Discussion Paper Series

No.145

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March 2006

Hitotsubashi University Research Unit for Statistical Analysis in Social Sciences
A 21st-Century COE Program

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An Analysis of Option Pricing in the Japanese Market

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1 Introduction

Stock index options made their debut in Japan in June 1989, when they were first introduced at the Osaka Securities Exchange . Since then, trading rules have been amended frequently, giving the market the structure it has today. Index options are now among the most actively traded options in Japan. Yet, because trading volumes are much smaller than in other markets such as those of the U.S., there are doubts whether the market is working efficiently, and one way in which to assess the peculiarities of the Japanese options market is to look at option pricing.

The earliest attempt to model option pricing was undertaken by Black and Scholes (1973). Since then, various models trying to improve upon the original Black and Scholes model (BS model) have been proposed. Most of these look at loosening the assumptions of the BS model. The principal objective of the proposed alternative models has been to describe the movement of the underlying asset price more accurately. As option prices synchronize with the underlying asset prices and as the volatility of the underlying asset price is an important factor for pricing, much effort has been expended on describing the volatility movement of the underlying asset precisely. Concretely, variance changing models such as ARCH, SV, and MS models have been estimated to describe the underlying asset distribution. However, the option price observed in the market may differ from the theoretical prices estimated by the models, since it is not simply determined by the movements of the underlying asset. Analyzing the rate of return of the Nikkei 225 options, Nishina and Nabil(1997), for example, showed that put-call parity does not hold. Bookstaber (1981) suggested that bias exists due to nonsimultaneity of the option market and the underlying asset market. Easley, O'Hara, and Srinivas (1998) pointed out that the trading volume of a stock option correlates with the future price of the underlying asset.

The main objective of this paper is to analyze the discrepancy of the market price from the theoretical price in the Japanese option market and investigate the option pricing mechanism in Japan. In the empirical analyses, the following points are emphasized as characteristics of the Japanese option data.

First, multiple options are traded on the same day in the market. It is necessary to classify the option price data not only by the traded days but also by the maturity days. As a result, the data set must be handled as a panel with a certain rotation structure. In addition, it is necessary to take the covariance of the price data into consideration when the price movements are explained by various factors such as survival period and the moneyness.

Second, in the data set, option prices frequently take a value of zero. Zero values indicate that the options were not traded on that day and such observations have been excluded from other empirical analysis. However, as such observations where the price is zero reflect the investors' decisions, exclusion of such data will lead to loss of information. More importantly, the estimation of any relationship may be biased due to this exclusion.

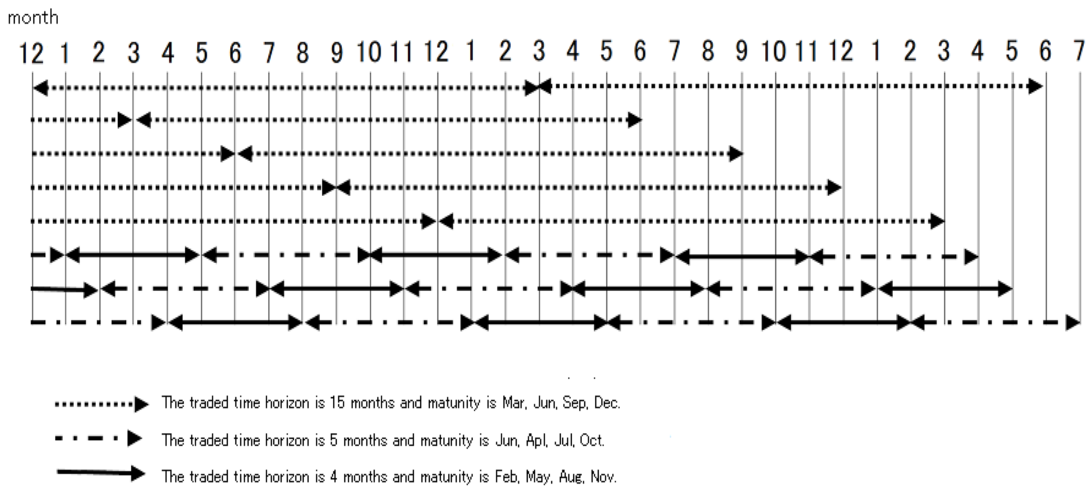
In the following, we investigate the characteristics of the option market in Japan by paying attention to these features of Japanese option data. The remainder of this paper is organized as follows. Section 2 provides a brief overview of the characteristics of the Japanese options market. It also briefly explains the calculation of theoretical option prices. Then, the distribution of the option prices is examined by looking at the summary statistics. In Section 3, the modeling of option pricing is considered taking the abovementioned characteristics of the Japanese data into account. Concretely, the variance structure is explicitly formulated and the treatment of 0-value data is discussed. Next, the estimation results based on the model are reported. Section 4 concludes the paper.

2 Data

2.1 A brief outline of the Japanese options market

First of all, let us briefly look at the features of the Japanese options market. Eight types of options

Figure 1: Creation and termination of the eight types of options in the Japanese options market



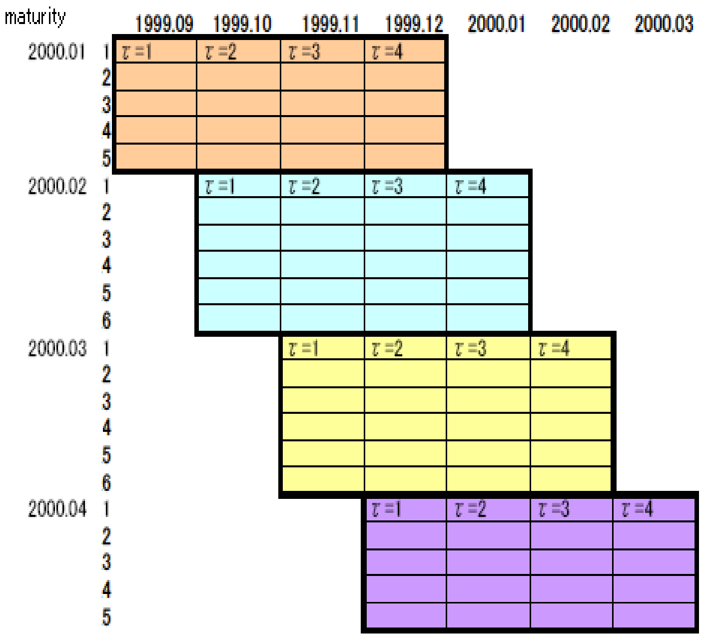
with different expiration dates are traded on the same day. Figure 1 illustrates the creation and termination of the options. There are three kinds of transaction time horizons:15 months, five months, and four months. When the exercise day of an option arrives, a new option is created on that day. Usually, for options with the same maturity, five strike prices are initially set symmetrically around the underlying asset price. Therefore, there are at least 40 options running on every trading day. However, the number of strike prices may increase when the underlying asset price exceeds the highest or lowest strike price. For our analysis, we simplify matters by creating monthly data based on the option price on the expiration day of each month. In addition, we confine the data for each option to the last four months before the expiration data ($\tau = 1, 2, 3, 4$). To do this we construct a set of time series data with a rotation structure, as illustrated in Figure 2. Note that in Figure 2, the number of strike price changes depends on the expiration day. When data for the last four months of an option are not available, the option is excluded from our analysis.

In this paper, we analyze the market prices of the Nikkei 225 call option (C_m) with maturity from January 2000 to April 2002. There are 28 maturities (months) in the arranged data and 242 options are traded when combined with different strike prices.

2.2 Estimation of C_s

In order to examine the peculiarities of the Japanese options market, we compare the actual price of Japanese options (C_m) with the theoretical price (C_s) predicted by various models, including the BS, GARCH, and EGARCH models. In the following discussion, the strike price of an option is written as K, the closing price of the underlying asset as S, the survival period as τ and moneyness ($= K/S$) as M. Furthermore, the number of strike prices for options in the i'th group maturity is denoted as

Figure 2: Monthly option data from January, 2000 to April, 2000



The number of each strike price k_i, are as follows: $k_1 = 5$, $k_2 = 6$, $k_3 = 6$, and $k_4 = 5$.

k_i.

Various models have been proposed for describing accurately the volatility of the underlying assets. Representative models among them are the GARCH model by Bollerslev (1986) that considers the persistence of volatility shocks and the EGARCH model by Nelson (1991) that considers asymmetry as well as the persistency of shocks. Comparisons of option pricing have been made between alternative the volatility fluctuation models of the underlying assets. For example, Crouhy (1994) and Duan and Zhang(2001) compared the ARCH type model and the BS model. They concluded that the predictive performance of the option price by the ARCH type model is better than by the BS model. [1]

In this paper, GARCH model, EGARCH model, and BS model are fitted to the Nikkei 225. Then, the predicted values of an option price at maturity are simulated using the estimated models. Then, from the simulated predicted values, the expected value of the option price is calculated as their average and it is considered as the estimated theoretical price C_s based on each model.

More concretely, the three models are estimated by the maximum likelihood method for the rate of return of the Nikkei 225 using 1000 business days' ($u = t - 999, \cdots, t$) data before the trading day. The GARCH (1, 1) model is:

$$R_{S,u} = \mu + \epsilon_u \tag{1}$$

$$\epsilon_u = \sigma_u z_u \qquad z_u \sim i.i.d.N(0,1)$$
$$\sigma_u^2 = \omega + \alpha\epsilon_{u-1}^2 + \beta\sigma_{u-1}^2. \tag{2}$$

For the EGARCH(1,1) model, the variance formulation (2) is replaced by:

$$\ln(\sigma_u^2) = \omega + \alpha[\theta z_{u-1} + \gamma(|z_{u-1}| - E(|z_{u-1}|)] + \beta\ln(\sigma_{u-1}^2). \tag{3}$$

Furthermore, for the constant volatility model, the variance is assumed to be:

$$\sigma_u^2 = \omega. \tag{4}$$

and this model is regarded as the BS model in this paper.

After the estimated model is transformed to the model where local risk neutrality is assumed (Duan 1995, Bauwens and Lubrano 2002), the underlying assets' prices at the maturity of the option are generated by the Monte Carlo simulation. Letting the strike price be K, the call option price at the maturity is calculated by:

$$\hat{C}_{s,tk} = max(S_{t+l} - K, 0), \tag{5}$$

where S_{t+l} is the underlying assets price at the maturity and l denotes the survival days of the option. The above procedure is repeated N_m times to calculate N_m option prices at the maturity. When the average of them is discounted at the transaction day, it becomes:

$$C_{s,tk} = (1+r)^{-l}\frac{1}{N_m}\sum_{j=1}^{N_m}\hat{C}_{s,tk,j}, \tag{6}$$

where r is a risk-free rate and is replaced by the call rate in the calculation. C_s is considered as the theoretical option price from the model. In the following calculations, $N_m = 10000$.

[1] There are researchers that have paid attention to the distribution of the error term in the ARCH model. Bollerslev (1987) and Watanabe (2000) showed that distributions with thicker tails better describe the movement of the underlying assets' rate of return than a normal distribution. However, Watanabe (2003) reported that the improvement of the description of the underlying asset price movements does not influence the prediction of the option price.

2.3 Summary statistics of $C_m - C_s$

As a preliminary analysis, this section examines the distribution of the difference between the market price and the theoretical option prices that are calculated according to (6) using the three models. The number of price observations is 968 in this study, and 74 among them record 0 market values; i.e.. $C_m = 0$. First, in order to see the difference of predictive accuracy of the three models, following the literature, the simulated option prices based on the three models are compared with the actual market prices in terms of RMSE (root mean squared error)

$$RMSE = \frac{1}{N}\sum_{j=1}^{N}\sqrt{(C_{m,j} - C_{s,j})^2}.$$

From Table 1, it is seen that the difference of the BS model is the smallest. That is, for the option with

Table 1: Comparison of call option pricing models (RMSE)

0 value data are excluded	BS	GARCH	EGARCH
	138.18	143.67	144.13
0 value data are included	BS	GARCH	EGARCH
	127.66	132.89	133.15

The number of observations is 968. After omitting 74 observations for which the price is recorded as zero, the number decreases to 894.

the survival period between one month and four months, the theoretical option prices calculated from the BS model are closer to the market prices than those from the GARCH model or the EGARCH model. This finding is different from conventional research results, though the values of RMSE are not so numerically different among the three models.

Then, let us investigate statistical properties of the difference between the market prices and the theoretical prices $(C_m - C_s)$. The distribution of $C_m - C_s$ will change by including $C_m = 0$ since the distribution of the rate of return of the option prices changes If C_s is a reasonable predicted value

Table 2: Summary statistics of $C_m - C_s$

	mean	sd	skew	kurt
BSmodel	-11.31	199.73	0.93	6.29
GARCHmodel	-43.34	191.06	0.63	5.64
EGARCHmodel	-43.76	197.70	0.40	5.54
$(C_{m,t} > 0)$	mean	sd	skew	kurt
BSmodel	-12.21	207.81	0.91	5.83
GARCHmodel	-46.70	198.44	0.66	5.29
EGARCHmodel	-47.35	205.32	0.44	5.18

of C_m, the distribution of $C_m - C_s$ is expected to be near to a normal distribution. From Table 2, however, the kurtosis is much larger than 3 showing that the distribution has a thicker tail than a normal distribution. When the observations with $C_m = 0$ are included, the mean and kurtosis become larger and the standard deviations become smaller than otherwise. Figure 3 shows the histogram of $(C_m - C_s)$ including $C_m = 0$. In the Japanese data, the distributions of $C_m - C_s$ are skewed positively for all of the three models. It is also observed that a large number of data points take the value of nearly 0.

3 Analysis of $C_m - C_s$

3.1 Model

In trading an option, investors have information on the theoretical price, the strike price, the survival periods, and the underlying asset's price. They can make their decisions based on this information. Because multiple options are traded on the same day, the market price C_m will be affected not only by the theoretical price C_s calculated from the underlying asset movements but also by the other option prices.

First, an investor makes a decision on buying or selling an option depending on the survival period and then makes a selection from several strike prices with the same maturity. When such an investor's decision behavior is considered, it would be reasonable to expect that the market price reflects the investor's behavior.

There has been little research on the difference between the market price and the theoretical price. Long and Officer (1997) analyzed the difference between the theoretical price calculated from the BS model and the market price, which they called the pricing error, using the individual equity option price from the Chicago Board Options Exchange in the United States. First, implied volatility (IV) of the at-the-money option is estimated as a linear function of the risk free rate, the trade volume, moneyness, lagged IV, and the survival period. Then, the volatility of each option is calculated from the estimated model and is substituted in the BS model to calculate the theoretical price. As a result, it is shown that the pricing error is large when IV is large, that the error shrinks when the trading volume increases, but that the error grows again as the trading volume increases further. Moreover, they concluded that the pricing error of the out-of-the-money option is larger than that of the in-the-money option, and the pricing error becomes smaller as the survival period becomes longer.

From Long and Officer (1997), it is suggested that the market price of an option is not solely determined by its theoretical price. In this research, we consider a simple model in which the market price depends on the moneyness and the survival period, in addition to the theoretical price: [2]

$$\begin{aligned} C_{m,itk\tau} &= C_{s,itk\tau} + \beta_0 M_{itk\tau} + \sum_{s=1}^{4} \beta_{\tau,s} D_{\tau,s} + \sum_{j=4}^{5} \beta_{op,j} D_{op,j} + u_{itk\tau} \\ & u_{itk\tau} = \epsilon_t + \epsilon_k + \epsilon_{itk\tau}. \end{aligned} \tag{7}$$

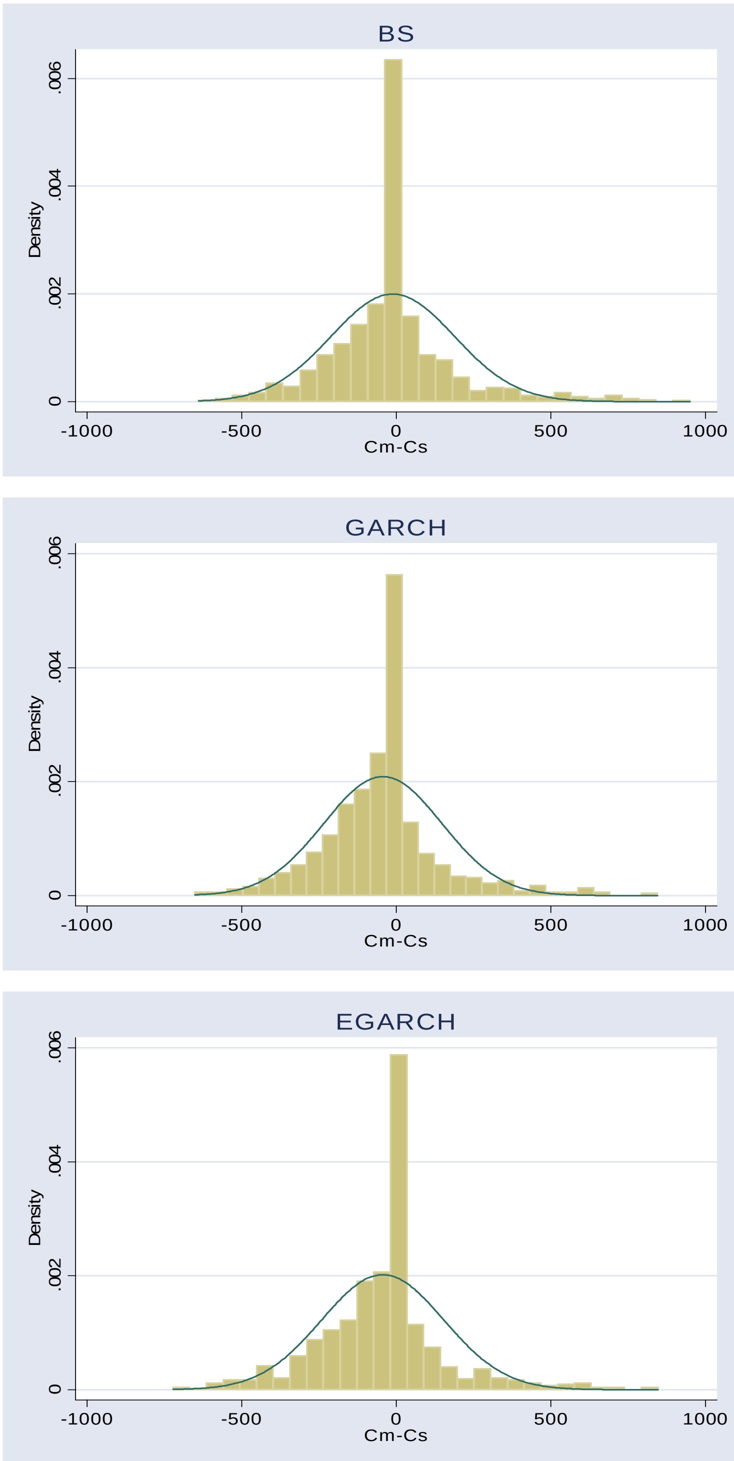
Here, $D_{\tau,s}$ is the dummy variable that takes one when the remaining period is s $(s = 1, 2, 3, 4)$ and $D_{op,j}(j = 4, 5)$ is the dummy variable that takes one when the option trading period is j months. The suffix i denotes a maturity $(i = 1, \cdots, 28)$, t is a trading day, k $(k = 1, \cdots, k_i)$is a strike price, and τ is a survival period, which $\tau = t - i + 1$. The error term $u_{itk\tau}$ is assumed to be decomposed into the error ϵ_t that depends on the transaction date, the error ϵ_k that depends on the strike price, and the error term $\epsilon_{itk\tau}$ that is purely random. It is assumed that ϵ_t, ϵ_k, and $\epsilon_{itk\tau}$ are mutually independent.

[2] The market price data is arranged as:

$$C_{m,1111}, \cdots, C_{m,1141}, C_{m,1212}, \cdots, C_{m,1444}, C_{m,2211}, \cdots, C_{m,2241}, \cdots, C_{m,2544}, \cdots$$

according to the maturity, the trading day, and the strike price.

Figure 3: Distribution of $C_m - C_s$ when the 0 value data are included



The variances of these three error terms are expressed respectively as:

$$
\begin{aligned}
Var(\epsilon_t) &= \sigma_t^2 && (8)\\
Var(\epsilon_k) &= \sigma_k^2 && (9)\\
Var(\epsilon_{itk\tau}) &= \sigma_{itk\tau}^2. && (10)
\end{aligned}
$$

In order to estimate (7) efficiently, we must take the covariance structure of $u_{itk\tau}$ into consideration. For its simplest case when only one exercise price exists for each maturity, i.e.. $k_i = 1$ for each i, the covariance matrix becomes:

$$
Var(u_{itk\tau}) = \Sigma =
$$

$$
\left(\begin{array}{ccccccccccccccccccccc}
\sigma^2 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 \\
\sigma_k^2 & \sigma^2 & \sigma_k^2 & \sigma_k^2 \\
\sigma_k^2 & \sigma_k^2 & \sigma^2 & \sigma_k^2 \\
\sigma_k^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 \\
0 & \sigma_t^2 & 0 & 0 & \sigma^2 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 \\
0 & 0 & \sigma_t^2 & 0 & \sigma_k^2 & \sigma^2 & \sigma_k^2 & \sigma_k^2 \\
0 & 0 & 0 & \sigma_t^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 & \sigma_k^2 \\
0 & 0 & 0 & 0 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 \\
0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma^2 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 \\
0 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & \sigma_k^2 & \sigma^2 & \sigma_k^2 & \sigma_k^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_t^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 & \sigma_k^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 \\
0 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma^2 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & \sigma_k^2 & \sigma^2 & \sigma_k^2 & \sigma_k^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_t^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 & \sigma_k^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 \\
\vdots & & & & \ddots & & & & \ddots & & & & \ddots & & & & \ddots \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma^2 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_t^2 & 0 & 0 & \sigma_t^2 & 0 & \sigma_k^2 & \sigma^2 & \sigma_k^2 & \sigma_k^2 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_t^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2 & \sigma_k^2 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_k^2 & \sigma_k^2 & \sigma_k^2 & \sigma^2
\end{array}\right). \tag{11}
$$

$$
\sigma^2 = \sigma_t^2 + \sigma_k^2 + \sigma_{itk\tau}^2. \tag{12}
$$

In this case, since only one strike price is assumed for each maturity, σ_k^2 appears commonly for the elements corresponding to the options with the same maturity. Moreover, σ_t^2 commonly appears for the elements corresponding to the options on the same trading day. For example, the option in the second month in the first maturity group and the option in the first month in the second maturity group are traded on the same days. Similarly, the option in the third month in the second maturity group, the option in the second month in the second maturity group and the option in the first month in the third maturity group are traded on the same days. Also, the option in the fourth month in the first maturity group, the option in the third month in the second maturity group, the option in the second month in the third maturity group and the option in the first month in the fourth group are traded on the same days. Thus, each option in one maturity group has the same trading days with three options belonging to neighboring three maturity groups. From these, the variance covariance matrix is seen to be expressed as (11) and (12) with a regular arrangement of σ_k^2 and σ_t^2.

In a more general situation when there are k_i exercise prices for the i-th maturity group among 28 maturity groups, the variance covariance structure becomes further complicated and is expressed as follows.

$$
\Sigma = \begin{pmatrix}
\Sigma^1 &&&&&&&& \\
\Sigma^1_2 & \Sigma^2 &&&&&&& \\
\Sigma^1_3 & \Sigma^2_2 & \Sigma^3 &&&&&& \\
\Sigma^1_4 & \Sigma^2_3 & \Sigma^3_2 & \Sigma^4 &&&&& \\
0 & \Sigma^2_4 & \Sigma^3_3 & \Sigma^4_2 & \Sigma^5 &&&& \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots &&& \\
\vdots & & 0 & \Sigma^i_4 & \Sigma^i_3 & \Sigma^i_2 & \Sigma^i && \\
\vdots & & & \ddots & \ddots & \ddots & \ddots & \ddots & \\
0 & \cdots & \cdots & \cdots & 0 & \Sigma^{25}_4 & \Sigma^{26}_3 & \Sigma^{27}_2 & \Sigma^{28}
\end{pmatrix} \tag{13}
$$

$$
\Sigma^i \quad = \quad
\begin{array}{cc}
 & \begin{array}{cccc} 1 & \cdots & \cdots & k_i \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ \vdots \\ k_i \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} &
\begin{pmatrix}
\sigma^2 & \sigma_t^2 & \cdots & \sigma_t^2 &&&&&&&&&&&& \\
\sigma_t^2 & \ddots & \ddots & \vdots &&&&&&&&&&&& \\
\vdots & \ddots & \ddots & \sigma_t^2 &&&&&&&&&&&& \\
\sigma_t^2 & \cdots & \sigma_t^2 & \sigma^2 &&&&&&&&&&&& \\
\sigma_k^2 & 0 & \cdots & 0 & \sigma^2 & \sigma_t^2 & \cdots & \sigma_t^2 &&&&&&&& \\
0 & \ddots & \ddots & \vdots & \sigma_t^2 & \ddots & \ddots & \vdots &&&&&&&& \\
\vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & \sigma_t^2 &&&&&&&& \\
0 & \cdots & 0 & \sigma_k^2 & \sigma_t^2 & \cdots & \sigma_t^2 & \sigma^2 &&&&&&&& \\
\sigma_k^2 & 0 & \cdots & 0 & \sigma_k^2 & 0 & \cdots & 0 & \sigma^2 & \sigma_t^2 & \cdots & \sigma_t^2 &&&& \\
0 & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots & \sigma_t^2 & \ddots & \ddots & \vdots &&&& \\
\vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & \sigma_t^2 &&&& \\
0 & \cdots & 0 & \sigma_k^2 & 0 & \cdots & 0 & \sigma_k^2 & \sigma_t^2 & \cdots & \sigma_t^2 & \sigma^2 &&&& \\
\sigma_k^2 & 0 & \cdots & 0 & \sigma_k^2 & 0 & \cdots & 0 & \sigma_k^2 & 0 & \cdots & 0 & \sigma^2 & \sigma_t^2 & \cdots & \sigma_t^2 \\
0 & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots & \sigma_t^2 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & \sigma_t^2 \\
0 & \cdots & 0 & \sigma_k^2 & 0 & \cdots & 0 & \sigma_k^2 & 0 & \cdots & 0 & \sigma_k^2 & \sigma_t^2 & \cdots & \sigma_t^2 & \sigma^2
\end{pmatrix}
\end{array} \tag{14}
$$

$$
\Sigma_2^i = \begin{array}{cc}
 & \begin{array}{cccccccccccc} 1 & \cdots & k_i & & & & & & & & & \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ k_{i+1} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} &
\begin{pmatrix}
0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0
\end{pmatrix}
\end{array} \tag{15}
$$

$$
\Sigma_3^i = \begin{array}{cc}
 & \begin{array}{cccccccccccc} 1 & \cdots & k_i & & & & & & & & & \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ k_{i+2} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} &
\begin{pmatrix}
0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0
\end{pmatrix}
\end{array} \tag{16}
$$

$$
\Sigma_4^i = \begin{array}{cc}
 & \begin{array}{cccccccccccc} 1 & \cdots & k_i & & & & & & & & & \end{array} \\
\begin{array}{c} 1 \\ \vdots \\ k_{i+3} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} &
\begin{pmatrix}
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \sigma_t^2 & \cdots & \sigma_t^2 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0
\end{pmatrix}
\end{array} \tag{17}
$$

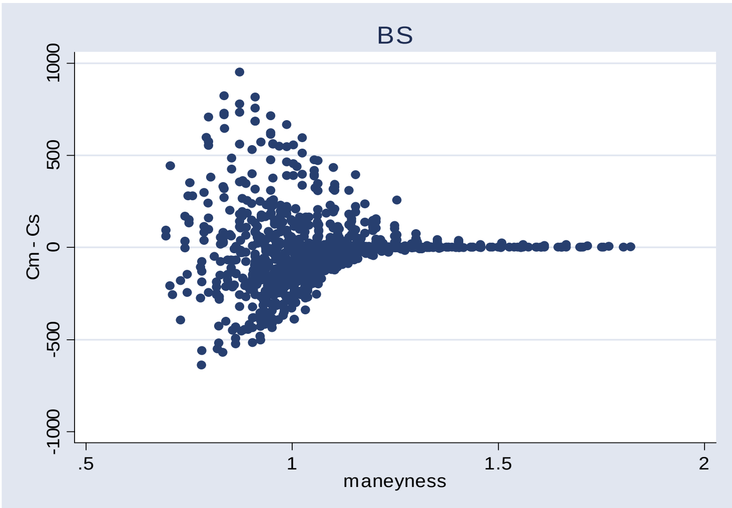
Let us consider the options with maturity i and the options with maturity j that are traded on the same day ($j = i+1, i+2, i+3$). Assume that the number of strike prices of the options with maturity i is k_i and the options with maturity j is k_j. Then, the variance matrix of the options in the ith maturity group, Σ_i, is a $4k_i \times 4k_i$ matrix and the covariance of the options on the same trading days $\Sigma_s^i (s = 2, 3, 4)$ becomes a $4k_j \times 4k_i$ matrix.

Σ_i consist of the blocks of the options with the same maturity. The diagonal blocks consists of the options traded on the same days and correspond to the components shown by σ^2 in equation (11). The sum of the variances of the three error terms enters as the diagonal elements of the diagonal blocks. The off-diagonal elements express the options with different strike prices but traded on the same day. The variance of the error term on the same trading day σ_t^2 enters for the off-diagonal elements. The diagonal elements of the off-diagonal blocks are σ_k^2 corresponding to the error terms with the same strike price and with the same maturity.

$\Sigma_s^i (s = 2, 3, 4)$ are the variance matrices of the options on the same trading day with different maturities. They consist of 16 component matrices, each of which is a $k_j \times k_i$ $(j = i + 1, i + 2, i + 3)$ matrix whose elements are all σ_t^2 corresponding to the single σ_t^2 or 0 corresponding to the scalar 0 in equation (11) . The $k_j \times k_i$ matrix denotes the variances of the options with different maturities but traded on the same day. The arrangement of the variance covariance matrices is basically the same as that of the equation (11).

Another characteristic of $C_m - C_s$, whose scatter diagram is shown in Figure 4, is that the variance

Figure 4: Scatter diagram of $C_m - C_s$ and moneyness



The BS model is used to calculate the theoretical price. Similar patterns are seen for the GARCH model and the EGARCH model.

becomes smaller as moneyness increases. Here, the adjustment for this heteroscedasticity is made simply by multiplying the moneyness M by both sides of (8). Then, the model to be estimated becomes:

$$MC_{m,itk\tau} = MC_{s,itk\tau} + \beta_0 M_{itk\tau}^2 + M \sum_{s=1}^{4} \beta_{\tau,s} D_{\tau,s} + M \sum_{j=4}^{5} \beta_{op,j} D_{op,s} + Mu_{itk\tau} \tag{18}$$

The variance of this model can be then expressed as:

$$diag(\tilde{M}) \times \Sigma \times diag(\tilde{M})' \tag{19}$$

where $\tilde{M}$ denotes the vector consisting of moneyness M and $diag(\tilde{M})$ is a matrix whose diagonal elements consist of the elements of $\tilde{M}$.

3.2 Treatment of 0-value data

Among the 968 observations of option prices that are used in this paper, 74 observations are with $C_m = 0$. In the conventional research, such data are excluded from the analysis. It is noted, however, that the investors' decisions are somehow reflected in these data and, therefore, exclusion of them will lead to loss of information. More importantly, exclusion may cause bias in the estimation. The difficulty is that the reasons why C_m become 0 are unknown. Also, the trading volume, which reflects the investor's demand, becomes 0 simultaneously with the price. Figure 5 shows the histogram of C_s including the data with $C_m = 0$ from May,1994 to April, 2002.

From the figure, it is seen that C_m becomes 0 even when C_s is large. In this paper, the following two possibilities of C_m being 0 are considered.

Table 3: Example : Data of 0 yen with BS model option price

maturity(K)	1994,05	(19000)	1996,03	(19000)	1998,05	(15000)
	C_m	C_s	C_m	C_s	C_m	C_s
$\tau = 1$	900	1739.79	330	620.58	1050	1029.87
$\tau = 2$	1720	2522.94	830	905.73	0	2626.33
$\tau = 3$	0	1973.88	0	1981.84	1790	2282.88
$\tau = 4$	1100	1211.30	2000	1890.98	1530	1143.91

maturity (K)	2000,07	(20500)	2001,12	(13500)	2002,03	(13000)
	C_m	C_s	C_m	C_s	C_m	C_s
$\tau = 1$	640	725.67	180	286.27	40	40.25
$\tau = 2$	835	917.69	35	5.92	0	11.43
$\tau = 3$	35	52.24	0	0.25	6	4.79
$\tau = 4$	0	2.77	1	0.23	0	0

First, the market price of an option that the investor thinks reasonable after considering the theoretical price, the strike price, and the survival period, can be negative when the investor requires a the risk premium. Since the market price cannot be negative, the option price is truncated at 0 yen under such a circumstance. That is, $C_m = 0$ could be observed when the market price that the investor considers appropriate (C^*_m) satisfies:

$$a) \quad C^*_{m,itk\tau} < 0. \tag{20}$$

The truncation (20) is not the sole reason there could be data with $C_m = 0$. Secondly, the transaction vanishes when either the investor's demand or supply becomes nil. This happens when the market price of an option deviates from the price that the investors think appropriate. That is, define the deviation as:

$$DV = C^*_{m,itk\tau} - \left(C_{s,itk\tau} + \beta_0 M_{itk\tau} + \sum_{s=1}^{4} \beta_{\tau,s} D_{\tau,s} + \sum_{j=4}^{5} \beta_{op,j} D_{op,s} \right). \tag{21}$$

Table 4 indicates several examples of 0-value data observed in the past. It suggests that even when $C_m = 0$, C^*_m might not be negative. It is plausible that C_m could be 0 when:

$$b) \qquad DV > a \qquad or \qquad DV < b(< 0). \tag{22}$$

The model here takes these two possibilities of deviations into consideration.

In summary, the model and the truncation mechanism are expressed as follows.

$$\begin{aligned} MC^*_{m,itk\tau} &= MC_{s,itk\tau} + \beta_0 M^2_{itk\tau} + M \sum_{s=1}^{4} \beta_{\tau,s} D_{\tau,s} + M \sum_{j=4}^{5} \beta_{op,j} D_{op,s} + M u_{itk\tau} \\ & u_{itk\tau} = \epsilon_t + \epsilon_k + \epsilon_{itk\tau} \\ & C_{m,itk\tau} = \begin{cases} C^*_{m,itk\tau} & \\ 0 & \text{if} \quad a) \quad or \quad b) \end{cases} . \end{aligned} \tag{23}$$

Figure 5: Distribution of C_s for the data with $C_m = 0$

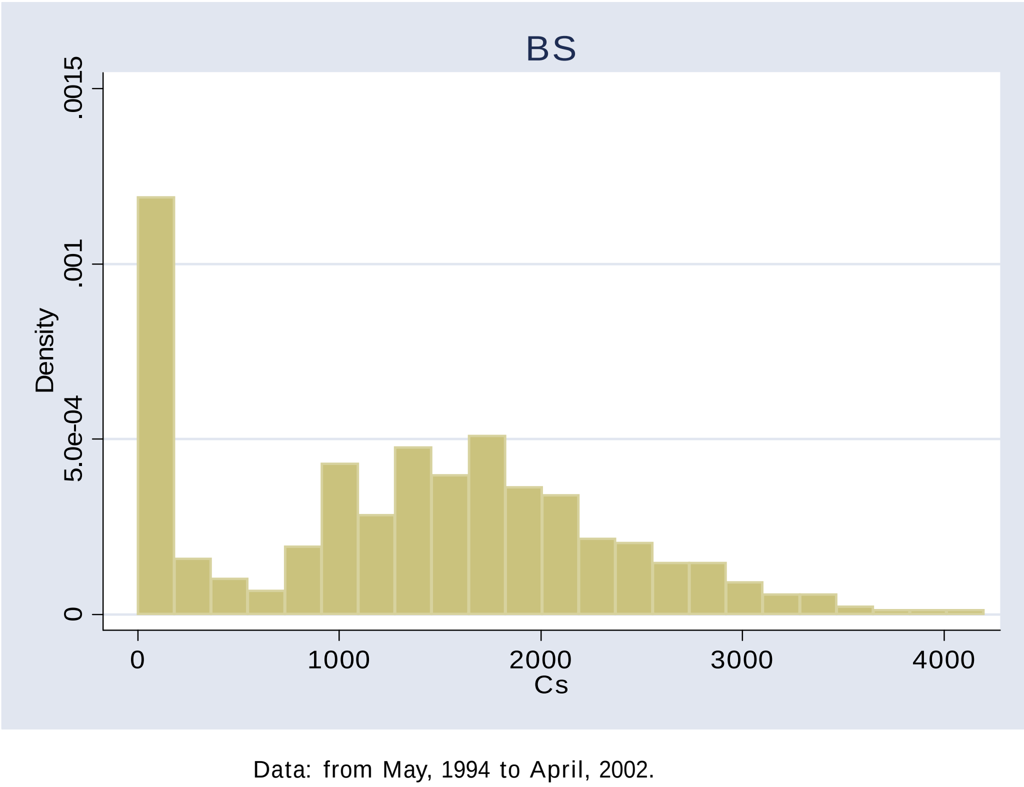


Figure 6 illustrates the truncation mechanisms of C_m for the given value of $C_s + f(S, K, \tau)$ in the case when $f(S, K, \tau)$ is a linear function of K/S with a homoscedastic error term.

Note that as this model has a complicated variance covariance structure as well as truncation conditions, the maximum likelihood estimations of the parameters of this model cannot be analytically obtained. If the variance structure was homoscedastic and the truncation mechanisms were expressed only by (20), the familiar Tobit model could be applied. Because of the complicated structure of the variance matrix and truncation mechanism, however, we are obliged to resort to a simulation method.

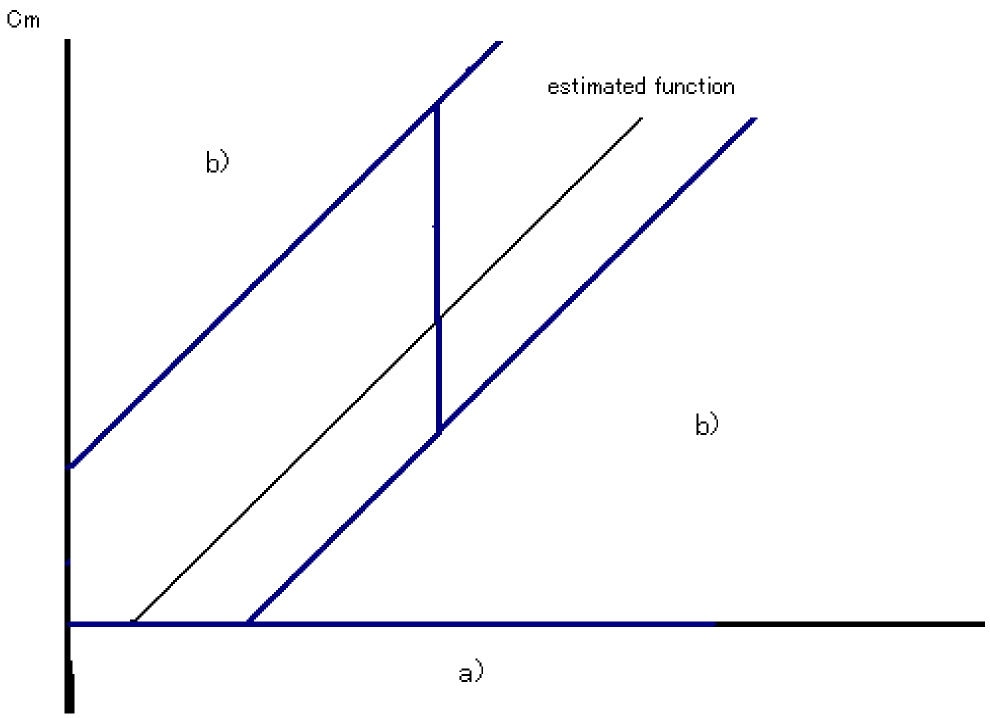
Let C_{m1} be a vector of the option prices in the market for which $C_m \neq 0$ and C^*_{m0} be a vector of the latent option prices corresponding to C_{m0} for which $C_m = 0$. Note here that C^*_{m0} is unobservable. Assume also that (C_{m1}, C^*_{m0}) distributes according to a multivariate normal distribution with the mean vector μ and the variance matrix Σ, respectively:

$$\mu = E \begin{pmatrix} C_{m1} \\ C^*_{m0} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_0 \end{pmatrix} \tag{24}$$

$$\Sigma = Var \begin{pmatrix} C_{m1} \\ C^*_{m0} \end{pmatrix} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \quad (\Sigma_{12} = \Sigma'_{21}). \tag{25}$$

Then, the distribution of C^*_{m0} conditional on C_{m1} is also a multivariate normal distribution whose

Figure 6: Truncation of C_m



density is:

$$f(C^*_{m0}|C_{m1}) = (2\pi)^{-\frac{k_0}{2}} \left(\frac{|\Sigma_{11}|}{|\Sigma|}\right)^{\frac{1}{2}} \exp\left(-\frac{Q_0}{2}\right), \tag{26}$$

$$\tag{27}$$

where

$$Q_0 = (C^*_{m0} - \nu_0)' \left(\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\right)^{-1} (C^*_{m0} - \nu_0)$$
$$\nu_0 = \mu_0 + \Sigma_{21}\Sigma_{11}^{-1}(C_{m1} - \mu_1).$$

We generate C^*_{m0} using this property. Concretely, the following steps are iterated.

1. Estimate the following model by the OLS using the observations for which $C_m \neq 0$.

$$MC_m = MC_s + \hat{\beta}^1_0 M^2 + M\sum_{s=1}^{4}\hat{\beta}^1_{\tau,s}D_{\tau,s} + M\sum_{j=4}^{5}\hat{\beta}^1_{op,j}D_{op,s} + M\hat{u}^1_{itk\tau} \tag{28}$$

2. Estimate $\sigma^2_t, \sigma^2_k, \sigma^2_{itk\tau}$ from the OLS residuals $\hat{u}^1_{itk\tau}$ using by OLS regressions.

3. Using the estimated variance covariance matrix $\hat{\Sigma}^1$ and $\hat{\nu}_o$, generate the data C^*_{m0}. Concretely,

the parameters in (26) are replaced by their estimates as:

$$\hat{\mu}_i^1 = C_s + \hat{\beta}_0^1 M + \sum_{s=1}^{4} \hat{\beta}_{\tau,s}^1 D_{\tau,s} + \sum_{j=4}^{5} \hat{\beta}_{op,j}^1 D_{op,s} \qquad i = 0,1 \tag{29}$$

and

$$\hat{\nu}_0^1 = \hat{\mu}_0 + \hat{\Sigma}_{21}\hat{\Sigma}_{11}^{-1}(C_{m1} - \hat{\mu}_1). \tag{30}$$

Then, generate random variables $\eta_{itk\tau}$ from the standard normal distribution, and set:

$$\hat{C}_{m0} = A\eta_{itk\tau} + \hat{\nu}_0^1. \tag{31}$$

where A is a matrix that satisfies:

$$\hat{\Sigma}^1 = A'A. \tag{32}$$

4. The truncation points in the second type truncation 2) are set to the minimum value of OLS residuals $\hat{u}_{itk\tau}^1$ for the lower bound and the maximum value of OLS residuals $\hat{u}_{itk\tau}^1$ for the upper bound (see Appendix). As a result, when $\hat{C}_{m0}$ satisfies one of the following conditions, we set $\hat{C}_{m0}^{*1} = \hat{C}_{m0}^*$:

$$1) \qquad \hat{C}_{m0} < 0 \tag{33}$$

$$2) \qquad M\hat{C}_{m0} - MC_s + \hat{\beta}_0^1 M^2 + M\sum_{s=1}^{4} \hat{\beta}_{\tau,s}^1 D_{\tau,s} + M\sum_{j=4}^{5} \hat{\beta}_{op,j}^1 D_{op,s} < M_{itk\tau}\hat{u}_{itk\tau}^{1,min} \tag{34}$$

$$M\hat{C}_{m0} - MC_s + \hat{\beta}_0^1 M^2 + M\sum_{s=1}^{4} \hat{\beta}_{\tau,s}^1 D_{\tau,s} + M\sum_{j=4}^{5} \hat{\beta}_{op,j}^1 D_{op,s} > M_{itk\tau}\hat{u}_{itk\tau}^{1,max} \tag{35}$$

5. After combining the generated data $\hat{C}_{m0}^{*1}$ with C_{m1}, conduct GLS.

6. Using the residual from GLS estimation, the variance matrix as well as other parameters(μ_i, ν_0) are reestimated and set to $\hat{\Sigma}^2, \hat{\mu}_i^2, \hat{\nu}_0^2$. These estimated parameters are used to generate $\hat{C}_{m0}^{*2}$.

7. The generated $\hat{\Sigma}^2$ and $\hat{C}_{m0}^{*2}$ are used to conduct GLS until convergence is achieved.

The obtained parameter estimates after convergence are regarded as the final estimates.

3.3 Estimation results

The OLS results using all observations including the one with $C_m = 0$ and the results of FGLS after convergence are shown in Table 4. The estimated model is:

$$\begin{aligned} M(C_{m,itk\tau} - C_{s,itk\tau}) &= \beta_0 M_{itk\tau}^2 + M\sum_{s=1}^{4} \beta_{\tau,s} D_{\tau,s} + M\sum_{j=4}^{5} \beta_{op,j} D_{op,s} + Mu_{itk\tau} \\ C_{m,itk\tau} &= \begin{cases} C_{m,itk\tau}^* \\ 0 & \text{if} \quad a) \quad or \quad b) \end{cases} \\ u_{itk\tau} &= \epsilon_t + \epsilon_k + \epsilon_{itk\tau}. \end{aligned} \tag{36}$$

The coefficient of determination is calculated for the model (36).

Table 4: Estimation results in OLS and FGLS

	OLS			FGLS		
	BS	GARCH	EGARCH	BS	GARCH	EGARCH
β_0	64.76*	103.65*	133.55*	45.86*	100.36*	138.56*
	(27.00)	(25.41)	(26.29)	(0.093)	(0.10)	(0.097)
$\beta_{\tau,1}$	−134.28*	−220.43*	−251.63*	−139.47*	−245.63*	−282.29*
	(33.85)	(31.86)	(32.96)	(0.088)	(0.10)	(0.097)
$\beta_{\tau,2}$	−95.48*	−172.51*	−205.71*	−95.10*	−194.47*	−233.48*
	(34.82)	(32.77)	(33.91)	(0.093)	(0.11)	(0.10)
$\beta_{\tau,3}$	−94.55*	−161.76*	−191.78*	−98.40*	−189.39*	−223.62*
	(35.35)	(33.27)	(34.42)	(0.090)	(0.10)	(0.099)
$\beta_{\tau,4}$	−83.60*	−139.77*	−172.24*	−89.12*	−163.08*	−200.48*
	(36.16)	(34.02)	(35.21)	(0.094)	(0.11)	(0.10)
$\beta_{op,4}$	47.46*	40.91*	42.37*	56.88*	56.65*	55.43*
	(14.42)	(13.57)	(14.04)	(0.083)	(0.091)	(0.086)
$\beta_{op,5}$	26.48*	22.95	22.54	20.17*	17.73*	16.73*
	(13.03)	(12.26)	(12.69)	(0.088)	(0.096)	(0.091)
σ_t^2	-	-	-	14477.50*	14011.60*	13984.91*
				(752.43)	(577.22)	(569.79)
σ_k^2	-	-	-	2673.17*	4317.85*	4606.87*
				(499.68)	(1247.85)	(1179.85)
σ_{itk}^2				22662.98*	16040.23*	18246.15*
				(4234.86)	(3004.94)	(5029.61)
R^2	0.021	0.046	0.051	0.024	0.049	0.054

The estimation was conducted using 968 data observations from January 2000 to April 2000. The number of truncated data is 74 and the omitted number of data of 0 yen is 894. The numerical value in parentheses shows the standard deviation. In each model, the frequency of iteration was four times.

When the two estimation results are compared, the sizes and the signs of the estimated coefficients are generally found to be similar. The results of FGLS have higher significance for the estimates and R^2 are slightly larger . In Table 6, β_0 and all $\beta_{op,j}(j = 4, 5)$ are estimated positively and significantly and $\beta_{\tau,s}(s = 1, 2, 3, 4)$ are negatively and significantly estimated in the three models. The estimated values of $\beta_{\tau,s}(s = 1, 2, 3, 4)$ increase as the survival period shortens. As $\beta_{op,j}(j = 4, 5)$ are significantly positive in all models, the option prices with different transaction periods behave differently. The option price with 15 months' transaction period has negative deviation from the theoretical price. As the trading time horizon shortens, the size of the negative deviation becomes smaller.

σ_t^2 is approximately between 14000 and 14500, σ_k^2 is between 2600 and 4600 , and σ_{itk}^2 is between 16000 and 22600. That is, σ_k^2 is relatively small and $\sigma_{itk\tau}^2$ is large, and they are significantly estimated. The size of the variance component depending on the transaction date is large. From these estimates, it is confirmed that heteroscedasticity exists and it depends on the transaction date as well as the strike price.

Table 5: Simulation results of 0 value

		BS	GARCH	EGARCH
NK225(74 obs.)	0 truncation	52	47	52
	+ truncation	0	0	0
	- truncation	22	27	22

0 truncation is the case of $C_m < 0$. + truncation is the case of $DV > a$.
- truncation is the case of $DV < b(< 0)$

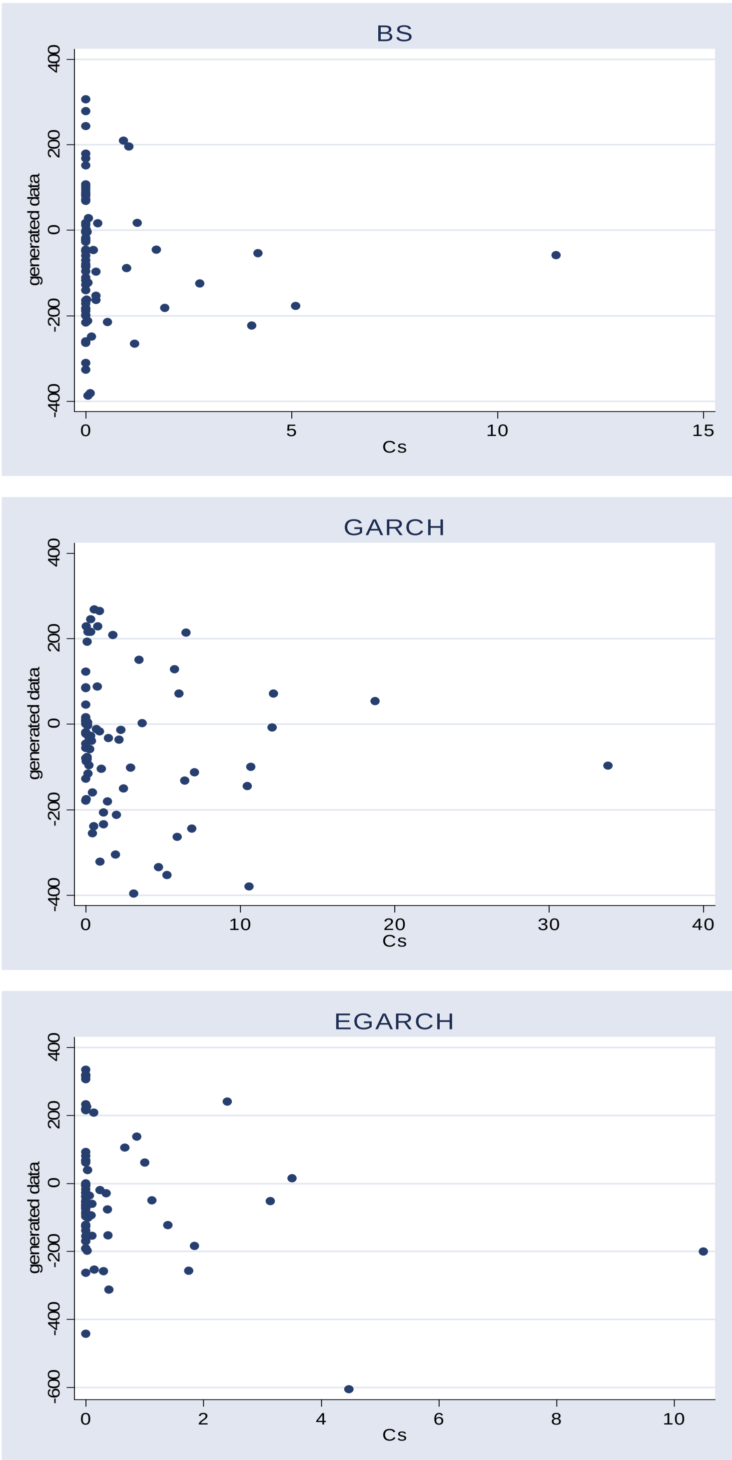
Table 5 classifies the truncation patterns for the simulated data. Table 5 shows that all of the potential market prices C_m^* are simulated as being truncated by either the condition a) (i.e.. $C_m < 0$)or b) (i.e.. $DV < b < 0$). That is, there is no potential market price C_m^* whose deviation DV is larger than a. Figure 7 depicts the plots of the generated data.

Finally, let us look at the distribution of $C_m - C_s$ when nil values of C_m are replaced by their estimated values from the FGLS estimation. First, it is observed from Table 6 that the mean shifts negatively by a large amount. This implies that the buyers of the options require a greater premium. The histogram of $C_m - C_s$ has changed, as illustrated in Figure 8. When nil values of C_m are replaced by their negative estimates, the distribution becomes more skewed to the right.

Table 6: Summary statistics for $C_m - C_s$ when $\hat{C}_{m0}^*$ are used

	mean	se	skewness	kurto
BS	-16.20	204.43	0.92	5.88
GARCH	-46.69	195.88	0.63	5.24
EGARCH	-46.32	203.21	0.41	5.14

Figure 7: Scatter diagram: $\hat{C}^*_{m0}$ vs. C_s



4 Conclusion

In this article, we investigated the Japanese option market and analyzed the difference between the market price and the theoretical price. The difference may be interpreted as a prediction error or a pricing error. Alternatively, it can be interpreted as the risk premium or the excess earnings. In either case, the difference will exist no matter what theoretical price is used, even when an accurate option theoretical price is obtained.

The estimation results showed that the difference depended on the moneyness, the strike price, and the survival period. Moreover, the variance of this difference depended on the strike price and the transaction data. As the survival period became longer, a negative bias of the difference increased. It is consistent with the earlier findings in the literature that the pricing error increases in its absolute value. Moreover, when the options with different trading periods were compared, the negative bias of the difference increased as trading period became longer. It implied that options with different maturities were priced differently. Though three models, i.e., BS, GARCH, and EGARCH models, were used to calculate the theoretical price, there were no notable differences in the signs of estimated coefficients and their relative sizes in the regression models of the difference between the market price and the theoretical price. As similar consequences were obtained from the three typical models, it is expected even if a model were used that could describe the movement of the underlying asset more accurately, the estimated relationship of the difference with the moneyness, the survival period, the trading period, and the striking price would not change drastically.

Some future research topics are as follows. First, the model was estimated by the FGLS method using simulation in this study. Though the normality of the error term is assumed, it is worth trying the other distributions that have heavier tails. Second, it is necessary to generalize the covariance structure of the model. If the difference is caused by investors' selection behavior, the correlation exists among options having different strike prices with the same maturity. Moreover, the correlation also exists among options with different maturities but on the same trading day. These factors were not taken into consideration and it is necessary to generalize the covariance structure to a more realistic one. Third, the option data analyzed in this paper are monthly data transformed from daily data. The maturity periods are limited between one month and four months. Since daily data are available and also the maturity periods in the market are as long as 15 months, it would be interesting to investigate the behavior of prices with shorter maturity (i.e., less than one month) using daily data or to investigate prices with longer maturities using the data with more than four months. Finally, the option market in Japan is said to be immature. A comparative study of the market with those is other countries would be interesting to see whether the results obtained in this paper are specific to Japan.

A Estimation of truncation point

To estimate the truncation point in the Tobit model, let us consider the likelihood function. We define:

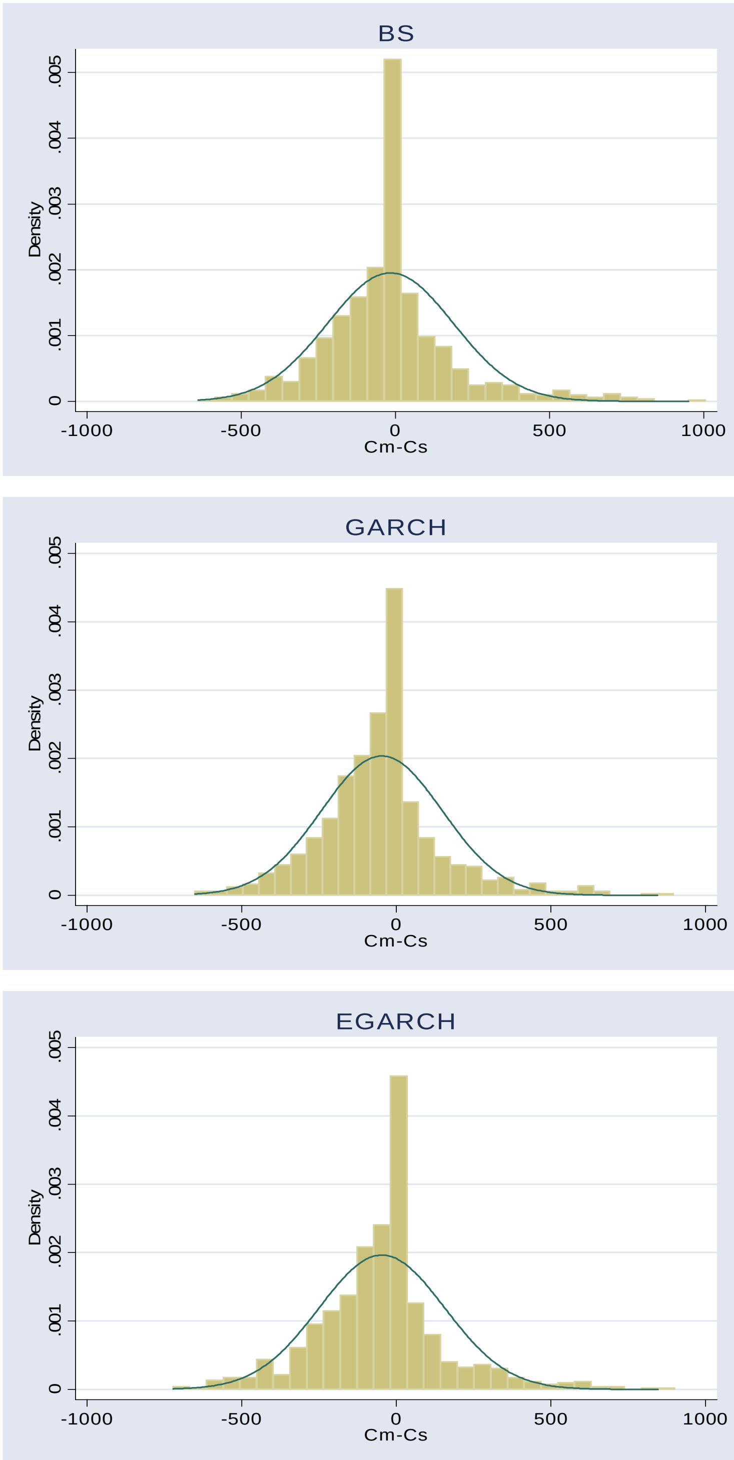
$$y_i = \begin{cases} y_i^* & \\ 0 & \text{if } y_i^* - \mu_i < b \end{cases} \tag{37}$$

$$y_i^* \sim N(\mu_i, \sigma^2), \tag{38}$$

where y_i^* is the latent variable and b is the truncation point. The likelihood function for a constant dispersion is:

$$\ln L = \sum_{b<y_i} -\frac{1}{2}\left(\ln(2\pi) + \ln\sigma^2 + \frac{(y_i-\mu)^2}{\sigma^2}\right)$$

Figure 8: Distribution of $C_m - C_s$ when $\hat{C}^*_{m0}$ are used



$$+\sum_{y_i=0} \ln\left(\int_{-\infty}^{b} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(y_i-\mu)^2}{2\sigma^2}\right\} dy_i\right). \tag{39}$$

By differentiating the likelihood function with respect to b, it becomes:

$$\begin{aligned}
\frac{\partial \ln L}{\partial b} &= \sum_{y_i=0} \frac{\partial}{\partial b} \ln \int_{-\infty}^{b} \sqrt{\frac{1}{2\pi\sigma^2}} \exp\left\{-\frac{(y_i-\mu_i)^2}{2\sigma^2}\right\} dy_i \\
&= \sum_{y_i=0} \frac{\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(b-\mu_i)^2}{2\sigma^2}\right\}}{\int_{-\infty}^{b} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(y_i-\mu_i)^2}{2\sigma^2}\right\} dy_i} > 0.
\end{aligned} \tag{40}$$

It can be seen that the likelihood function is monotonically increasing with respect to b. In addition, since the data whose values are smaller than b do not exist, the minimum value of the untruncated data becomes the estimated value of b. That is:

$$\hat{b} = min(y_i - \mu_i | y_i \text{ is observed}). \tag{41}$$

Next, consider the heteroscedasticity of y_i and;

$$y_i = \begin{cases} y_i^* & \\ 0 & \text{if } \frac{y_i^*-\mu_i}{\sigma_i} < b. \end{cases} \tag{42}$$

where

$$y_i^* \sim N(\mu_i, \sigma_i). \tag{43}$$

Truncation point b is defined for a normalized y_i^*. For simplification the data are converted as;

$$\tilde{y}_i = \begin{cases} \tilde{y}_i^* & \\ 0 & \text{if } \tilde{y}_i^* < b, \end{cases} \tag{44}$$

where

$$\tilde{y}_i^* = \frac{y_i^* - \mu}{\sigma_i} \sim N(0,1). \tag{45}$$

The Jacobean of this transformation is;

$$\frac{d\tilde{y}_i}{dy_i} = \frac{1}{\sigma_i}, \tag{46}$$

and the likelihood function for the transformed data can be shown as

$$\begin{aligned}
\ln L &= \sum_{b<y_i} -\frac{1}{2}\left[\ln(2\pi) + (\tilde{y}_i)^2\right] \\
&\quad + \sum_{y_i=0} \ln\left(\int_{-\infty}^{b} \sqrt{\frac{1}{2\pi}} \exp\left[-(\tilde{y}_i)^2\right] dy_i^*\right).
\end{aligned} \tag{47}$$

This likelihood function is monotonically increasing with respect to b, and the value of b is estimated as:

$$\hat{b} = \min(\tilde{y}_i | \ y_i \text{ is observed}). \tag{48}$$

In a general case where the correlation among y_i exists, and y_i is treated as in this paper, we must consider:

$$
\begin{aligned}
y_i &= \begin{cases} y_i^* & \\ 0 & \text{if } \frac{y_i^*-\mu_i}{\sigma_i} < b \end{cases} & (49)\\
Y &\sim N(\mu, \Sigma), & (50)
\end{aligned}
$$

where $Y = \{y_1, y_2, \ldots, y_{n-1}, y_n\}\prime$. In this case, the likelihood function cannot be expressed as simply as in equation (47). When the correlation exists, the likelihood function is expressed by:

$$
\begin{aligned}
\ln L &= \sum_{y_i > b_i} \left(-\frac{n}{2}\ln(2\pi) - \frac{1}{2}|\Sigma| - \frac{1}{2}(Y-\mu)'\Sigma^{-1}(Y-\mu) \right) \\
&\quad + \sum_{y_i=0} \ln \int_{-\infty}^{b_1} \cdots \int_{-\infty}^{b_n} (2\pi)^{-\frac{n}{2}}|\Sigma|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2}(Y-\mu)'\Sigma^{-1}(Y-\mu) \right\} dy_1 \ldots dy_n
\end{aligned} \tag{51}
$$

where $b_i = b\sigma_i + \mu_i$. It is difficult to maximize such a likelihood function directly with respect to the parameters because the integration of the second term is complicated. We have transformed y_i in the same way as above:

$$
\begin{aligned}
\tilde{y}_i &= \begin{cases} y_i^* & \\ 0 & \text{if } \tilde{y}_i^* < b \end{cases} & (52)\\
\tilde{Y}^* &= \tilde{\Sigma}^{-\frac{1}{2}}(Y^* - \mu) \sim N(0, R) & (53)
\end{aligned}
$$

$$
\tilde{\Sigma} = \begin{pmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{pmatrix}, \tag{54}
$$

where $R = \tilde{\Sigma}^{-\frac{1}{2}}\Sigma\tilde{\Sigma}^{-\frac{1}{2}}$. The corresponding Jacobean is:

$$
\frac{d\tilde{Y}}{dY} = \tilde{\Sigma}^{-\frac{1}{2}}. \tag{55}
$$

The likelihood function of the transformed variable $\tilde{Y}$ is:

$$
\begin{aligned}
\ln L &= \sum_{\tilde{y}_i > b} |\tilde{\Sigma}^{-\frac{1}{2}}| \left(-\frac{n}{2}\ln(2\pi) - \frac{1}{2}|R| - \frac{1}{2}\tilde{Y}'R^{-1}\tilde{Y} \right) \\
&\quad + \sum_{\tilde{y}_i=0} \ln \int_{-\infty}^{b} \cdots \int_{-\infty}^{b} |\tilde{\Sigma}^{-\frac{1}{2}}|(2\pi)^{-\frac{n}{2}}|R|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2}\tilde{Y}'R^{-1}\tilde{Y} \right\} d\tilde{y}_1 \ldots d\tilde{y}_n.
\end{aligned} \tag{56}
$$

At this time, where the likelihood function of equation (51) is differentiated w.r.t. b, it can be easily seen that:

$$
\frac{\partial \ln L}{\partial b} = \sum_{y_i=0} \frac{|\tilde{\Sigma}^{-\frac{1}{2}}|(2\pi)^{-\frac{n}{2}}|R|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2}\tilde{Y}'R^{-1}\tilde{Y} \right\}}{\int_{-\infty}^{b} \cdots \int_{-\infty}^{b} |\tilde{\Sigma}^{-\frac{1}{2}}|(2\pi)^{-\frac{n}{2}}|R|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2}\tilde{Y}'R^{-1}\tilde{Y} \right\} d\tilde{y}_1 \ldots d\tilde{y}_n} > 0. \tag{57}
$$

That is, the likelihood function is monotonically increasing with b. Clearly from the definition of b, we can choose:

$$\hat{b} \quad = \quad \min(\tilde{y}_i | \ \tilde{y}_i \text{ is observed}), \tag{58}$$

in order to maximize the likelihood function.

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