AFRGDB\_V1.0 database, to measure the external accuracy of the AFRGDB\_V1.0 gravity database.

Figure 4 illustrates the differences at the new data points on land between the new data and the AFRGDB\_V1.0 gravity database values. The white pattern indicates differences below  $10$  mgal in magnitude. These differences range between  $-82.7$  mgal and  $82.5$  mgal with an average of about  $3.2$  mgal and a standard deviation of  $16.3$  mgal. This indicates that the external accuracy of the AFRGDB\_V1.0 is about  $16$  mgal, which is rather a very good value compared to the very large gaps in the land gravity data set used to generate the AFRGDB\_V1.0 gravity database.

Figure 5 shows a histogram of the differences at the new data points on land between the new data and the AFRGDB\_V1.0 gravity database values. Figure 5 shows Gaussian normal distribution with high precision index.

More interesting details on the internal and external evaluation of the AFRGDB\_V1.0 can be found in (Abd-Elmotaal et al., 2017b).

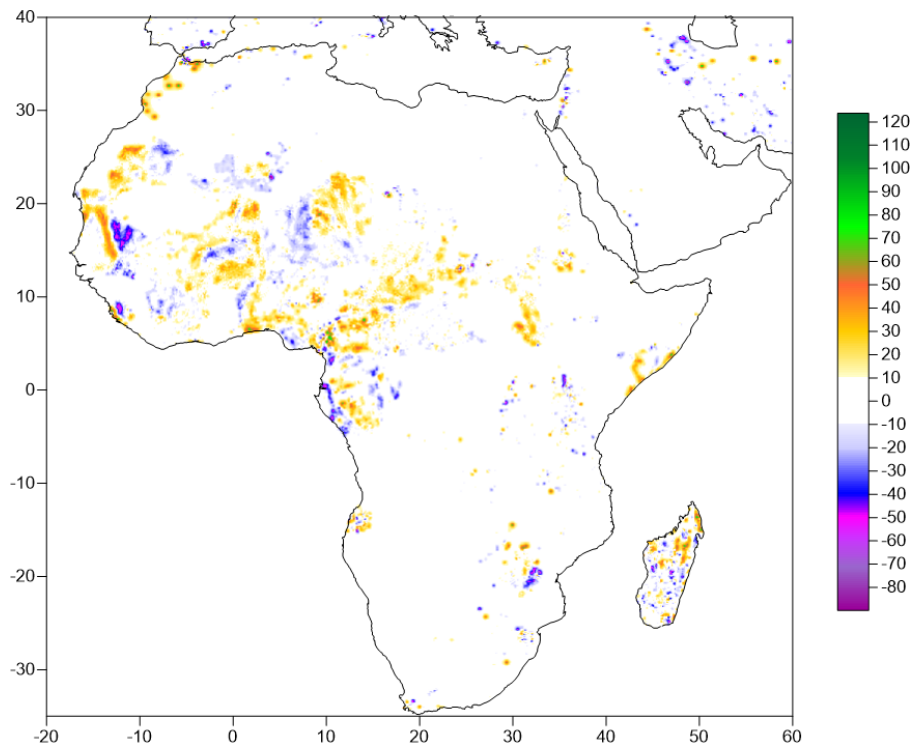


Fig. 4: External validation of the AFRGDB\_V1.0 gravity data base. Contour interval: 10 mgal (after Abd-Elmotaal et al., 2017b).

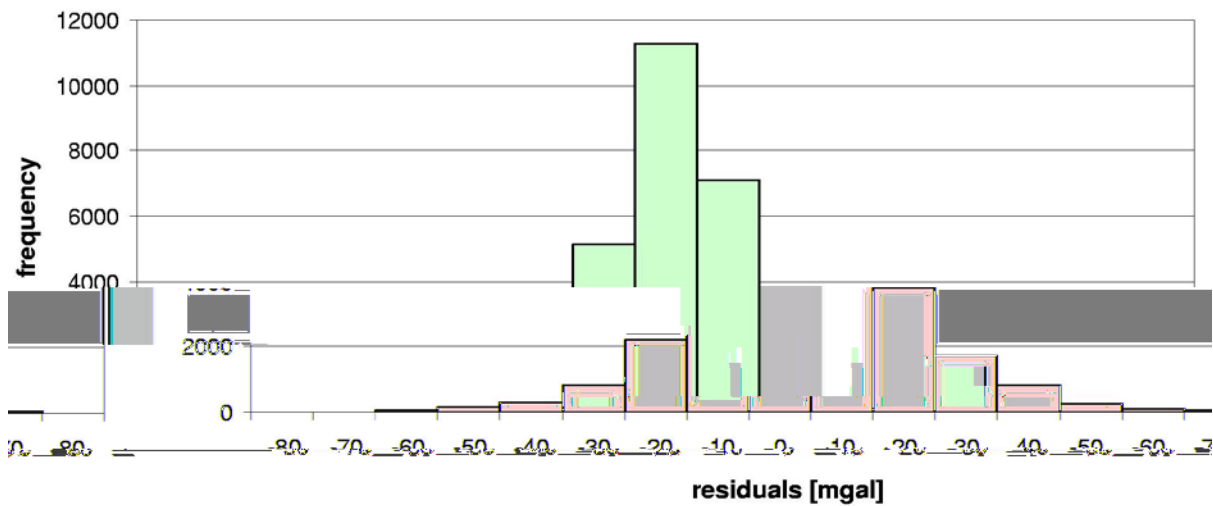


Fig. 5: Histogram of the external validation of the AFRGDB\_V1.0 gravity data base on land (after Abd-Elmotaal et al., 2017b).



## 2.6 Establishing an Optimum Gravity Interpolation Technique for Large Data Gaps

As illustrated in Figs. 1 and 2, the current gravity database for Africa contains significantly large data gaps. These large data gaps affect the interpolation precision of the reduced gravity anomalies needed for the determination of the earth's mathematical surface for Africa. Hence a capable interpolation technique that can be used for a proper gravity interpolation within large data gaps has been developed (cf. Abd-Elmotaal and Kühtreiber, 2017).

An artificial gap has been made in the land gravity data set of Africa. This artificial gap is located in the high lands of Ethiopia covering the window ( $5^{\circ} \leq \phi \leq 15^{\circ}; 35^{\circ} \leq \lambda \leq 40^{\circ}$ ). This gap contains 1837 gravity stations. The free-air gravity anomalies in the gap area range between -120.3 mgal and 157.1 mgal with an average of about 16.3 mgal and a standard deviation of 39.6 mgal. Figure 6 shows the distribution of the gap points (in red) within the land data (in green).

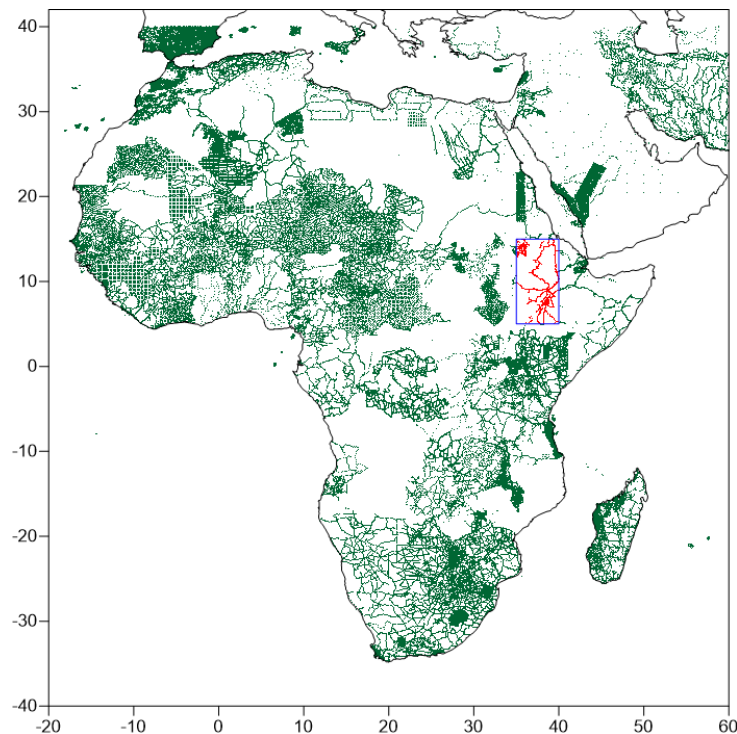


Fig. 6: Distribution of the gap (in red) and land (in green) gravity data for Africa (after Abd-Elmotaal and Kühtreiber, 2017).

The artificial gap area has significantly rough topography. Figure 7 shows the DTM for the gap area (after Abd-Elmotaal et al., 2017a). It shows the rough topography of the gap area. The heights within the gap area range between -26 m and 4483 m with an average of 1605 m and a standard deviation of about 700 m.

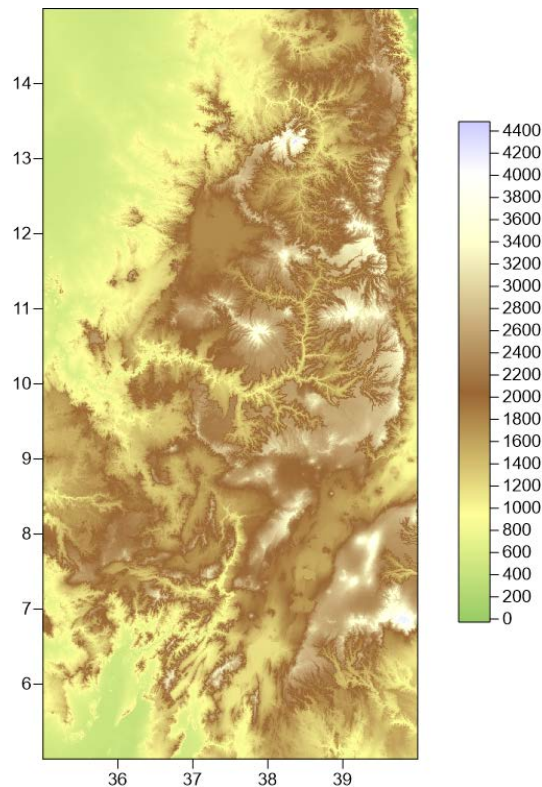


Fig. 7: The DTM for the artificial gap (after Abd-Elmotaal et al., 2017a). Units in [m].

The remaining data set, comprising land (without the original data at the artificial gap), shipborne and altimetry has been used to interpolate the gravity values at the gap points. Then a comparison between the interpolated and the data values has been carried-out to determine the accuracy of the used interpolation technique. The unequal weight least-squares prediction (with the optimum curvature parameter at the origin; cf. Abd-Elmotaal and Kührtreiber, 2016) with an underlying grid at the gap area computed by the satellite-only GO\_CONS\_GCF\_2\_DIR\_R5 model till degree and order 300, has been proposed as the developed interpolation approach. The window technique, suggested by Abd-Elmotaal and Kührtreiber (2003) to get rid of the double consideration of the topographic-isostatic masses within the data window in the framework of the remove-restore technique, has been used for the reduction process.

Figure 8 shows the difference between the data and interpolated anomalies for the artificial gap using the developed interpolation approach. These differences range between -102.45 mgal and 60.49 mgal with an average of -0.60 mgal and a standard deviation of about 20.70 mgal. The white pattern in Fig. 8 indicates differences below 20 mgal in magnitude. 68.9% of the data points have differences below 20 mgal.

Figure 8 shows nearly no correlation with topography (compare Figs. 7 and 8). This indicates that the developed interpolation approach (unequal weight least-squares prediction with underlying grid at the data gaps employing the window remove-restore technique) is





capable for interpolation purposes within large data gaps. This is also indicated by the nearly zero mean and the small standard deviation of the differences.

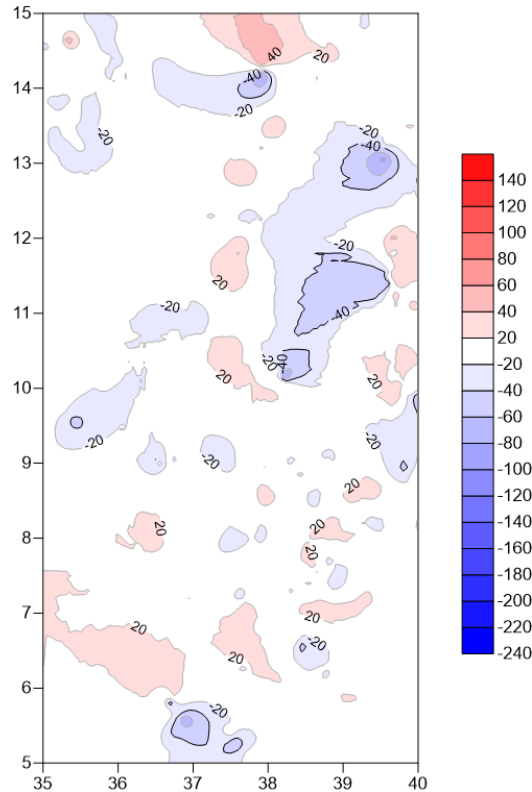


Fig. 8: Difference between the data and interpolated anomalies for the artificial gap using the developed interpolation approach (after Abd-Elmotaal and Kühnreiber, 2017). Units in [mgal].

## 2.7 Establishment of the Gravity Database of Africa (AFRGDB\_V2.0)

The currently available gravity data are treated in somewhat different way to establish the AFRGRV\_V2.0 gravity database for Africa. This treatment is described in the followings.

The currently available land gravity data set consists of 154,037 gravity data points. In order to enhance the behaviour of the empirical covariance function, the land data have been filtered on a  $1' \times 1'$  grid (i.e., in each cell of  $1' \times 1'$ , only one data point, the closest to the cell-center, has been selected). The number of land data after the grid filtering became 127,067 points.

A smart gross-error detection scheme has been carried out on the land data set (Abd-Elmotaal and Kühnreiber, 2014). That gross-error detection scheme uses the least-squares prediction technique. The gross-error detection technique estimates first the gravity anomaly value at the computational point using the values of the surrounding stations excluding the computational point. Comparing the estimated and data values defines a possible gross-error.



Accordingly, the effect of the computational point on the surrounding stations is examined. Data points which show real gross-error behaviour are removed from the database. The number of land data after the gross-error removal became 126,202 points.

Figure 9 shows the distribution of the land data set (after grid filtering and gross-error removal). It illustrates that the land data still contain very large data gaps. The free-air gravity anomalies on land range between -163.2 mgal and 465.5 mgal with an average of about 9.8 mgal and a standard deviation of 40.9 mgal.

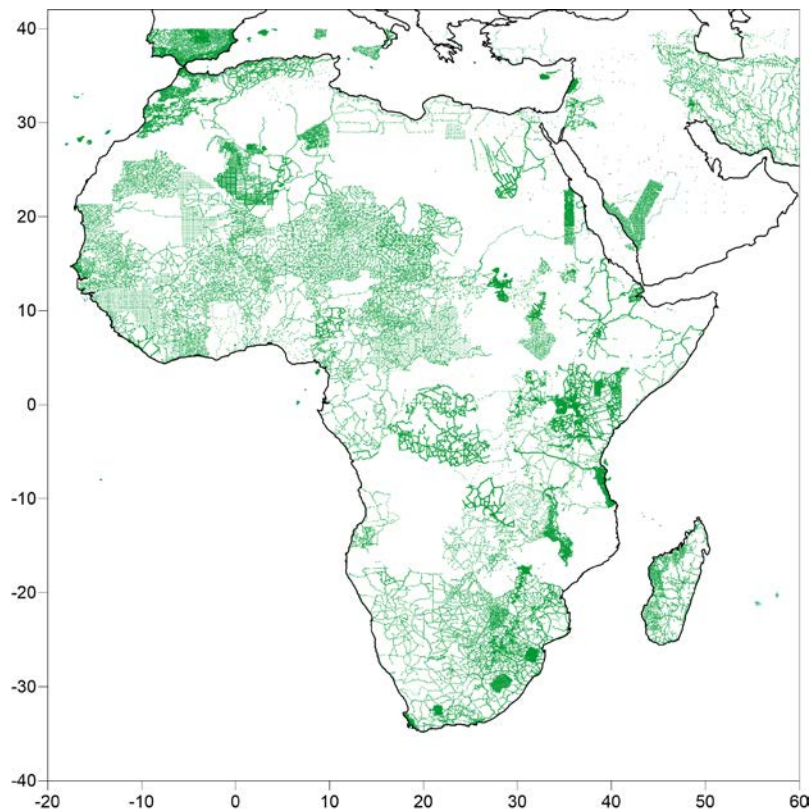


Fig. 9: Distribution of the land gravity data for Africa used to create the AFRGDB\_V2.0 gravity database (after Abd-Elmotaal et al., 2017c).

The currently available shipborne gravity data set, after a preliminary gross-error detection scheme developed by Abd-Elmotaal and Makhloof (2013), consists of 971,945 gravity data points. The applied preliminary gross-error approach is based on the least-squares prediction technique. It estimates the gravity anomaly value at the computational point using the values of the surrounding stations excluding the computational point. Hence, a comparison between the estimated and data values is used to define a possible blunder. The gross-error technique works in an iterative scheme till the standard deviation of the discrepancy between the data and estimated values is less than 1.5 mgal.

In order to enhance the behaviour of the empirical covariance function as well as to decrease the domination of the shipborne data, the shipborne data have been filtered on a  $3' \times 3'$  grid. The number of shipborne data after the grid filtering became 148,858 points. A



sophisticated gross-error detection scheme, similar to that applied on the land data (Abd-Elmotaal and Kühtreiber, 2014), has been carried out on the shipborne data set. The number of shipborne data after the sophisticated gross-error removal became 148,674 points.

Figure 10 shows the distribution of the shipborne gravity data (after grid filtering and gross-error removal). It illustrates a better distribution than that of the land data. The remaining gaps of the shipborne data are partially filled with the altimetry-derived gravity anomalies. The shipborne free-air gravity anomalies range between -238.3 mgal and 354.4 mgal with an average of about -6.2 mgal and a standard deviation of 34.9 mgal.

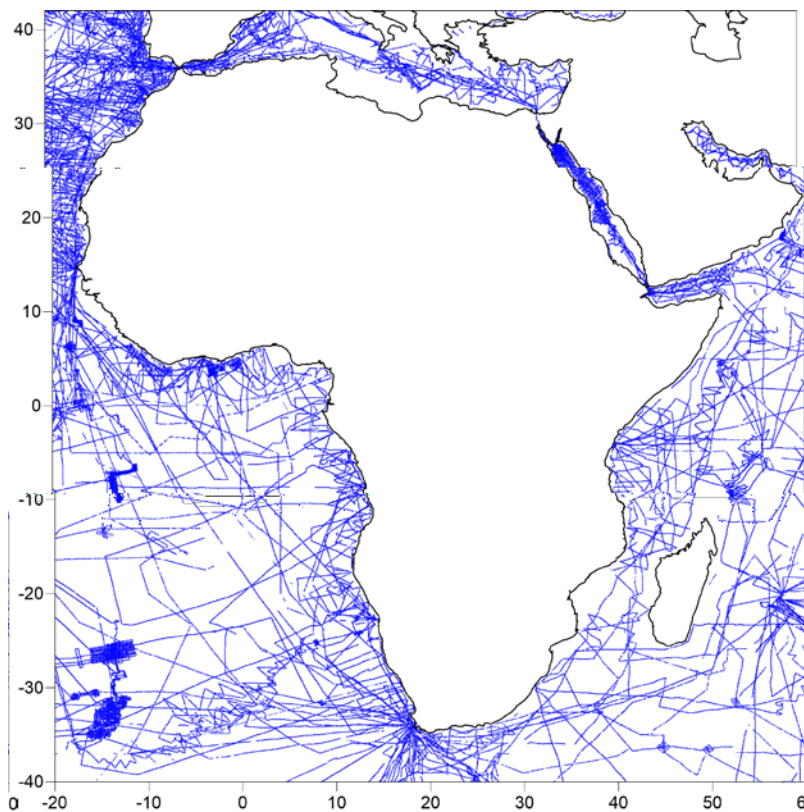


Fig. 10: Distribution of the shipborne gravity data for Africa used to create the AFRGDB\_V2.0 gravity database (after Abd-Elmotaal et al., 2017c).

The currently available altimetry-derived gravity anomaly data set, which were constructed from the average of 44 repeated cycles of GEOSAT by the National Geophysical Data Center NGDC ([www.ngdc.noaa.gov](http://www.ngdc.noaa.gov)), after applying a preliminary gross-error detection technique similar to that applied on the shipborne data, consists of 119,249 gravity data points. A combination between the shipborne and altimetry data took place (Abd-Elmotaal and Makhloof, 2014). This combination causes some gaps along altimetry tracks when the altimetry data don't match with the shipborne data (cf. Fig. 11).

In order to enhance the behaviour of the empirical covariance function as well as to decrease the domination of the altimetry-derived data, the altimetry-derived data have been



filtered on a  $3' \times 3'$  grid. The number of altimetry-derived data after the grid filtering became 70,732 points. A sophisticated gross-error detection scheme, similar to that applied on the land data, has been carried out on the altimetry-derived data set. The number of altimetry-derived data after the sophisticated gross-error removal became 70,589 points.

Figure 11 shows the distribution of the available altimetry data (after grid filtering and gross-error removal). It illustrates, more or less, a regular distribution. The altimetry free-air gravity anomalies range between -172.2 mgal and 156.6 mgal with an average of 4.1 mgal and a standard deviation of 18.2 mgal.

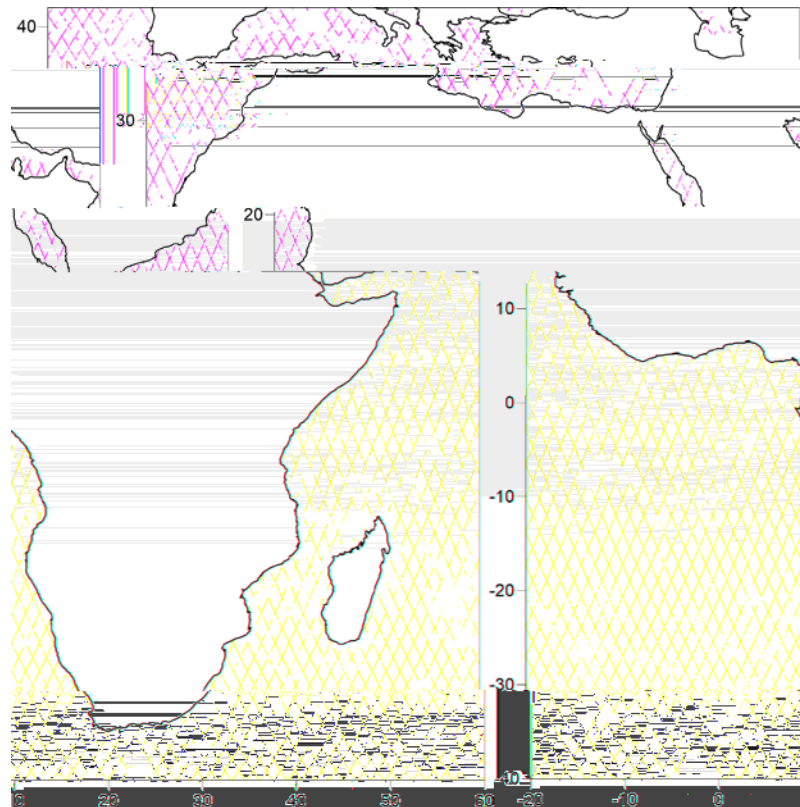


Fig. 11: Distribution of the altimetry gravity data for Africa used to create the AFRGDB\_V2.0 gravity database (after Abd-Elmotaal et al., 2017c).

In order to reduce the free-mobility of the interpolation solution in the large data gaps, an underlying grid on a  $15' \times 15'$  resolution has been created using the GOCE DIR-R5 model complete to degree and order 300. A number of 48,497 underlying grid points have been created. Figure 12 shows the distribution of the underlying grid points. The underlying grid free-air gravity anomalies range between -175.1 mgal and 190.0 mgal with an average of 3.3 mgal and a standard deviation of 27.3 mgal.

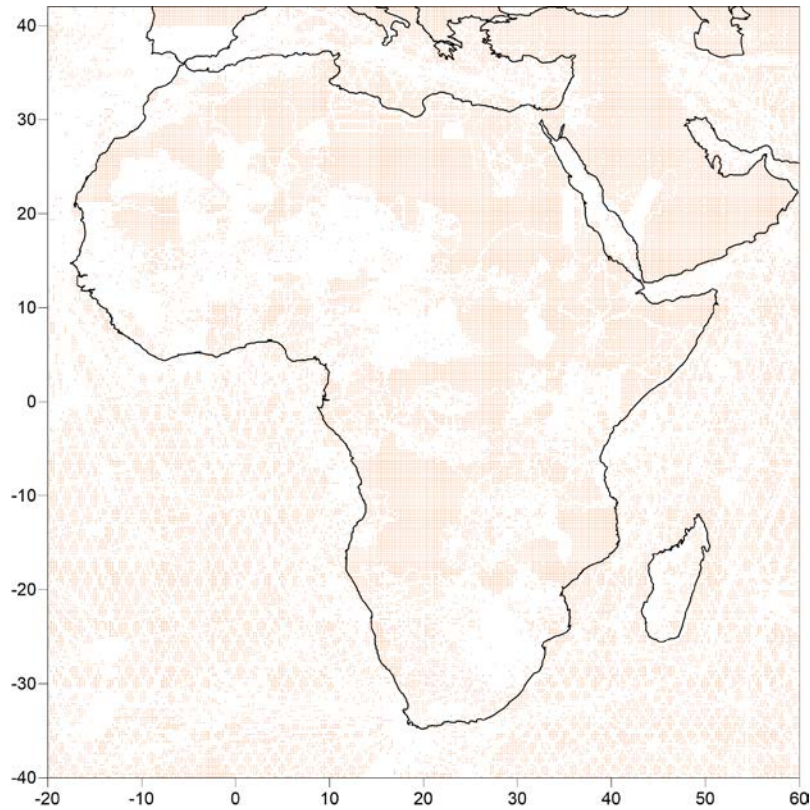


Fig. 12: Distribution of the underlying grid data for Africa used to create the AFRGDB\_V2.0 gravity database (after Abd-Elmotaal et al., 2017c).

The window technique has been applied to reduce the gravity anomalies for Africa. The EGM2008 complete to degree and order 1800 has been used, where the harmonic coefficients of the topographic-isostatic masses for the African data window ( $-42^\circ \leq \phi \leq 44^\circ; -22^\circ \leq \lambda \leq 62^\circ$ ) complete to degree and order 1800 have been computed by the technique developed by Abd-Elmotaal and Kührtreiber (2015). The reduced anomalies range between -230.6 mgal and 318.8 mgal with an average of 0.8 mgal and a standard deviation of 11.7 mgal.

An unequal weight least-squares interpolation technique takes place to interpolate the reduced gravity anomalies on a  $5' \times 5'$  grid. The standard deviations of the different data sets have been taken as shown in Table 1.

Table 1: Used standard deviations of the different data sets for creating the AFRGDB\_V2.0.

Gravity type	Std
Land	1 mgal
Shipborne	3 mgal
Altimetry	5 mgal
Underlying	20 mgal



Figure 13 shows the  $5' \times 5'$  AFRGDB\_V2.0 African free-air gravity anomaly database, after performing the window restore step. The free-air gravity anomalies range between  $-243.0$  mgal and  $468.0$  mgal with an average of  $3.0$  mgal and a standard deviation of  $31.9$  mgal.

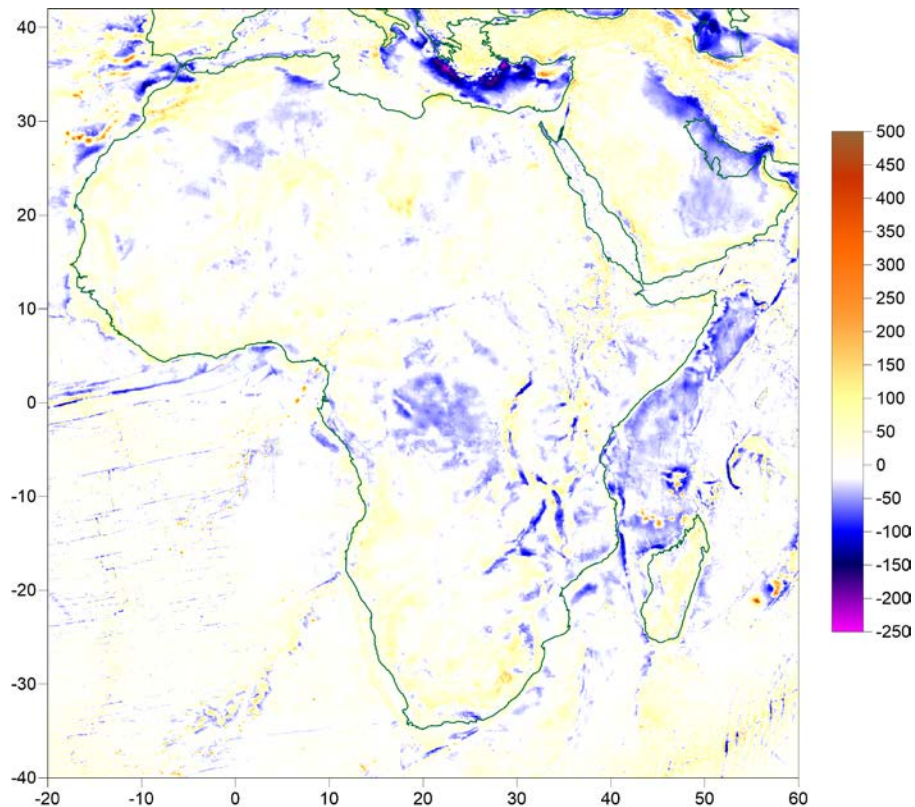


Fig. 13: The  $5' \times 5'$  African free-air gravity anomaly database AFRGDB\_V2.0 (after Abd-Elmotaal et al., 2017c). Units in [mgal].

As an estimation of the internal quality of the established free-air gravity anomaly database of Africa AFRGDB\_V2.0, Fig. 14 shows the residuals between the measured and the database values at the data points used to create the database (those of Figs. 9, 10, 11). These residuals range between  $-50.80$  mgal and  $55.71$  mgal with an average of  $-0.37$  mgal and a standard deviation of  $5.56$  mgal. Figure 14 shows that most of the area (81.9% of the data points) have residuals below 5 mgal (the white pattern).

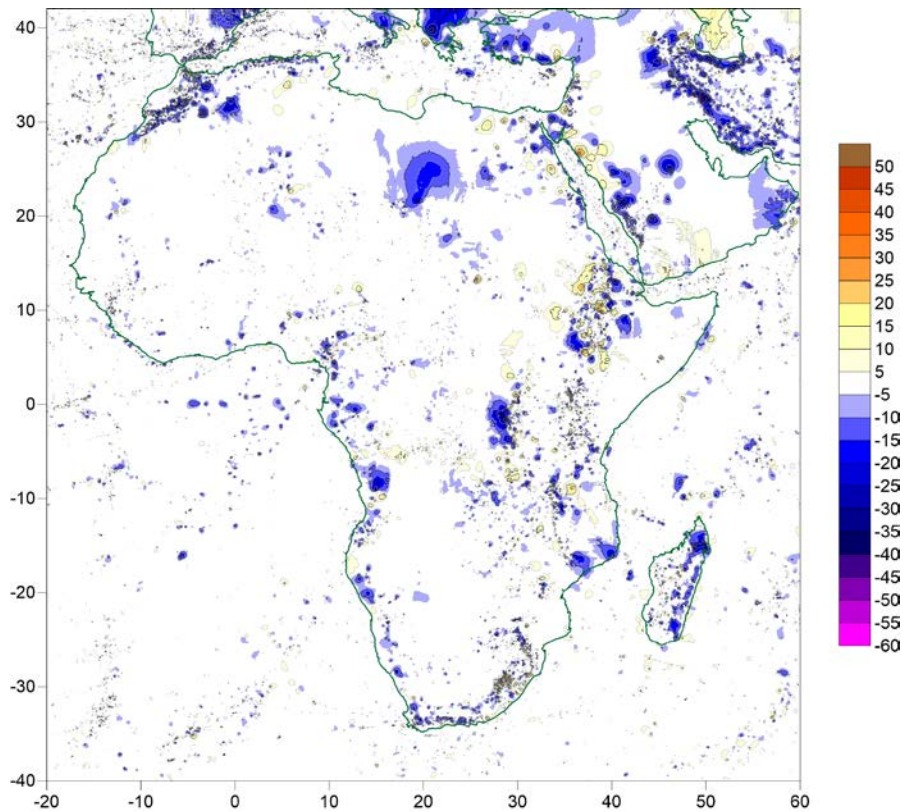


Fig. 14: Residuals at the data points between measured values and the  $5' \times 5'$  African free-air gravity anomaly database AFRGDB\_V2.0 (internal precision). Contour interval: 5 mgal (after Abd-Elmotaal et al., 2017c).

The external check of the AFRGDB\_V2.0 has been estimated by comparing the values of the measured and database gravity anomalies for those points which were deselected by the grid-filtering technique (a set of 898,236 points). Figure 15 shows the external check of the AFRGDB\_V2.0 gravity database. These residual values range between -66.97 mgal and 67.25 mgal with an average of -0.59 mgal and a standard deviation of 6.99 mgal. Figure 15 shows that most of the area (71.9% of the data points) have residuals below 5 mgal (the white pattern).

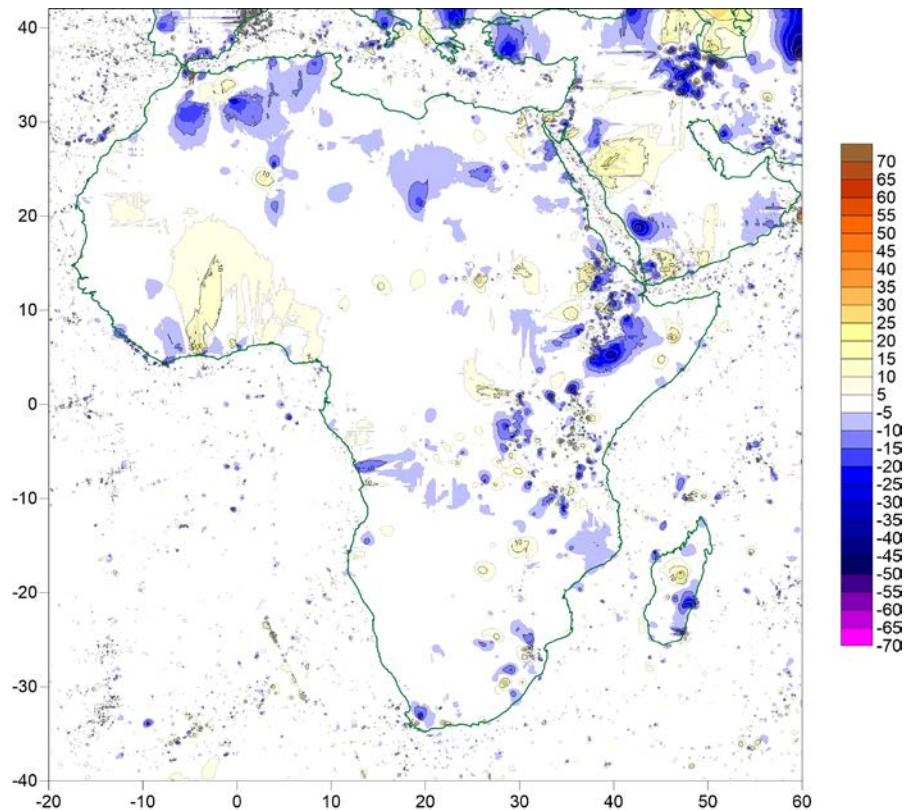


Fig. 15: Residuals at the points which were not used to create the database between measured values and the  $5' \times 5'$  African free-air gravity anomaly database AFRGDB\_V2.0 (external accuracy). Contour interval: 5 mgal (after Abd-Elmotaal et al., 2017c).

## 2.8 Effect of Unclassified Land Depressions on Gravity and Geoid

The determination of the gravimetric geoid is based on the magnitude of the gravity observed at the topographic surface of the earth. In order to satisfy Laplace's equation, the masses between the surface of the earth and the geoid must be removed or shifted inside the geoid. Then the gravity values have to be reduced to the geoid, forming the boundary values on the boundary surface. Gravity reduction techniques using unclassified DTM usually presume that negative elevations are reserved for ocean stations. This is usually wrong for land depressions. In case of Qattara Depression in Egypt/Africa, the elevations are negative, i.e., suited below sea level. This leads to an obvious error in the topographic-isostatic reduction using, for example, TC-program employing unclassified DTM by assuming water masses filling the depression instead of air, besides computing at the non-existing sea level instead of computing at the actual negative surface of the topography. The case of Qattara Depression has been studied as a prototype of the effect of the unclassified land depressions on gravity reduction and geoid determination.

Qattara Depression is a land depression in the north west of Egypt and is a part of the Egyptian Western Desert (cf. Fig. 16). It lies below sea level and is covered with salt pans,





sand dunes and salt marshes. The depression covers an area of about 19,605 km<sup>2</sup>, a size twice as large as Lebanon.

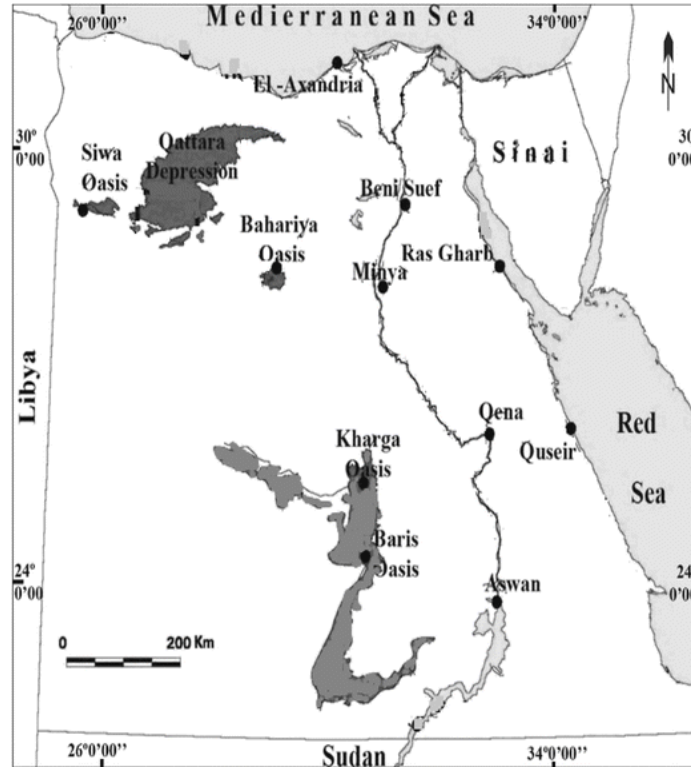


Fig. 16: Location of Qattara Depression (after Abd-Elmotaal and Kühtreiber, 2018).

Figure 17 illustrates the detailed 1" × 1" DTM for Qattara Depression. It shows that the minimum elevation below sea level inside the depression reaches 220 m.

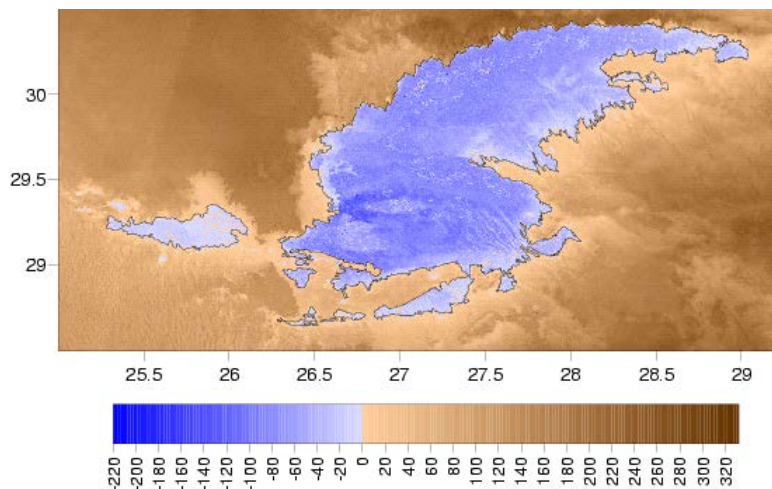


Fig. 17: Detailed 1" × 1" DTM for Qattara Depression (after Abd-Elmotaal and Kühtreiber, 2018).



The effect of Qattara Depression on the gravity anomalies, employing the window remove-restore technique, is shown in Fig. 18. It shows that the total effect of Qattara Depression on the gravity anomalies is limited to the area of the depression and reaches 20 mgal. This effect for the 4745 points on the  $15' \times 15'$  grid covering the whole Egyptian window ( $19^\circ \leq \phi \leq 35^\circ; 22^\circ \leq \lambda \leq 40^\circ$ ) ranges between -0.59 mgal and 20.58 mgal with an average of 0.08 mgal and a standard deviation of 1.08 mgal.

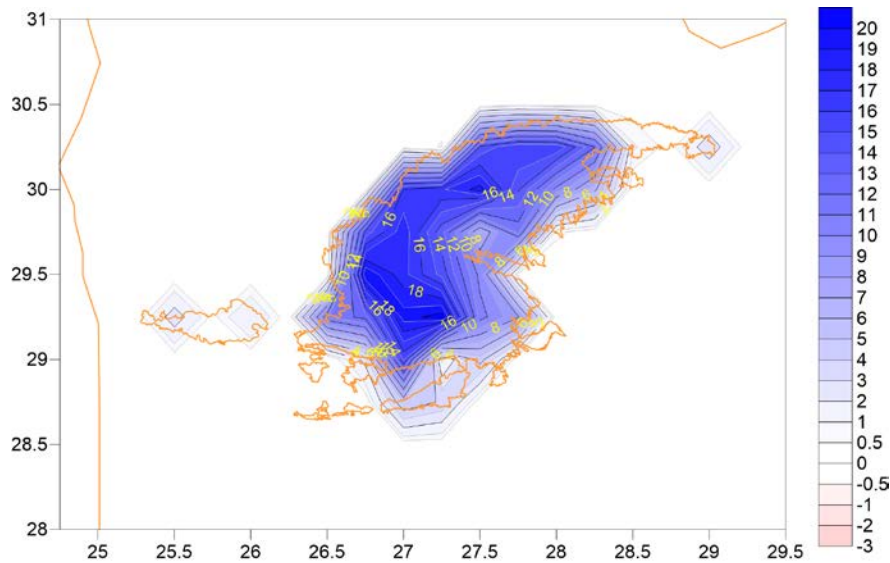


Fig. 18: Effect of Qattara Depression on the gravity anomalies (after Abd-Elmotaal and Kühtreiber, 2018). Units in [mgal].

Figure 19 shows the total effect of Qattara Depression on the geoid undulations employing the window remove-restore technique. Figure 19 shows a regional effect of Qattara Depression on the geoid undulations, which reaches 1.1 m at the area of Qattara depression and decreases radially with distance from the depression. It vanishes after nearly 1000 km from the depression. The effect of Qattara Depression on geoid undulation for the 4745 points on the  $15' \times 15'$  grid covering the whole Egyptian window ( $19^\circ \leq \phi \leq 35^\circ; 22^\circ \leq \lambda \leq 40^\circ$ ) ranges between 0.027 m and 1.123 m with an average of 0.093 m and a standard deviation of 0.093 m.

This study has developed an appropriate technique which is capable to determine the effect of a land depression on gravity reduction and geoid undulation. More details about the used methodology, the developed technique and the detailed results of the components of the effect of the land depression on both gravity anomalies and geoid undulation can be found in (Abd-Elmotaal and Kühtreiber, 2018).

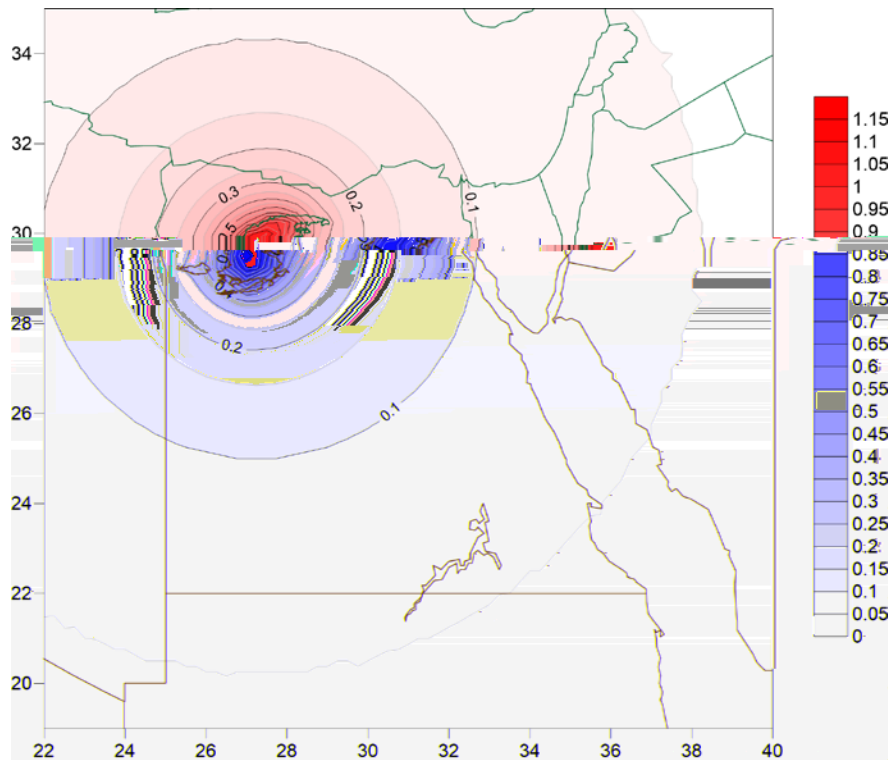


Fig. 19: Effect of Qattara Depression on the geoid undulations (after Abd-Elmotaal and Kühtreiber, 2018). Units in [m].

## 2.9 Establishment of the Updated Gravity Database of Africa (AFRGDB\_V2.2)

The same gravity data used to create the AFRGDB\_V2.0 database are used to create an updated gravity database called AFRGDB\_V2.2. The establishment of the AFRGDB\_V2.2 gravity database for Africa has been carried out using RTM-reduced anomalies, considering all topographic elements within the African data window, employing a weighted least-squares prediction technique. The land gravity data get the highest precision, while the shipborne and altimetry gravity data get a moderate precision. The data gaps are filled with gravity anomalies derived from the GOCE DIR\_R5 global reference model, complete to degree and order 280, getting the lowest precision within the prediction technique. The weighted least-squares prediction technique is thus carried out to estimate gridded gravity anomalies. The AFRGDB\_V2.2 gravity database on a uniform  $5' \times 5'$  grid has been established by the developed process after performing the proper restoring step.

Figure 20 shows the updated  $5' \times 5'$  AFRGDB\_V2.2 African free-air gravity anomaly database. The free-air gravity anomalies range between  $-238.3$  mgal and  $512.0$  mgal with an average of  $3.2$  mgal and a standard deviation of  $31.7$  mgal.

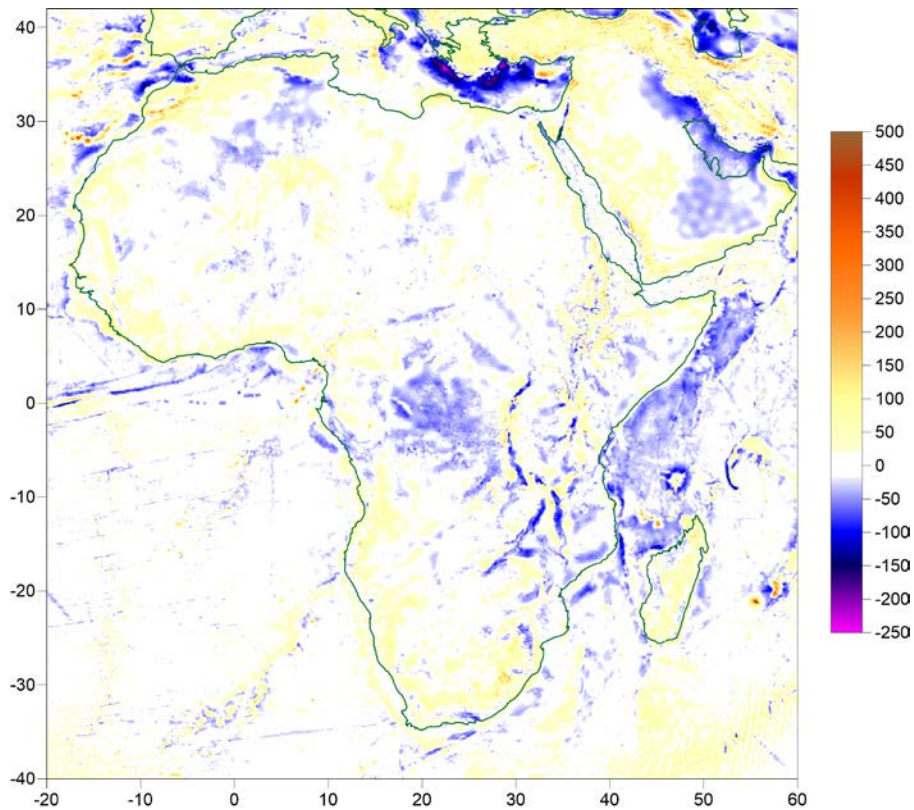


Fig. 20: The 5'  $\times$  5' African free-air gravity anomaly updated database AFRGDB\_V2.2 (after Abd-Elmotaal et al., 2018). Units in [mgal].

Figure 21 shows the difference between the old AFRGDB\_V2.0 and the updated AFRGDB\_V2.2. These differences are nearly centered with a standard deviation of 9.8 mgal. The differences are correlated with the topographic height.

Figure 22 shows the residuals between the measured and the database values at the data points used to create the database (the internal precision). These residuals range between -51.09 mgal and 61.99 mgal with an average of -0.35 mgal and a standard deviation of 5.67 mgal. Figure 22 shows that most of the area (81.2% of the data points) have residuals below 5 mgal (the white pattern).

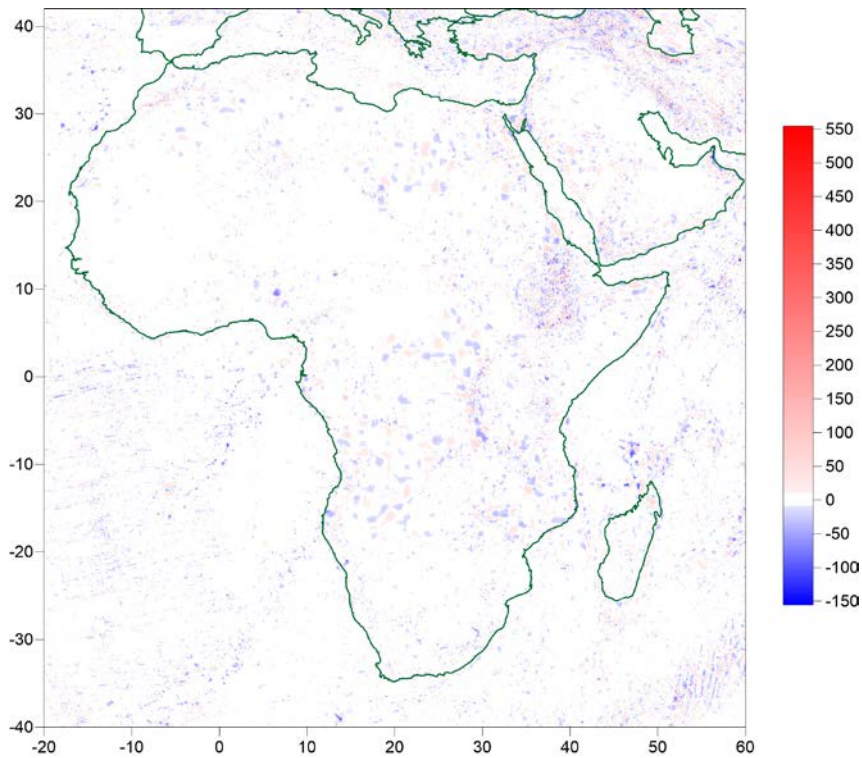


Fig. 21: Difference between AFRGDB\_V2.0 and AFRGDB\_V2.2 (after Abd-Elmotaal et al., 2018). Units in [mgal].

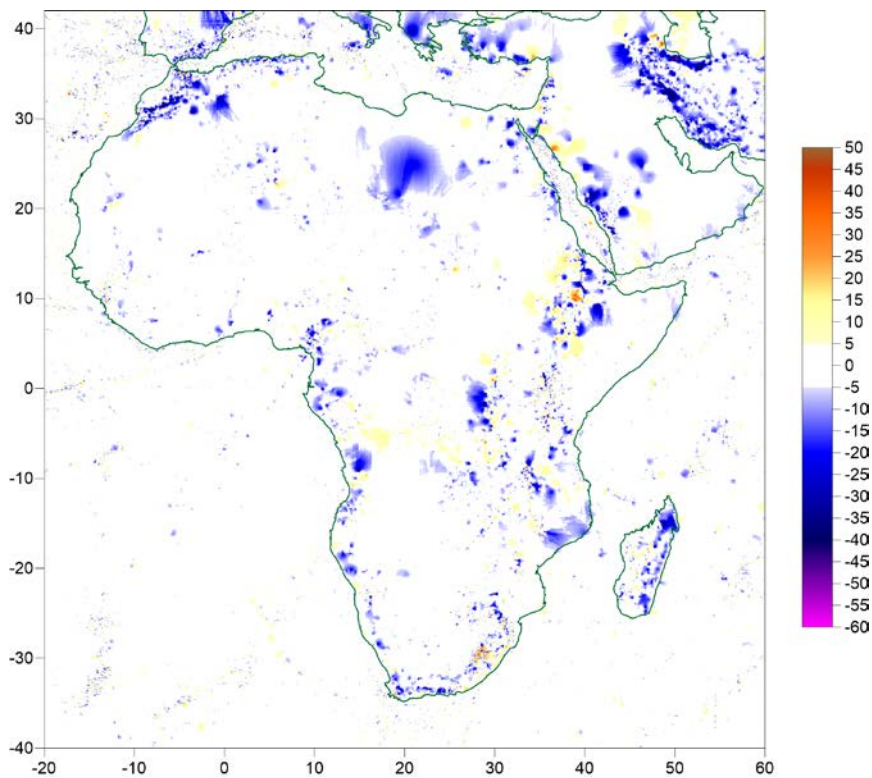


Fig. 22: Internal precision of the updated gravity database AFRGDB\_V2.2 (after Abd-Elmotaal et al., 2018). Contour interval: 5 mgal.



The external check of the AFRGDB\_V2.2 has been estimated by comparing the values of the measured and database gravity anomalies for those points which were deselected by the grid-filtering technique (a set of 898,247 points). Figure 23 shows the external check of the AFRGDB\_V2.2 gravity database. These residual values range between -65.21 mgal and 65.10 mgal with an average of -0.55 mgal and a standard deviation of 7.29 mgal. Figure 23 shows that most of the area (69.1% of the data points) have residuals below 5 mgal (the white pattern).

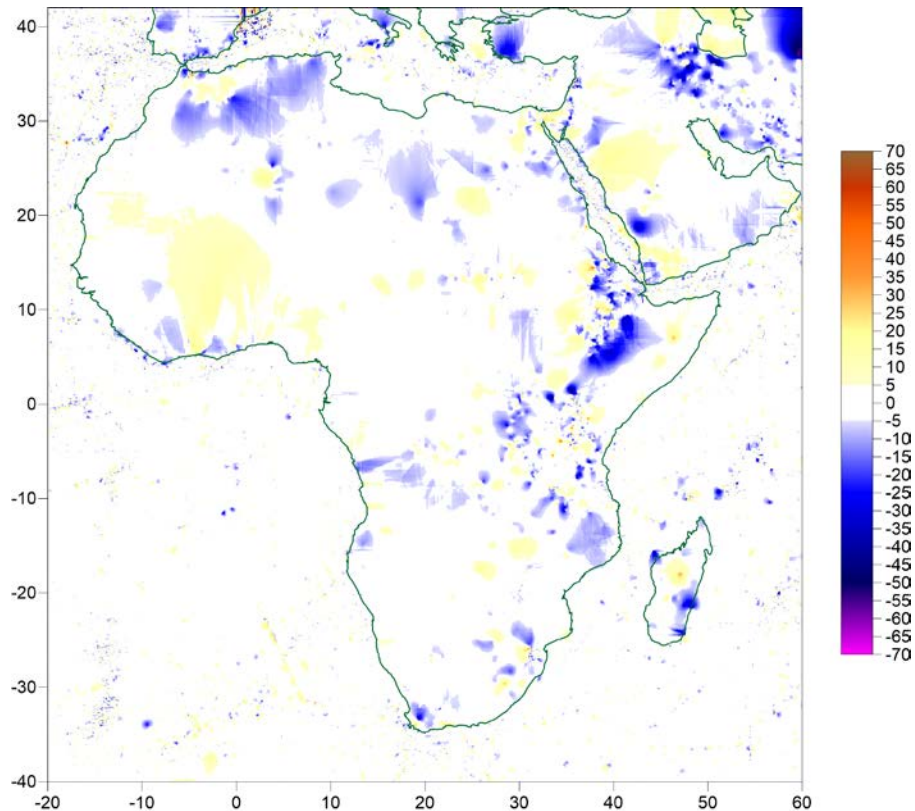


Fig. 23: Residuals at the points which were not used to create the database between measured values and the 5'  $\times$  5' African free-air gravity anomaly database AFRGDB\_V2.2 (external accuracy) (after Abd-Elmotaal et al., 2018). Contour interval: 5 mgal.

The validation of the updated version of the gravity database AFRGDB\_V2.2 shows similar quality as the previous AFRGDB\_V2.0 database. However, the computation efforts and time for the updated database are much less compared to those of the previous database.

### 3. Publications

A number of papers are published within the course of the project. Here follows a list of these publications among other references cited in this report.



1. Abd-Elmotaal, H. and Kühtreiber, N. (2003) Geoid Determination Using Adapted Reference Field, Seismic Moho Depths and Variable Density Contrast. *Journal of Geodesy*, Vol. 77, 77–85, DOI: 10.1007/s00190-002-0300-7.
2. Abd-Elmotaal, H. and Kühtreiber, N. (2014) Automated Gross Error Detection Technique Applied to the Gravity Database of Africa. General Assembly of the European Geosciences Union (EGU), Vienna, Austria, April 27 – May 2, 2014.
3. Abd-Elmotaal, H. and Kühtreiber, N. (2015) On the Computation of the Ultra-High Harmonic Coefficients of the Topographic-Isostatic Masses within the Data Window. General Assembly of the European Geosciences Union (EGU), Vienna, Austria, April 12 – 17, 2015.
4. Abd-Elmotaal, H. and Kühtreiber, N. (2016) Effect of the Curvature Parameter on Least-Squares Prediction within Poor Data Coverage: Case Study for Africa. General Assembly of the European Geosciences Union (EGU), Vienna, Austria, April 17 – 22, 2016.
5. Abd-Elmotaal, H. and Kühtreiber, N. (2017) Optimum Gravity Interpolation Technique for Large Data Gaps: Case Study for Africa. General Assembly of the European Geosciences Union (EGU), Vienna, Austria, April 23 – 28, 2017.
6. Abd-Elmotaal, H. and Makhloof, A. (2013) Gross-Errors Detection in the Shipborne Gravity Data Set for Africa. Geodetic Week, Essen, Germany, October 8–10, 2013 ([http://www.uni-stuttgart.de/gi/research/Geodaetische\\_Woche/2013/session02/Abd-Elmotaal-Makhloof.pdf](http://www.uni-stuttgart.de/gi/research/Geodaetische_Woche/2013/session02/Abd-Elmotaal-Makhloof.pdf)).
7. Abd-Elmotaal, H. and Makhloof, A. (2014) Combination between Altimetry and Shipborne Gravity Data for Africa. 3<sup>rd</sup> International Gravity Field Service (IGFS) General Assembly, Shanghai, China, June 30 – July 6, 2014.
8. Abd-Elmotaal, H., Makhloof, A., Abd-Elbaky, M. and Ashry, M. (2017a) The African 3" × 3" DHM and its Validation. *International Association of Geodesy Symposia Journal*, DOI: 10.1007/1345\_2017\_19.
9. Abd-Elmotaal, H., Seitz, K., Abd-Elbaky, M. and Heck, B. (2013a) Comparison among Three Harmonic Analysis Techniques on the Sphere and the Ellipsoid. *Journal of Applied Geodesy*, 1–18, DOI: 10.1515/jag-2013-0008.
10. Abd-Elmotaal, H., Seitz, K., Kühtreiber, N. and Heck, B. (2015) Establishment of the Gravity Database AFRGDB\_V1.0 for the African Geoid. *International Association of Geodesy Symposia Journal*, Vol. 144, 131–138, DOI: 10.1007/1345\_2015\_51.
11. Abd-Elmotaal, H., Seitz, K., Kühtreiber, N. and Heck, B. (2017b) Evaluation of the African Gravity Database AFRGDB\_V1.0. *International Association of Geodesy Symposia Journal*, DOI: 10.1007/1345\_2017\_16.
12. Abd-Elmotaal, H., Seitz, K., Kühtreiber, N. and Heck, B. (2017c) AFRGDB\_V2.0: The Gravity Database for the Determination of the Earth's Mathematical Surface in Africa. Joint Scientific Assembly of the International Association of Geodesy (IAG) and International Association of Seismology and Physics of the Earth's Interior (IASPEI), Kobe, Japan, July 30 – August 4, 2017.



13. Abd-Elmotaal, H. and Kühtreiber, N. (2018) Effect of Unclassified Land Depressions on Gravity and Geoid in Africa: Case Study for Qattara Depression. General Assembly of the European Geosciences Union (EGU), Vienna, Austria, April 7 – 13, 2018.
14. Abd-Elmotaal, H., Kühtreiber, N., Seitz, K. and Heck, B. (2018) The AFRGDB\_V2.2 Updated Gravity Database for Africa. International Symposium on Gravity, Geoid and Height Systems 2 (GGHS2018), Copenhagen, Denmark, September 17–21, 2018.

#### **4. Summary Financial Report**

<b>Item</b>	<b>Receipts</b>	<b>Expenditures</b>
<b>IUGG grant</b>	10,000 \$	
<b>Travel</b>		10,882 \$
<b>Research</b>		1,500 \$
<b>Teaching</b>		1,000 \$
<b>Planning/Coordination</b>		500 \$
<b>Sum</b>	<b>10,000 \$</b>	<b>13,882 \$</b>