An adjoint sensitivity study of extratropical transition: Floyd (1999) and a few comments about adjoint sensitivity fields

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Motivation: improved understanding of the dynamics and targeted observing of ET.

- Identification of the key antecedent weather systems and dynamical processes responsible for the transition of
- Formulation and use of an adjoint-derived initial condition perturbations to confirm synoptic interpretation.
- Description of some recent work understand the adjustment of of adjoint sensitivity fields towards "balance"

Hurricane Floyd 1999

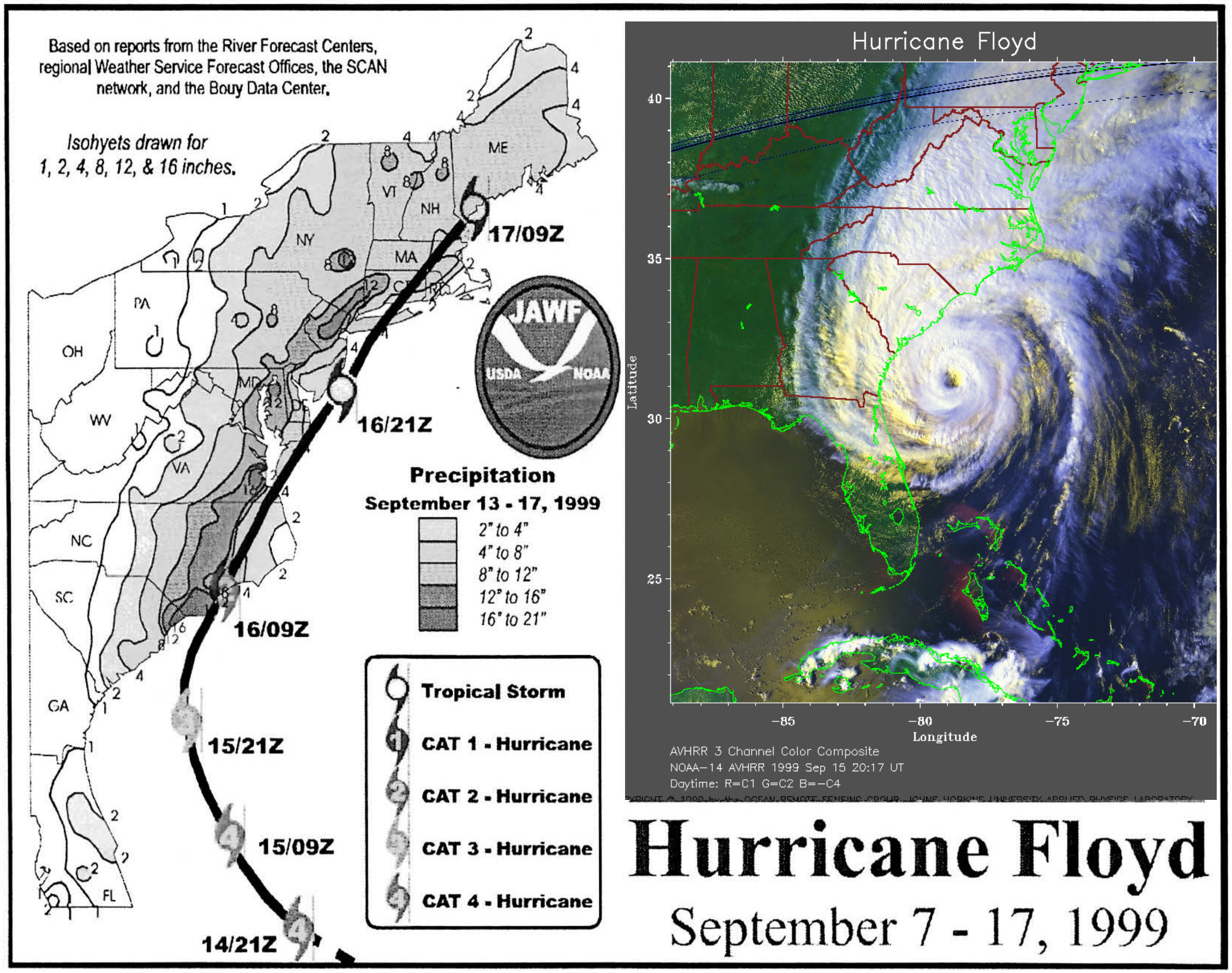


FIG. 1. Schematic of the track and storm total precipitation associated with Hurricane Floyd as reproduced from the NASS (1999).

Hurricane Floyd 1999

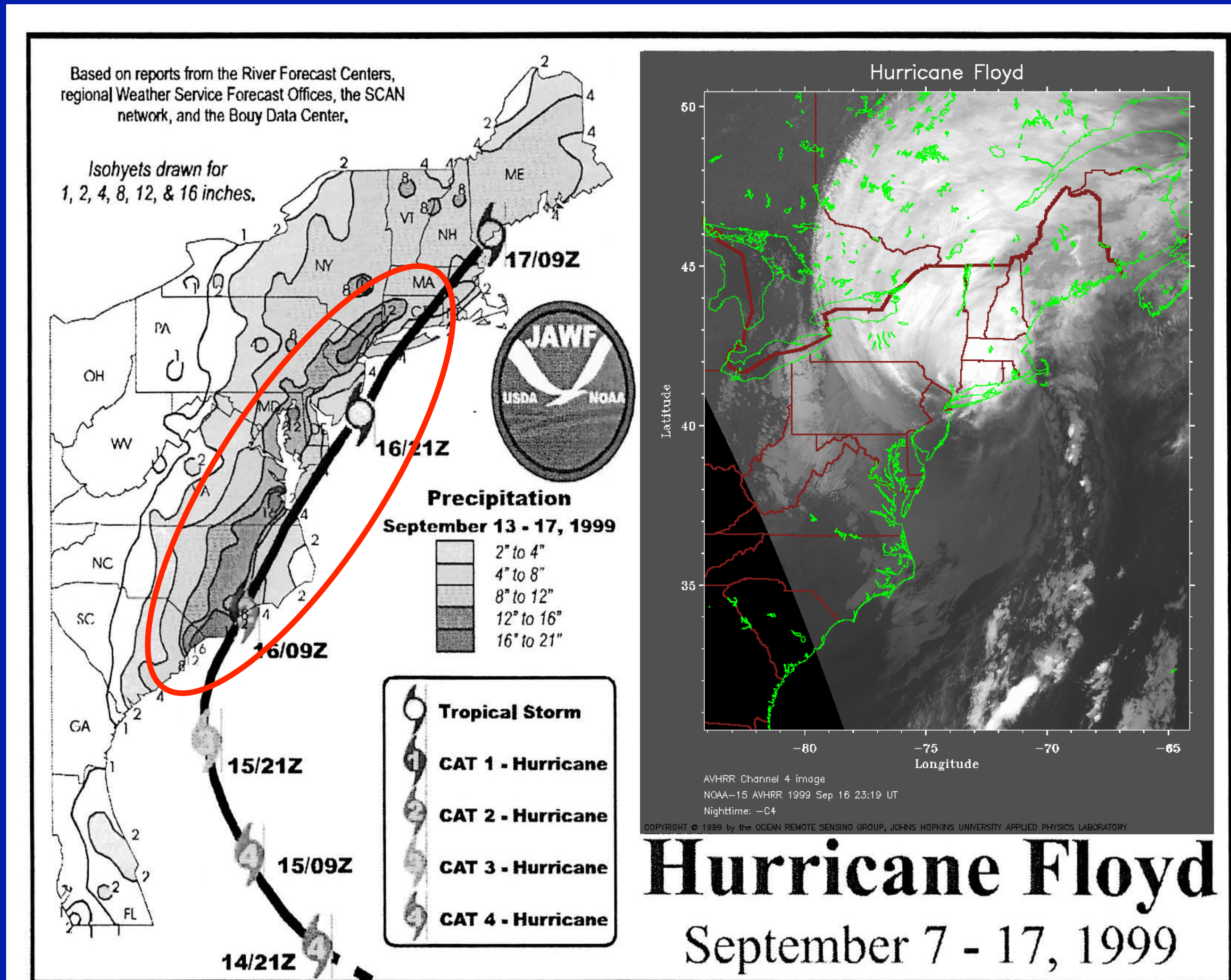
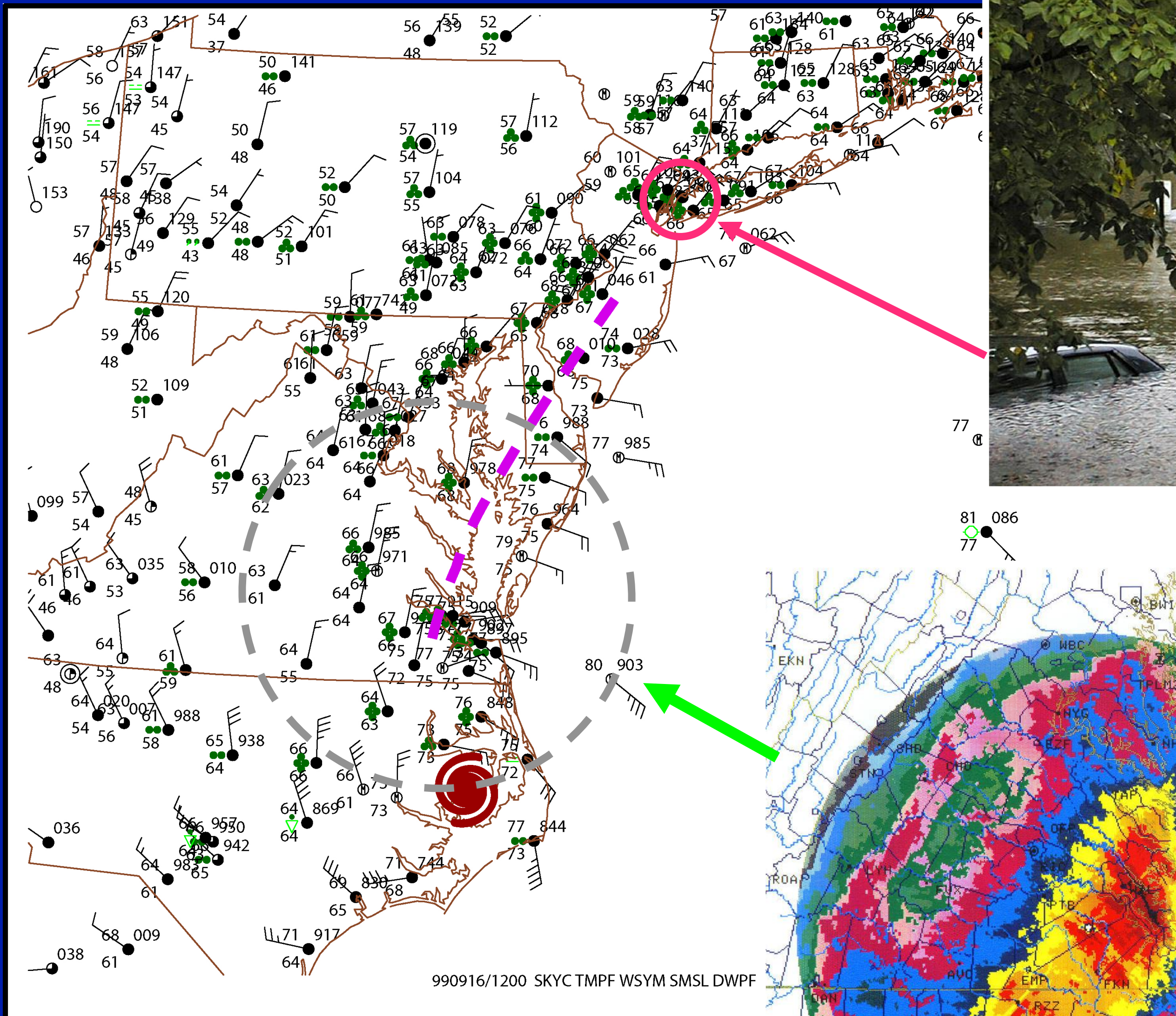


FIG. 1. Schematic of the track and storm total precipitation associated with Hurricane Floyd as reproduced from the NASS (1999).

Prior studies: Atallah and Bosart (2003) and Colle (2003)



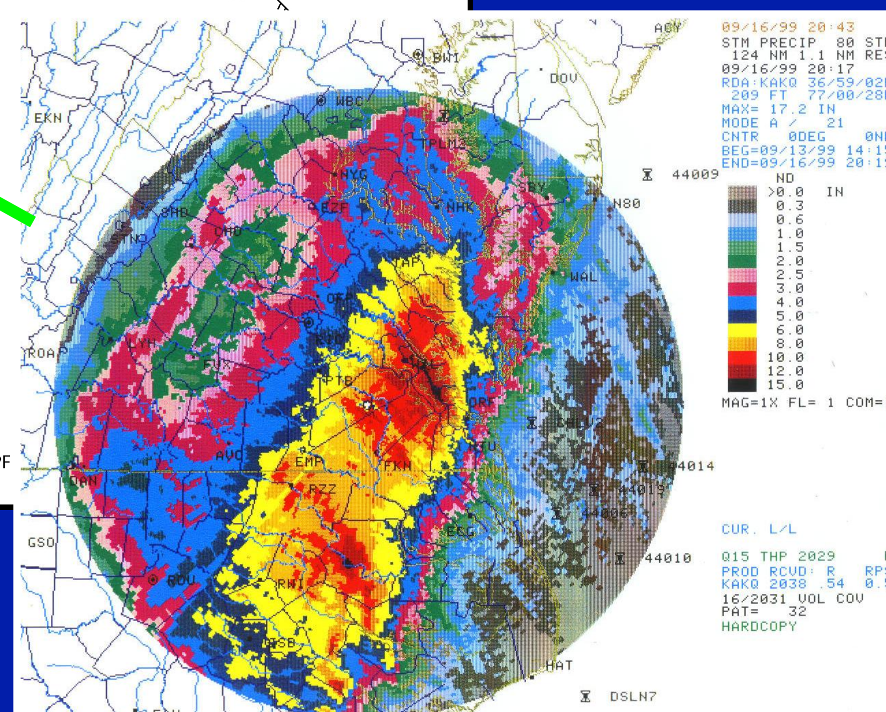
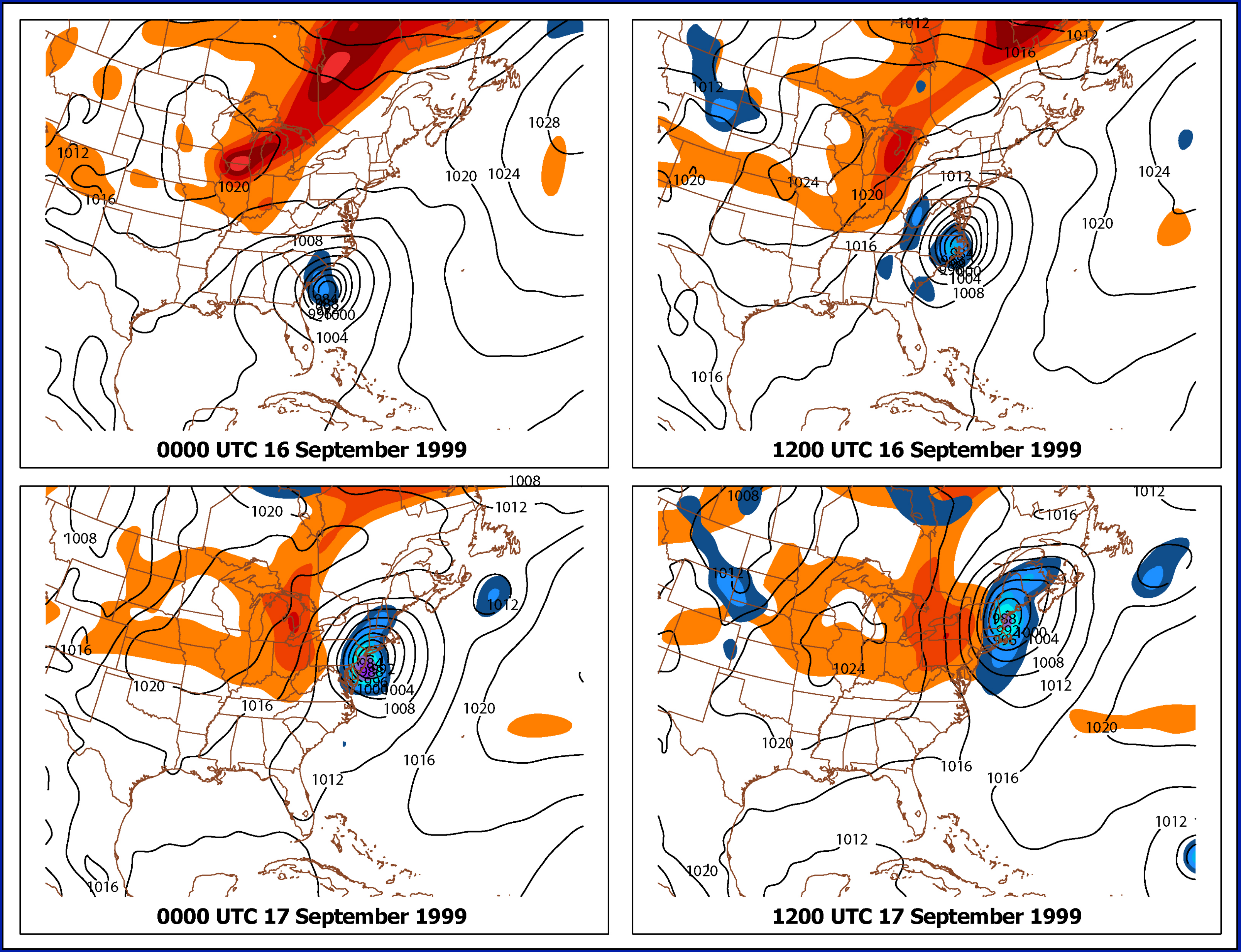


image from Little Falls, NJ Fire/Rescue

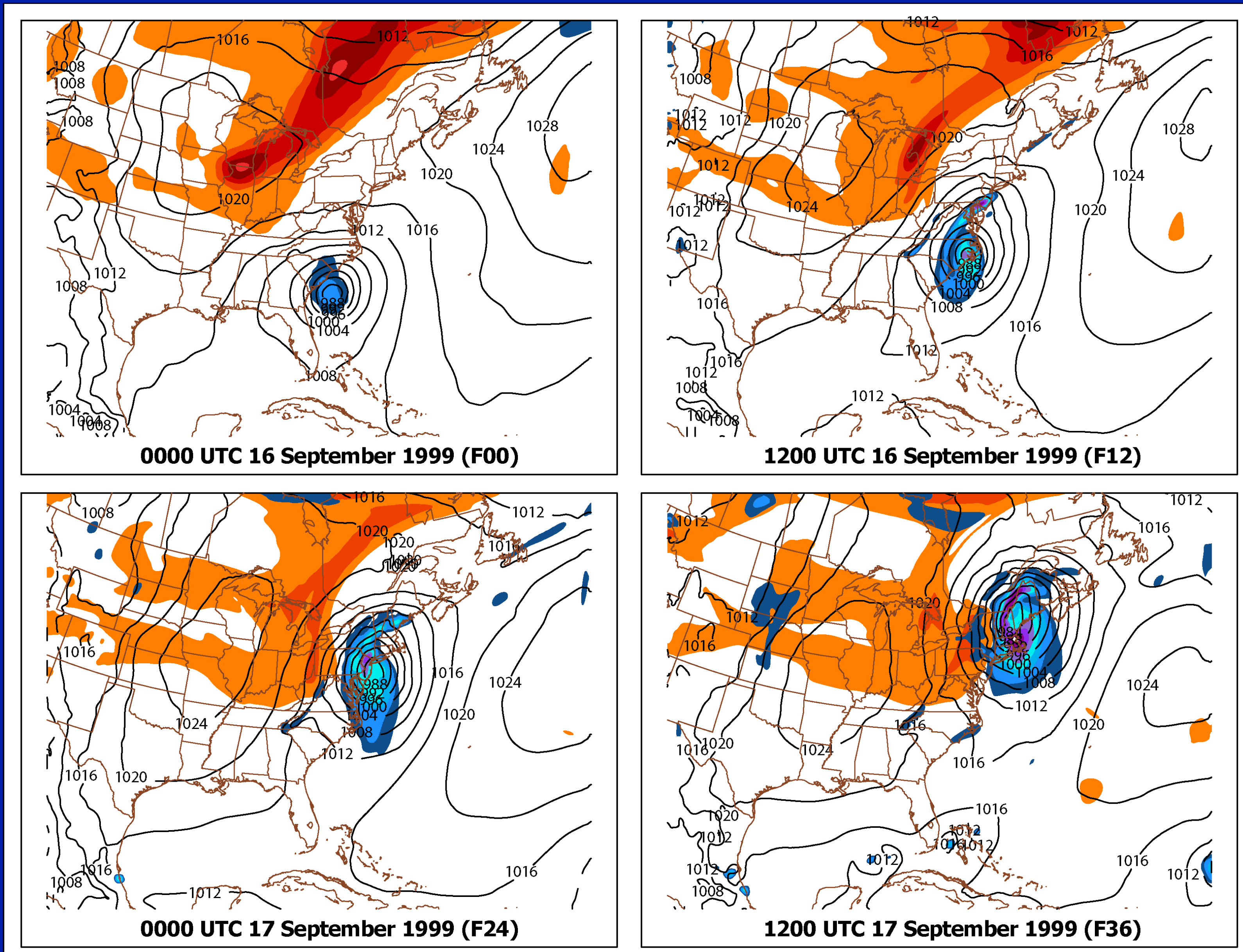
36h evolution of upper and lower tropospheric PV for Floyd (1999)

ANALYSIS

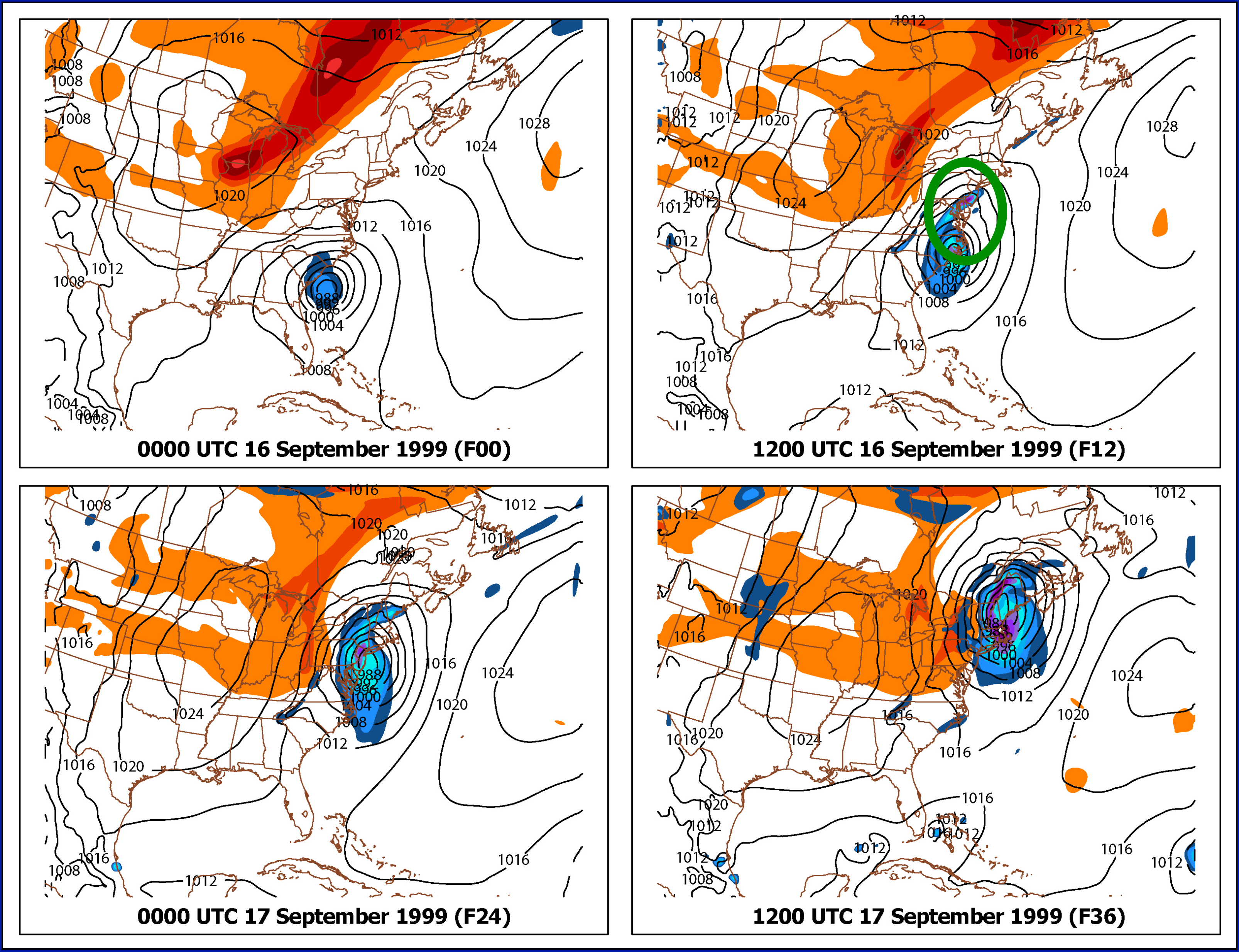


36h evolution of upper and lower tropospheric PV for Floyd (1999)

FORECAST

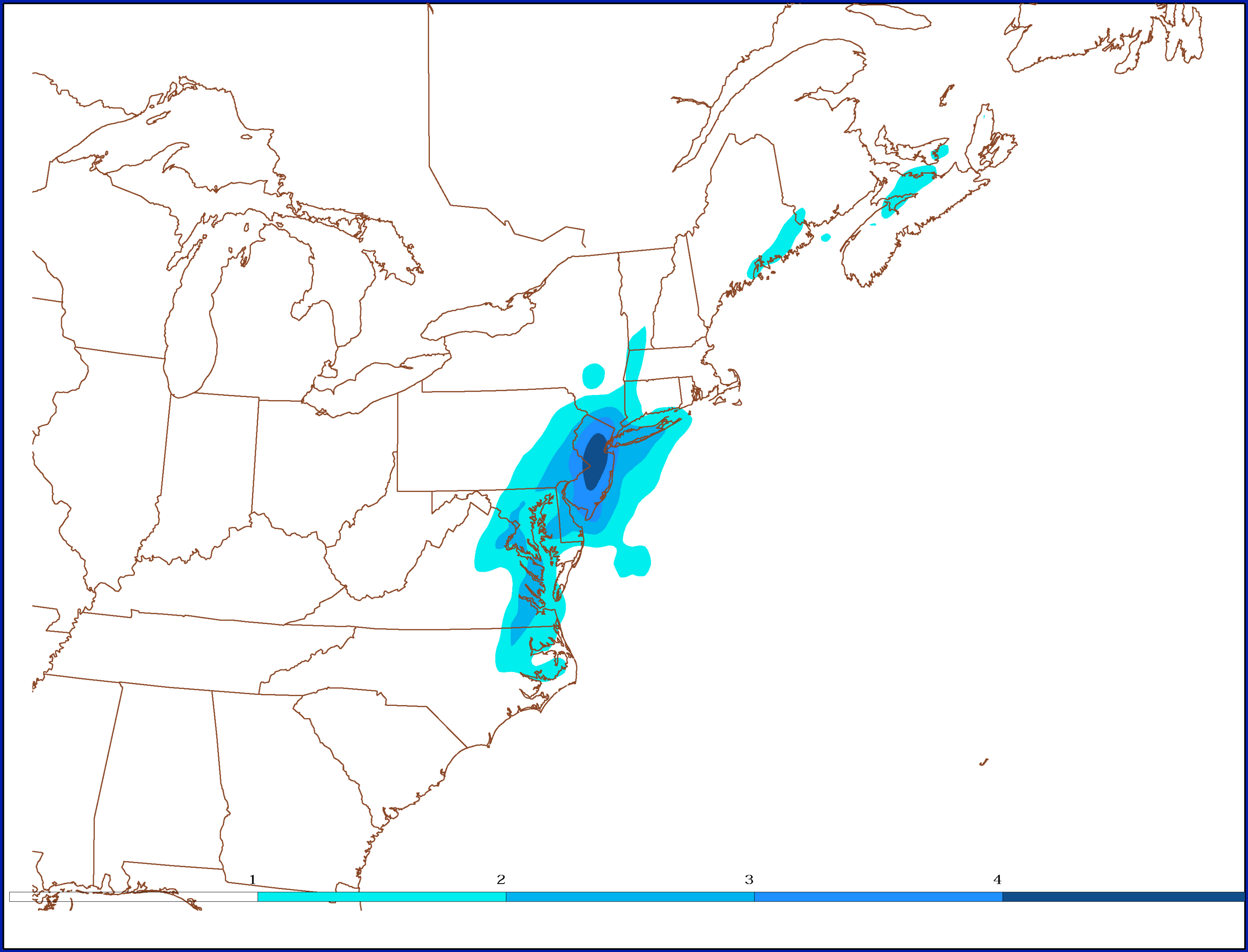


36h evolution of upper and lower tropospheric PV for Floyd (1999)

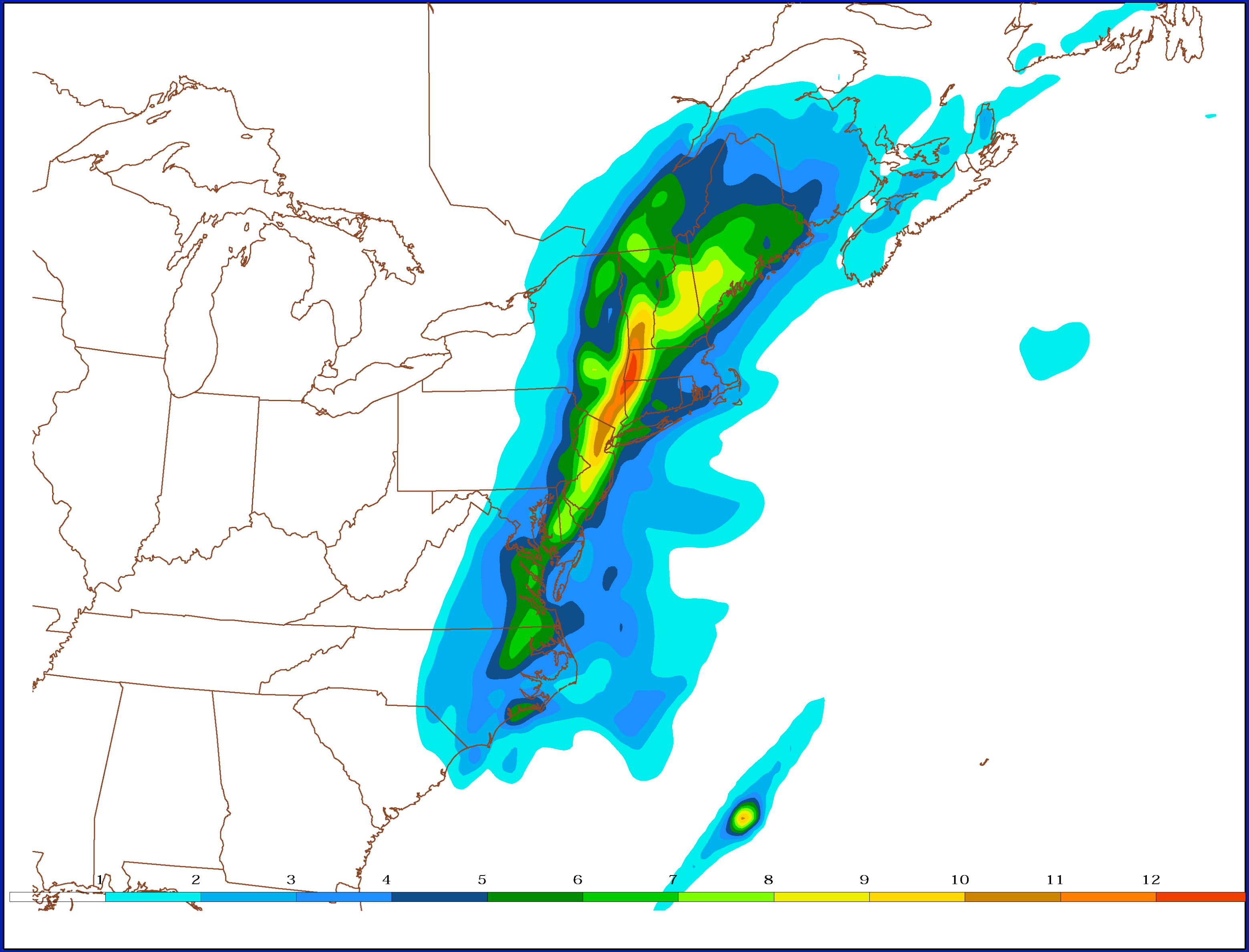


FORECAST

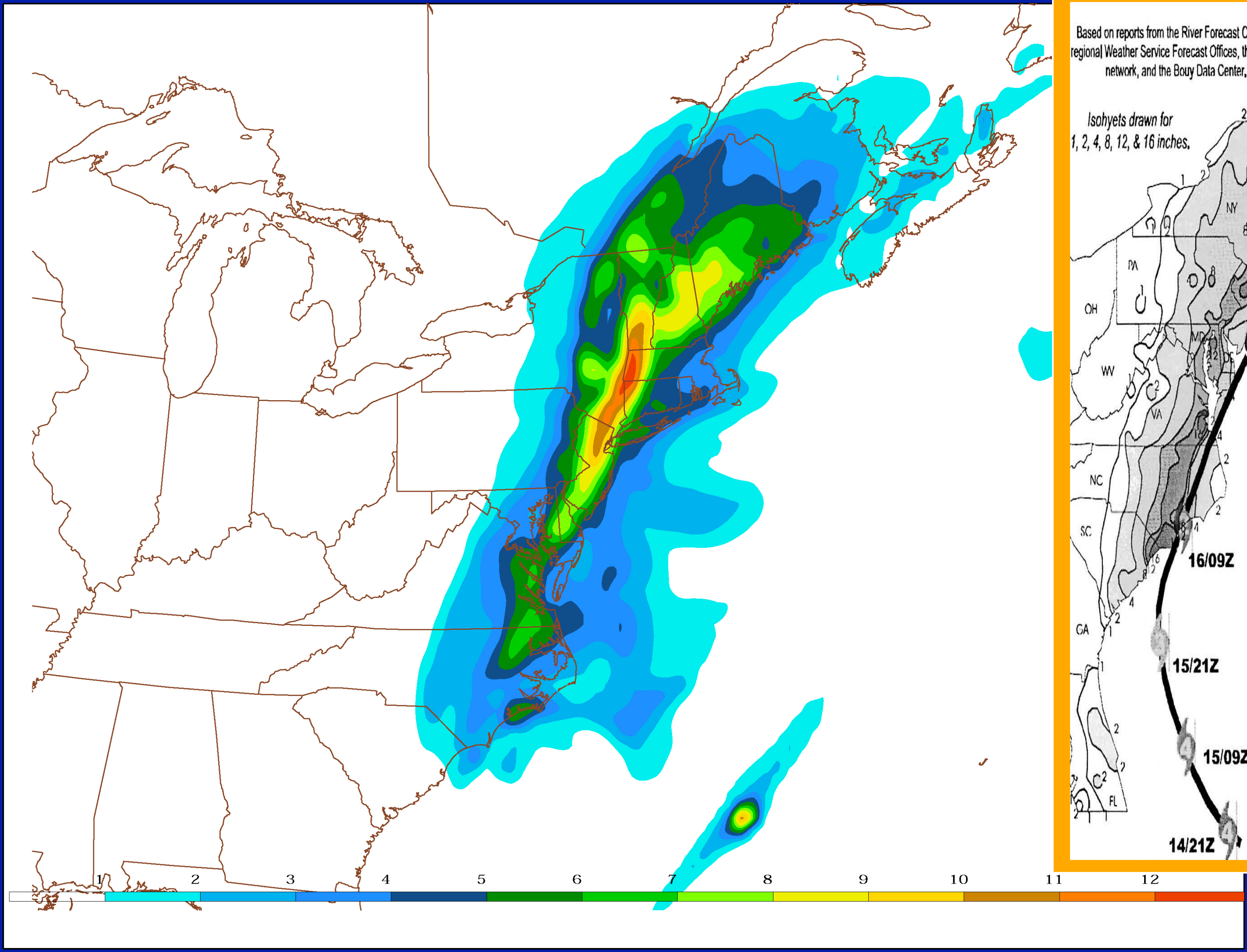
6h forecasted precipitation (inches) at F15

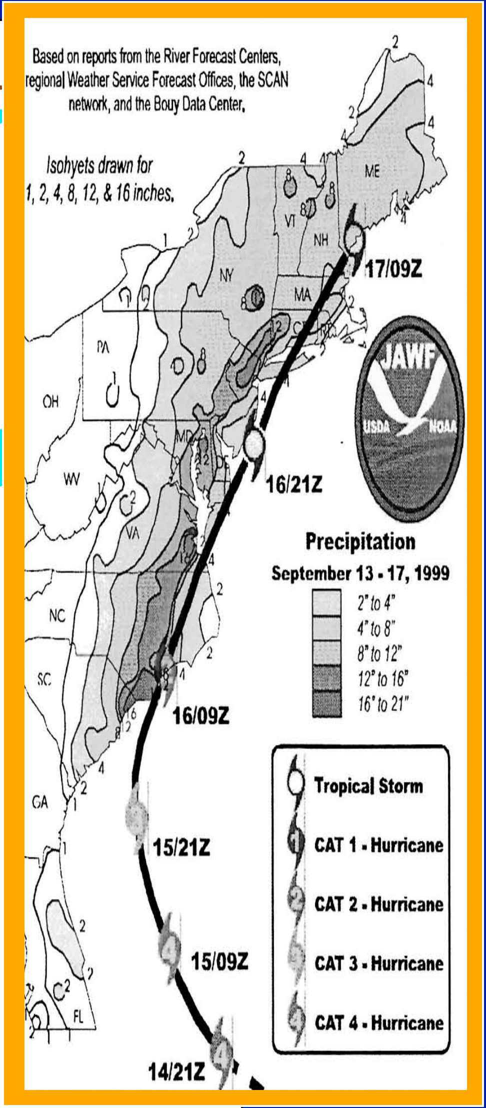


36h forecasted precipitation (inches) at F36



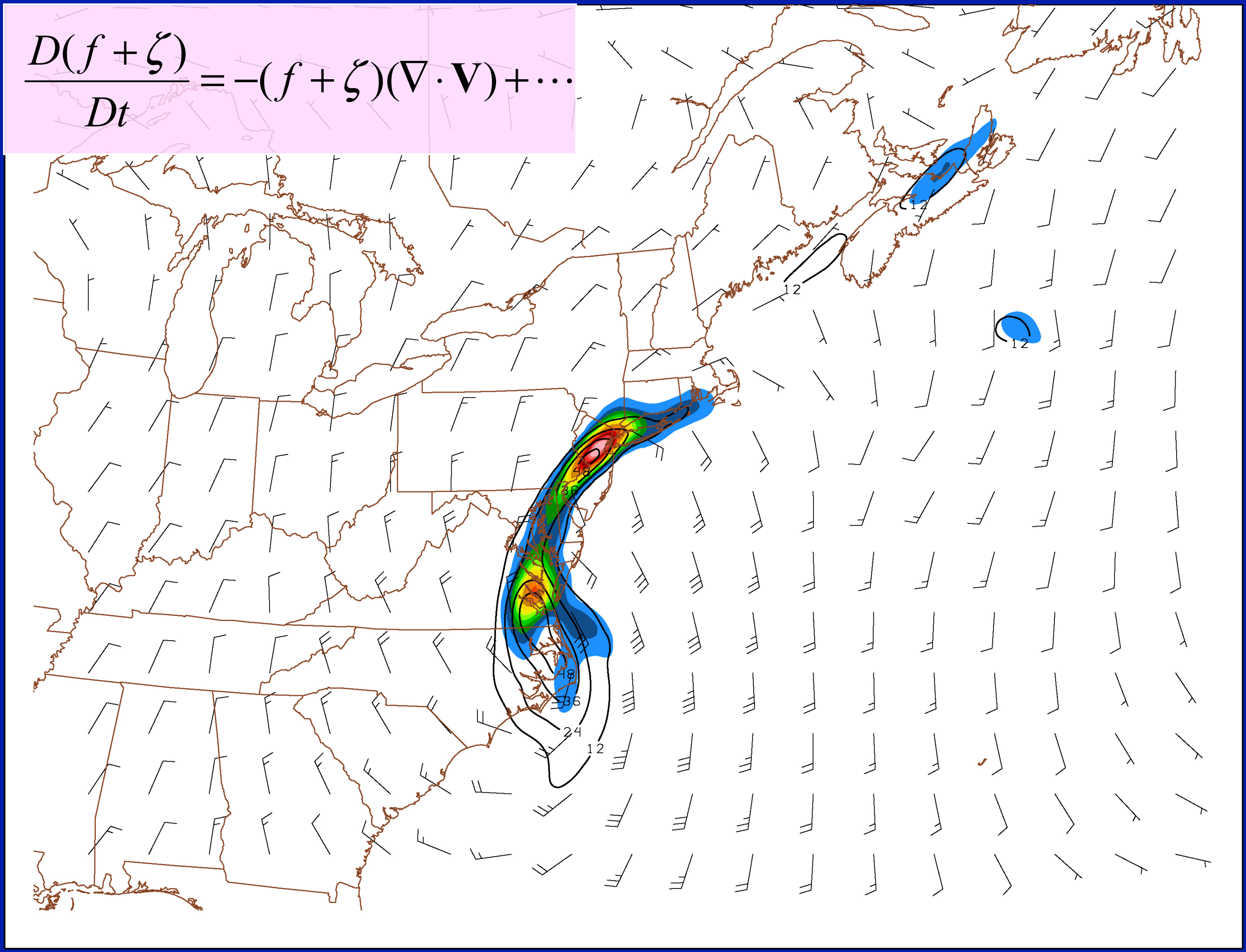
36h forecasted precipitation (inches) at F36





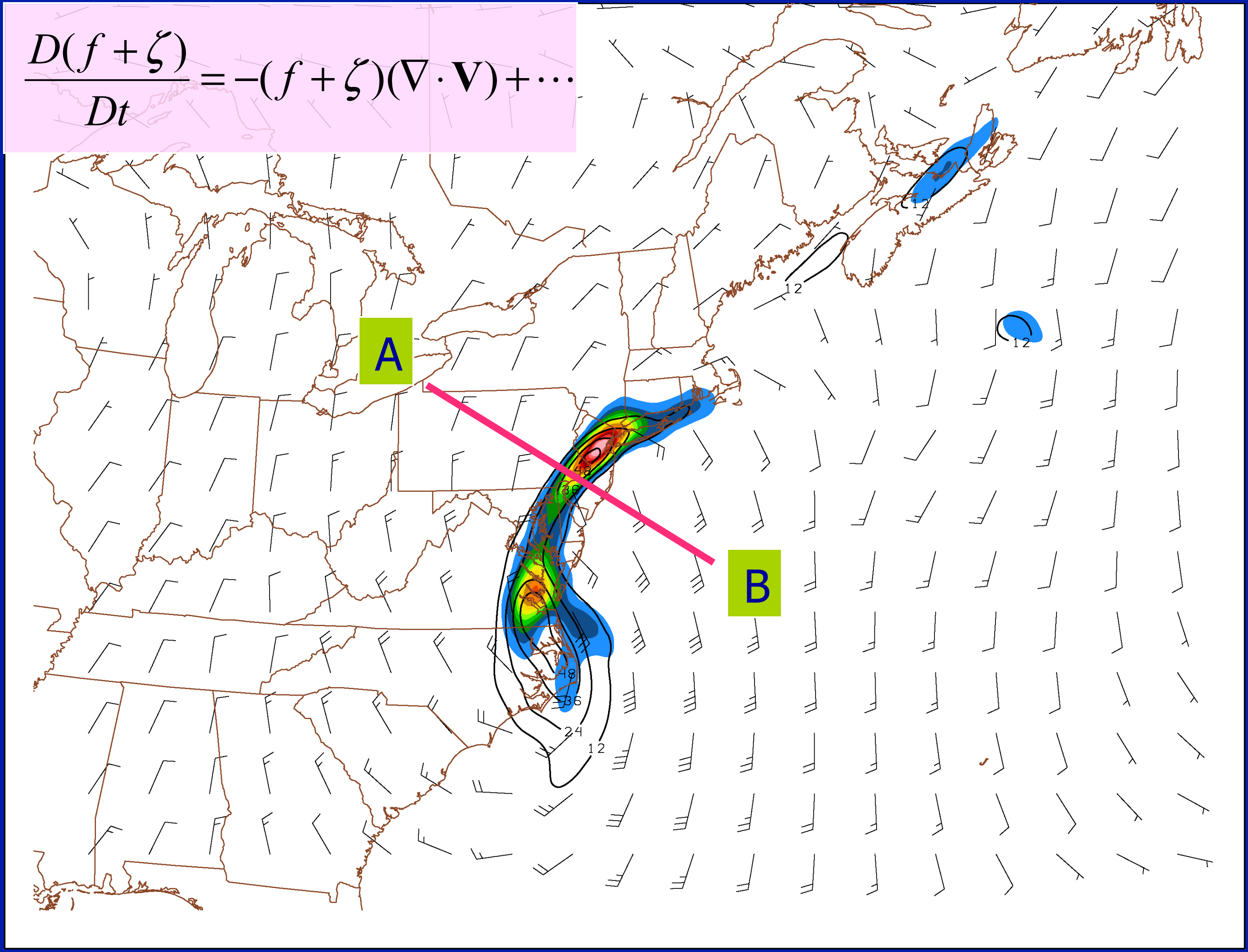
Vorticity generation (fill) and vorticity at F15

$$\frac{D(f+\zeta)}{Dt} = -(f+\zeta)(\nabla \cdot \mathbf{V}) + \cdots$$

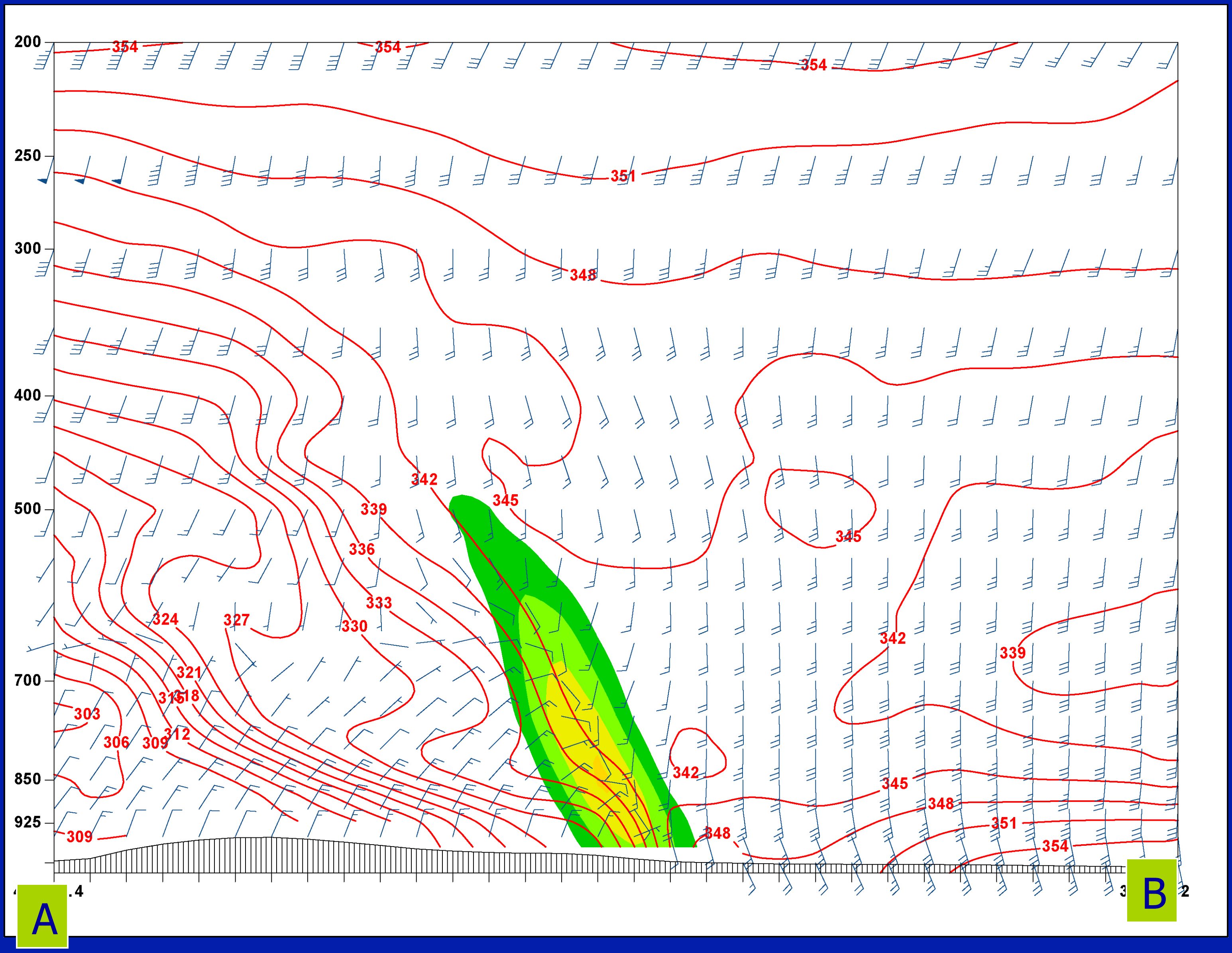


Vorticity production (fill) and vorticity at F15

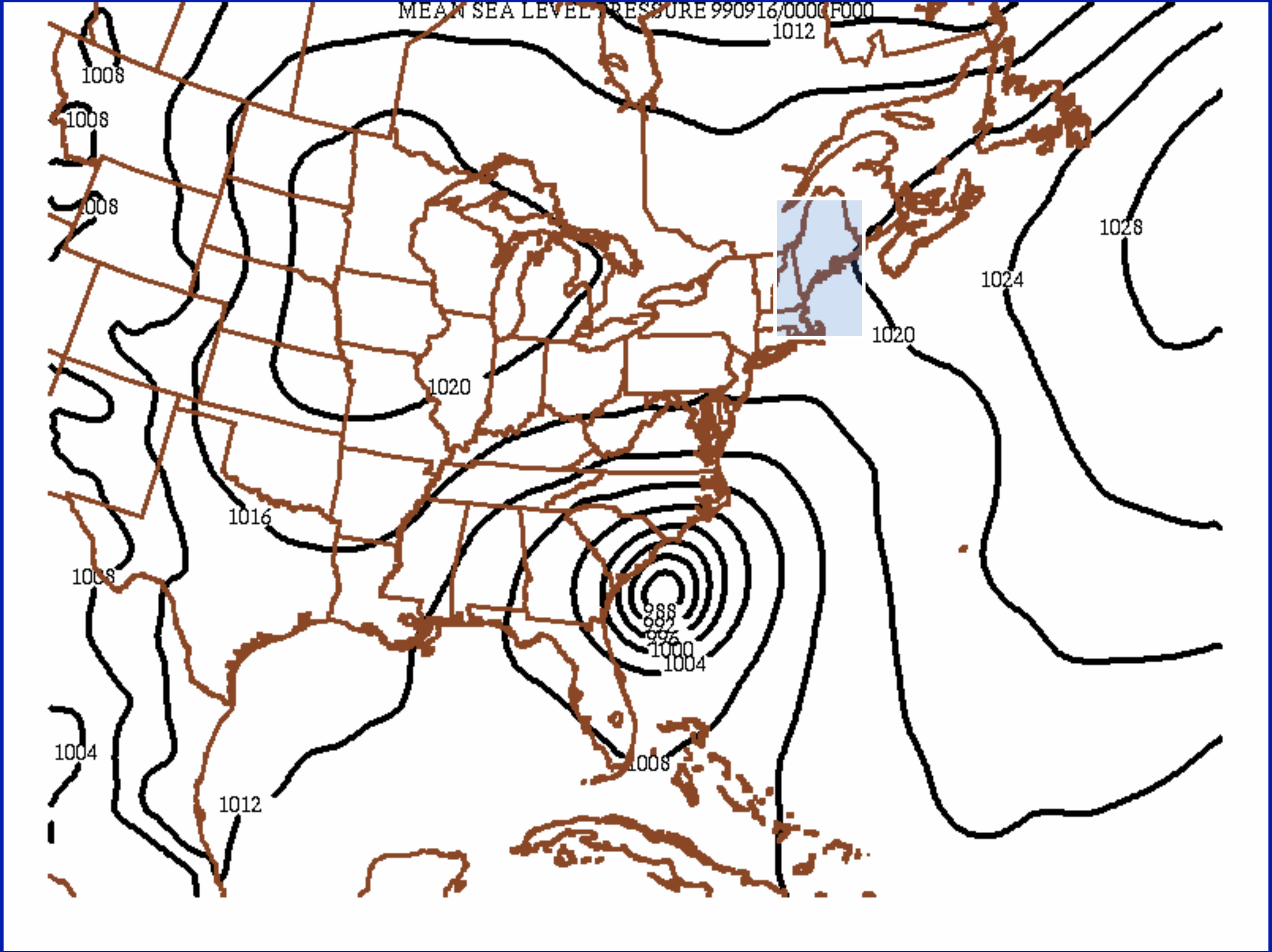
$$\frac{D(f+\zeta)}{Dt} = -(f+\zeta)(\nabla \cdot \mathbf{V}) + \cdots$$



Vorticity and q_e at F15



36h mean sea level pressure evolution



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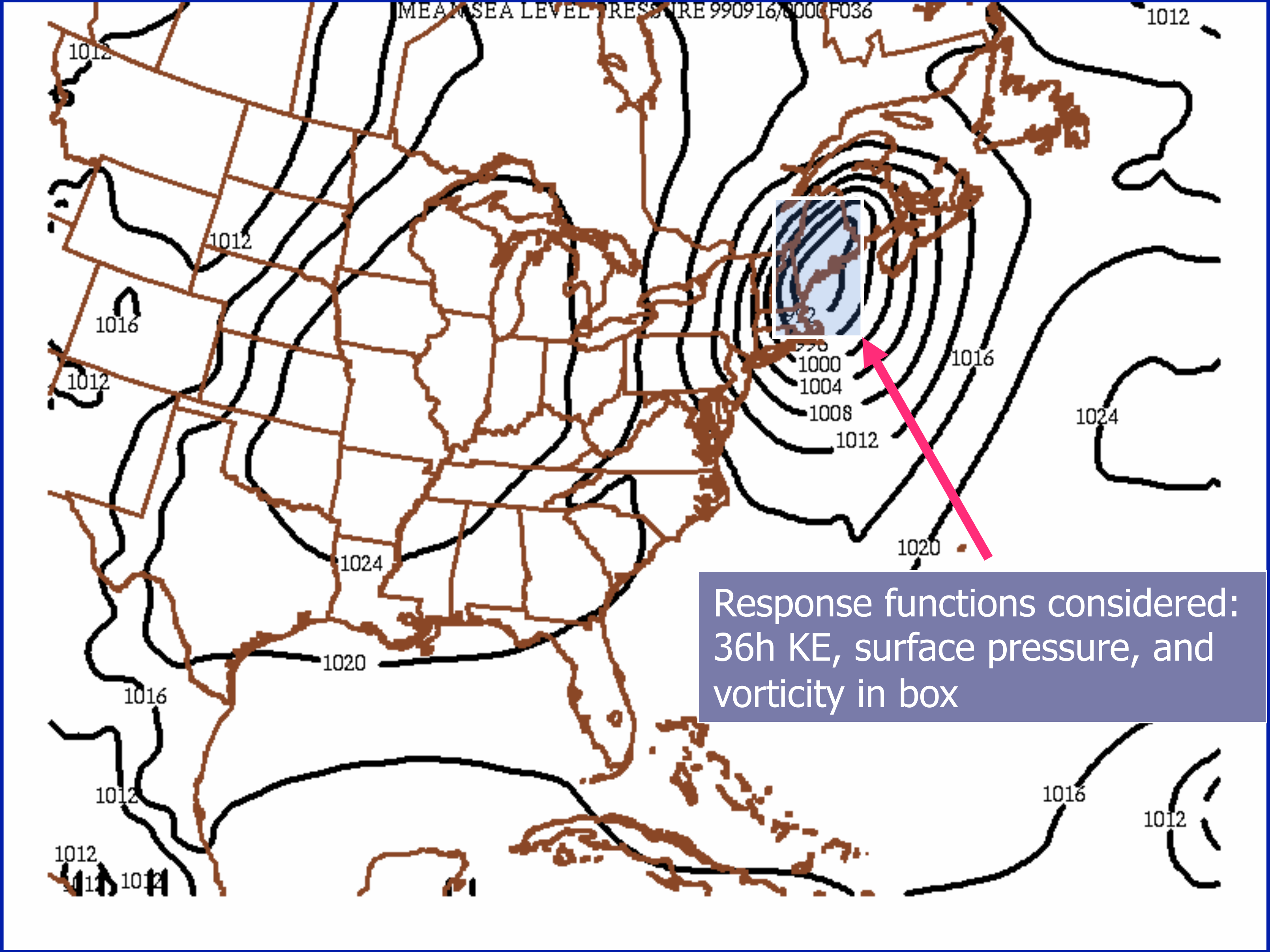


Response functions considered: 36h KE, surface pressure, and vorticity in box

Adjoint-derived sensitivity studies

- Adjoint-derived sensitivity studies involve the objective evaluation of the sensitivity of a specific function of the output ($\mathbf{x}_f$) of an NWP model (called the "response function", R) to changes in the model initial or boundary conditions:

$$\boxed{\frac{\partial R}{\partial \mathbf{x}_a}}$$

- Given this "*sensitivity gradient*", one can compute the changes to R (dR) attributed to small, but otherwise ***arbitrary***, initial or boundary perturbations ($d\mathbf{x}_a$):

$$\boxed{\delta R = \left\langle \frac{\partial R}{\partial \mathbf{x}_a}, \delta \mathbf{x}_a \right\rangle}$$

Relationship between the nonlinear model and its adjoint

$$\mathbf{x}_a \rightarrow \boxed{\text{nonlinear model}} \rightarrow \mathbf{x}_f \rightarrow R(\mathbf{x}_f)$$

$$\frac{\partial R}{\partial \mathbf{x}_a} \leftarrow \boxed{\text{adjoint model}} \leftarrow \frac{\partial R}{\partial \mathbf{x}_f}$$

Modeling System Used

- MM5 Adjoint Modeling System (Zou et al. 1997) with non-linear model state vector:

$$\mathbf{x} = (\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{T}, \mathbf{p}', \mathbf{q}_\mathbf{v})$$

- All sensitivities were calculated by integrating the adjoint model "backwards" using dry dynamics, about a moist basic state.

- The corresponding adjoint model state vector is:

$$\frac{\partial R}{\partial \mathbf{x}} = \left(\frac{\partial R}{\partial \mathbf{u}}, \frac{\partial R}{\partial \mathbf{v}}, \frac{\partial R}{\partial \mathbf{w}}, \frac{\partial R}{\partial \mathbf{T}}, \frac{\partial R}{\partial \mathbf{p}'}\right)$$

Sensitivities and their interpretation

- Sensitivities of surface pressure, KE, and vorticity were calculated with respect to vorticity
- At a particular point, sensitivity to vorticity indicates how the response function (TC intensity) would change if the forecast trajectory vorticity were perturbed at that point.
- “Physical processes that produce large tendencies in regions of strong adjoint sensitivity are significant to the feature … represented by the forecast aspect, [R].” Langland et al. (1995)

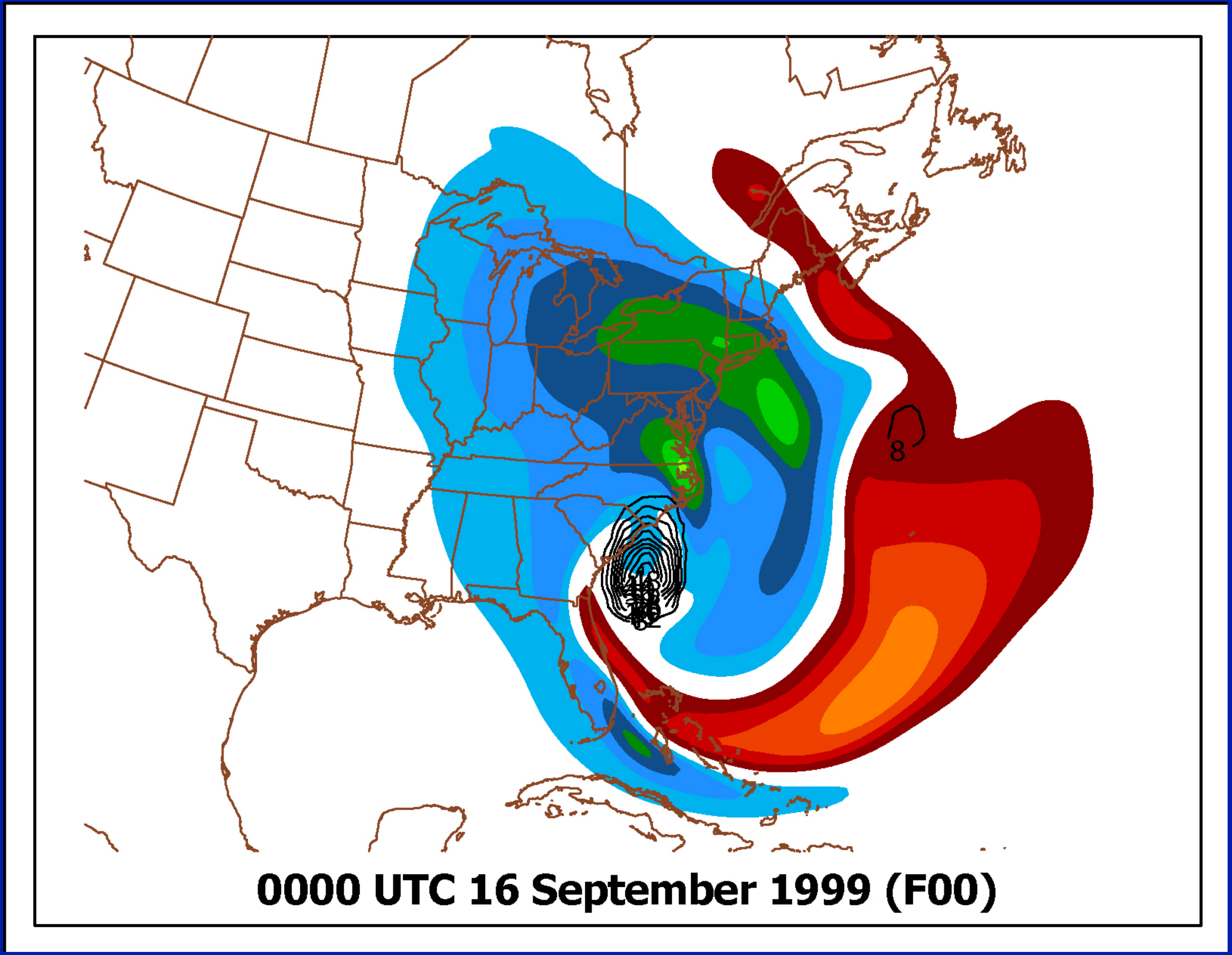
Interpretation of adjoint fields

- When dynamical interpretation of sensitivities is provided, often what is offered is merely the observation that the distribution of sensitivities is coincident with a synoptic feature.
- Coincidence of adjoint sensitivities with a synoptic feature alone is insufficient to attribute dynamical significance to the feature.
- While adjoint sensitivities do not provide information regarding whether particular physical processes actually occurred within a basic state forecast trajectory, these sensitivities do provide information concerning the effect of possible perturbations to the basic state.
- It is this particular characteristic of adjoint sensitivities that makes them a potentially powerful tool in synoptic case studies.

Interpretation of adjoint fields

- The key to interpretation of an adjoint sensitivity of a response function with respect to some function of the model state, $f(\mathrm{x})$, $\dfrac{\partial R}{\partial f(\mathbf{x})}$, is the coincidence of the adjoint sensitivity field, with the tendency in that function, $\dfrac{Df(\mathbf{x})}{Dt}$.

s=.925 initial vorticity and 36h KE sensitivity to initial vorticity



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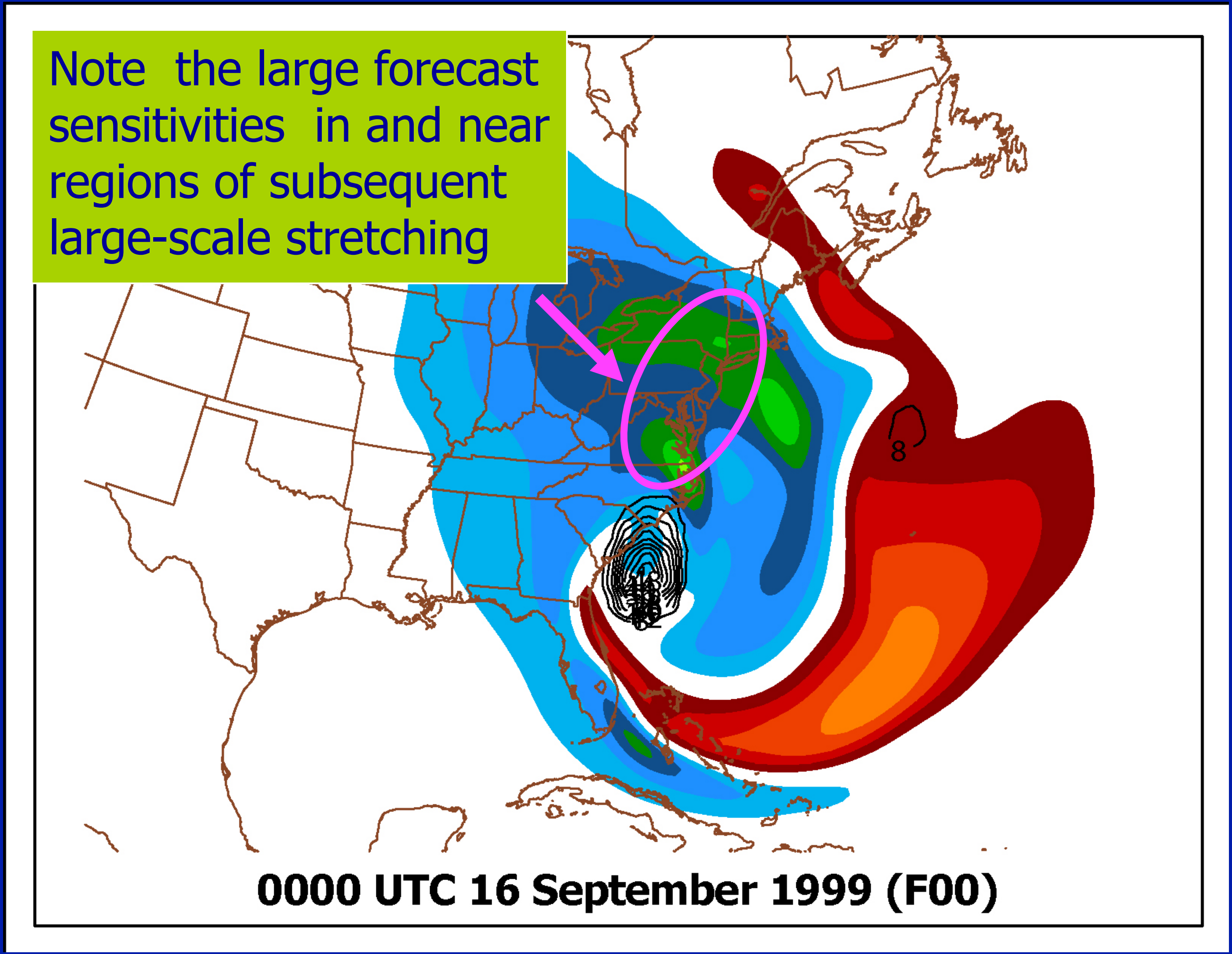

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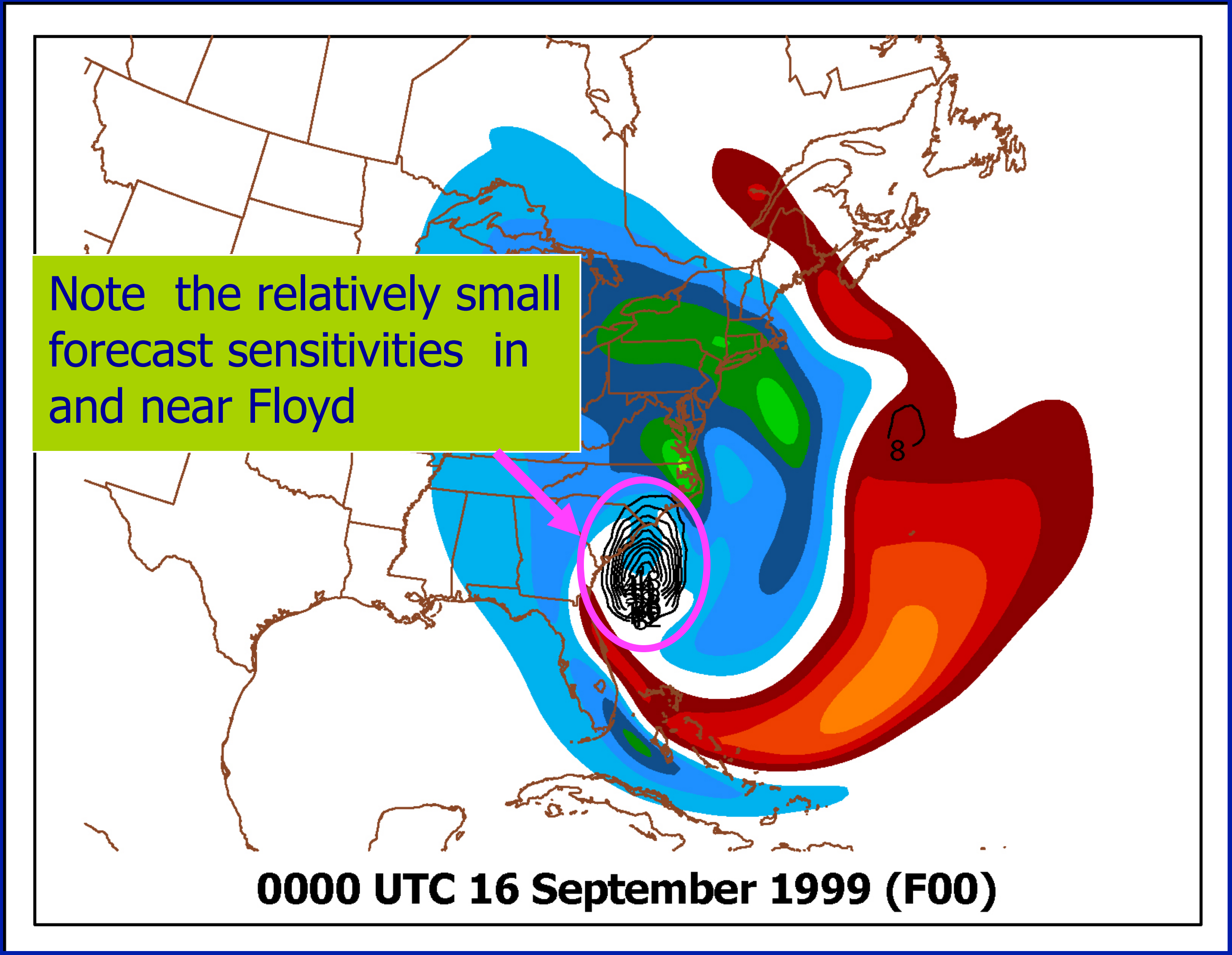
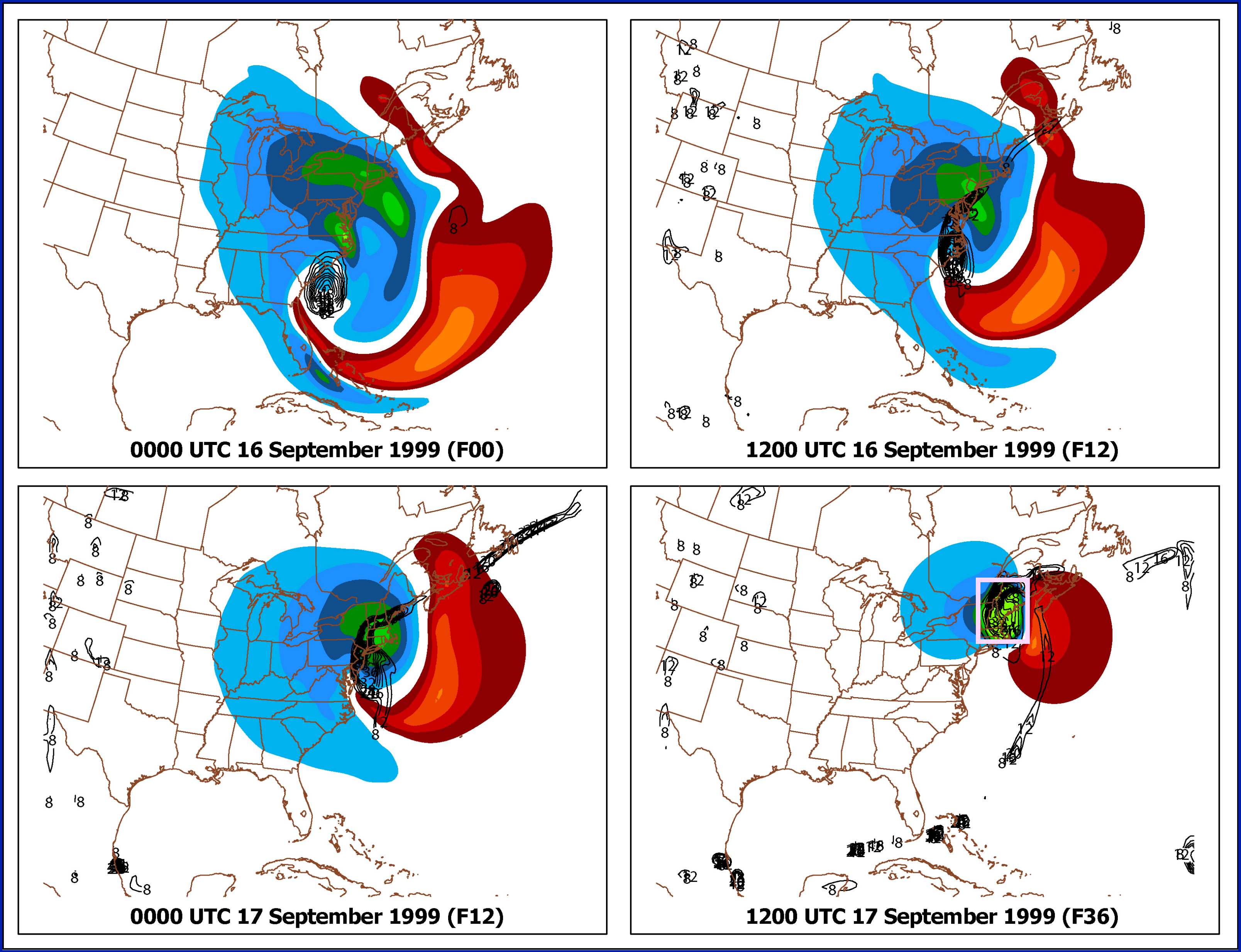
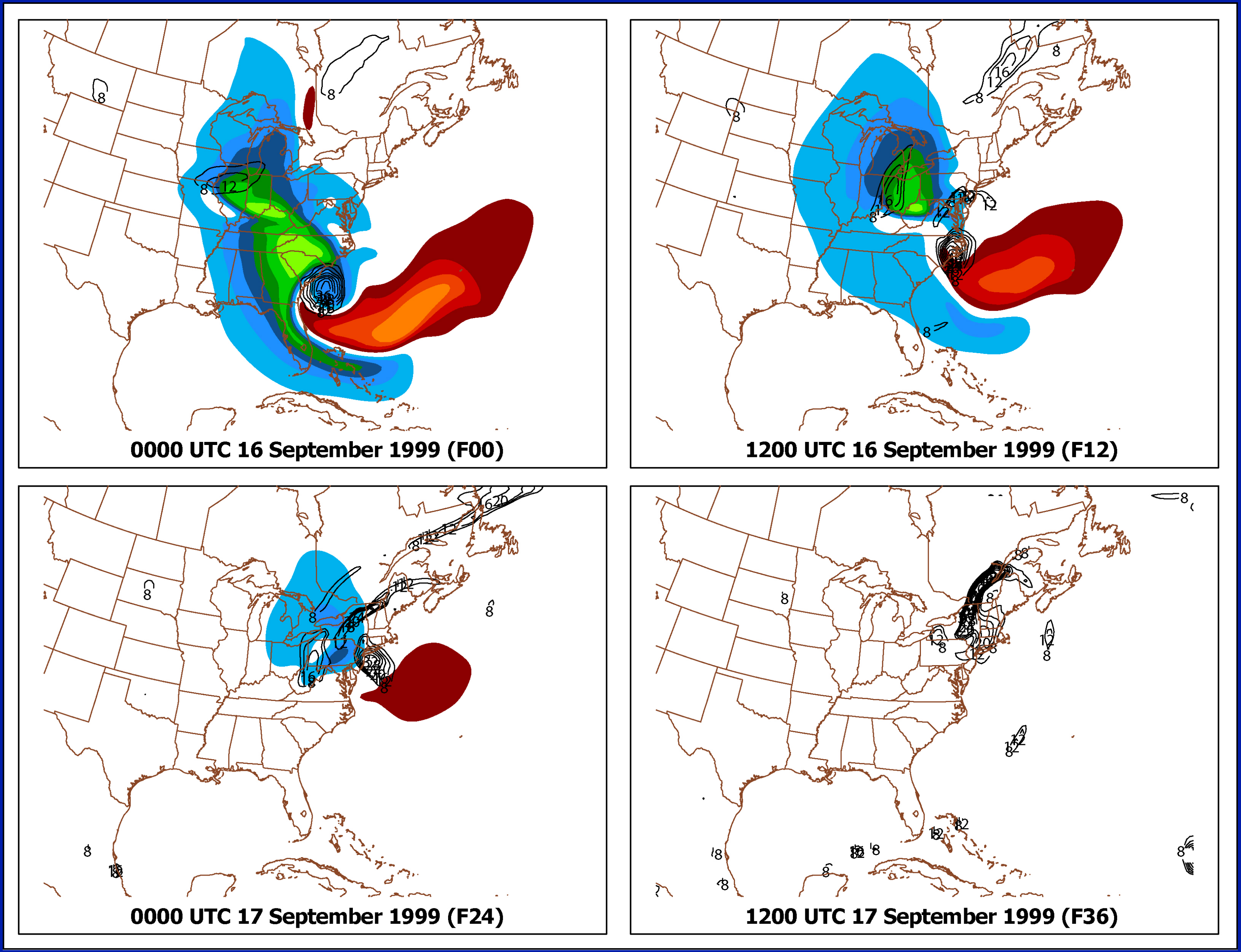


36h evolution of s = .925 vorticity and KE sensitivity to vorticity



36h evolution of s=.425 vorticity and KE sensitivity to vorticity



Adjoint-derived analysis perturbation

To test the suggestion that enhanced coastal frontogenesis would lead to a more intense transitioned cyclone, the control analysis is perturbed using a perturbation constructed from the adjoint-derived forecast sensitivity. A new simulation is then run.

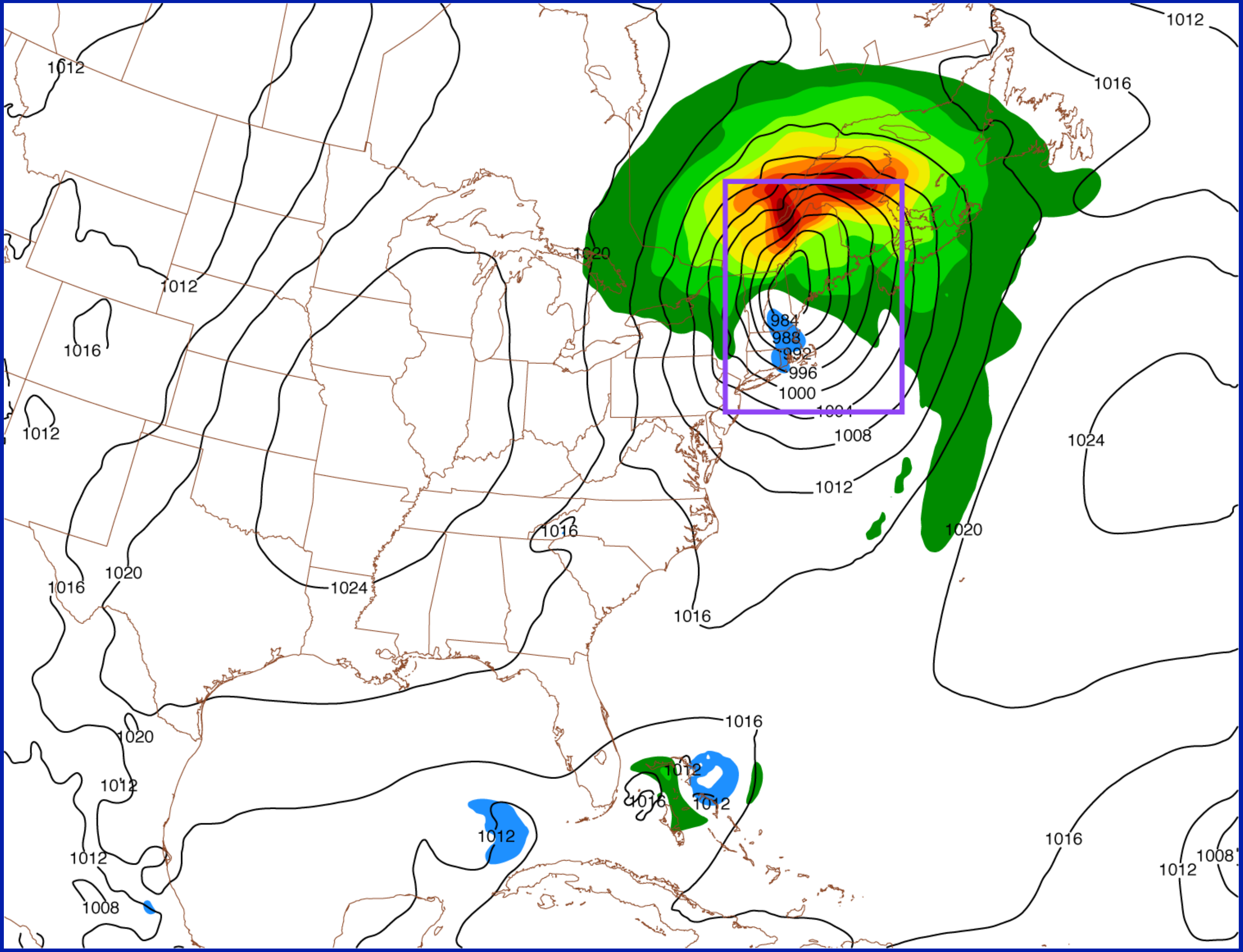
$$\mathbf{x}_a^{pert} = \mathbf{x}_a^{cntl} - \mathbf{W}\frac{\partial R}{\partial \mathbf{x}_a}$$

where

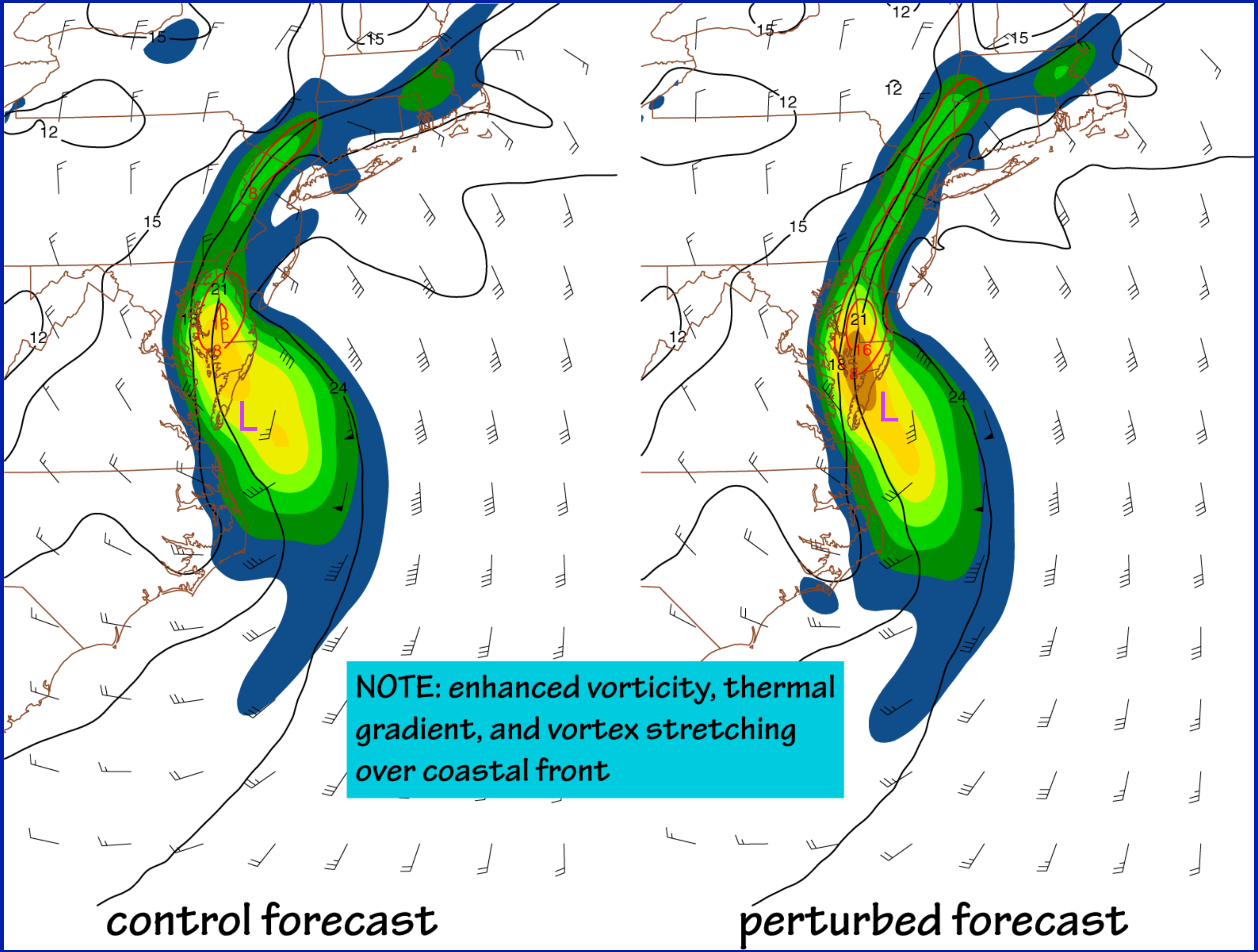
$$\mathbf{W} = \frac{\mathbf{C}}{\max\left|\dfrac{\partial R}{\partial \mathbf{x}_a}\right|}$$

The initial perturbations have the same structure as the sensitivity fields and are designed to decrease the surface pressure. Perturbations are made to either, but not both, the wind and temperature fields.

MSLP of control and change in MSLP between perturbed and control runs

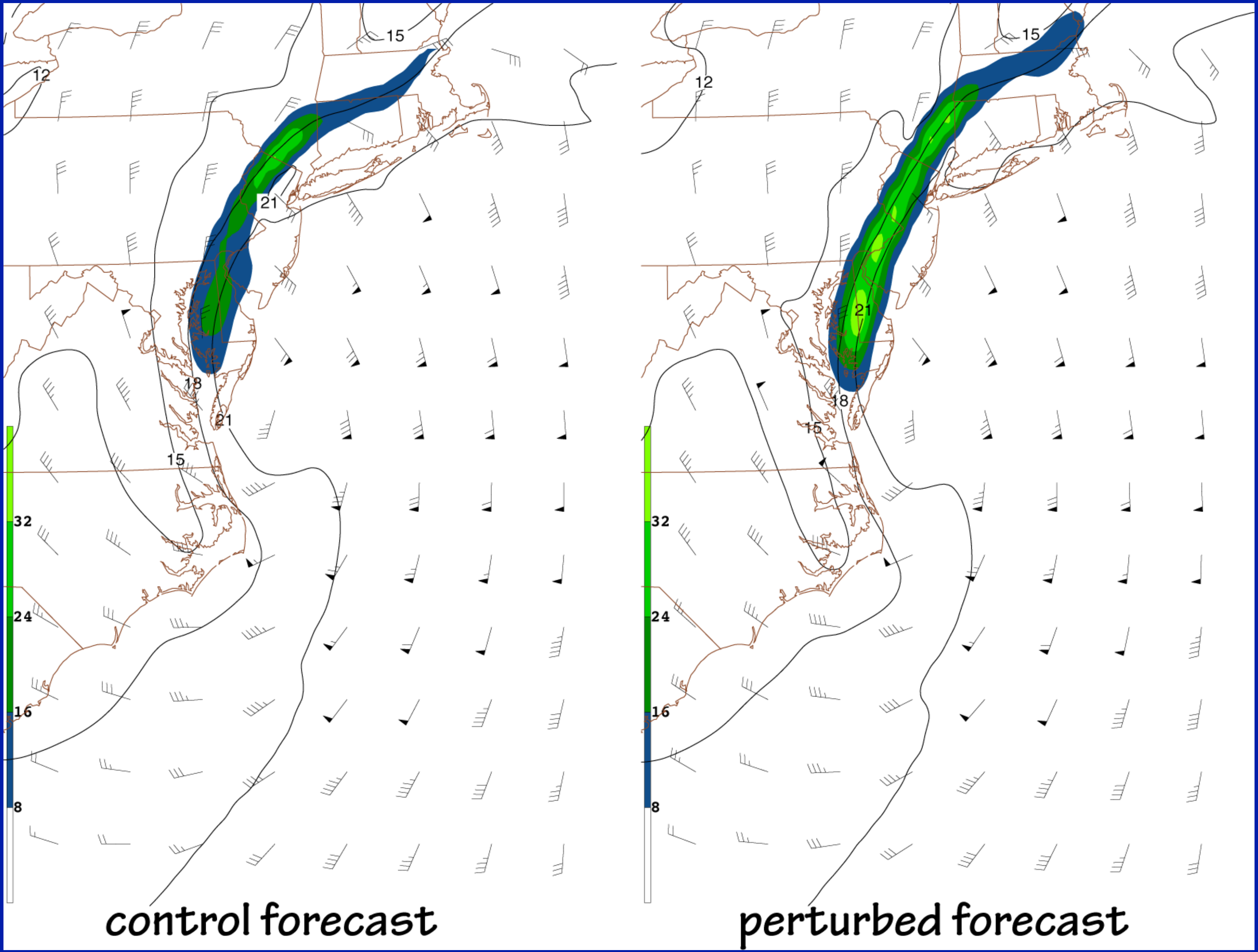


950 hPa vorticity (fill), vortex stretching (contour), and SFC wind and temperature (black) at F18



NOTE: *enhanced vorticity, thermal gradient, and vortex stretching over coastal front*

950 hPa frontogenesis (fill), temperature (contour), and wind (knots) at F18

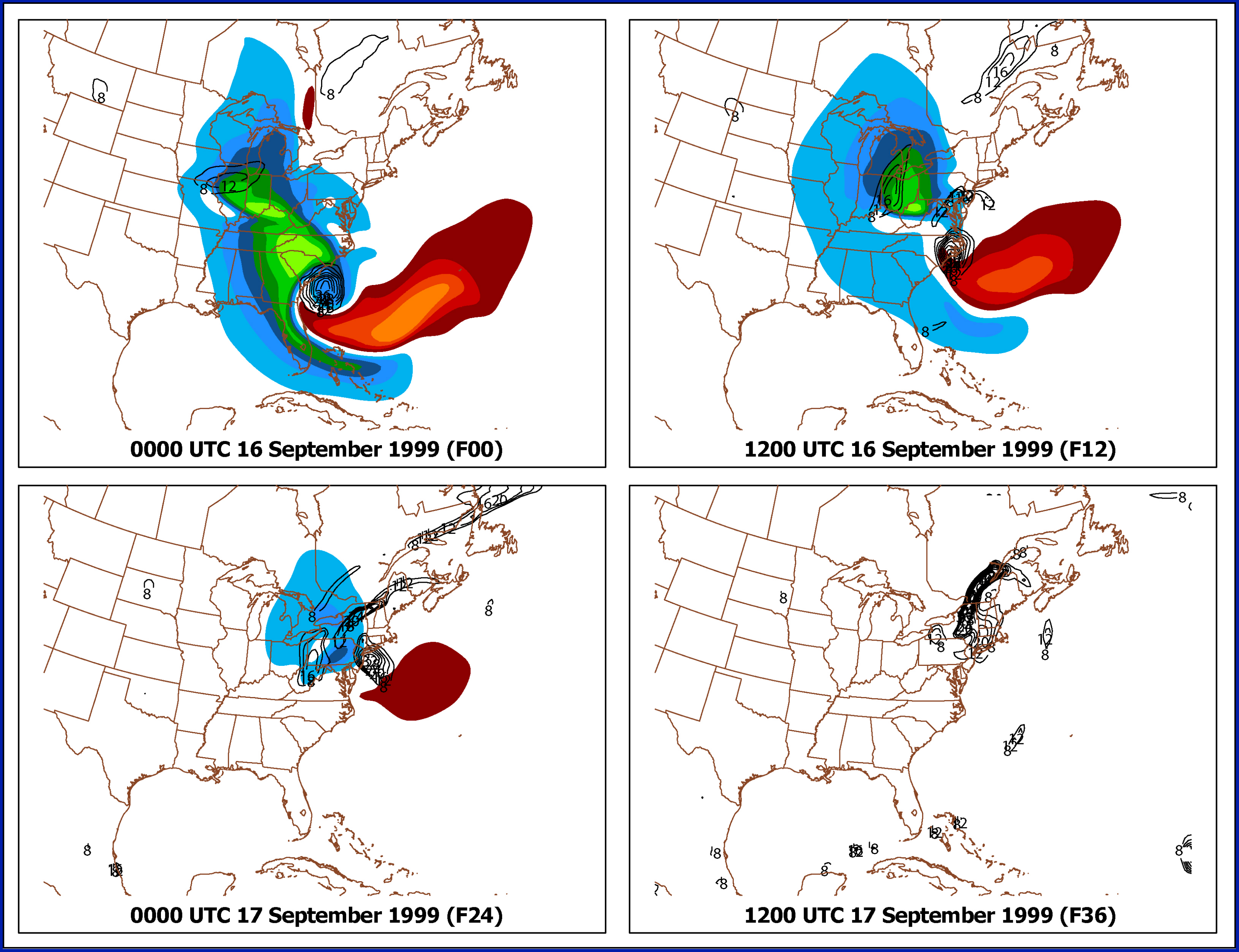


Summary

For the surface pressure response function considered:

- 36 h prior to ET, Hurricane Floyd was situated in a region of small (but positive) sensitivity to vorticity.
- The large sensitivities identify the coastal front as a "locus of frontogenesis and [concomitant] cyclonic vorticity generation".
- The vorticity associated with the coastal front is a significant contributor to the subsequent intensity of the transitioned cyclone over Maine.
- The upper trough associated with this transition was initially located in near a region of large sensitivity and remained in this region for much of the forecast.

36h evolution of s=.425 vorticity and KE sensitivity to vorticity



Future directions

- Use of sensitivities coupled with the adjoint of an assimilation scheme to develop targeted observing strategies for these and related events.
- Use of sensitivities to model two-dimensional wind field to calculate sensitivities to deformation associated with the flow remote from the coastal front.
- Use of these sensitivities in refining adaptive observing strategies for ET and for "surgical" PV impact studies
- Investigation of consistency of presented results with other response functions (i.e., PV, circulation, "total energy")

An ‘ideal’ targeting stategy

- Previous approaches include:
  - DLM steering variance
  - ETKF forecast variance reduction
  - SVs with TE or Var norms
  - Other adjoint-based methods
- An ideal strategy should
  - Be based upon a forecast aspect relevant to the problem at hand → ease of use/interpretation
  - Blend dynamical characteristics of the flow with statistical uncertainty
  - Provide a measure of forecast uncertainty (reduction)
  - Account for how data will be assimilated
  - Allow for timely deployment of observational platform(s)

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{x}_b)$$

$\mathbf{x}_b, \mathbf{y} \rightarrow$ assimilation system $\rightarrow \mathbf{x}_a$

$\dfrac{\partial R}{\partial \mathbf{x}_b}, \dfrac{\partial R}{\partial \mathbf{y}} \leftarrow$ adjoint of assimilation system $\leftarrow \dfrac{\partial R}{\partial \mathbf{x}_a}$

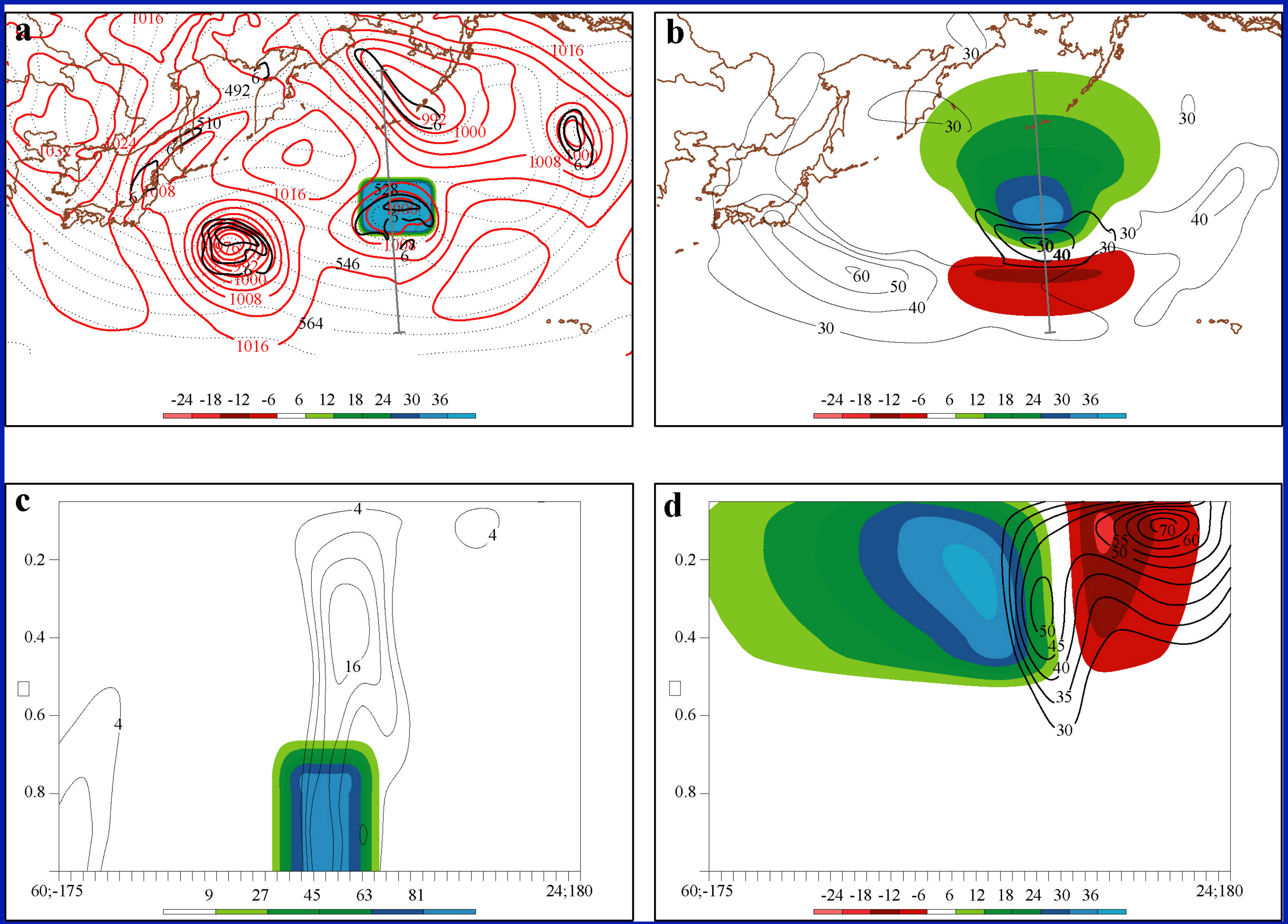
$$\frac{\partial R}{\partial \mathbf{y}} = \mathbf{K}^{\mathrm{T}} \frac{\partial R}{\partial \mathbf{x}_a}$$

and

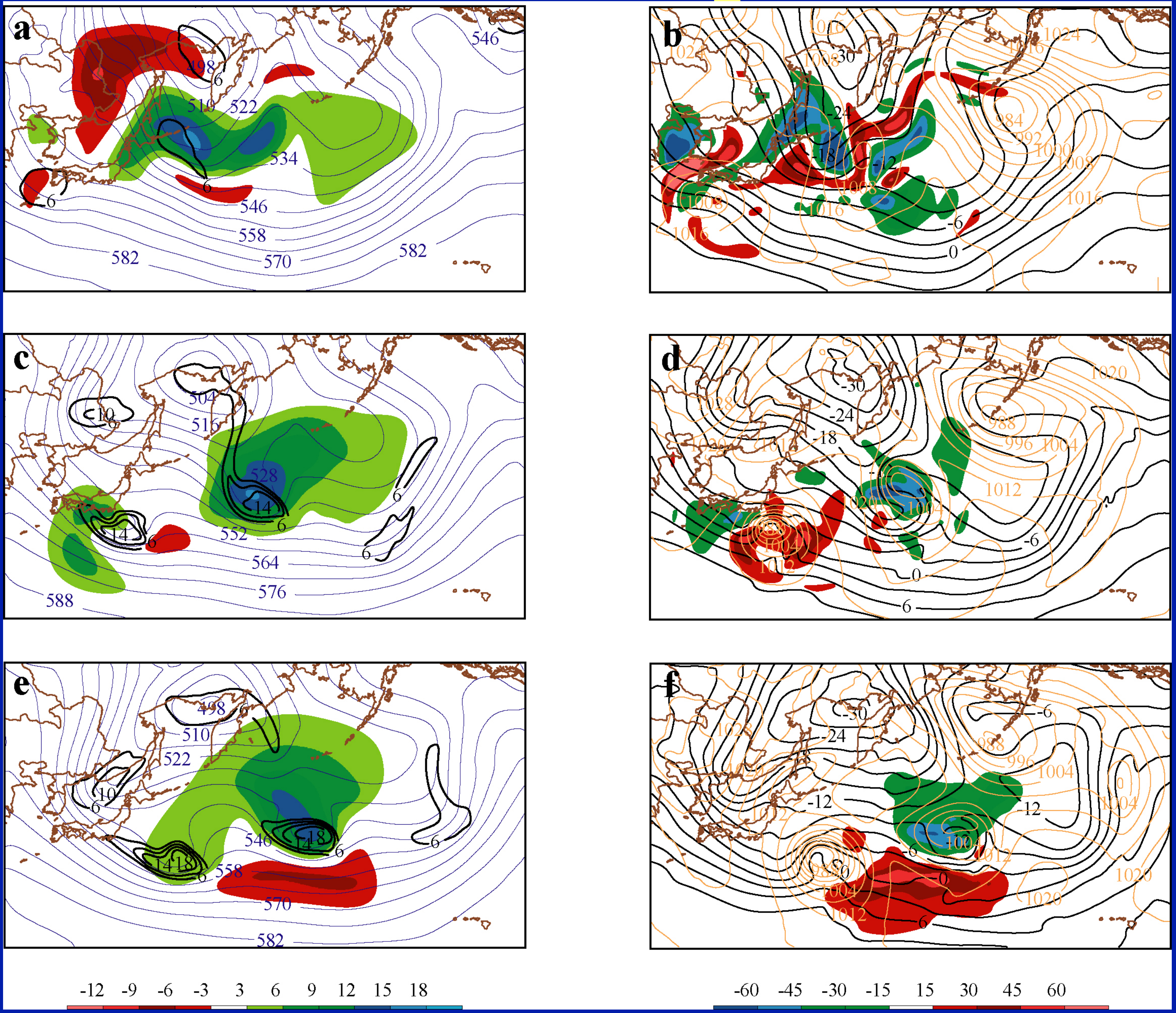
$$\frac{\partial R}{\partial \mathbf{x}_b} = (\mathbf{I} - \mathbf{H}^{\mathrm{T}}\mathbf{K}^{\mathrm{T}}) \frac{\partial R}{\partial \mathbf{x}_a}$$

Langland and Baker (2004)

Sensitivity experiments



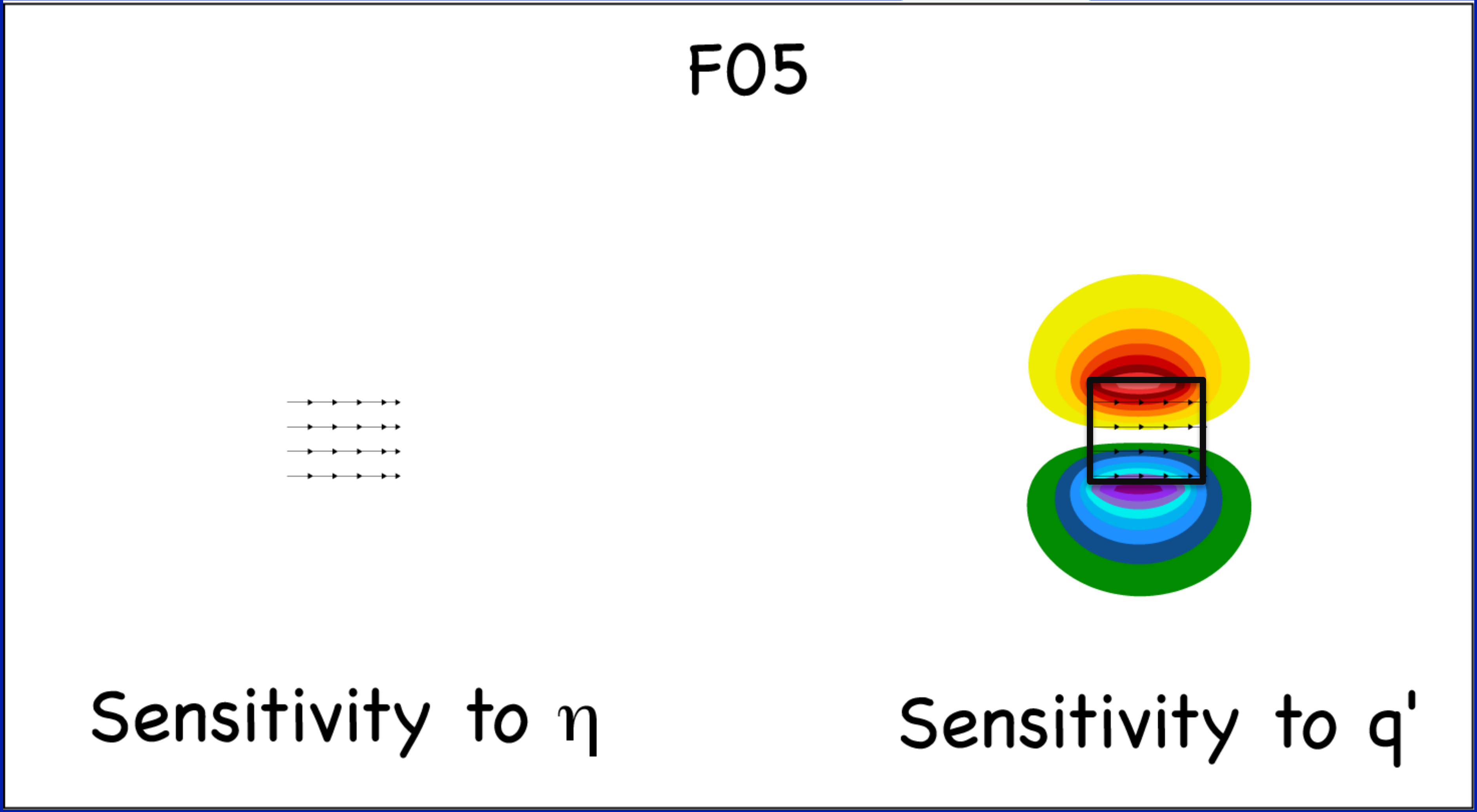
Sensitivities to R_2 to z and T



Forward and adjoint shallow water systems

	forward equation	adjoint equation
zonal momentum	$\frac{\partial u'}{\partial t} = fv' - g\frac{\partial \eta}{\partial x}$	$-\frac{\partial \hat{u}}{\partial t} = f\hat{v} + H\frac{\partial \hat{\eta}}{\partial x}$
meridional momentum	$\frac{\partial v'}{\partial t} = fu' - g\frac{\partial \eta}{\partial y}$	$-\frac{\partial \hat{v}}{\partial t} = +f\hat{u} + H\frac{\partial \hat{\eta}}{\partial y}$
mass continuity	$\frac{\partial \eta}{\partial t} = -H(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y})$	$-\frac{\partial \hat{\eta}}{\partial t} = g(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y})$
conserved variable	$q' = \frac{\zeta'}{H} - \frac{f\eta}{H^2}$	$\hat{\eta}_b = \hat{\eta} - \frac{g}{f}(\frac{\partial \hat{v}}{\partial x} - \frac{\partial \hat{u}}{\partial y})$
inversion	$\nabla^2\eta_b - \frac{f^2}{gH}\eta_b = \frac{Hf}{g}q'(x,y,t=0)$	$\nabla^2\hat{q} - \frac{f^2}{gH}\hat{q} = \frac{Hf}{g}\hat{\eta}_b$
potential vorticity	$q' = -\frac{1}{H}\frac{\partial u'}{\partial y} + \frac{1}{H}\frac{\partial v'}{\partial x} - \frac{f\eta}{H^2}$	$\hat{u} = \frac{1}{H}\frac{\partial \hat{q}}{\partial y}$ $\hat{v} = -\frac{1}{H}\frac{\partial \hat{q}}{\partial x}$ $\hat{\eta} = -\frac{f}{H^2}\hat{q}$
combined	$\frac{\partial^2 \eta}{\partial t^2} - c^2\nabla^2\eta + f^2\eta = -fH^2q'(x,y,t=0)$	$\frac{\partial^2 \hat{\eta}}{\partial t^2} - c^2\nabla^2\hat{\eta} + f^2\hat{\eta} = f^2\hat{\eta}_b(x,y,t=t_f)$

Initial condition for adjoint: $\hat{u} = 1$



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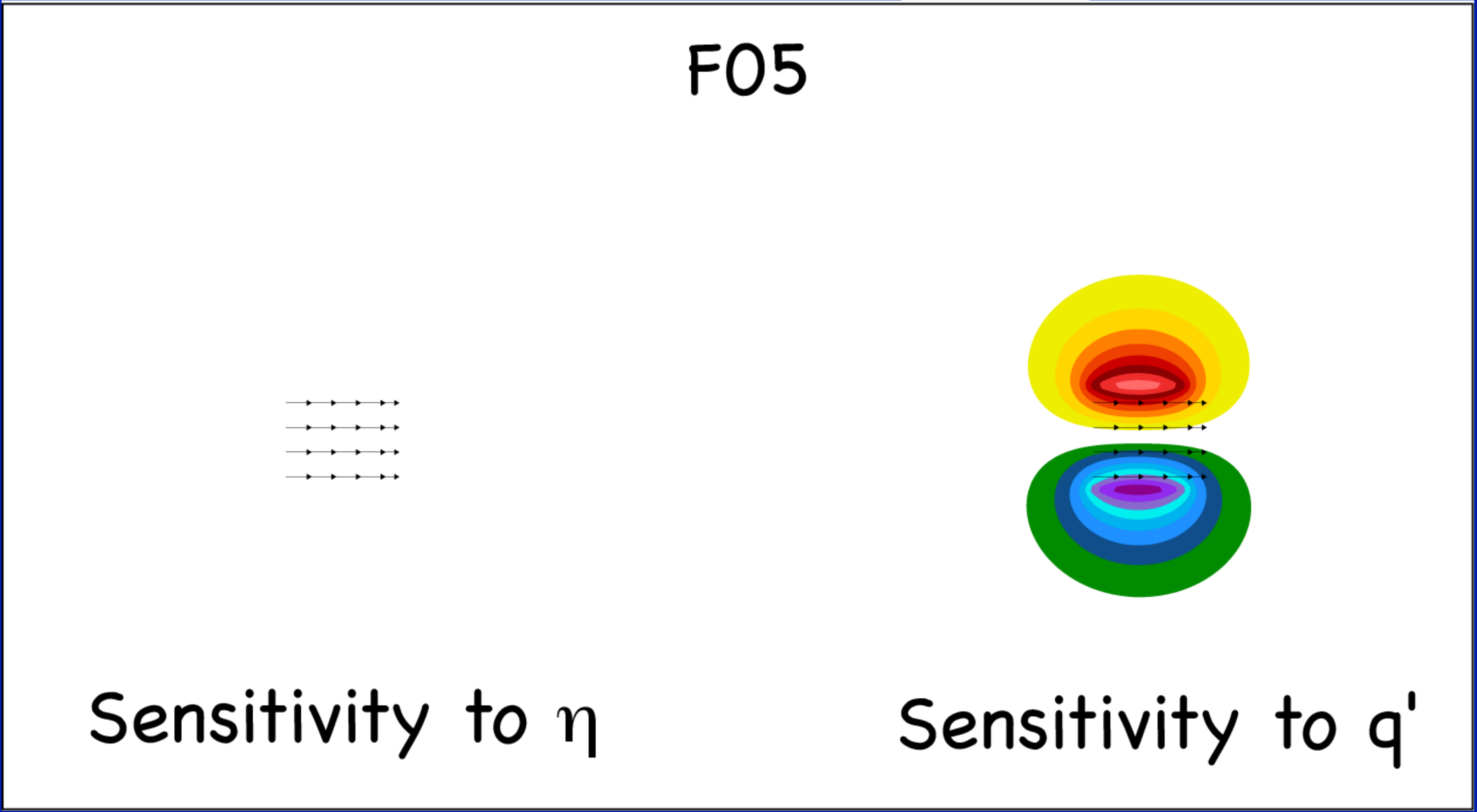
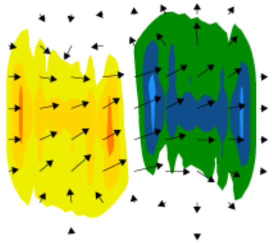
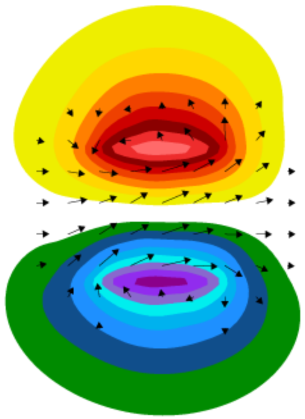


F04

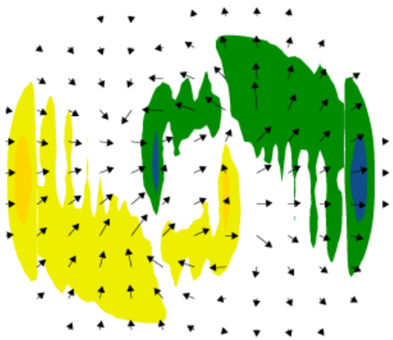


Sensitivity to η

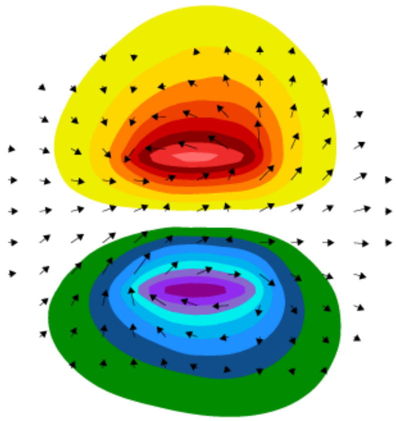


Sensitivity to q'

F03

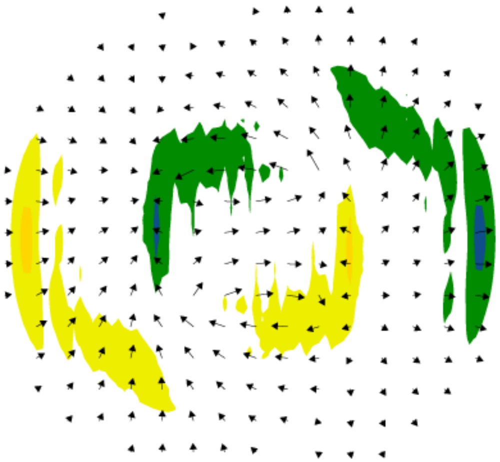


Sensitivity to η

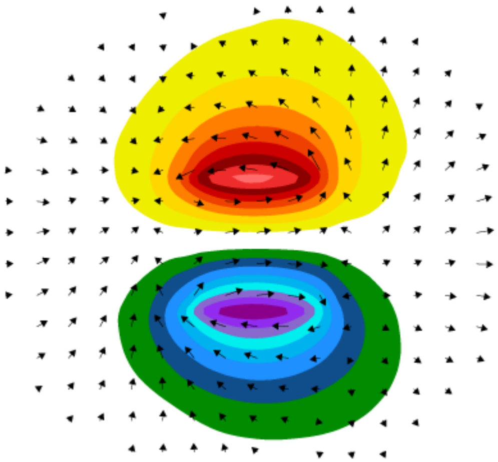


Sensitivity to q'

F02

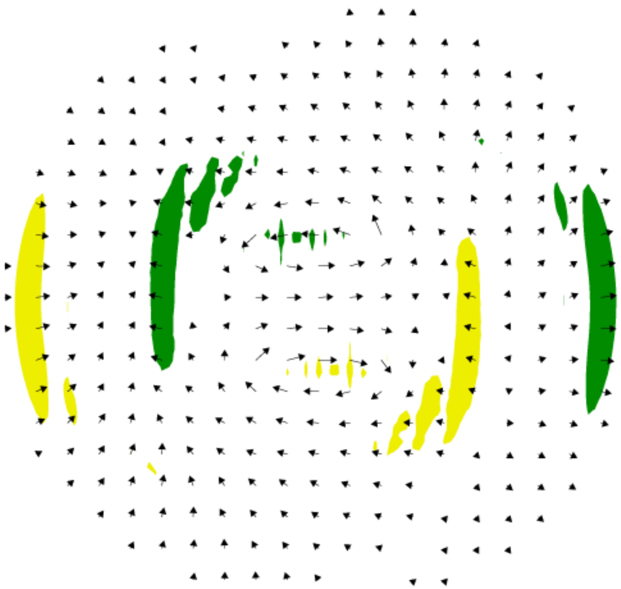


Sensitivity to η

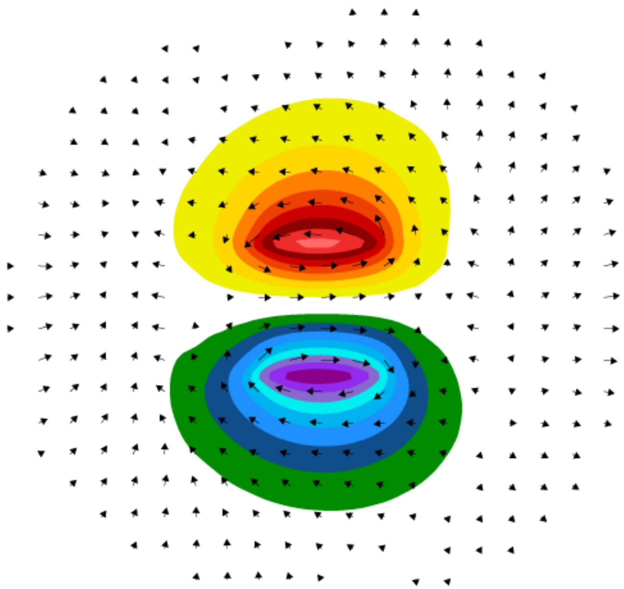


Sensitivity to q'

F01

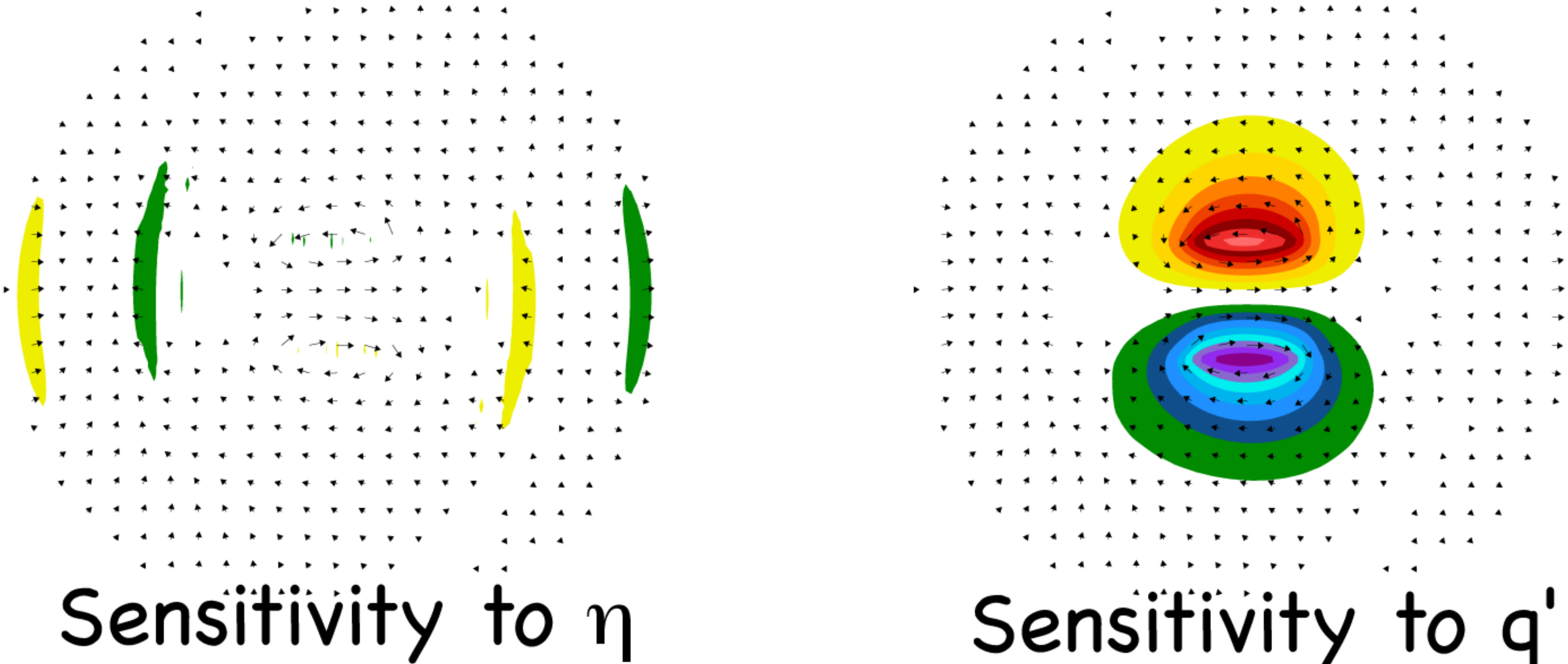


Sensitivity to η



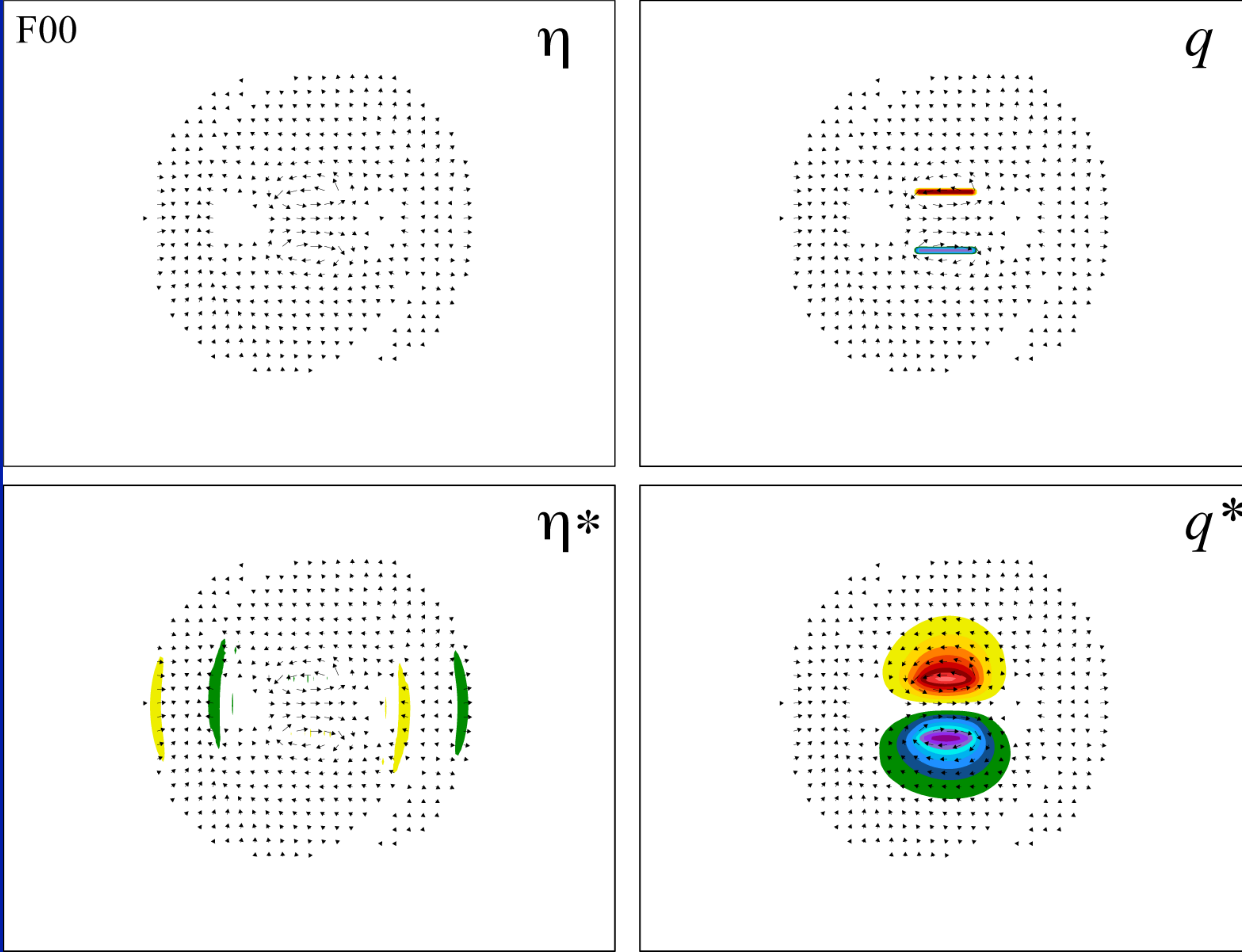
Sensitivity to q'

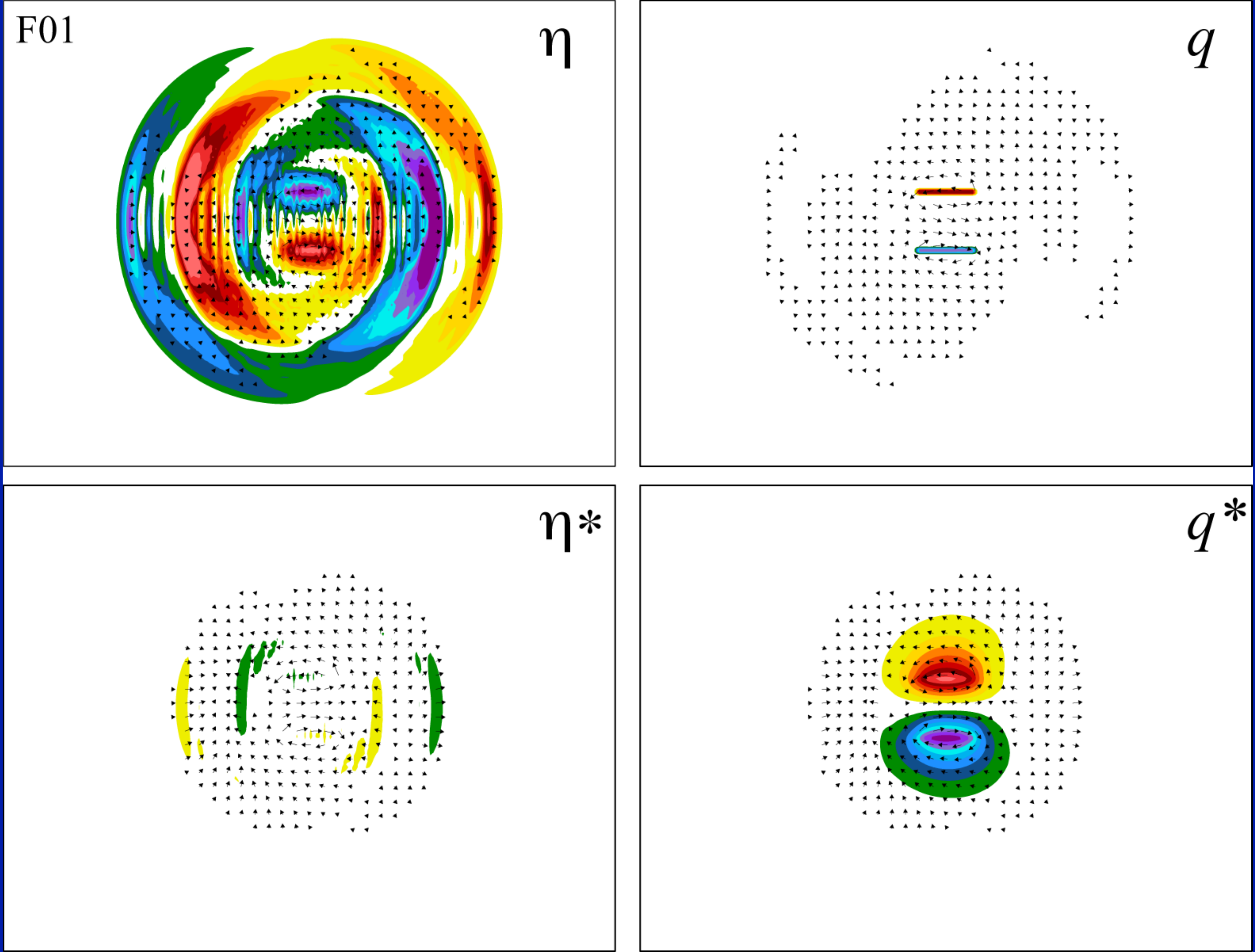
FOO



Geostrophic adjustment

- Forward (linearized) model integrated forward using sensitivity gradients as initial conditions . . .


F00
η
q
η*
q*



F01
η
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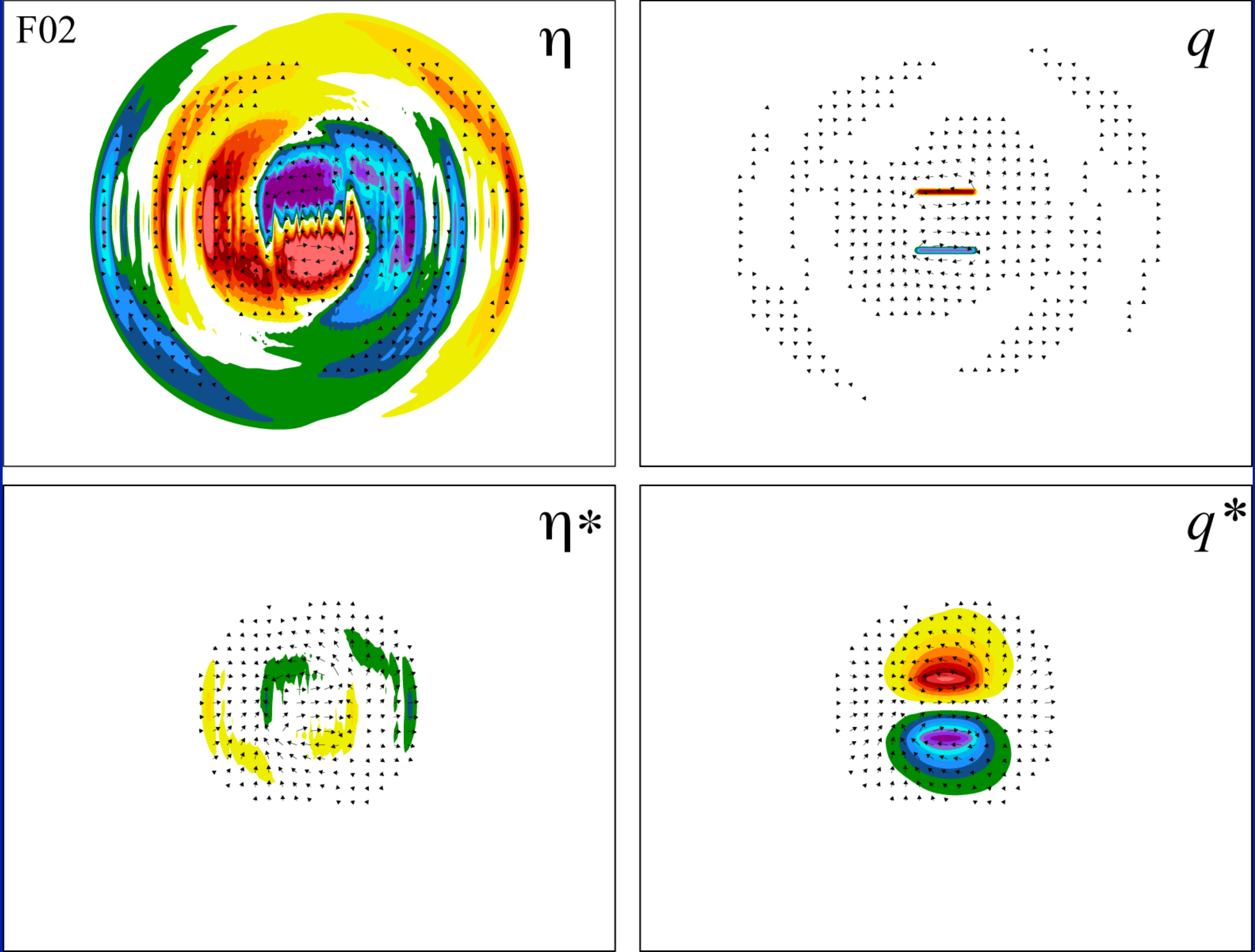
η*
q*


F02
η
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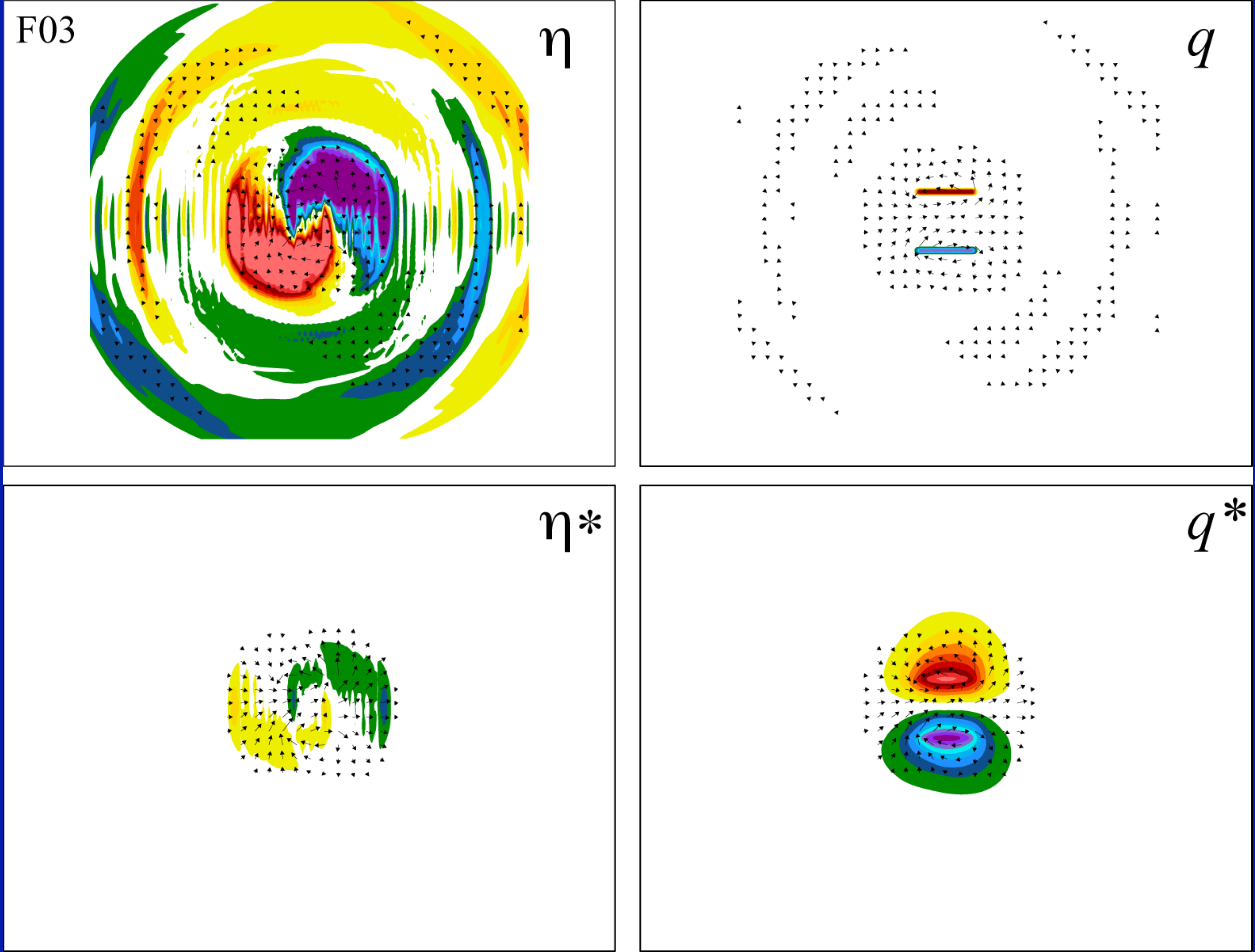

F03
η
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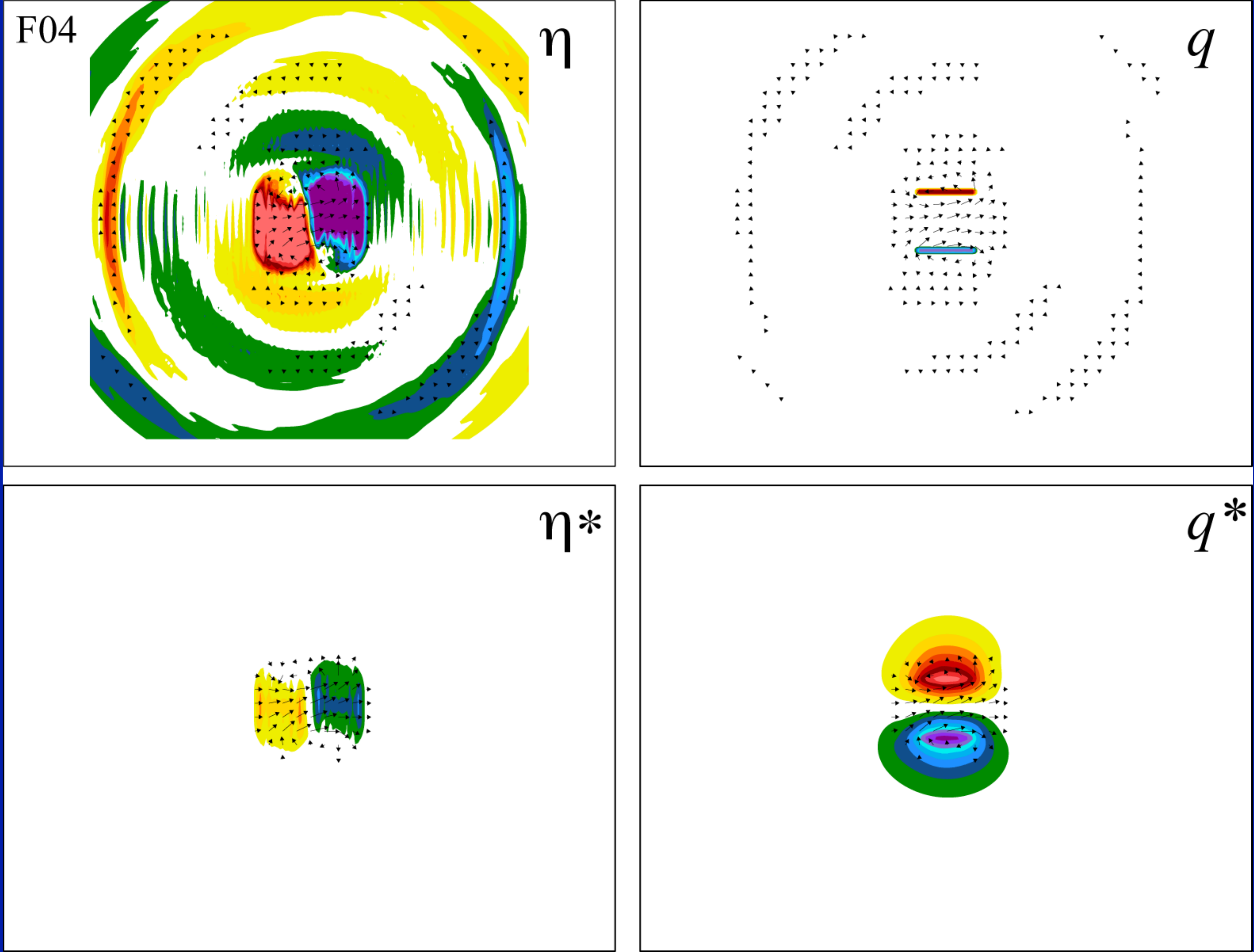

F04
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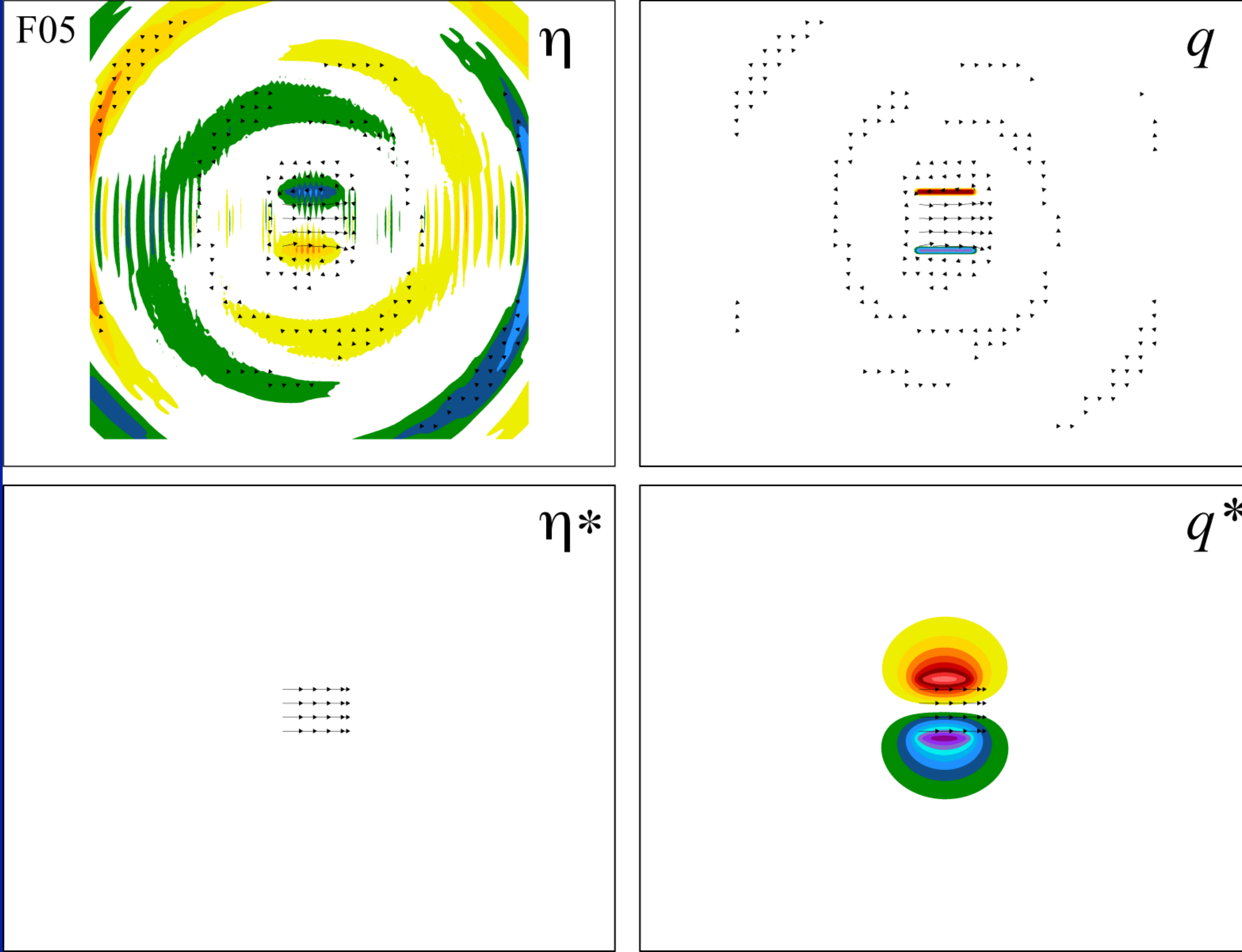

F05
η
q
η*
q*

Summary (2)

- Sensitivities to PV appear to be locally "conserved" – while sensitivities to wind (vorticity) and height "evolve".
- Scale of PV sensitivity is large, while the scale of the height sensitivity is small
- Perturbations derived from the sensitivity gradients "remember" the future PV distribution associated with the response function.