commentary and analysis

Comments on "The Differentiation between Grid Spacing and Resolution and Their Application to Numerical Modeling"
Reply

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Abstract

The comments of Grasso are extended to show that the minimum wavelength that is resolved by a numerical model is typically longer than the shortest unfiltered wave in the model. This result occurs because the numerical grid is unable to accurately describe the amplitudes and horizontal derivatives of short wavelengths in the numerical solution. The numerical scheme may also introduce phase and dispersion errors that cause short wavelengths to be inaccurate.

1. Introduction

Grasso (2000) provides good discussion of the distinction between the resolution of a numerical model and the model's grid resolution or grid increment. These points are well motivated. While numerical modelers typically understand these differences, some users of model output may not truly understand the distinction and may overestimate the actual resolution of a numerical model as a result. As Grasso (2000) points out, forecasters should be aware that the smallest wavelength resolvable by a numerical model is typically longer than twice the numerical grid spacing. He states that waves of at least $4\Delta x$ may be resolved, after making the assumption that shorter wavelengths have been removed from the numerical solution to prevent nonlinear instability or for other reasons. In this paper, we extend Grasso's comments to show that assuming a grid resolution of even $4\Delta x$ may be quite optimistic. As a result, the shortest wavelength that can be effectively resolved by a gridpoint numerical model is usually considerably longer than $4\Delta x$, which is representative of the shortest unfiltered wavelength allowed by typical numerical models as discussed by Grasso (2000).

2. Discussion

Here we define a numerical model's effective resolution as the minimum wavelength the model can describe with some required level of accuracy (not defined). Errors in either amplitude or phase (or both) will limit the effective resolution of the numerical solution. This definition of effective resolution is closely tied to the concept of the numerical scheme's accuracy, which can be related to both the spatial variation of the structure of the error at a given time level and the behavior of the amplitude of the error as a function of time (Anderson 1995). As noted by Grasso (2000), the numerical grid is unable to represent the amplitude of small wavelength features accurately. Although the theoretically minimum resolvable wavelength is $2\Delta x$, the amplitude of a $2\Delta x$ wave may be zero if such a wave is completely out of phase with the numerical grid. For a $4\Delta x$ wave, only 71% of the amplitude is resolved if the solution is $\pi/4$ out of phase with the grid (assuming without loss of generality that the solution is harmonic). These results show that the amplitude resolution of the true solution may be less than optimum at short wavelengths.

Similar errors result in estimating horizontal derivatives using finite differences. For example, using centered finite differences, the ratio of the numerical first derivative to the true derivative for a harmonic wave is $(\sin k\Delta x)/(k\Delta x)$ where $k = 2\pi/L$ is the wavenumber and L is the wavelength (Pielke 1984). For $\Delta x = 2$, the resulting numerical derivative has an amplitude of zero relative to the true derivative; for a $4\Delta x$ wave, the ratio of the amplitudes is $2/\pi$. Therefore, the numerical derivative is approximately 64% of the true value. Errors in estimating derivatives are introduced by the time and space differencing of the particular finite difference scheme, which may result in the numerical model having a lower effective resolution than the theoretical grid resolution of $2\Delta x$. As an example, the numerical solution of the linear advection equation

$$\frac{\partial f}{\partial t} + c\frac{\partial f}{\partial x} = 0, \qquad (1)$$

using a simple leapfrog scheme,

$$f_i^{n+1} = f_i^{n-1} + c\frac{\Delta t}{\Delta x}\left(f_{i+1}^n - f_{i-1}^n\right) \qquad (2)$$

with centered space differencing, shows a numerical phase response that is a function of wavenumber. In Eq. (2), f is the dependent variable, c is the phase speed, Δt is the time step, and Δx is the horizontal grid spacing. Superscripts n refer to the time level, and subscripts i refer to the spatial grid index. It can be seen easily from Eq. (2) that a $2\Delta x$ wave is stationary since $f_{i+1}^n - f_{i-1}^n = 0$. Although the theoretical resolution of the numerical grid in this example is $2\Delta x$, a true solution with that wavelength is clearly not resolved accurately because it does not propagate correctly, despite the fact that its amplitude may be well described. Haltiner and Williams (1980) show that the phase error of the leapfrog scheme as applied to the linear advection equation is worse at smaller wavelengths and is also a function of the Courant number $c\Delta t/\Delta x$. For example, a wavelength of $4\Delta x$ with a Courant number of 0.8 has phase speed that is 74% of the true solution. Numerical schemes may also have amplitude responses that are a function of wavenumber (Mesinger and Arakawa 1976; Haltiner and Williams 1980), resulting in a further decrease in the ability of the numerical solution to resolve the solution amplitude at short wavelengths. Pielke (1984) provides a similar analysis of phase and amplitude errors as a function of wavelength for various numerical schemes. Mesinger and Arakawa (1976) also show that phase errors can be introduced by the nature of the time differencing. For example, numerical solutions of the oscillation equation

$$\frac{df}{dt} = i\omega f \qquad (3)$$

using the leapfrog time differencing scheme are accelerating compared to the analytical solutions, despite the absence of horizontal space differencing. If a numerical scheme is inaccurate at short wavelengths due to such model-induced phase or amplitude errors, it can be claimed that those particular features are not adequately resolved, despite the theoretical resolution of the numerical grid.

A related type of phase error is due to inaccurate dispersion caused by the spatial arrangement of variables on the grid, which affects the frequency response of the numerical solution as a function of wavenumber. The one-dimensional shallow water equations provide the simplest case for discussing this type of dispersion in numerical schemes (Mesinger and Arakawa 1976; Arakawa and Lamb 1977). Their results show that the dispersion characteristics of numerical inertial gravity wave solutions have frequency responses that differ from the true analytical solution of the linearized equations, and that the nature of the difference depends on the specific Arakawa grid (Winninghoff 1968; Arakawa 1972) that is used. For example, Fig. 1 shows the nondimensional frequency response ν/f for the simplest one-dimensional case based on finite difference solutions of the linearized shallow water equations where f is the Coriolis parameter. In Fig. 1, the nondimensional frequency is plotted against the number of grid intervals per wavelength, N, rather than against the nondimensional wavenumber (described below) that is more typical for presentation of these results (Mesinger and Arakawa 1976; Arakawa and Lamb 1977). The quantity N can also be defined as $N = 2\pi/\alpha$, where $\alpha = k\Delta x = 2\pi\Delta x/L$ is the nondimensional horizontal wavenumber. The nondimensional frequency has a maximum at $N = 4$ on both the A grid and the D grid where the group velocity of the solution (which is proportional to $\partial\omega/\partial\alpha$) is therefore zero. This result indicates that inertial gravity waves of this wavelength will have zero group velocity, unlike the analytical solution. For $N < 4$, the group velocity is actually negative on the A and D grids, which implies energy propagation in a direction opposite to the direction of the true group velocity. On the B and C grids the situation is somewhat better; however, the ratio of the dimensionless frequency to the true analytical frequency drops off more rapidly at smaller wavelengths. At $N = 4$ the numerical frequency is approximately 93% of the true frequency on the B and C grids, indicating that the numerical group velocity is slower than that of the actual solution. Using two-dimensional solutions of the shallow water equations, similar re-

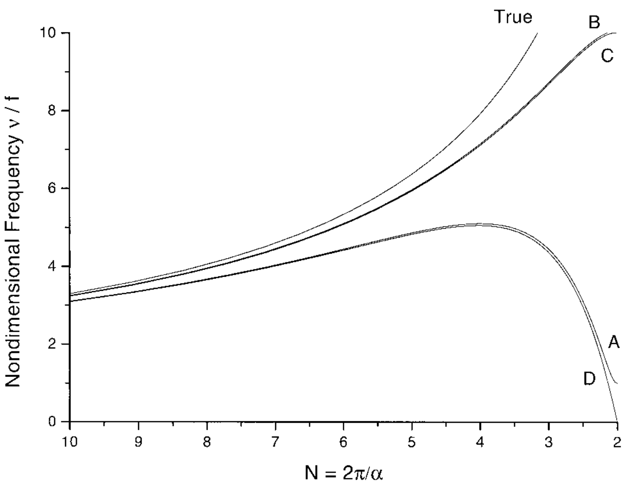

FIG. 1. Nondimensional inertial gravity wave frequency ν/f of the numerical solution of the one-dimensional shallow water equations on four different Arakawa grids labeled A through D. Here, N is the number of grid intervals per wavelength, f is the Coriolis parameter, and $\alpha = k\Delta x$ is the nondimensional wavenumber where $k = 2\pi/L$ and L is wavelength. Adapted from Mesinger and Arakawa (1976).

sults can be found that also show that the dispersive properties of the numerical solutions are less accurate at small grid resolutions. For detailed discussions of these results including their derivation, see Mesinger and Arakawa (1976), Arakawa and Lamb (1977), and Haltiner and Williams (1980). These results indicate the numerical dispersion at short grid intervals may be very unsatisfactory on some grids, and that therefore important geostrophic adjustment processes may not be well represented at certain scales (i.e., small grid intervals). This problem occurs in addition to the poor amplitude resolution at small grid scales described above. In addition to the problems described here, the numerical solution may also introduce high-frequency noise (Mesinger and Arakawa 1976).

3. Conclusions

For a finite difference numerical model, the resolution of the numerical solution at short wavelengths may be unsatisfactory for several reasons. Due only to the geometric relationship between the numerical grid and the true solution, as many as 10 grid points may be required to assure a reasonable (e.g., greater than 95%) representation of the true solution's amplitude and its first horizontal derivative. The accuracy of numerical solutions of small-scale features may be further hampered due to both phase and amplitude errors in the numerical solution (which are generally a function of N for finite difference schemes) and related dispersion errors introduced by the spatial arrangement of variables on the computational grid. As a result, the shortest wavelength that can be effectively resolved by a gridpoint numerical model may be considerably longer than $4\Delta x$, which is representative of the shortest unfiltered wavelength allowed by typical numerical models as discussed by Grasso (2000).

Acknowledgments. The views expressed in this article are those of the author and do not reflect the official policy or position of the U.S. Air Force, Department of Defense, or the U.S. government.

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Comments on "The Differentiation between Grid Spacing and Resolution and Their Application to Numerical Modeling"

Grasso (2000) suggests that the terminology "*grid resolution* and *grid spacing* should not be used interchangeably." Following Pielke (1991), who proposed a definition of resolution in which "resolution within a numerical model should refer to at least four times the grid interval," Grasso states that waves must have "a scale of at least $4\Delta x$" to be "resolved." In formulating these definitions Grasso and Pielke attempt to highlight the important fact that the error in all numerical methods increases as the number of grid points per wavelength decreases, and that these errors can become quite large in the limiting case in which the wavelength approaches $2\Delta x$. I agree with Grasso and Pielke that the numerical analyst should never lose sight of the difficulties generated by poor numerical resolution, but I do not agree that the problems associated with $2\Delta x$ waves require us to create imprecise and artificial distinctions in the English language.

In formulating their definitions, Grasso (2000) and Pielke (1991) seem to assume that "resolve" means "to deal with successfully." Nevertheless, another meaning for resolve, which actually precedes the "to deal with successfully" entry in the *Merriam-Webster OnLine: WWWebster Dictionary* (http://www.m-w.com) is "break up, separate, to distinguish between or make independently visible adjacent parts." There is therefore no basis in standard English usage for Grasso's objection to the phrases in Wicker and Wilhelmson (1995) that read "fine-mesh simulation with 250-m horizontal resolution" or " the vertical resolution was rather coarse (Δx = 500 m)." In both cases, *resolution* can be replaced by *separation* without any change in meaning or loss in clarity.

One might nevertheless argue that it is appropriate to define specialized scientific terminology allowing the cognoscente to speak with more precision, but unfortunately the definitions proposed by Grasso and Pielke are not precise. According to their definitions, "grid resolution" is not necessarily $4\Delta x$, but rather some unspecified multiple of Δx at which the speaker feels waves are well represented in the numerical model. This lack of specificity is inherent in any definition that attempts to separate those waves that are successfully simulated from those whose representation is inadequate because there is no particular scale that sharply divides these two cases. Grasso seems to assume that the candidate scales are all integer multiples of the grid spacing when he states that filtering all waves of wavelength $3\Delta x$ and shorter to prevent nonlinear instability "means that the smallest resolvable wave is at least $4\Delta x$, since $2\Delta x$ and $3\Delta x$ waves have been removed." In fact, any reasonable sized domain will support lots of waves with wavelengths between $3\Delta x$ and $4\Delta x$, such as the $3.5\Delta x$ wave, which corresponds to a disturbance in which two wavelengths occupy an interval of $7\Delta x$.

As a consequence of the rich spectrum of waves representable on the numerical mesh, there is no dramatic change in numerical performance as the wavelength of a disturbance just begins to exceed $4\Delta x$. The behavior of a $3.9\Delta x$ wave will be very similar to that of a $4.1\Delta x$ wave in any given application. Moreover, the actual error associated with disturbances at wavelengths equal to some specific multiple of Δx varies from problem to problem and from numerical method to numerical method. For example, in contrast to most spatial differencing schemes, a sixth-order compact difference approximation to the advection equation will propagate a $3.5\Delta x$ wave with almost no phase speed error (Durran 1999, pp. 86–88).

In summary, I note that standard English usage permits *grid resolution* to be used in the sense of *grid separation* or equivalently *grid spacing*. I suggest adding an adjective (e.g., good numerical resolution) when the speaker wishes to convey a value judgment.

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Reply

I would like to thank Mr. Durran and Mr. Walters for their extended comments on the difference between grid spacing and resolution by Grasso (2000).

Mr. Walters provided further detailed information and references to support statements about the difference between grid spacing and resolution. The objection by Mr. Durran is well noted; however, he may have misunderstood the focus of the recent article.

The main focus of the article was to distinguish between the length scale between grid points and the length scale of information simulated on a gridded domain. Consider a numerical domain with 5-km horizontal grid spacing and 100-m vertical grid spacing. Let the domain extend from the surface to 20 km in height and 200 km in each horizontal direction. Suppose the domain is initialized horizontally homogeneously with a sounding that has 3000 J kg^{-1} of convective available potential energy and vertical shear of 10^{-2} s^{-1}. Allow a warm bubble to trigger convection. This domain will support a convective updraft from the boundary layer to the tropopause. Even though the convective updraft exists within the domain, a supercell thunderstorm is said to be poorly resolved.

One can also distinguish the footprint of a satellite image and the resolution of features in the imagery. Consider an infrared satellite image whose footprint is 12 km. Suppose further that thunderstorms are seen in the image. In this situation thunderstorms are also said to be poorly resolved in the imagery.

I would like to reiterate that the main focus of the recent article was to discuss the difference between grid spacing and resolution. Although no definition of resolution was offered in Grasso (2000), Mr. Durran is correct in stating that no precise definition exists for resolution in regards to numerical modeling.

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corrigenda

In section 3a of the article "Loss of Life in the United States Associated with Recent Atlantic Tropical Cyclones," by E. N. Rappaport (*Bull. Amer. Meteor. Soc.*, **81,** 2065–2073), the number of drowning deaths was reported incorrectly. Drowning accounted for 488 of the 600 fatalities associated with Atlantic tropical cyclones during 1970–99.

In the August issue of the *Bulletin*, Kevin E. Trenberth was erroneously listed as an editor for *Earth Interactions.* Dr. Michael Manton replaced Dr. Trenberth as an *Earth Interactions* editor in 2000. Dr. Manton's contact information is

Dr. Michael Manton, Bureau of Meteorology Research Centre, GPO Box 1289K, Melbourne 3001, Australia.

The *Bulletin* apologizes for this error.