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CONSTRUCTION OF A CROSSING SYMMETRIC, REGGE BEHAVED AMPLITUDE FOR LINEARLY RISING TRAJECTORIES

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A B S T R A C T

A representation of the scattering amplitude, containing an average Regge behaviour and crossing symmetry for linearly rising trajectories, is proposed. It obeys superconvergence sum rules at all t, exhibits in a clear way the Regge poles vs. resonances duality and demands families of parallel daughters.

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Crossing has been the first ingredient used to make Regge theory a predictive concept in high energy physics. However, a complete and satisfactory way of imposing crossing and crossed channel unitarity is still lacking. We can look at the recent investigations on the properties of Reggeization at $t=0$ as giving a first encouraging set of results along this line of thinking [1]. A technically different approach, based on superconvergence, has been also recently investigated [2], and the possibility of a self-consistent determination of the physical parameters, through the use of sum rules, has been stressed.

In this note we propose a quite simple expression for the relativistic scattering amplitude, that obeys the requirements of Regge asymptotics and crossing symmetry in the case of linearly rising trajectories. Its explicit form is suggested by the work of Ref. [3] and contains only a few free parameters [*)].

Our expression contains automatically Regge poles in families of parallel trajectories (at all t) with residue in definite ratios. It furthermore satisfies the conditions of superconvergence [4] and exhibits in a nice fashion the duality between Regge poles and resonances in the scattering amplitude.

The first example we want to discuss is the scattering $\pi\pi \to \pi\omega$, whose convenient properties have been already stressed in Ref. [3]. We introduce the invariant amplitude $A(s,t,u)$ through the definition of the T matrix

$$T = \varepsilon_{\mu\nu\rho\sigma}\, e_\mu\, P_\nu\, P_{2\rho}\, P_{3\sigma} \cdot A(s,t,u) \tag{1}$$

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*) We shall mostly work here in the approximation of real, linear trajectories and consequently of narrow resonances. We briefly discuss the effects of a non-zero imaginary part in the trajectory function which, in any case, we demand to have a linearly rising real part.

where P_i are the pion momenta and e_μ is the ω polarization vector. $A(s,t,u)$ has only dynamical singularities as it is free of kinematical ones. It is also completely symmetric in the three Mandelstam variables.

It was found in Ref. [3] that a "good" parametrization of A at high s and fixed t could be written as:

$$A(s,t,u) \underset{s\to\infty}{\simeq} \frac{\bar{\beta}}{\pi}\,\Gamma(1-\alpha(t))\,(-\alpha(s))^{\alpha(t)-1} + (s \leftrightarrow u) \tag{2}$$

with $\bar{\beta}$ = constant. We use the word "good" in the sense that Eq. (2), when used as an imput, is able to reproduce itself quite consistently through the use of superconvergence sum rules.

What is the amplitude for non-asymptotic values of s? If Eq. (2) was exact after some $\bar{s}$, analyticity in the s plane (at fixed t) would require it to be valid at all s and Eq. (2) is certainly a solution of superconvergence. However, Eq. (2) does not satisfy s,t crossing as this demands poles in s such as those induced in t by the $\Gamma(1-\alpha(t))$ factor. On the other hand these poles in s could in principle destroy the asymptotic behaviour (2) through the introduction of fixed singularities. The lowest moment sum rules are just imposing that this is not happening at the nearest negative integers. Furthermore, we expect that the presence of bumps in the low energy region will produce (through analyticity) a modification of the high energy form which will not be as smooth as Eq. (2), but will rather show oscillations in s.

Consequently, we take out the factor $(-\alpha(s))^{\alpha(t)-1}$ and we symmetrize Eq. (2) multiplying by a factor $\Gamma(1-\alpha(s))$ and dividing by $\Gamma(2-\alpha(s)-\alpha(t))$ in order to have the correct asymptotic behaviour. After symmetrization in s,t,u we have

$$A(s,t,u) = \frac{\bar{\beta}}{\pi}\left[B(1-\alpha(t),1-\alpha(s)) + B(1-\alpha(t),1-\alpha(u)) + B(1-\alpha(s),1-\alpha(u))\right] \tag{3}$$

where we have introduced the Euler B function

$$B(x,y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$$

Notice that, in Eq. (3), $\bar{\beta}$ must be a constant if we want to have a Regge-like behaviour which, together with crossing, also demands the $1/(\Gamma(\alpha))$ t dependence of the reduced residue function. Equation (3) in fact is hard to modify if one demands an $s^{\alpha-1}$ behaviour in all channels. The only simple generalization of Eq. (3) seems to consist in the addition of non-leading and similarly structured terms like $B(m-\alpha(t), n-\alpha(s))$ with m, $n \geq 1$.

We now discuss some properties of Eq. (3) in detail

1) Behaviour for large positive s and fixed t

The first two terms (we shall come to the last one in a moment) give:

$$A = \frac{\beta(t)}{\sin\pi\alpha(t)}\left[-\frac{\sin\pi(\alpha(s)+\alpha(t))}{\sin\pi\alpha(s)}\,\frac{\Gamma(\alpha(s)+\alpha(t)-1)}{\Gamma(\alpha(s))} + \frac{\Gamma(1-\alpha(u))}{\Gamma(2-\alpha(u)-\alpha(t))}\right] \tag{4}$$

The second term is purely real (for positive s) and goes like $(\alpha(s))^{\alpha(t)-1}$. The first term is the one that corresponds to the Regge term $(-\alpha(s))^{\alpha(t)-1}$ and has both a real and an imaginary part. Some trivial algebra shows that, from the whole Eq. (4), we have a real piece like

$$\beta(t)\,\frac{1-\cos\pi\alpha(t)}{\sin\pi\alpha(t)}\,[\alpha(s)]^{\alpha(t)-1} \quad ; \quad \beta(t) = \bar{\beta}/\Gamma(\alpha(t)) ,$$

as in the Regge theory, while the s discontinuity is all contained in the form

$$A \underset{s\to\infty}{\sim} -\beta(t)\,\cot\alpha(s)\,[\alpha(s)]^{\alpha(t)-1} \tag{5}$$

If $\mathrm{Im}\,\alpha$ is strictly zero, Eq. (5) gives just poles in s and $\mathrm{Im}\,A$ is a sequence of δ functions. If $\mathrm{Im}\,\alpha$ is different from zero and, for instance, increases with s (this happens if the total width does not vary strongly with s), $\mathrm{Im}\,A$ will describe bumps for moderate values of s, but will finally tend to $\beta(t)(\alpha(s))^{\alpha(t)-1}$ as in the Regge theory [this is due to $\cot \alpha(s) \to -i$]. Of course, the parametrization of Eq. (3) can be taken as such only for linearly rising trajectories in which case $(\alpha(s))^{\alpha(t)}$ is equivalent to $(\frac{s}{s_0})^{\alpha(t)}$. However, we only need a leading term in $\alpha(s)$ going linearly in s, and this does not imply $\mathrm{Im}\,\alpha = 0$. If $\mathrm{Im}\,\alpha \neq 0$ one probably gets, besides moving poles, other singularities (cuts?) as well.

2) Singularities in the various channels

Equation (3) has quite nice analytic features. It has cuts in all the three Mandelstam variables starting from the 2π threshold, where α begins to show an imaginary part. However, if we restrict to real linear trajectories, our expression has only poles whenever α passes through an integer bigger than 0. Furthermore, because of the $\Gamma(2-\alpha(s)-\alpha(t))$ denominator, no double pole appears, in the sense that the residue in a pole is a polynomial in the other variable.

At first glance our expression shows poles at even values of α as well, in contrast with invariance principles. As these are always non-leading terms, one can in general eliminate them by the addition of non-leading expressions as explained in the beginning. More amusing to notice is the fact that, at least in this reaction, the elimination of spurious singularities can be achieved with a single condition on the trajectory $\alpha(t)$. Take in fact $\alpha(t) = 2$. The residue at the pole, produced there by $\Gamma(1-\alpha(t))$ is simply proportional to $\alpha(u)+\alpha(s)$.

We then demand

$$\alpha(s)+\alpha(u) = 0 \qquad \text{for} \qquad \alpha(t)=2 \tag{6}$$

which after some easy algebra gives (always for linear trajectories)

$$\alpha(s)+\alpha(t)+\alpha(u)=2 \tag{7}$$

Equation (7) can easily be transformed into the prediction

$$\alpha(-2m_\rho^2+m_\omega^2+3m_\pi^2)=\alpha(-0.53\,\mathrm{BeV}^2)=0 \tag{8}$$

which was derived in Ref. [2] from the sum rules. The reader can verify that Eq. (7) is enough to cancel all the undesired poles at the even integer values of α . A further interesting consequence of Eq. (7) concerns the third term of Eq. (3) which could in principle violate the Regge behaviour. Instead, using (7), that term can be rewritten as:

$$\frac{\beta(t)}{\sin\pi\alpha(s)}\;\frac{\Gamma(\alpha(s)+\alpha(t)-1)}{\Gamma(\alpha(s))} \tag{9}$$

which is still Regge behaved. The whole Eq. (3) can be rewritten in the form

$$A=\beta(t)\,\frac{\Gamma(\alpha(s)+\alpha(t)-1)}{\Gamma(\alpha(s))}\left[\frac{1-e^{i\pi\alpha(s)}}{\sin\pi\alpha(s)}+\frac{1-e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\right] \tag{10}$$

which shows the automatic cancellation of the poles at the even integer values of α . By use of (7) one can also write (3) in the very symmetric form

$$A=\frac{\bar{\beta}}{\pi^2}\,\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))\Gamma(1-\alpha(u))\left[\sin\pi\alpha(s)+\sin\pi\alpha(t)+\sin\pi\alpha(u)\right] \tag{11}$$

<u>As a second example</u> let us consider the process $\pi\eta \to \pi\rho$. According to our prescription the invariant amplitude A defined as in Eq. (1) will be given by:

$$A_{\pi\eta\to\pi\rho} = \frac{\bar{\beta}_1}{\pi}\left[B(1-\alpha_{A_2}(s), 1-\alpha_\rho(t)) + B(1-\alpha_{A_2}(u), 1-\alpha_\rho(t)) - B(1-\alpha_{A_2}(s), 1-\alpha_{A_2}(u))\right] \tag{21}$$

where $\pi\pi \to \eta\rho$ is the t channel. Imposing to find no poles at even integer value for $\alpha_\rho(t)$ we obtain:

$$\alpha_{A_2}(s) + \alpha_{A_2}(u) + \alpha_\rho(t) = 2 \tag{22}$$

Imposing absence of poles at the odd integers for α_{A_2} we find again Eq. (22). This demands

$$\alpha'_\rho = \alpha'_{A_2} \tag{23}$$

Using $m_\rho^2 = 0.6$ GeV^2 and Eq. (7) we obtain

$$\alpha_{A_2}(0) = 1 - \frac{1}{2}\,\frac{3m_\rho^2 - m_\omega^2 - m_\pi^2 + m_\eta^2}{3m_\rho^2 - m_\omega^2 - 3m_\pi^2} \simeq 0.36 \tag{24}$$

We thus predict $m_{A_2} = 1350$ MeV.

<u>As a third example</u> one could try to build up a scattering amplitude for $s+s \to s+s$ (s being a scalar particle with the vacuum quantum numbers) and try to ask dominance of a leading trajectory passing by the particle itself. This is seen to be impossible with a positive slope of α, since the equation similar to (7) gives $\alpha(0) = 1$.

It is possible to extend the above considerations to the more interesting case of $\pi\pi$ scattering and to obtain a crossing symmetric amplitude in the approximation of ρ and f trajectory dominance and disregarding the Pomeranchuk, according to a now accepted philosophy [8),11)]. We find consistency only if $\alpha_\rho = \alpha_f = \alpha$ and $\alpha(0) \simeq 1/3$. Furthermore, we can predict $\pi\pi$ scattering lengths in terms of $g^2_{\rho\pi\pi}$ and obtain (apart from the Pomeranchuk contribution)

$$a_0 = 5/2\, a_2 = -1.25\, m_\pi^{-1}$$

Further details as well as applications of this scheme to more complicated cases will be considered elsewhere.

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FOOTNOTES AND REFERENCES

1) For a general review of these problems see L. Bertocchi, Proceedings of the Heidelberg international conference on elementary particles, North Holland Pub. Co., (1967).

2) Such an approach was proposed independently by M. Ademollo, H.R. Rubinstein, G. Veneziano and M.A. Virasoro, Phys.Rev.Letters 19, 1402 (1967) and Phys.Letters 27B, 99 (1968), and by S. Mandelstam, Phys.Rev. 166, 1539 (1968). Further developments and a number of references to related works can be found in Ref. [3].

3) M. Ademollo, H.R. Rubinstein, G. Veneziano and M.A. Virasoro, Weizmann Institute preprint (1968), submitted to Phys.Rev.

4) For superconvergence we mean both the original sum rules proposed by V. de Alfaro, S. Fubini, G. Furlan and C. Rossetti, Phys.Letters 21, 576 (1966), and the more recent generalized superconvergence (finite energy) sum rules [see Ref. [3] for detailed references]. A unified treatment of all superconvergence sum rules has been given by S. Fubini, Nuovo Cimento 52A, 224 (1967).

5) H.R. Rubinstein, A. Schwimmer, G. Veneziano and M.A. Virasoro, Weizmann Institute preprint (1968), submitted to Phys.Rev.Letters; see also Ref. [3].

6) G. Cosenza, A. Sciarrino and M. Toller, University of Rome preprint Nr. 158 and Trieste preprint.

7) C. Schmid, Phys.Rev.Letters 20, 689 (1968).

8) H. Harari, Phys.Rev.Letters 20, 1395 (1968).

9) G. Cocconi et al., Phys.Rev. 138B, 165 (1965). Having linearly rising trajectories we are also consistent with the Cerulus-Martin bound. See C.B. Chiu and C.I. Tan, Phys.Rev. 162, 1701 (1967).

10) We know that, at $t = 0$, a simple (irreducible) Lorentz pole does obey factorization [see Ref. [1]]. It seems also plausible to conjecture (M. Toller, private communication) that this is the only case in which factorization is fulfilled. Since our expression does not probably correspond to a single Lorentz pole, non-leading terms might be needed in order to have factorization. We thank M. Toller for a discussion on this point.

11) It may be, however, that one runs into difficulties in adding the Pomeranchuk contribution at the end in a crossing symmetric way. An alternative interesting possibility would be to consider it as originated somehow by the other trajectories (through their non-resonating parts) and not as an independent object. This problem which certainly requires further study, is closely connected to that of the nature of the Pomeranchuk singularity.