

# A Catalogue of Sturm-Liouville Differential Equations

W. Norrie Everitt

*Dedicated to all scientists who, down the long years,  
have contributed to Sturm-Liouville theory.*

**Abstract.** This catalogue commences with sections devoted to a brief summary of Sturm-Liouville theory including some details of differential expressions and equations, Hilbert function spaces, differential operators, classification of interval endpoints, boundary condition functions and the Liouville transform.

There follows a collection of more than 50 examples of Sturm-Liouville differential equations; many of these examples are connected with well-known special functions, and with problems in mathematical physics and applied mathematics.

For most of these examples the interval endpoints are classified within the relevant Hilbert function space, and boundary condition functions are given to determine the domains of the relevant differential operators. In many cases the spectra of these operators are given.

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