Electromagnetic & Sensor Systems Department

The Relationship Between Atmospheric Boundary Layer Structure and Refractivity

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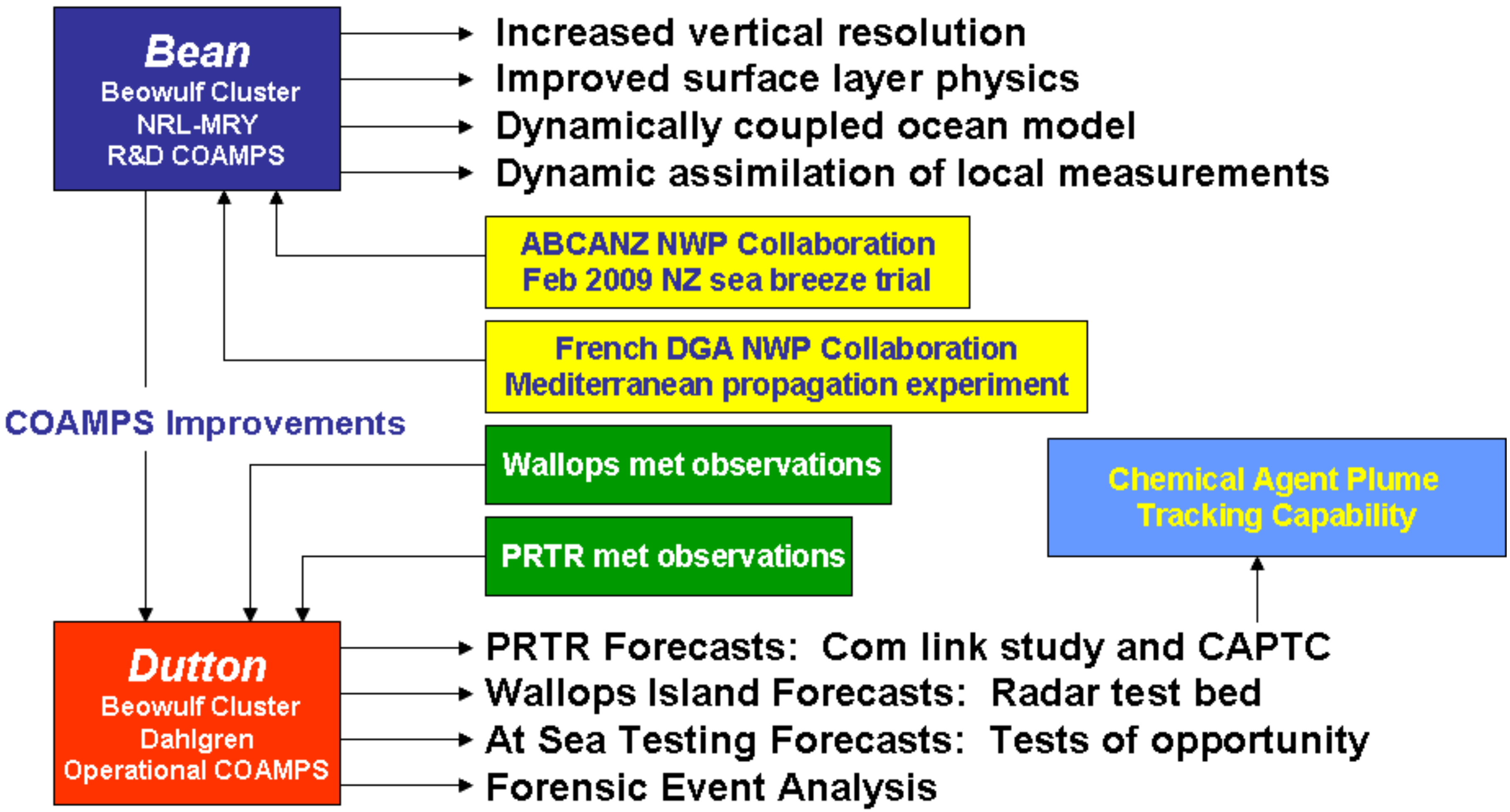
  - Radio Frequency Propagation Analyst

- **William Thornton, Q32**
  - Computer Programmer
  - Radio Frequency Propagation Analyst

Our R&D Structure



Refraction

- Atmospheric refraction bends radio frequency energy away from intended destinations.

- The direction of refraction is dependent on the vertical thermodynamic structure of the atmospheric boundary layer.
  - surface layers
  - mixing layers
  - internal boundary layers
  - entrainment layers

- Within 100km of the coast, mesoscale circulations can produce significant refraction

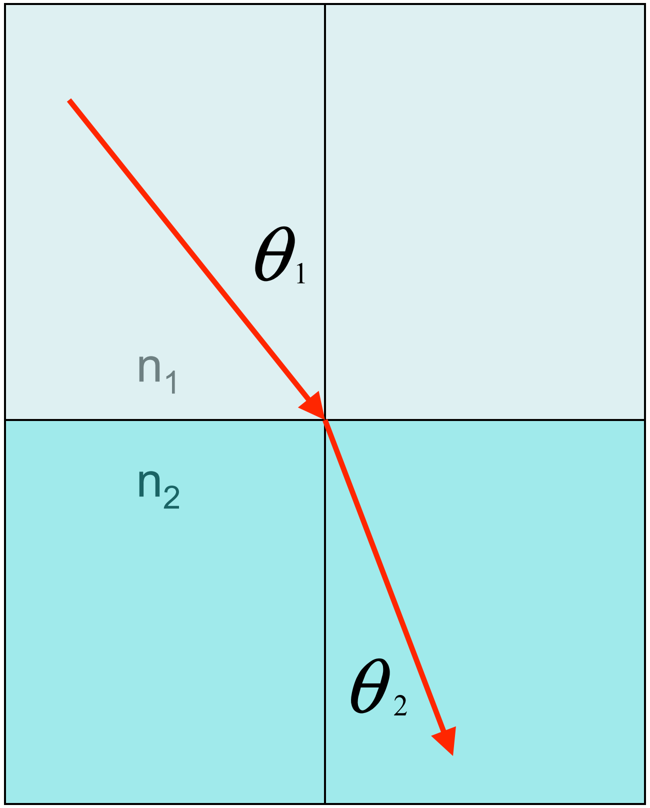
- Refraction can introduce 10^3 deficits on applicable radio frequency engineering solutions

Snells Law

Dutch scientist Willebrørd Snell (1591–1626), who first stated the law in a manuscript in 1621. In French, however, the same law is often called “la loi de Descartes” because it was René Descartes (1596–1650) who first put the law into widespread circulation in his Discourse on Method, published in 1637.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$n \equiv \text{index of refraction}$$



Index of Refraction

$$n \equiv \text{index of refraction} = \sqrt{\varepsilon_r \mu_r}$$

$$\varepsilon_r \equiv \text{relative permittivity}$$

$$\mu_r \equiv \text{relative permeability}$$

$$n = \frac{c}{v}$$

$$c \equiv \text{speed of light in a vacumm}$$

$$\text{v} \equiv \text{phase speed in the medium}$$

Refractivity

$$n \approx 1.000300 \text{ in the atmosphere}$$

$$N \equiv \text{refractivity}$$

$$N = (n-1)10^6$$

$$N = \left(\frac{77.6}{T}\right)\left(P + \frac{4810e}{T}\right)$$

$$p \equiv \text{atmospheric pressure (mb)}$$

$$T \equiv \text{atmospheric temperature (K)}$$

$$e \equiv \text{vapor pressure (mb)}$$

Modified Refractivity

$$M \equiv \text{modified refractivity}$$

$$N + \text{an earth curvature term}$$

$$M = N + \frac{z}{\mathrm{R_e}} 10^6$$

$$z \equiv \text{height above the surface}$$

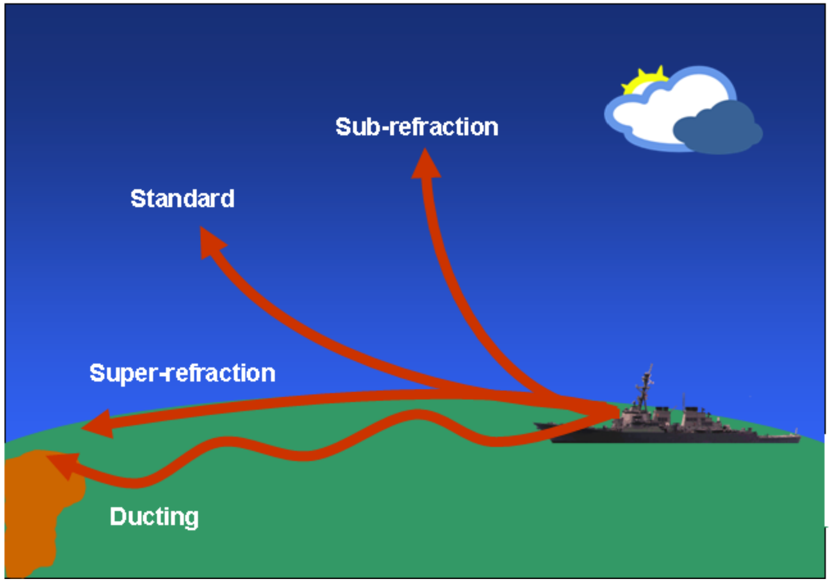
$$\mathrm{R_e} \equiv \text{radius of the earth}$$

$$M = N + 0.157 z$$

Refractivity and Boundary Layer Structure

The vertical gradient of modified refractivity defines the radio frequency refraction regime.

Behavior (dM/dz) m^{-1}	Range 1	Range 2
Standard	= 0.118	
Ducting	< 0.0	
Super-refractive	0.0	0.079
Normal	0.079	0.157
Sub-refractive		> 0.157



Standard: *Short lived in the littorals. Radar energy gently curves away from earth curvature.*

Super-refraction: *Radar energy follows the curvature of the earth. Extended radar horizon and folded land clutter.*

Trapping: *Radar energy trapped in a shallow duct formed by the sea surface and a positive vertical gradient of refractivity above the surface. Extended and separated areas of sea clutter.*

Sub-refraction: *Radar energy abruptly curves away from earth curvature. Ameliorating engineering costs are very high.*

Refractivity and Boundary Layer Structure

Introduce the Conserved Variables

$$M = \left(\frac{77.6}{T}\right)\left(P + \frac{4810e}{T}\right) + 0.157z$$

$$w = 0.622\left(\frac{e}{p-e}\right) \approx 0.622\left(\frac{e}{p}\right) \qquad \theta = T\left(\frac{1000}{p}\right)^{0.286}$$

$$M = \left(\frac{77.6}{\theta\left[\frac{p}{1000}\right]^{0.286}}\right)\left(P + \frac{4810\frac{wP}{0.622}}{\theta\left[\frac{p}{1000}\right]^{0.286}}\right) + 0.157z$$

$w \equiv$ water vapor mixing ratio (kg kg^{-1})

$\theta \equiv$ potential temperature (K)

Refractivity and Boundary Layer Structure

The Vertical Gradient

$$\frac{dM}{dz} = \frac{dp}{dz}\left(\frac{1.336X10^{7}w}{\theta^{2}p^{0.572}} + \frac{399.54}{\theta\, p^{0.286}}\right)$$

$$+\frac{dw}{dz}\left(\frac{3.106X10^{7}P^{0.428}}{\theta^{2}}\right)$$

$$-\frac{d\theta}{dz}\left(\frac{6.212X10^{7}w\,p^{0.428}}{\theta^{3}} + \frac{559.6\,p^{0.714}}{\theta^{2}}\right)$$

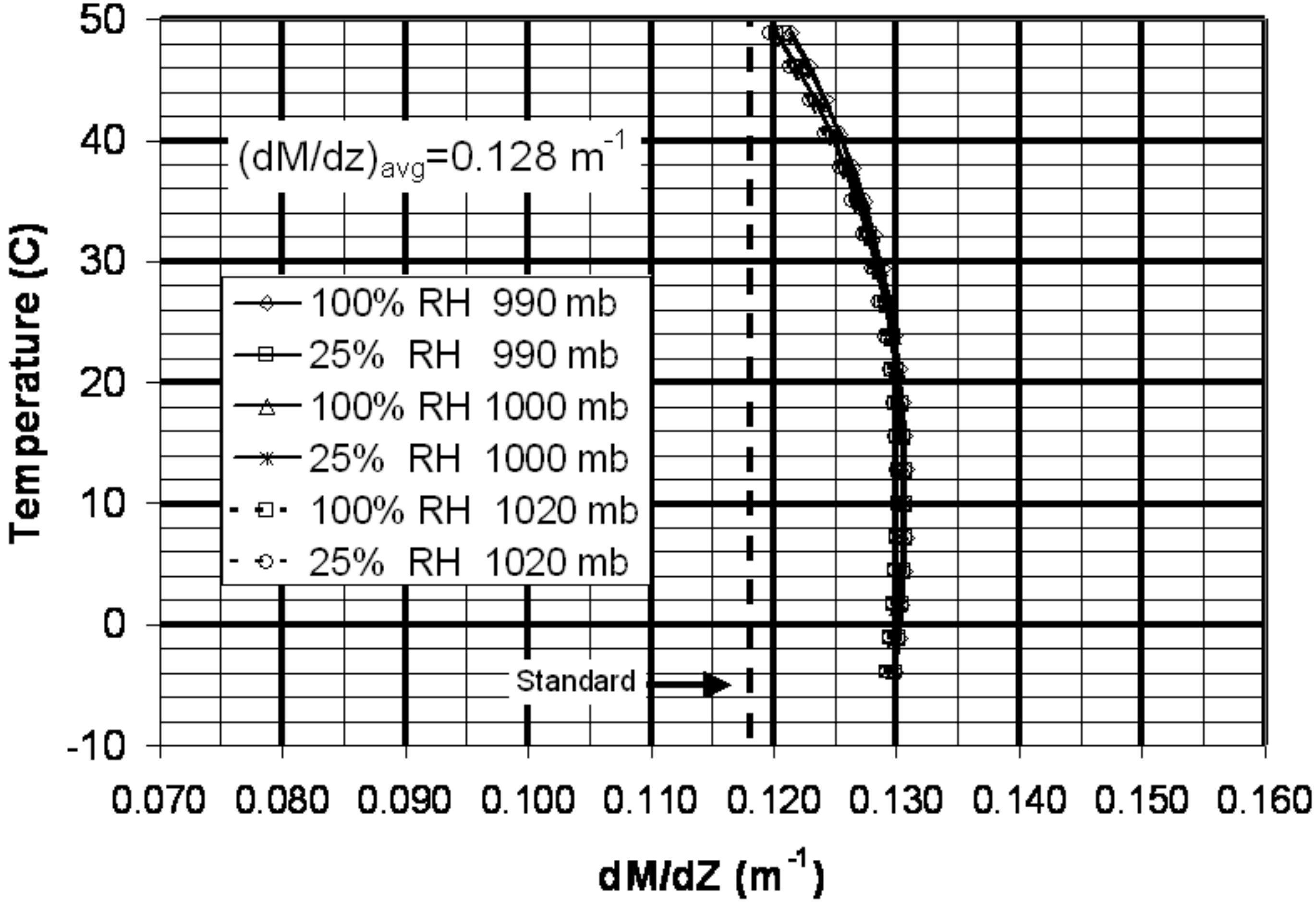
$$+0.157 \quad m^{-1}$$

Well Mixed Layer

$$\frac{d\theta}{dz} = \frac{dw}{dz} = 0$$

$$\frac{dM}{dz} = \frac{dp}{dz}\left(\frac{1.336X10^{7}w}{\theta^{2}p^{0.572}} + \frac{3.995X10^{2}}{\theta p^{0.286}}\right) + 0.157$$

Well Mixed Layer



Refractivity and Boundary Layer Structure

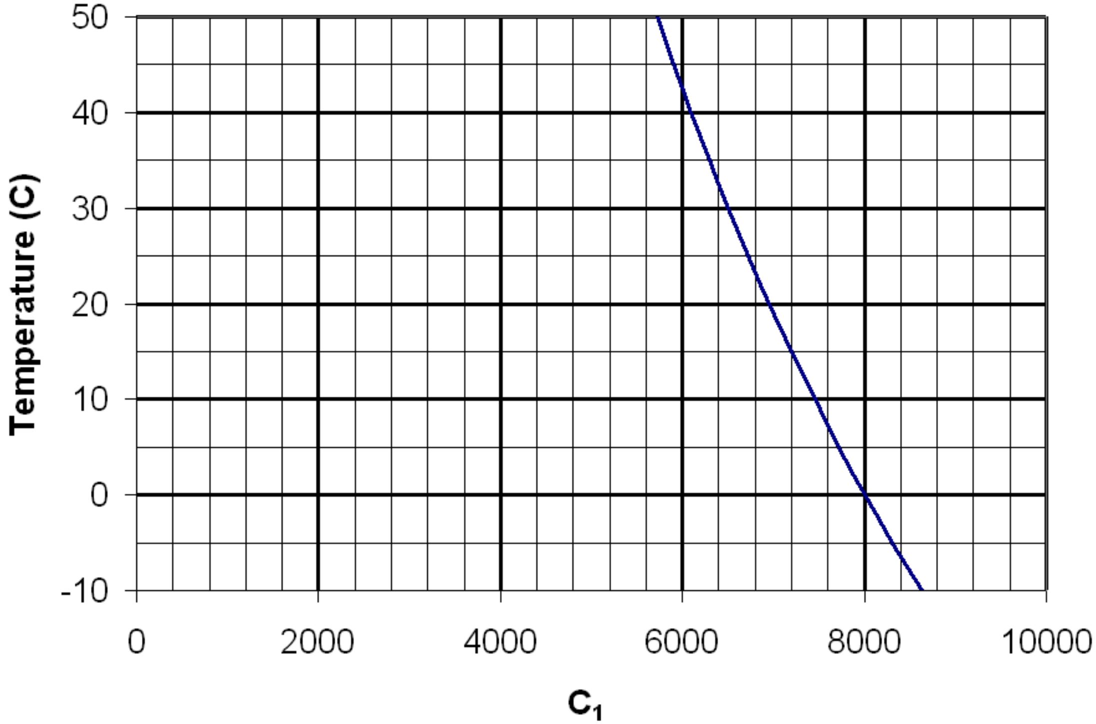
$$\frac{dM}{dz} \approx 0.128$$

$$+\frac{dw}{dz}\left(\frac{5.97X10^5\,p}{T^2}\right)$$

$$-\frac{d\theta}{dz}\left(\frac{p^{1.286}}{T^3}\left[2.57X10^5 w + 10.76T\right]\right)$$

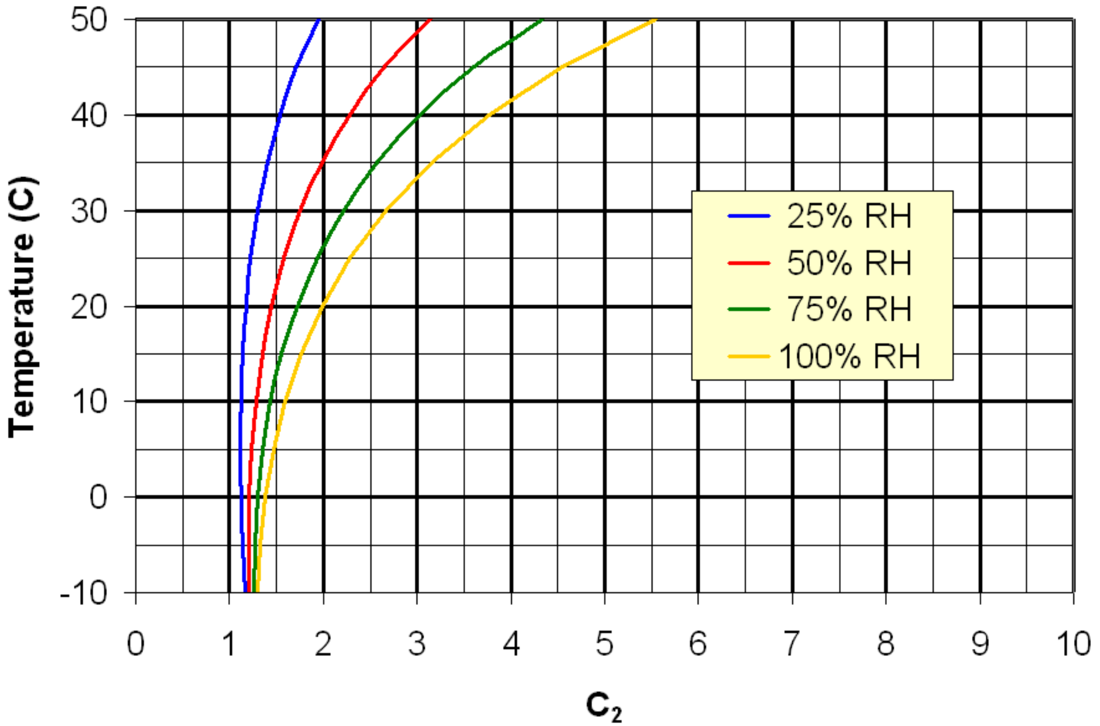
Refractivity and Boundary Layer Structure

$$\frac{dM}{dz} \approx 0.128 + c_1 \frac{dw}{dz} - c_2 \frac{d\theta}{dz}$$



Refractivity and Boundary Layer Structure

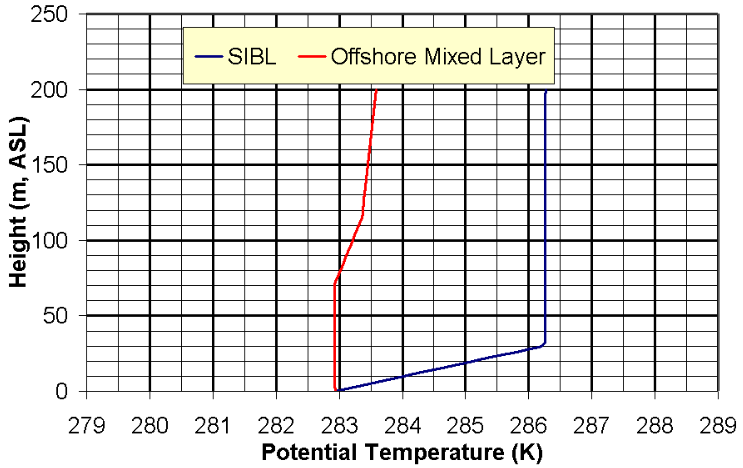
$$\frac{dM}{dz} \approx 0.128 + c_1 \frac{dw}{dz} - c_2 \frac{d\theta}{dz}$$

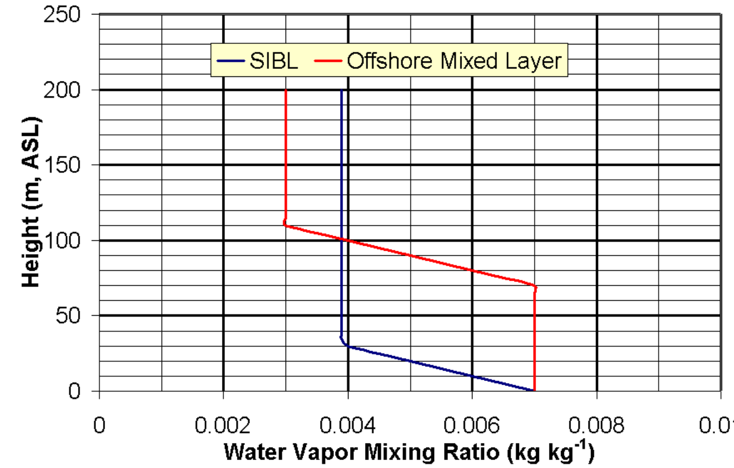


Refractivity and Boundary Layer Structure

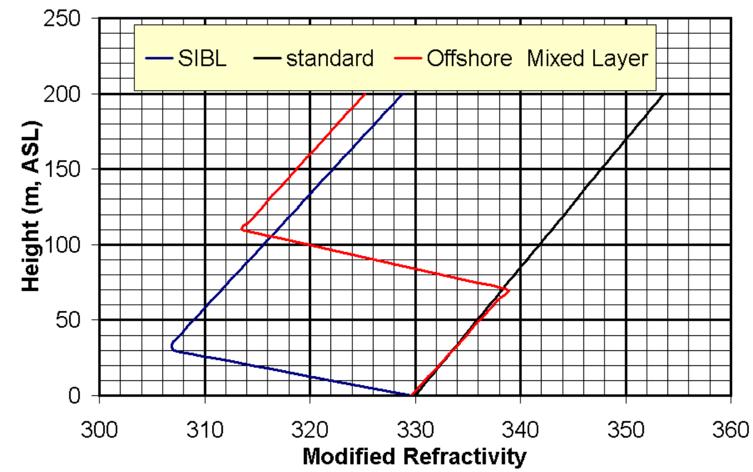
Stable Internal Boundary Layers (SIBL)

Offshore Flow of Warm and Drier Air over a Colder Sea Surface





$$\frac{dM}{dz} \approx 0.128 + c_1 \frac{dw}{dz} - c_2 \frac{d\theta}{dz}$$

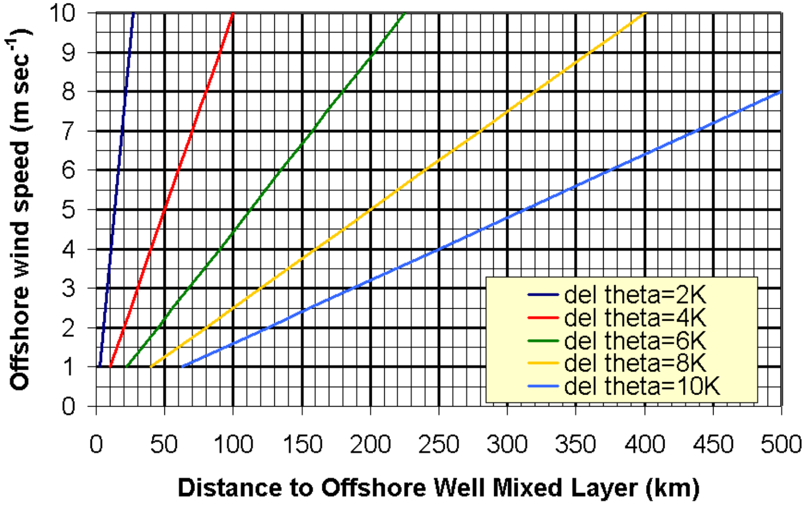


SIBLs eventually advect into well mixed layers a distance (d) Offshore.

Refractivity and Boundary Layer Structure

Evolution of stable internal boundary layers over a cold sea
Smedman, Bergstrom, Grisogono, Journal of Geophysical Research, January, 1997

$$d \approx \frac{5625V}{f}\left(\frac{\Delta\theta}{\theta_r}\right)^2$$



$d \equiv$ offshore distance to mixed layer

$V \equiv$ velocity

$f \equiv$ Coriolis parameter $\approx 10^{-4}$

$\Delta\theta \equiv$ potential temperature difference across SIBL

$\theta_r \equiv$ surface potential temperature

On the formation of a stably stratified internal boundary layer by advection of warm air over a cooler sea

Mulhearn, Boundary Layer, Meteorology, 1981

$$h \approx 0.0146x\left(\frac{gx}{v^2}\left[\frac{(\theta_1 - T_0)}{T_r} - 0.1\frac{(T_{s1} - T_0)}{T_r}\right]\right)^{-0.47}$$

$h \equiv$ height of SIBL

$x \equiv$ offshore distance

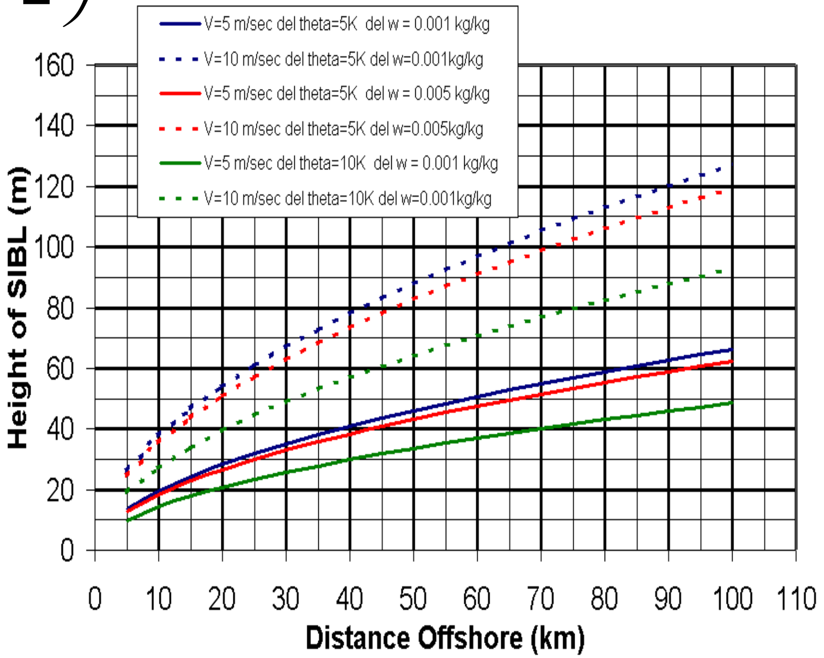
$v \equiv$ wind speed

$\theta_1 \equiv$ land surface potential temperature

$T_0 \equiv$ SST

$T_{s1} \equiv$ land surface potential dewpoint temperature

$T_r \equiv$ average SIBL temperature



Refractivity and Boundary Layer Structure

On the formation of a stably stratified internal boundary layer by advection of warm air over a cooler sea

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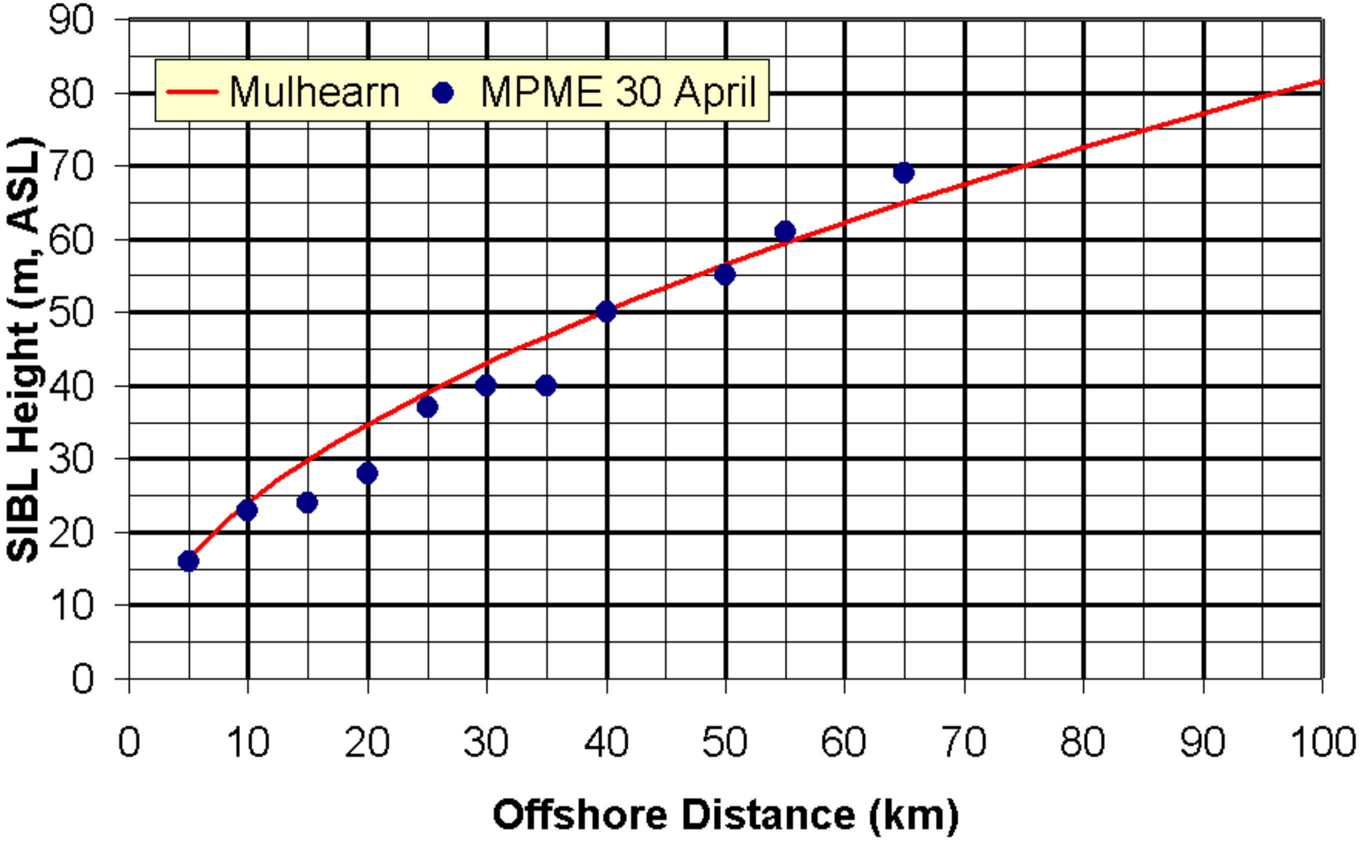
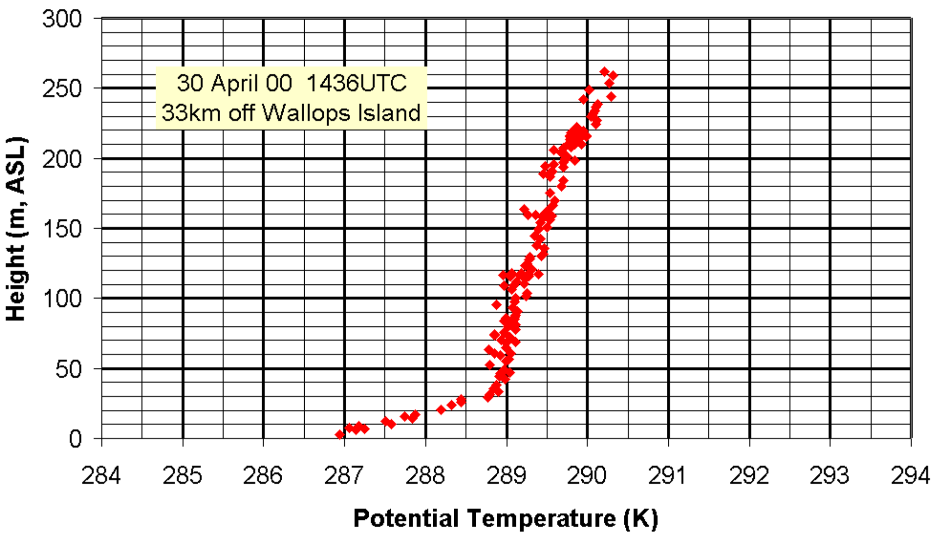
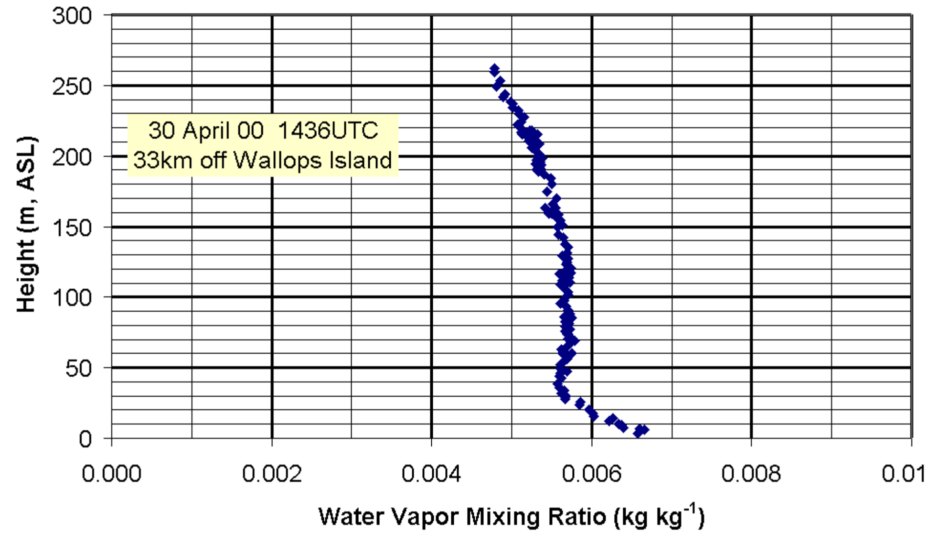
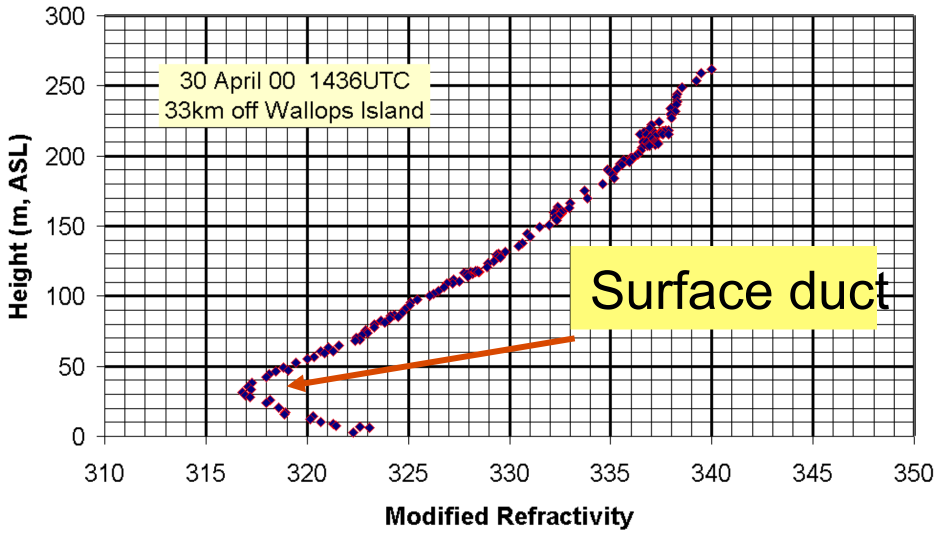


SIBL Height is Duct Height

Refractivity and Boundary Layer Structure







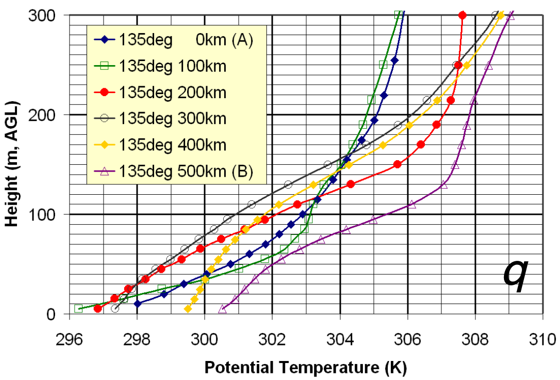
Measured data off Wallops Island

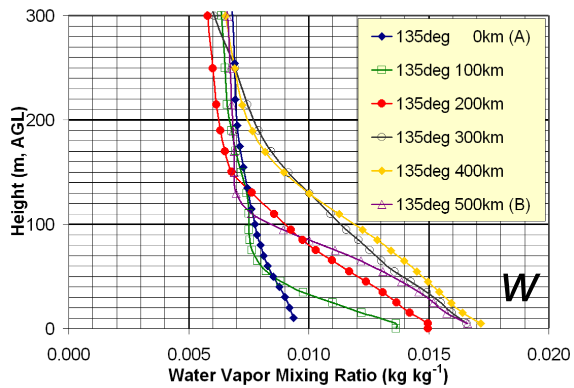
Offshore flow of warm air over cooler ocean

$$\frac{dM}{dz} \approx 0.128 + c_1 \frac{dw}{dz} - c_2 \frac{d\theta}{dz}$$

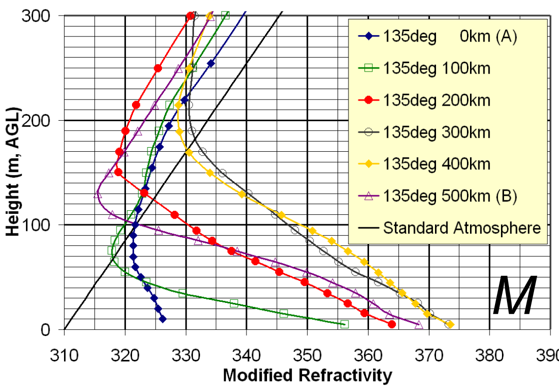
Refractivity and Boundary Layer Structure

Stable Internal Boundary Layers

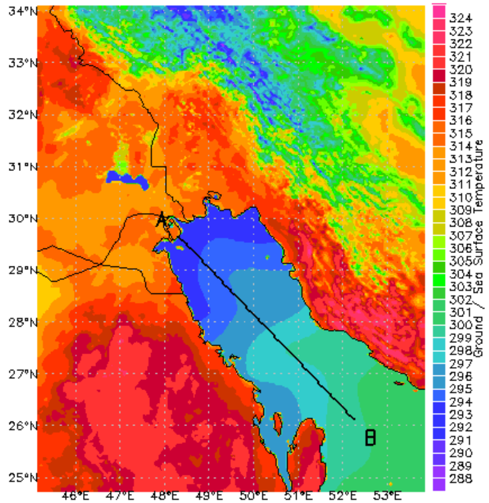




$$\frac{dM}{dz} \approx 0.128 - c_1 \frac{d\theta}{dz} + c_2 \frac{dw}{dz}$$



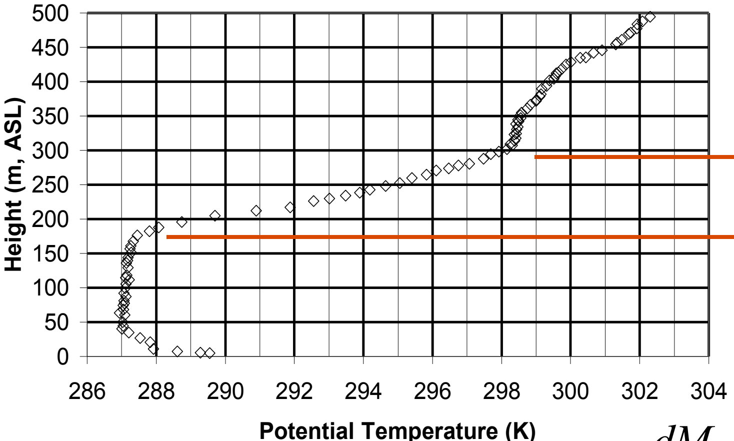
1100UTC on 14 May, 2009



- COAMPS® profiles every 100km from A to B
- dq/dz > 0, warmer air advecting up and over colder air at the sea surface
- dw/dz < 0, drier air advecting up and over saturated air at the sea surface
- dM/dz < 0, advection ducts, bi-linear ducts, or surface ducts
- Advection ducts can extend hundreds of km offshore

Refractivity and Boundary Layer Structure

Entrainment Layers



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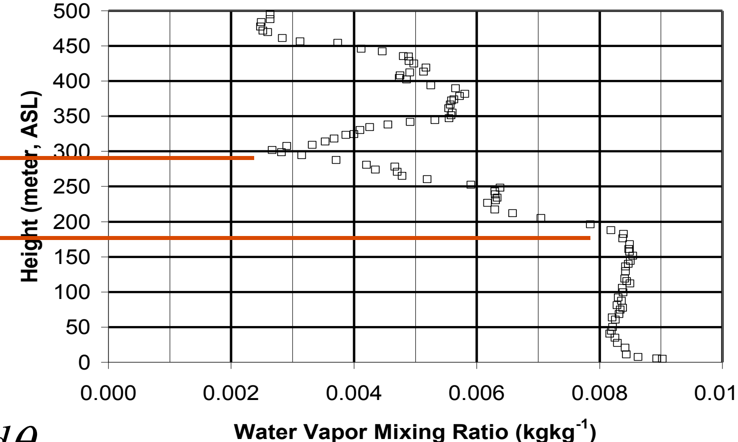
Free Atmosphere

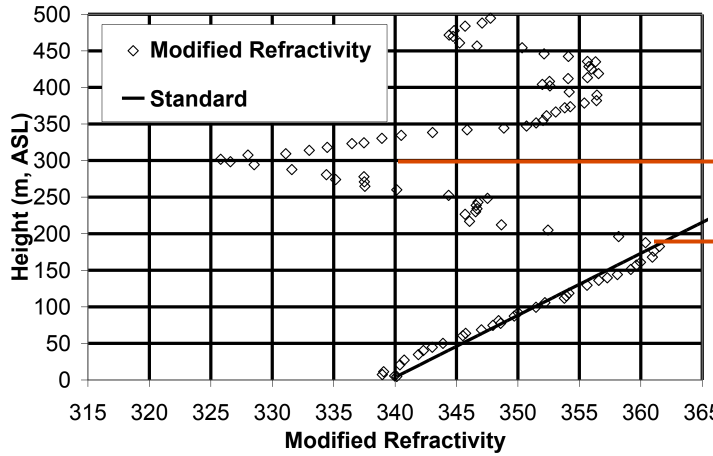
Entrainment Layer

Mixed Layer



W

$$\frac{dM}{dz} \approx 0.128 + c_1 \frac{dw}{dz} - c_2 \frac{d\theta}{dz}$$



Free Atmosphere

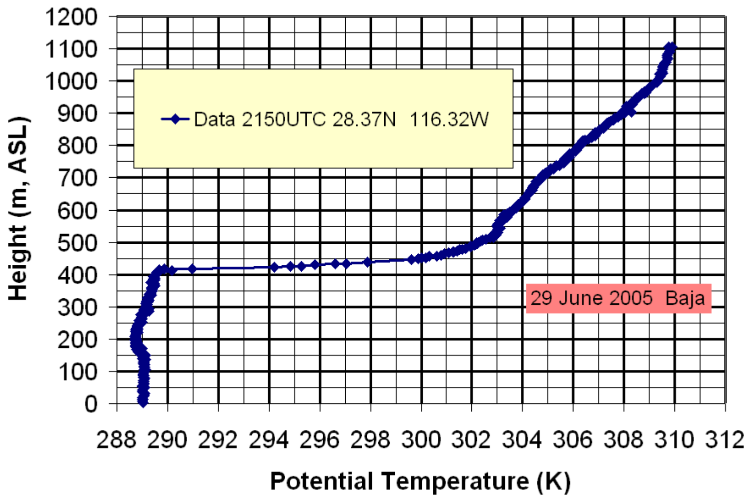
Entrainment Layer

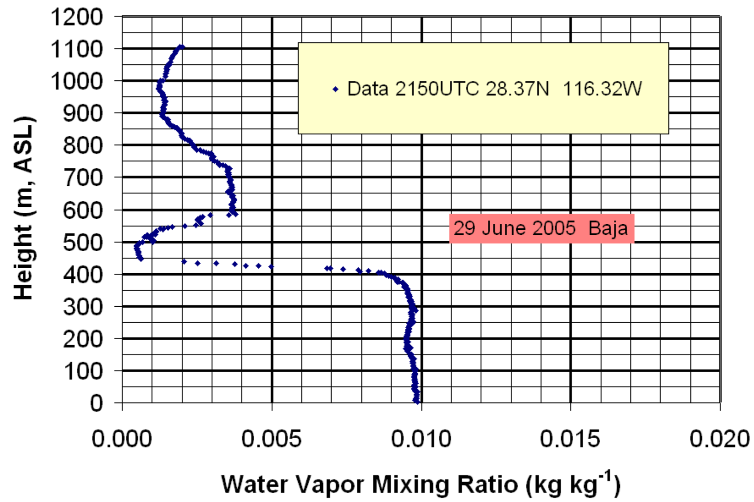
ELT

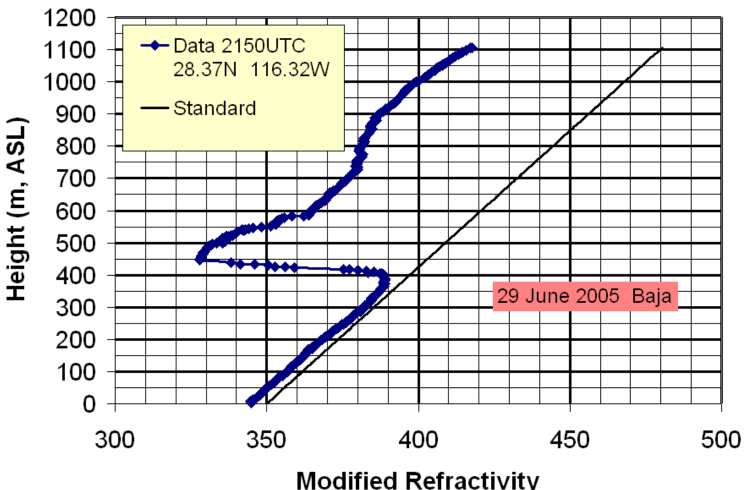
Mixed Layer

Refractivity and Boundary Layer Structure

Sea Breeze Circulations





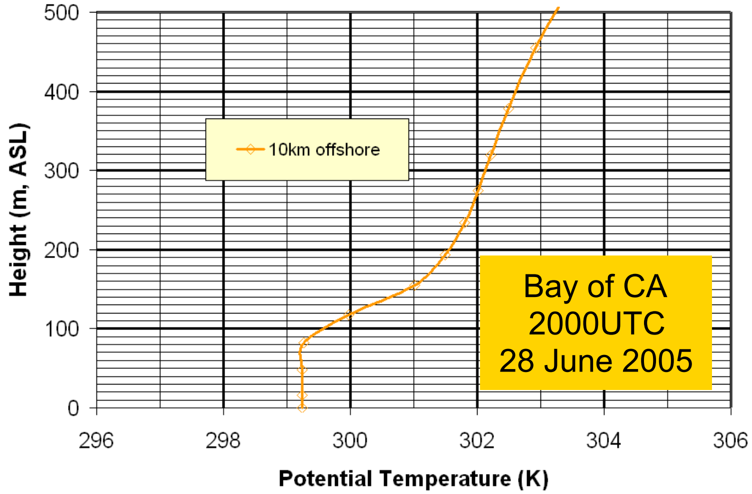


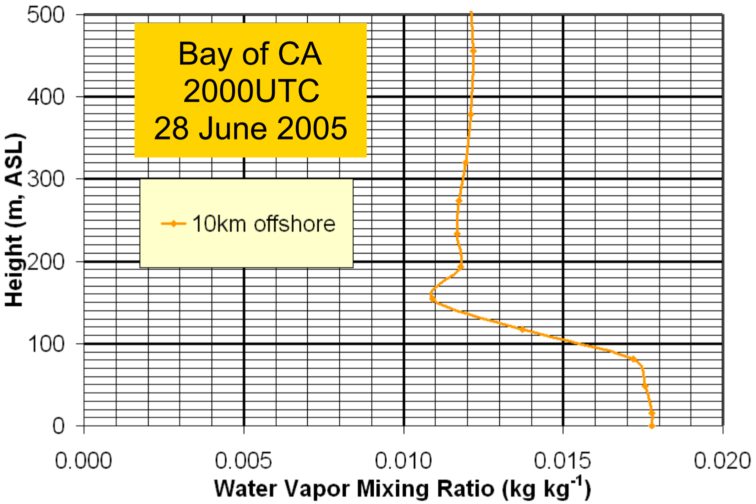
- Well mixed layer up to 400m, ASL
- 50m deep entrainment layer
- dq/dz > 0 in the stable entrainment layer
- Dry tongue above the entrainment layer
- dw/dz < 0 enhanced by dry tongue
- dM/dz < 0 in the entrainment layer
- Sub-refractive layer above entrainment layer
- Entrainment layers are breeding grounds for radio frequency ducts

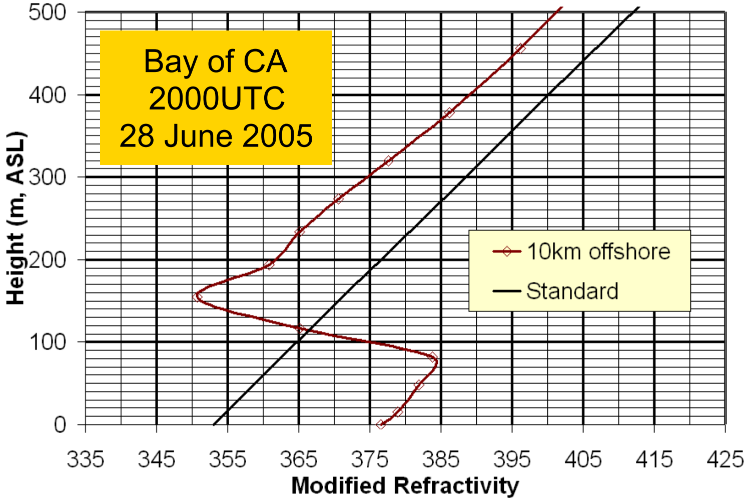
R. E. Marshall T41 NSWCDL

Refractivity and Boundary Layer Structure

Sea Breeze Circulations



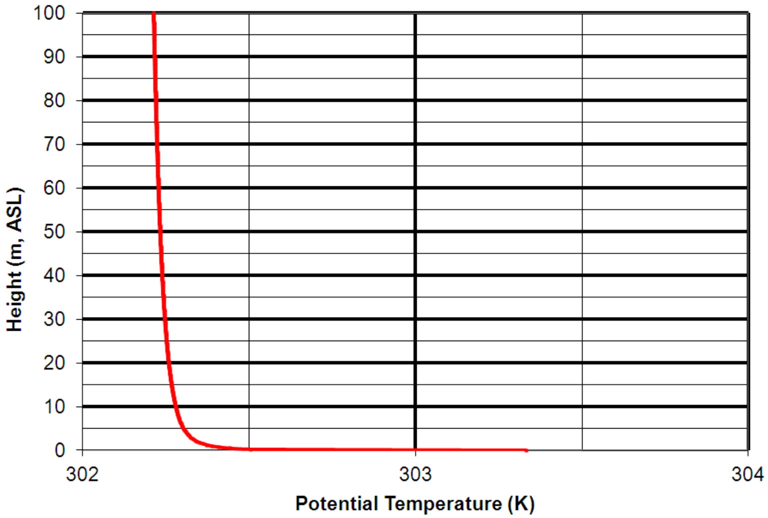


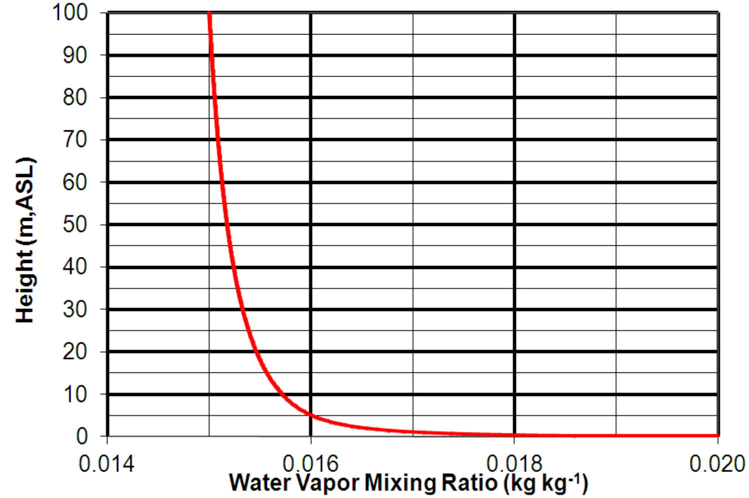


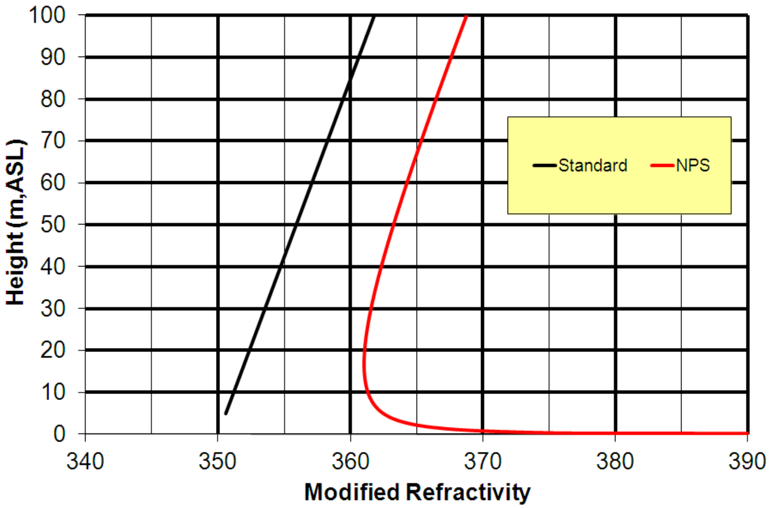
- COAMPS ® modeling
- Well mixed layer up to 80m, ASL
- 80m deep entrainment layer
- dq/dz > 0 in the stable entrainment layer
- Dry tongue above the entrainment layer
- dw/dz < 0 enhanced by dry tongue
- dM/dz < 0 in the entrainment layer
- Sub-refraction above the entrainment layer

Refractivity and Boundary Layer Structure

Surface Layer







Surface Layer Model (Evaporation Duct Model)

- atmospheric surface layer turbulence model for a thermally stratified layer
- based on Monin-Obukhov similarity theory
- assumes horizontal homogeneity of thermodynamic and wind variables
- predicts the vertical profiles of wind speed, pressure, temperature, moisture and modified refractivity from the sea surface to the top of the atmospheric surface layer

Refractivity and Boundary Layer Structure

Engineering Significance of Refractivity

$$\frac{P_r}{P_t} = \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} F^4$$

Two way propagation factor in the Radar equation

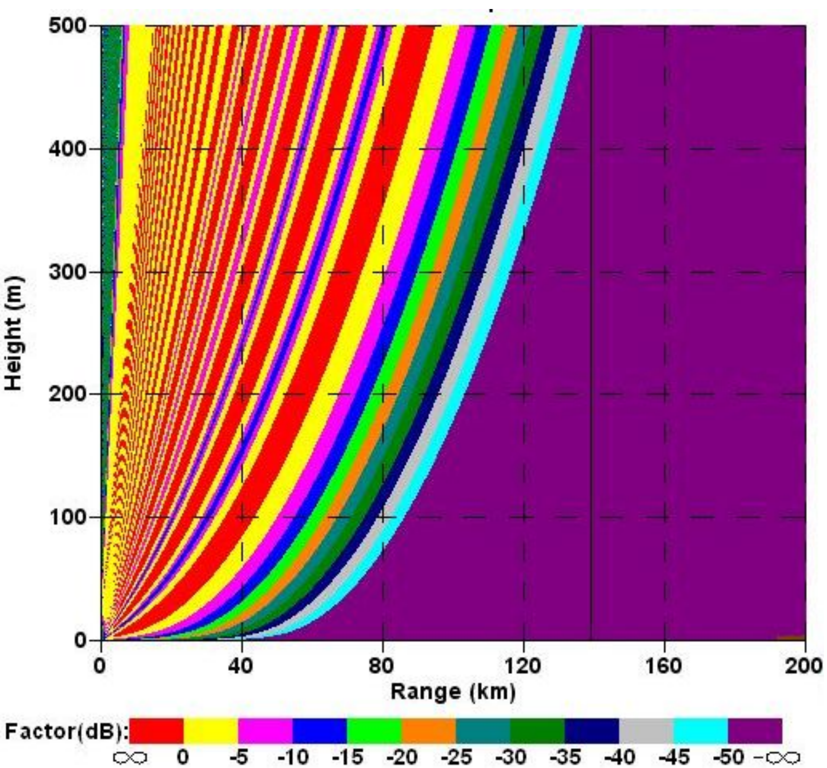
$$\frac{P_r}{P_t} = \frac{G_T G_R \lambda^2}{(4\pi R)^2} F^2$$

One way propagation factor in the Communications link equation

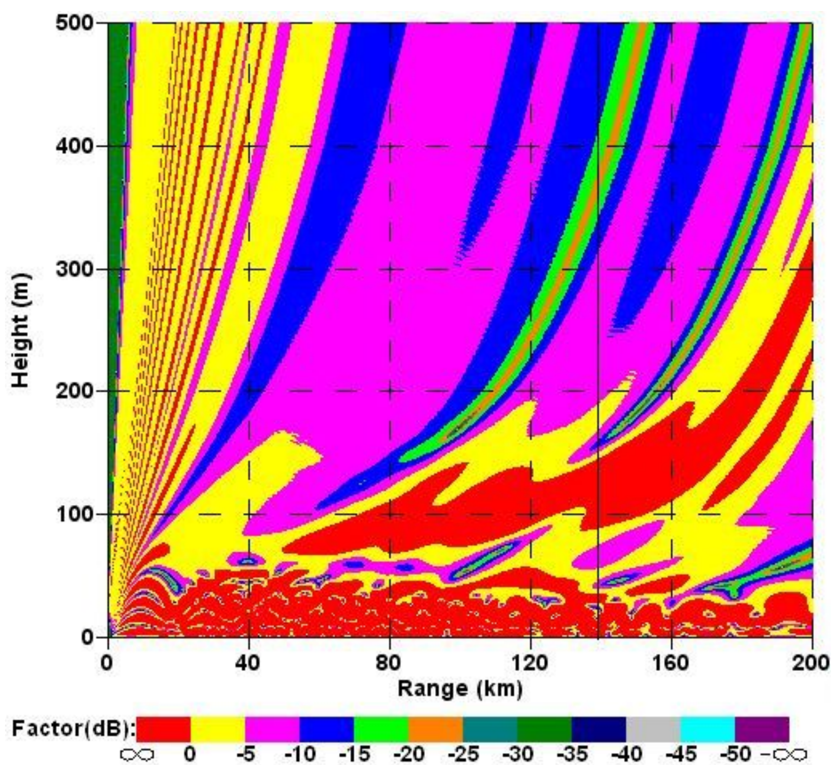
- Propagation factor due to non standard refraction
- 0dB in free space
- F^2 potentially greater than +/- 30dB in real near surface atmospheres

Refractivity and Boundary Layer Structure

Engineering Significance of Refractivity

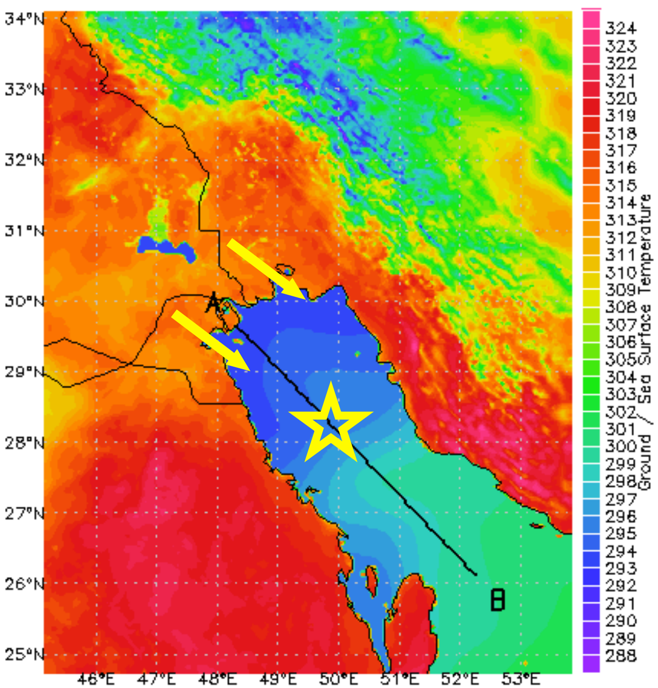


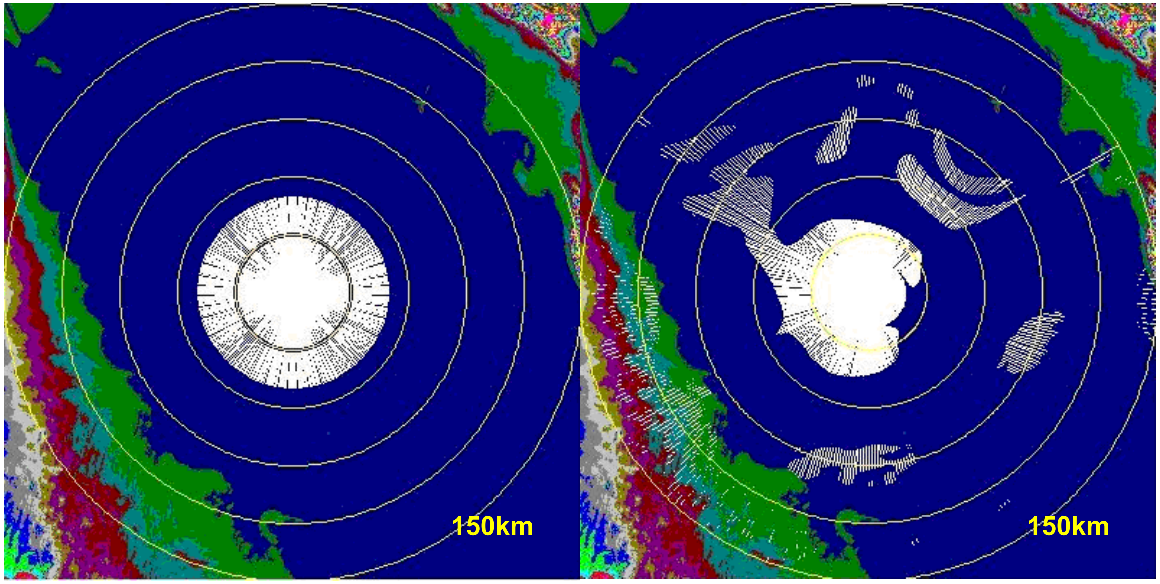
Standard Atmosphere (multipath nulls)



Strong Ducting

Engineering Significance of Refractivity



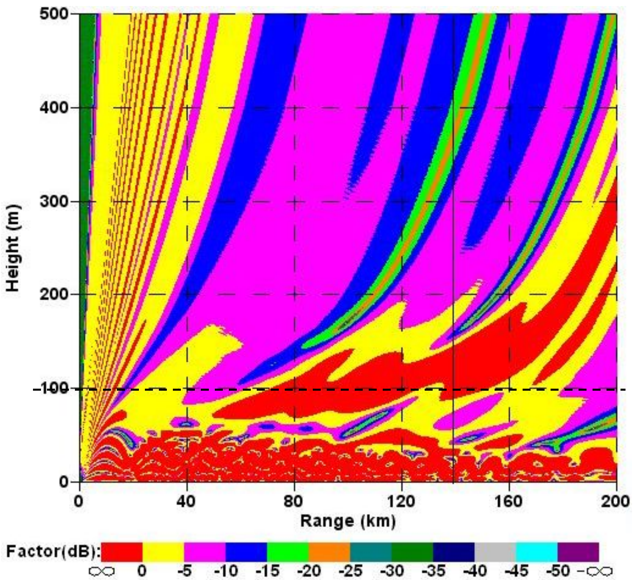


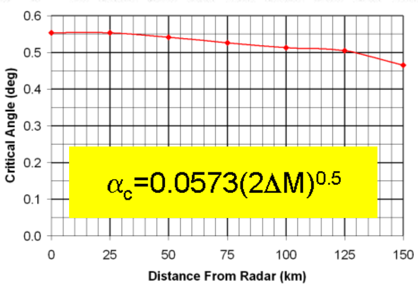
Notional S-band radar detection areas in white of a notional target at 100m ASL. The image on the left is an AREPS model in a standard atmosphere. The image on the right is a COAMPS®/AREPS model for 1100UTC on 14 May 2009

Engineering Significance of Refractivity

S band notional radar in the Persian Gulf

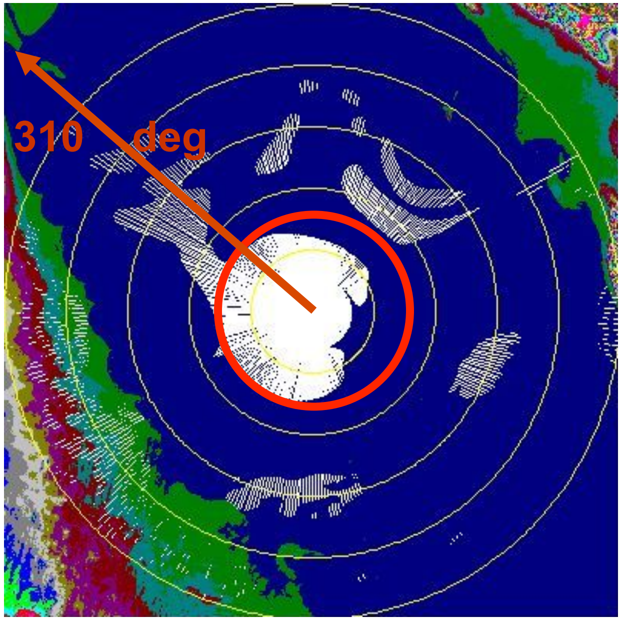
1100UTC, 14 May 2009 (validated refractivity field)





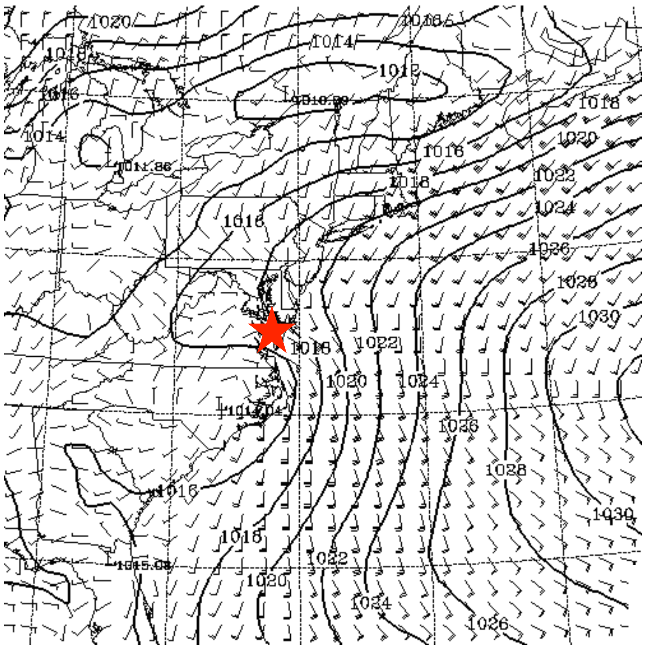
- Energy escapes the duct as critical angle (a_c) decreases with range

- Critical angle (a_c) increases with duct strength (DM)

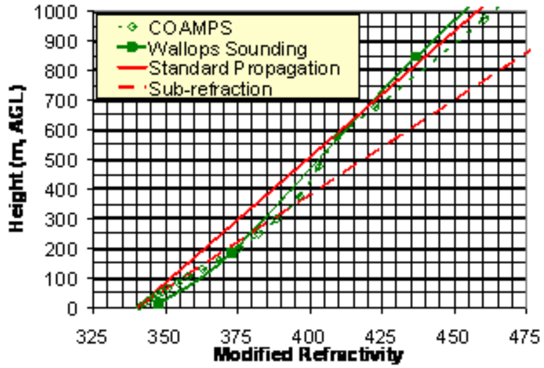


Refractivity and Boundary Layer Structure

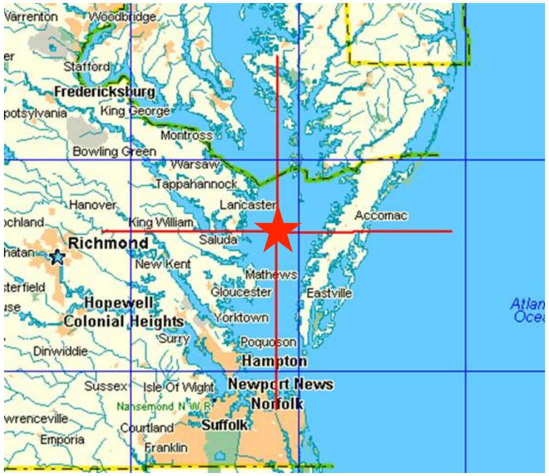
Engineering Significance of Sub-refraction

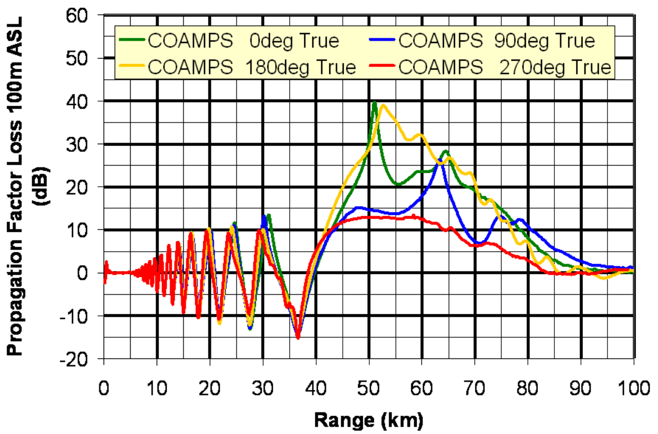


EDAS 15 APR 2006
Place a ship based radar
In Chesapeake Bay
★



Wallops Synoptic
Sounding Comparison





Sub-refraction creates expensive engineering demands

Refractivity and Boundary Layer Structure

Summary

- Refractivity in the PBL can significantly influence radio frequency system performance.
- Refractivity is directly related to PBL thermodynamic structure.
- Mesoscale NWP has become a powerful tool for understanding the four dimensional engineering demands placed on radio frequency systems at specific locations.
- The potential exists for a 0 to 72 hour globally locatable radio frequency system performance tool.