

# Bottom-Up Parsing

CS143

Lecture 8

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Slide design by Prof. Alex Aiken, with modifications

# Bottom-Up Parsing

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- Bottom-up parsing is more general than top-down parsing
  - And just as efficient
  - Builds on ideas in top-down parsing
- Bottom-up is the preferred method
- Concepts today, algorithms next time

# An Introductory Example

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- Bottom-up parsers don't need left-factored grammars
- Revert to the “natural” grammar for our example:  
$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$
- Consider the string:  $\text{int} * \text{int} + \text{int}$

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

## The Idea

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Bottom-up parsing reduces a string to the start symbol by inverting productions:

int \* int + int

$T \rightarrow \text{int}$

int \* T + int

$T \rightarrow \text{int} * T$

T + int

$T \rightarrow \text{int}$

T + T

$E \rightarrow T$

T + E

$E \rightarrow T + E$

E

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

## Observation

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- Read the productions in reverse (bottom to top)
- This is a reverse rightmost derivation!

int \* int + int

$T \rightarrow \text{int}$

int \* T + int

$T \rightarrow \text{int} * T$

T + int

$T \rightarrow \text{int}$

T + T

$E \rightarrow T$

T + E

$E \rightarrow T + E$

E

# Important Fact #1

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Important Fact #1 about bottom-up parsing:

A bottom-up parser traces a rightmost derivation in  
reverse

# A Bottom-up Parse

$E \rightarrow T + E \mid T$

$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

int \* int + int

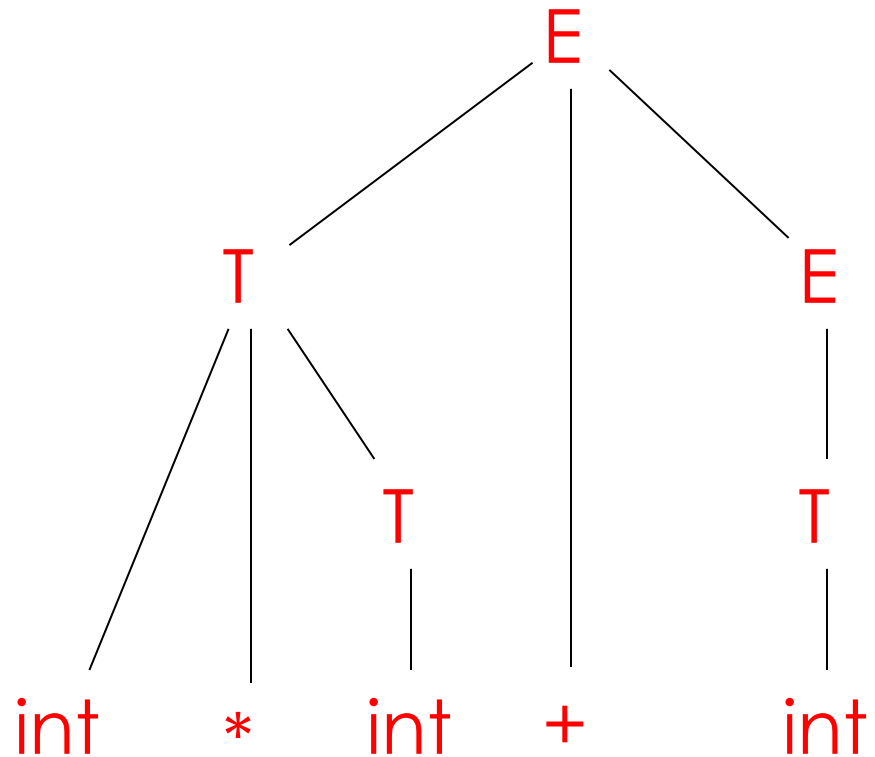
int \* T + int

T + int

T + T

T + E

E



# A Bottom-up Parse in Detail (1)

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int \* int + int

int \* int + int

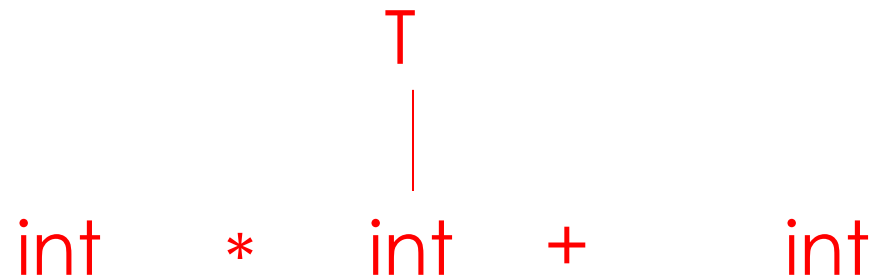


# A Bottom-up Parse in Detail (2)

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int \* int + int

int \* T + int



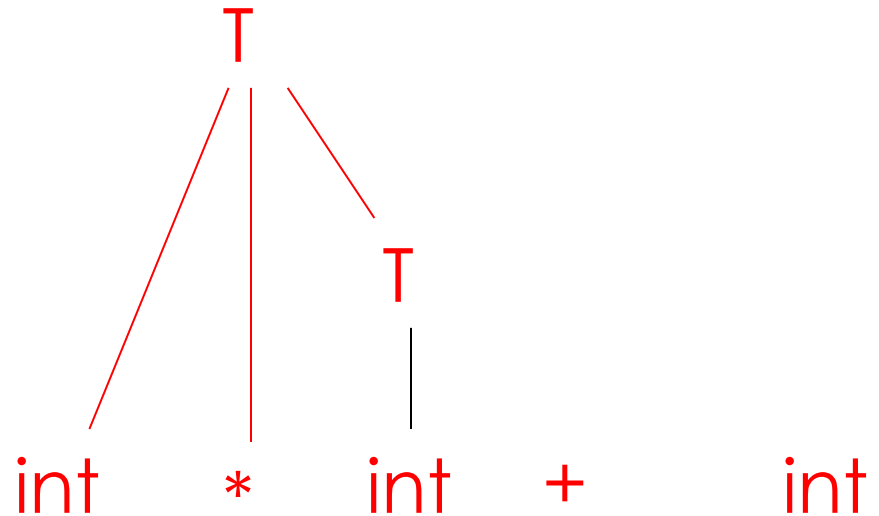
# A Bottom-up Parse in Detail (3)

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int \* int + int

int \* T + int

T + int



# A Bottom-up Parse in Detail (4)

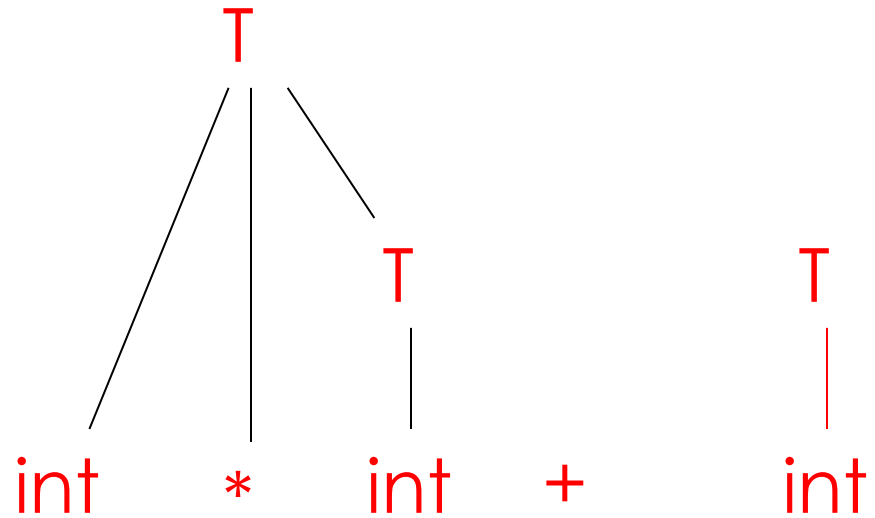
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int \* int + int

int \* T + int

T + int

T + T



# A Bottom-up Parse in Detail (5)

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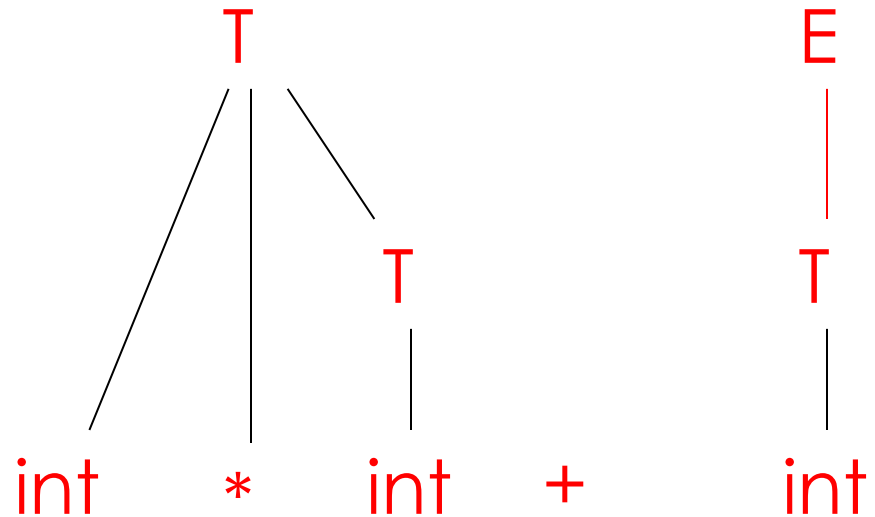
int \* int + int

int \* T + int

T + int

T + T

T + E



# A Bottom-up Parse in Detail (6)

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int \* int + int

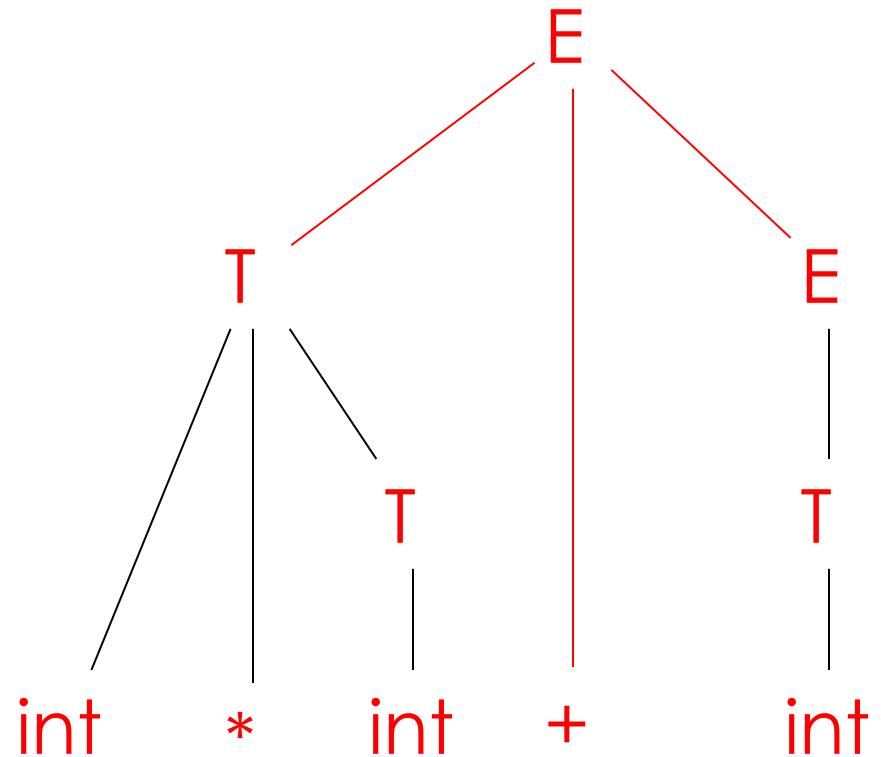
int \* T + int

T + int

T + T

T + E

E



# Where Do Reductions Happen?

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Important Fact #1 has an interesting consequence:

- Let  $\alpha\beta\omega$  be a step of a bottom-up parse
- Assume the next reduction is by  $X \rightarrow \beta$
- Then  $\omega$  is a string of terminals

Why? Because  $\alpha X \omega \rightarrow \alpha\beta\omega$  is a step in a right-most derivation

# Notation

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- Idea: Split string into two substrings
  - Right substring is as yet unexamined by parsing (a string of terminals)
  - Left substring has terminals and non-terminals
- The dividing point is marked by a |
  - The | is not part of the string
- Initially, all input is unexamined  $|x_1x_2 \dots x_n$

# Shift-Reduce Parsing

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Bottom-up parsing uses only two kinds of actions:

Shift

Reduce



# Shift

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- Shift: Move | one place to the right
  - Shifts a terminal to the left string

ABC|xyz  $\Rightarrow$  ABCx|yz

# Reduce

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- Apply an inverse production at the right end of the left string
  - If  $A \rightarrow xy$  is a production, then

$$Cbxylijk \Rightarrow CbAlijk$$

# The Example with Reductions Only

---

int \* int | + int

reduce  $T \rightarrow \text{int}$

int \* T | + int

reduce  $T \rightarrow \text{int} * T$

T + int |

reduce  $T \rightarrow \text{int}$

T + T |

reduce  $E \rightarrow T$

T + E |

reduce  $E \rightarrow T + E$

E |

# The Example with Shift-Reduce Parsing

---

| int \* int + int

shift

int | \* int + int

shift

int \* | int + int

shift

int \* int | + int

reduce  $T \rightarrow \text{int}$

int \* T | + int

reduce  $T \rightarrow \text{int} * T$

T | + int

shift

T + | int

shift

T + int |

reduce  $T \rightarrow \text{int}$

T + T |

reduce  $E \rightarrow T$

T + E |

reduce  $E \rightarrow T + E$

E |

# A Shift-Reduce Parse in Detail (1)

---

| int \* int + int

int \* int + int  
↑

# A Shift-Reduce Parse in Detail (2)

---

| int \* int + int

int | \* int + int

int \* int + int  
↑

# A Shift-Reduce Parse in Detail (3)

---

| int \* int + int

int | \* int + int

int \* | int + int

int \* int + int  
          ↑

# A Shift-Reduce Parse in Detail (4)

---

| int \* int + int

int | \* int + int

int \* | int + int

int \* int | + int

int \* int + int  
                  ↑



# A Shift-Reduce Parse in Detail (5)

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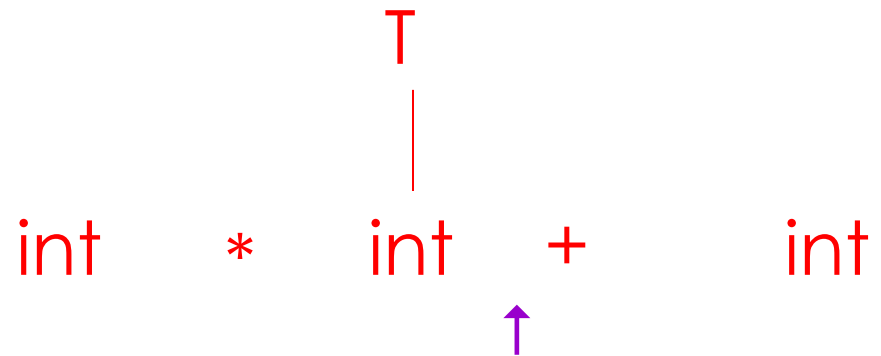
| int \* int + int

int | \* int + int

int \* | int + int

int \* int | + int

int \* T | + int



# A Shift-Reduce Parse in Detail (6)

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| int \* int + int

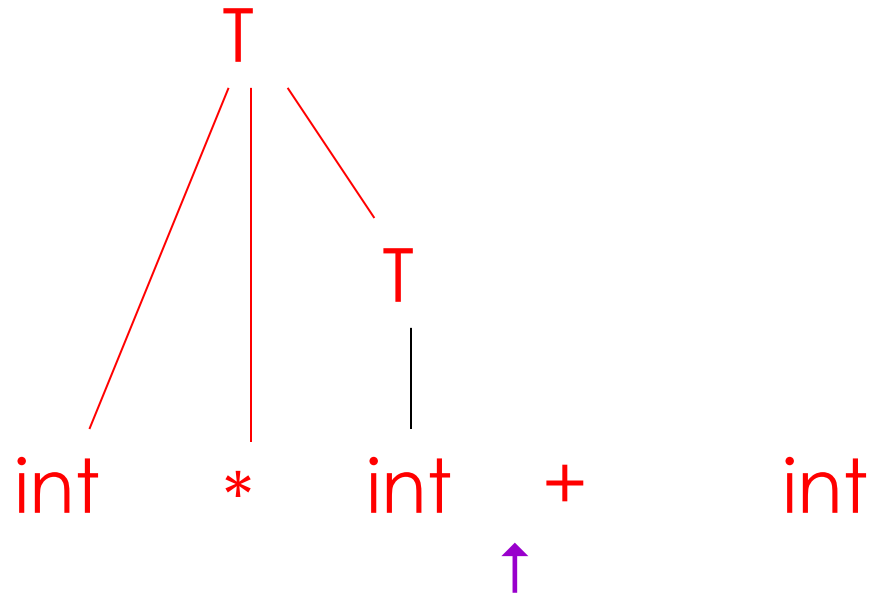
int | \* int + int

int \* | int + int

int \* int | + int

int \* T | + int

T | + int



# A Shift-Reduce Parse in Detail (7)

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| int \* int + int

int | \* int + int

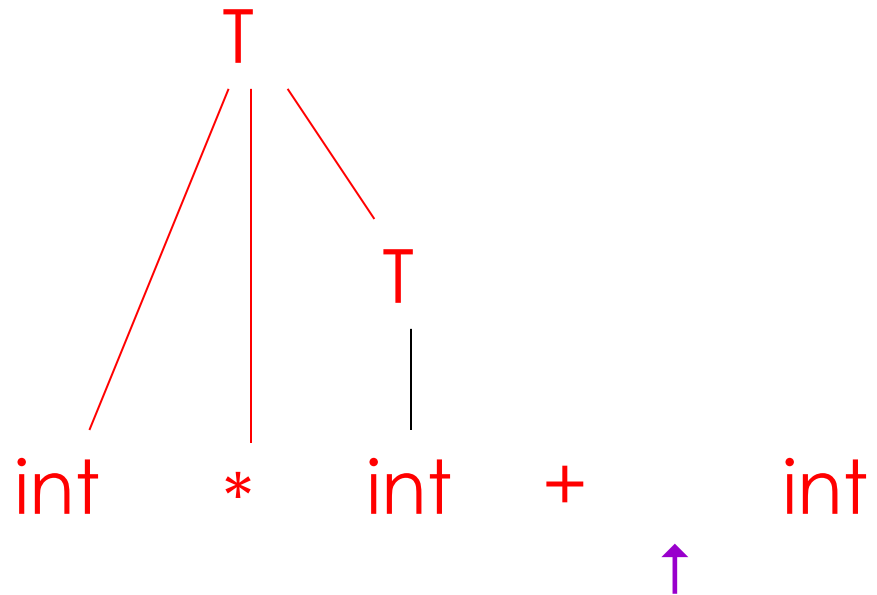
int \* | int + int

int \* int | + int

int \* T | + int

T | + int

T + | int



# A Shift-Reduce Parse in Detail (8)

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| int \* int + int

int | \* int + int

int \* | int + int

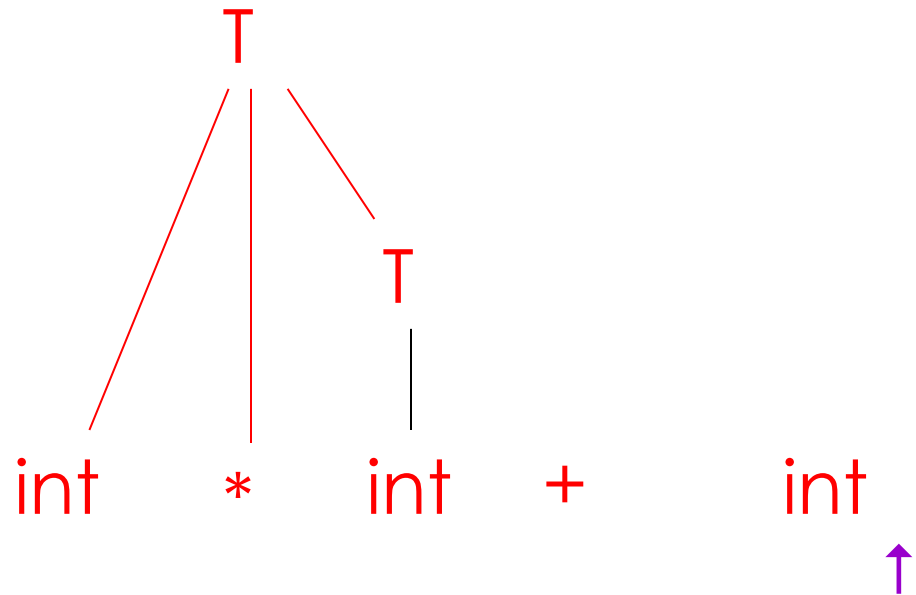
int \* int | + int

int \* T | + int

T | + int

T + | int

T + int |



# A Shift-Reduce Parse in Detail (9)

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| int \* int + int

int | \* int + int

int \* | int + int

int \* int | + int

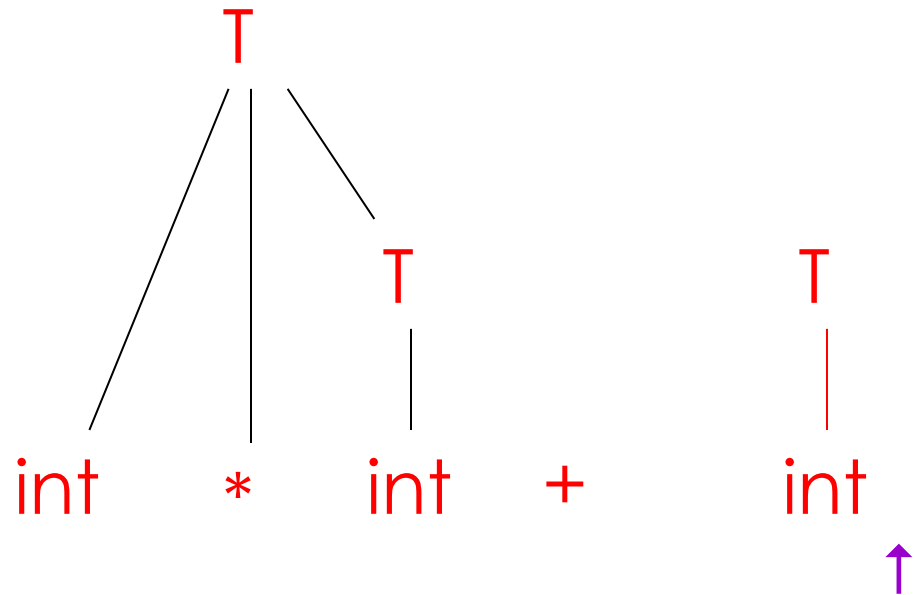
int \* T | + int

T | + int

T + | int

T + int |

T + T |



# A Shift-Reduce Parse in Detail (10)

| int \* int + int

int | \* int + int

int \* | int + int

int \* int | + int

int \* T | + int

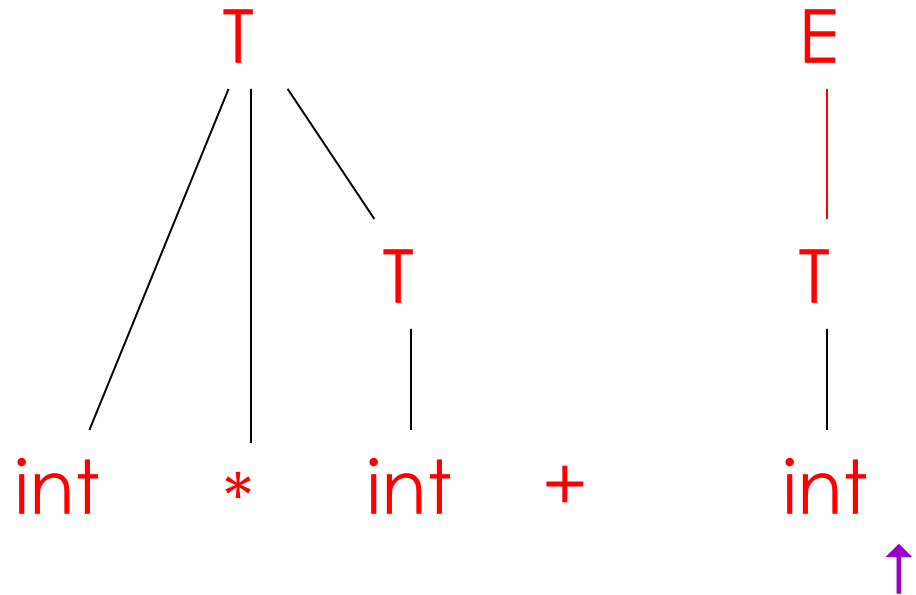
T | + int

T + | int

T + int |

T + T |

T + E |



# A Shift-Reduce Parse in Detail (11)

| int \* int + int

int | \* int + int

int \* | int + int

int \* int | + int

int \* T | + int

T | + int

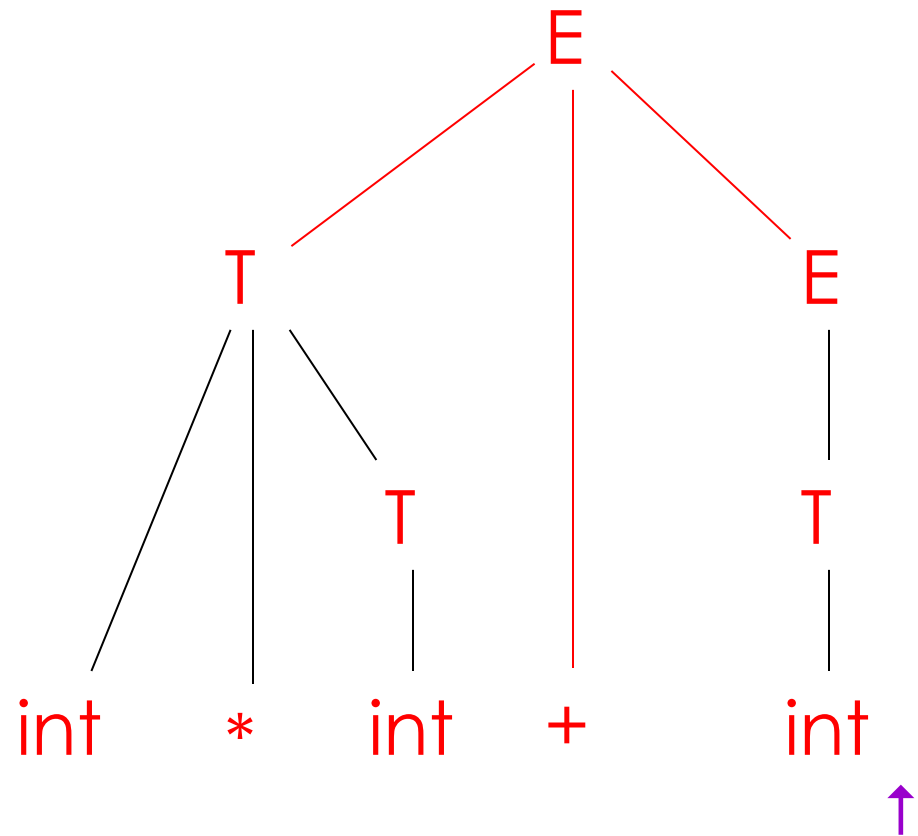
T + | int

T + int |

T + T |

T + E |

E |



# The Stack

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- Left string can be implemented by a stack
  - Top of the stack is the |
- Shift pushes a terminal on the stack
- Reduce pops 0 or more symbols off of the stack (production rhs) and pushes a non-terminal on the stack (production lhs)



# Conflicts

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- In a given state, more than one action (shift or reduce) may lead to a valid parse
- If it is legal to shift or reduce, there is a shift-reduce conflict
- If it is legal to reduce by two different productions, there is a reduce-reduce conflict
- You will see such conflicts in your project!
  - More next time . . .

# Key Issue

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- How do we decide when to shift or reduce?
- Example grammar:  
$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$
- Consider step  $\text{int} \mid * \text{int} + \text{int}$ 
  - We could reduce by  $T \rightarrow \text{int}$  giving  $T \mid * \text{int} + \text{int}$
  - A fatal mistake!
    - No way to reduce to the start symbol  $E$

# Definition: Handles

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- Intuition: Want to reduce only if the result can still be reduced to the start symbol

- Assume a rightmost derivation

$$S \rightarrow^* \alpha X \omega \rightarrow \alpha \beta \omega$$

- Then  $X \rightarrow \beta$  in the position after  $\alpha$  is a handle of  $\alpha \beta \omega$
- Can and must reduce at handles.

## Handles (Cont.)

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- Handles formalize the intuition
  - A handle is a string that can be reduced and also allows further reductions back to the start symbol (using a particular production at a specific spot)
- We only want to reduce at handles
- Note: We have said what a handle is, not how to find handles

## Important Fact #2

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Important Fact #2 about bottom-up parsing:

In shift-reduce parsing, handles appear only at the top of the stack, never inside

# Why?

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- Informal induction on # of reduce moves:
- True initially, stack is empty
- Immediately after reducing a handle
  - right-most non-terminal on top of the stack
  - next handle must be to right of right-most non-terminal, because this is a right-most derivation
  - Sequence of shift moves reaches next handle

# Summary of Handles

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- In shift-reduce parsing, handles always appear at the top of the stack
- Handles are never to the left of the rightmost non-terminal
  - Therefore, shift-reduce moves are sufficient; the **|** need never move left
- Bottom-up parsing algorithms are based on recognizing handles

# Recognizing Handles

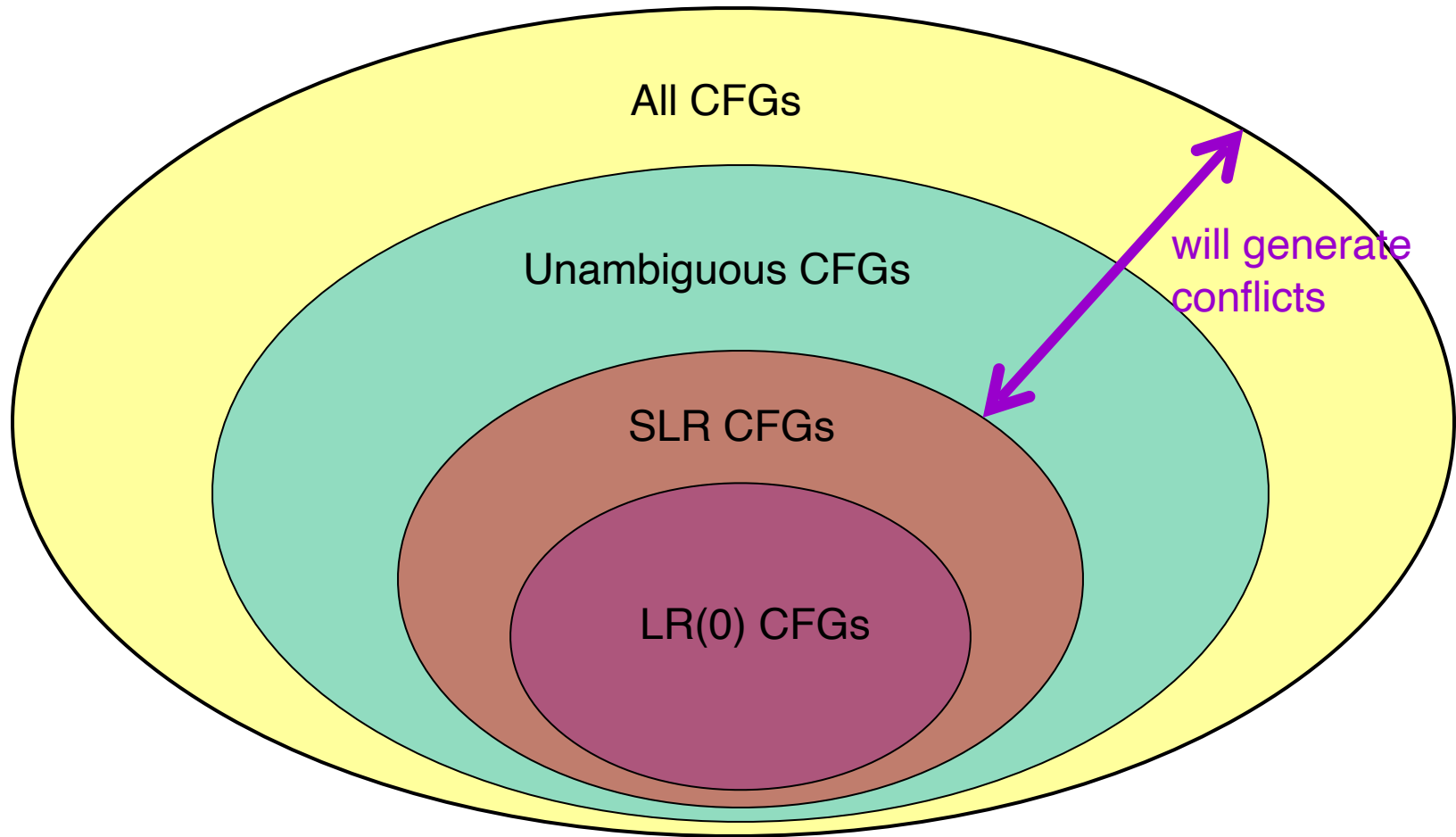
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- There are no known efficient algorithms to recognize handles
- Solution: use heuristics to guess which stacks are handles
- On some CFGs, the heuristics always guess correctly
  - For the heuristics we use here, these are the SLR grammars
  - Other heuristics work for other grammars



# Grammars

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# Viabale Prefixes

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- It is not obvious how to detect handles
- At each step the parser sees only the stack, not the entire input; start with that . . .

$\alpha$  is a viable prefix if there is an  $\omega$  such that  $\alpha|\omega$  is a state of a shift-reduce parser

# Huh?

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- What does this mean? A few things:
  - A viable prefix does not extend past the right end of the handle
  - It's a viable prefix because it is a prefix of the handle
  - As long as a parser has viable prefixes on the stack no parsing error has been detected

## Important Fact #3

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Important Fact #3 about bottom-up parsing:

For any SLR grammar, the set of viable prefixes is a regular language

## Important Fact #3 (Cont.)

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- Important Fact #3 is non-obvious
- We show how to compute automata that accept viable prefixes

# Items

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- An item is a production with a “.” somewhere on the rhs, denoting a focus point
- The items for  $T \rightarrow (E)$  are
  - $T \rightarrow \cdot(E)$
  - $T \rightarrow (\cdot E)$
  - $T \rightarrow (E \cdot)$
  - $T \rightarrow (E) \cdot$

## Items (Cont.)

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- The only item for  $X \rightarrow \varepsilon$  is  $X \rightarrow \cdot$ .
- Items are often called “LR(0) items”

# Intuition

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- The problem in recognizing viable prefixes is that the stack has only bits and pieces of the rhs of productions
  - If it had a complete rhs, we could reduce
- These bits and pieces are always prefixes of rhs of productions



$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

## Example

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Consider the input (int)

- Then (E | ) is a state of a shift-reduce parse
- (E is a prefix of the rhs of  $T \rightarrow (E)$ 
  - Will be reduced after the next shift
- Item  $T \rightarrow (E.)$  says that so far we have seen (E of this production and hope to see )

# Generalization

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- The stack may have many prefixes of rhs's  
 $\text{Prefix}_1 \text{Prefix}_2 \dots \text{Prefix}_{n-1} \text{Prefix}_n$
- Let  $\text{Prefix}_i$  be a prefix of rhs of  $X_i \rightarrow \alpha_i$ 
  - $\text{Prefix}_i$  will eventually reduce to  $X_i$
  - The missing part of  $\text{Prefix}_{i-1}$  of  $\alpha_{i-1}$  starts with  $X_i$
  - i.e. there is a  $X_{i-1} \rightarrow \text{Prefix}_{i-1} X_i \beta$  for some  $\beta$
- Recursively,  $\text{Prefix}_{k+1} \dots \text{Prefix}_n$  eventually reduces to the missing part of  $\alpha_k$

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

## An Example

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Consider the string  $(\text{int} * \text{int})$ :

$(\text{int} * \mid \text{int})$  is a state of a shift-reduce parse

From top of the stack:

“ $\epsilon$ ” is a prefix of the rhs of  $E \rightarrow T$

“(” is a prefix of the rhs of  $T \rightarrow (E)$

“ $\epsilon$ ” is a prefix of the rhs of  $E \rightarrow T$

“ $\text{int} *$ ” is a prefix of the rhs of  $T \rightarrow \text{int} * T$

$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

## An Example (Cont.)

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The stack of items

$$T \rightarrow \text{int} * .T$$

$$E \rightarrow .T$$

$$T \rightarrow (.E)$$

Says

We've seen  $\text{int} *$  of  $T \rightarrow \text{int} * T$

We've seen  $\epsilon$  of  $E \rightarrow T$

We've seen  $($  of  $T \rightarrow (E)$

# Recognizing Viable Prefixes

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Idea: To recognize viable prefixes, we must

- Recognize a sequence of partial rhs's of productions, where
- Each sequence can eventually reduce to part of the missing suffix of its predecessor

# An NFA Recognizing Viable Prefixes

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1. Add a new start production  $S' \rightarrow S$  to  $G$
2. The NFA states are the items of  $G$ 
  - (Including the new start production)
3. For item  $E \rightarrow \alpha.X\beta$  add transition
$$E \rightarrow \alpha.X\beta \xrightarrow{X} E \rightarrow \alpha X.\beta$$
4. For item  $E \rightarrow \alpha.X\beta$  and production  $X \rightarrow \gamma$  add
$$E \rightarrow \alpha.X\beta \xrightarrow{\varepsilon} X \rightarrow .\gamma$$

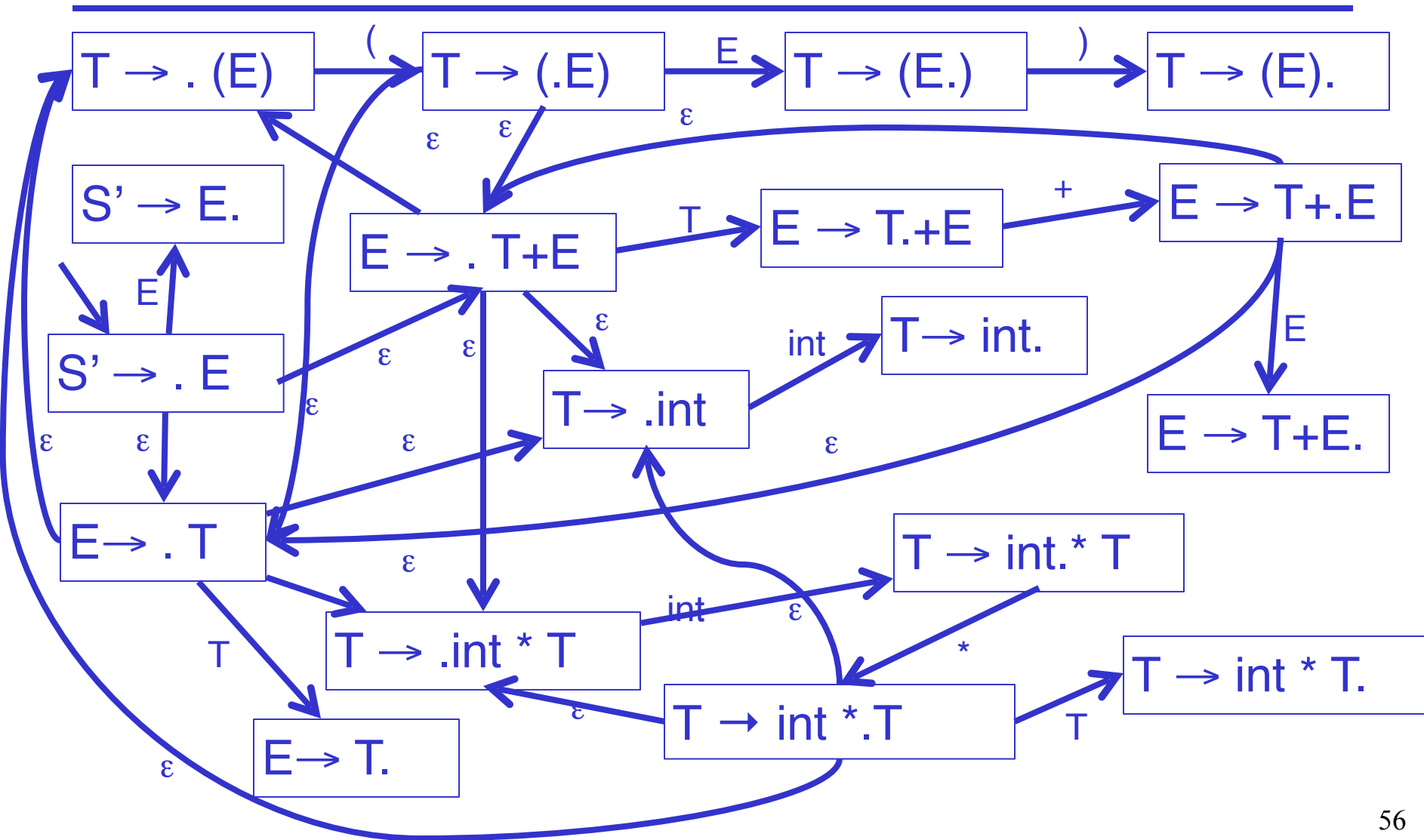
## An NFA Recognizing Viable Prefixes (Cont.)

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5. Every state is an accepting state
6. Start state is  $S' \rightarrow .S$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

# NFA for Viable Prefixes

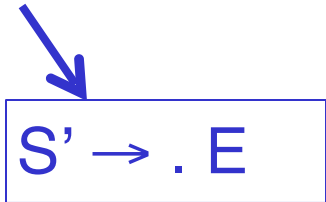




$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

# NFA for Viable Prefixes

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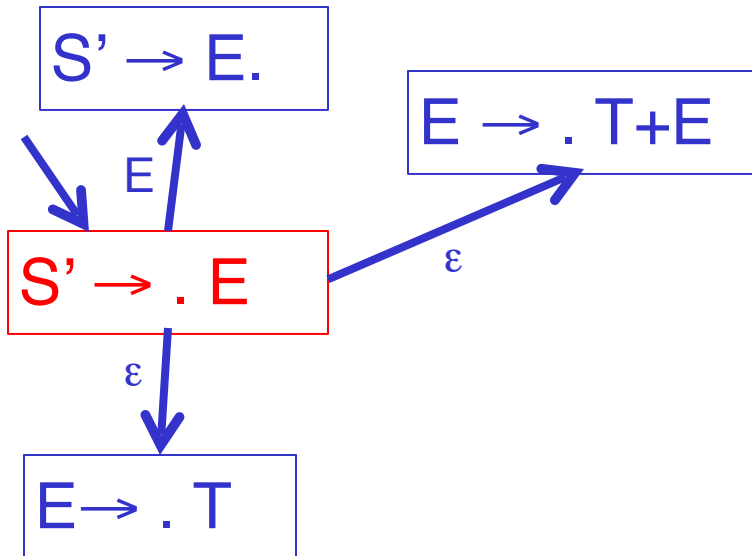


$S' \rightarrow \cdot E$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

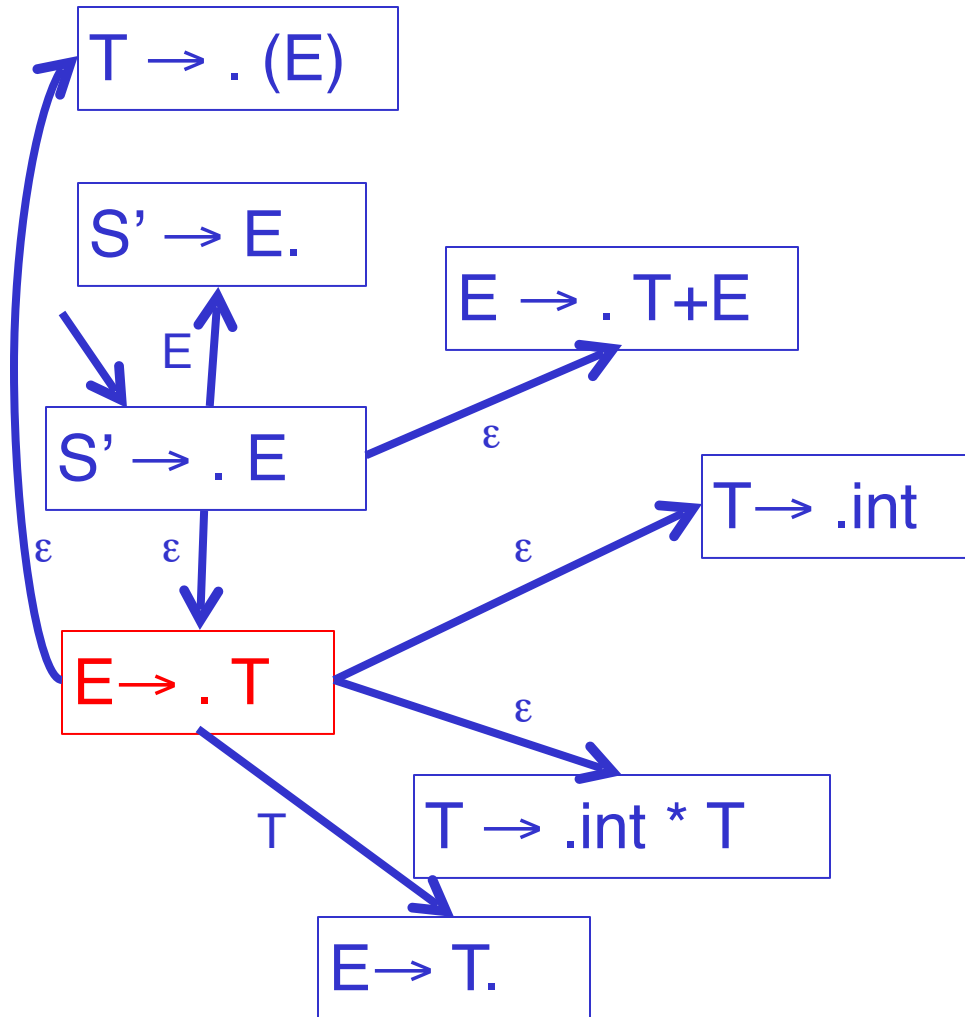
# NFA for Viable Prefixes

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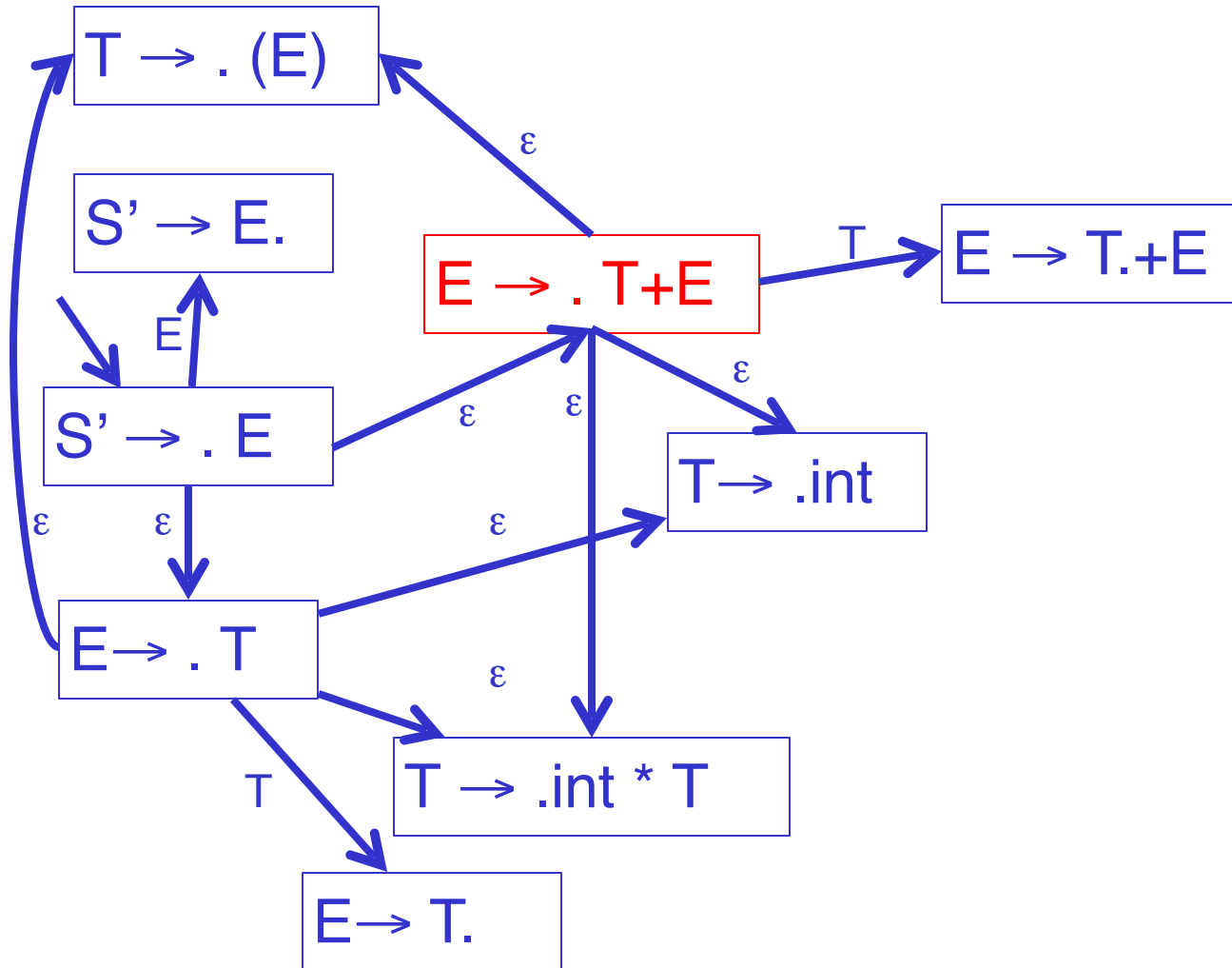
$E \rightarrow T + E \mid T$   
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## NFA for Viable Prefixes



$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

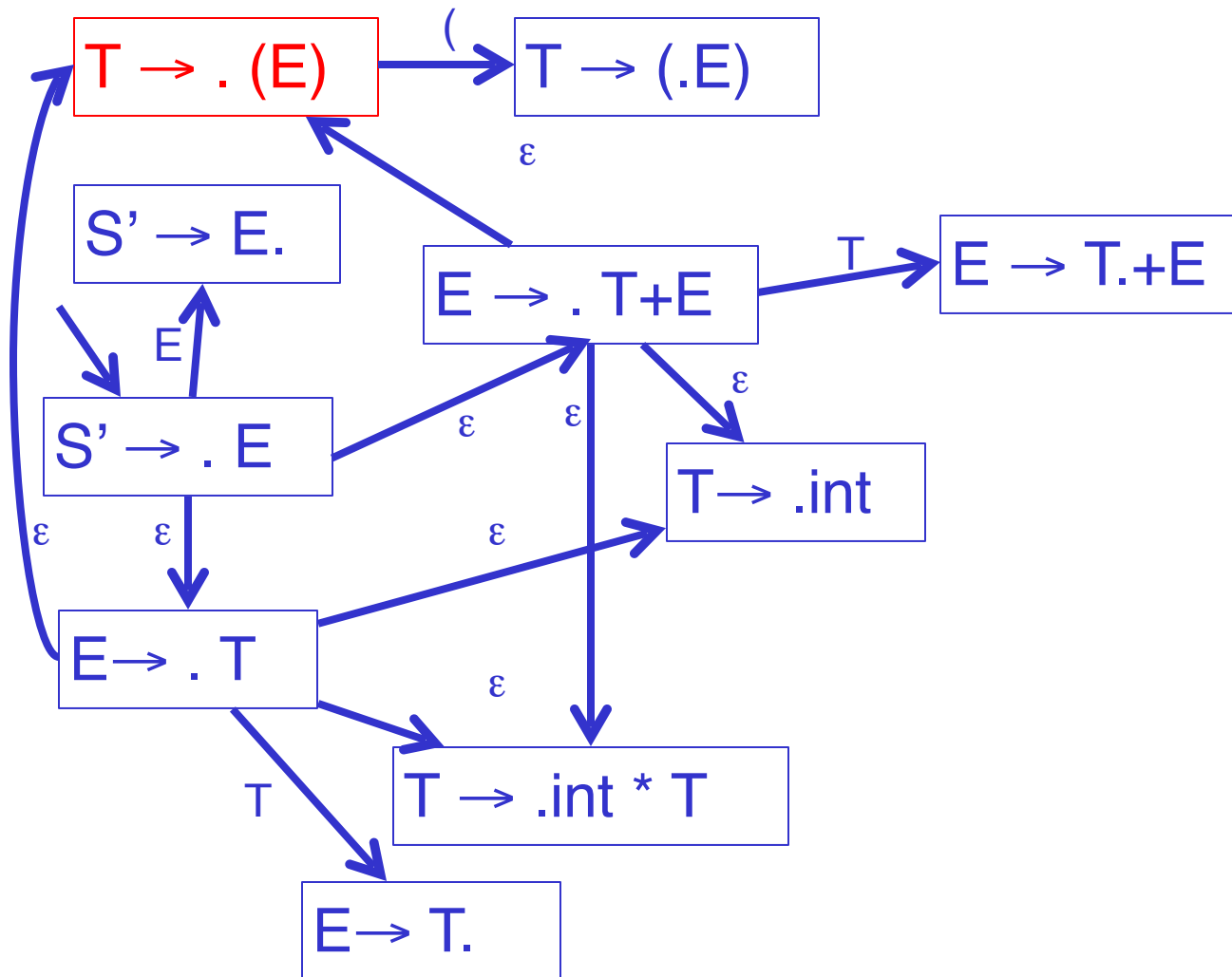
## NFA for Viable Prefixes



$$E \rightarrow T + E \mid T$$

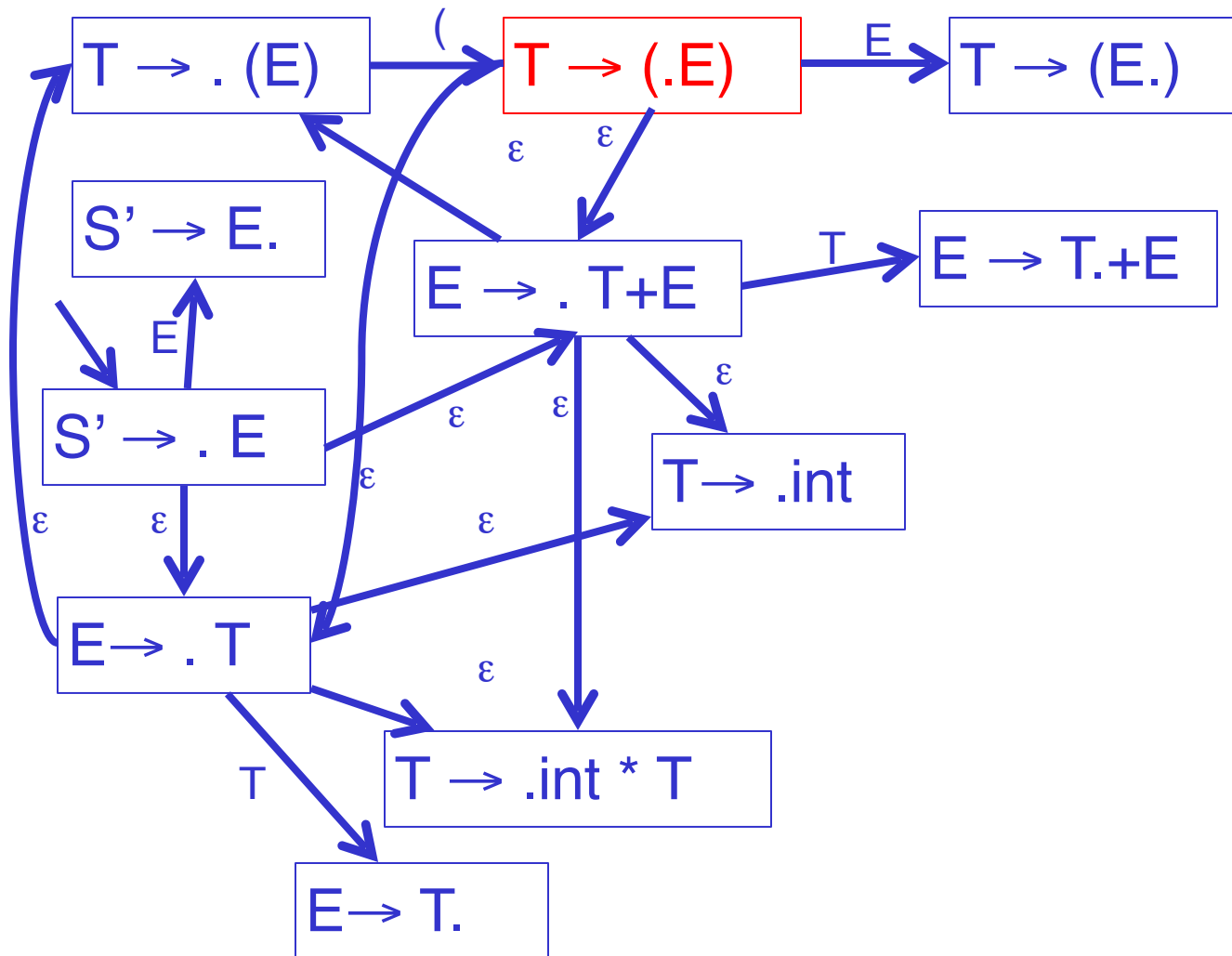
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

# NFA for Viable Prefixes



$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

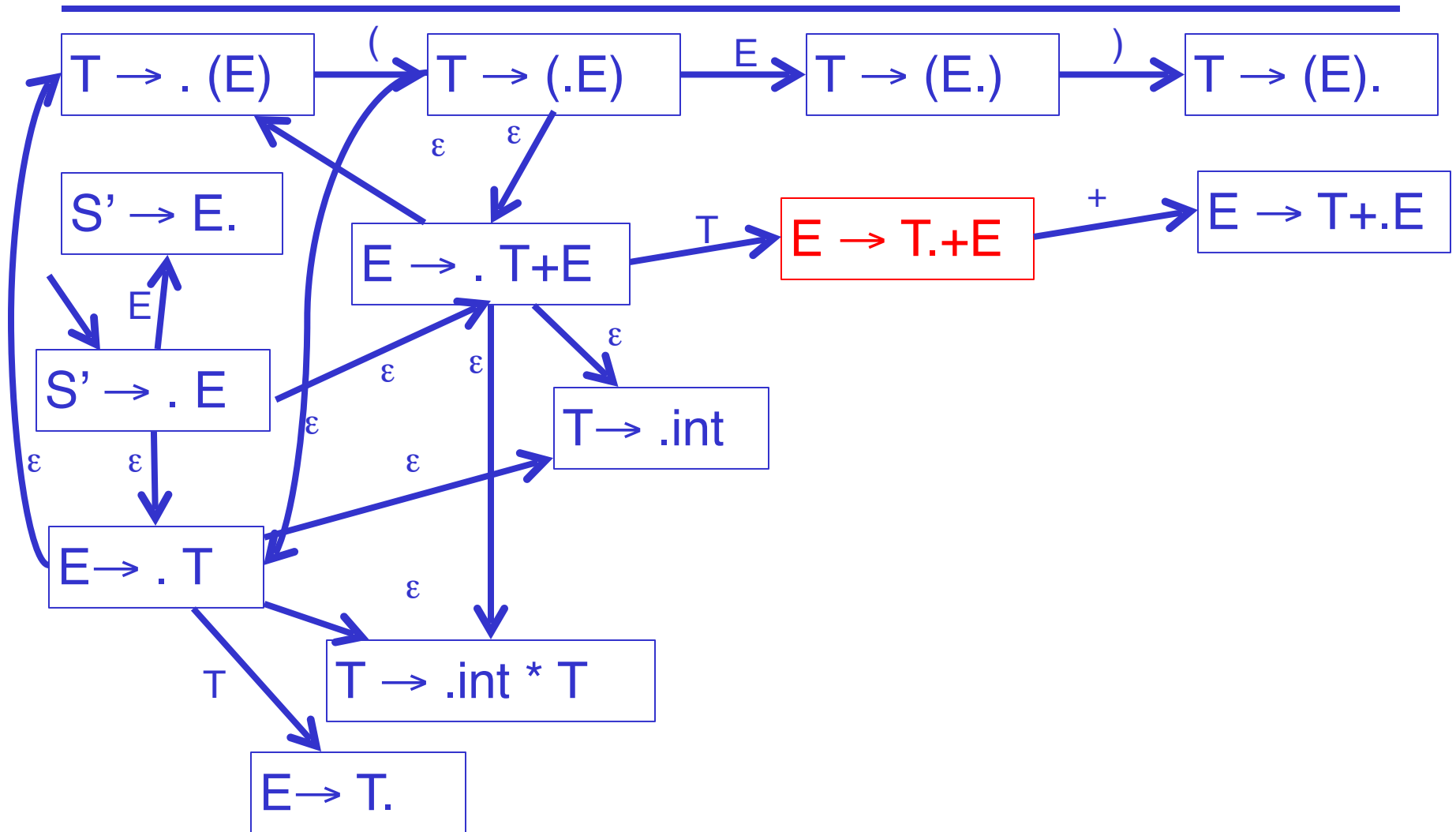
# NFA for Viable Prefixes





$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

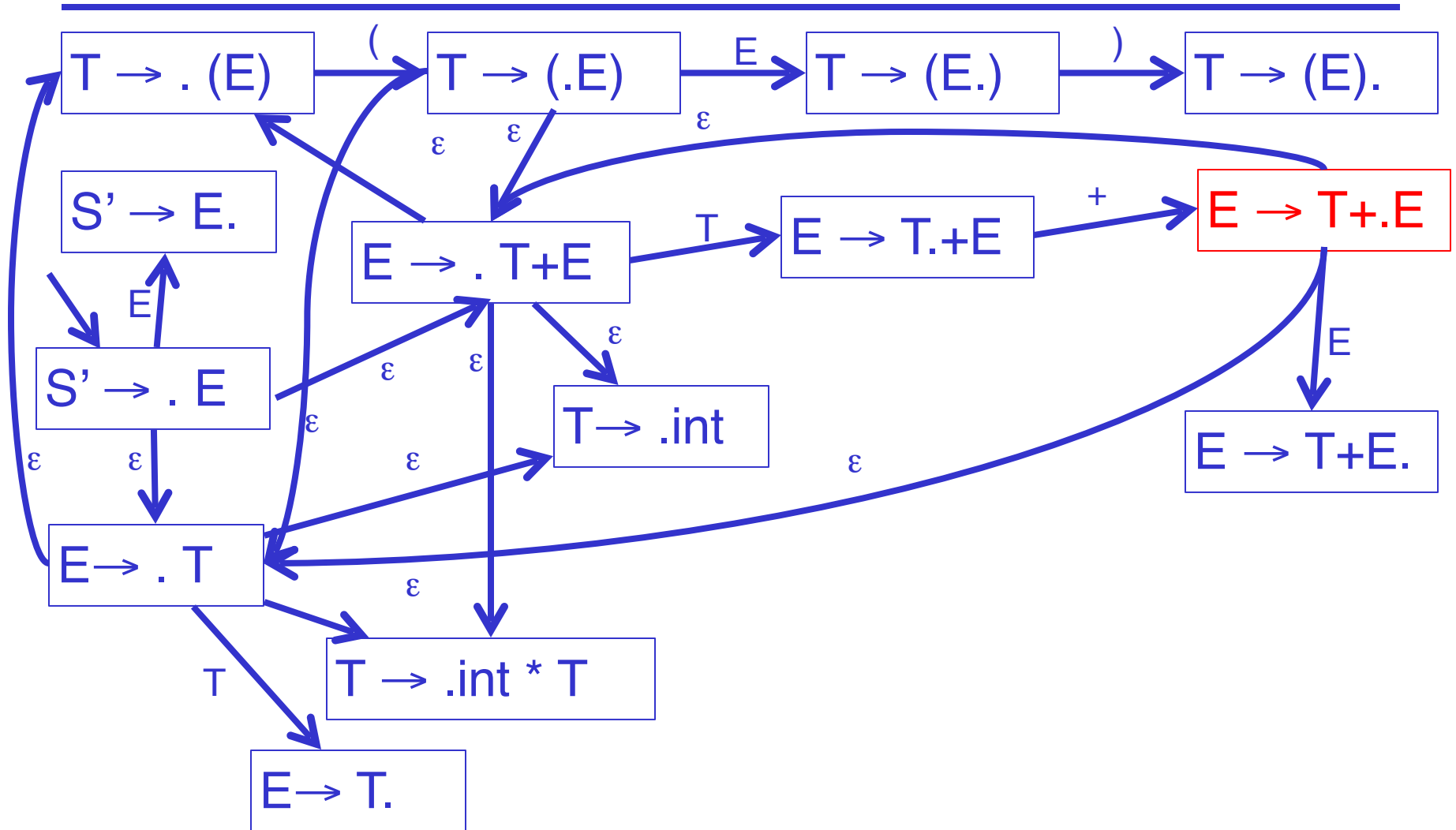
# NFA for Viable Prefixes





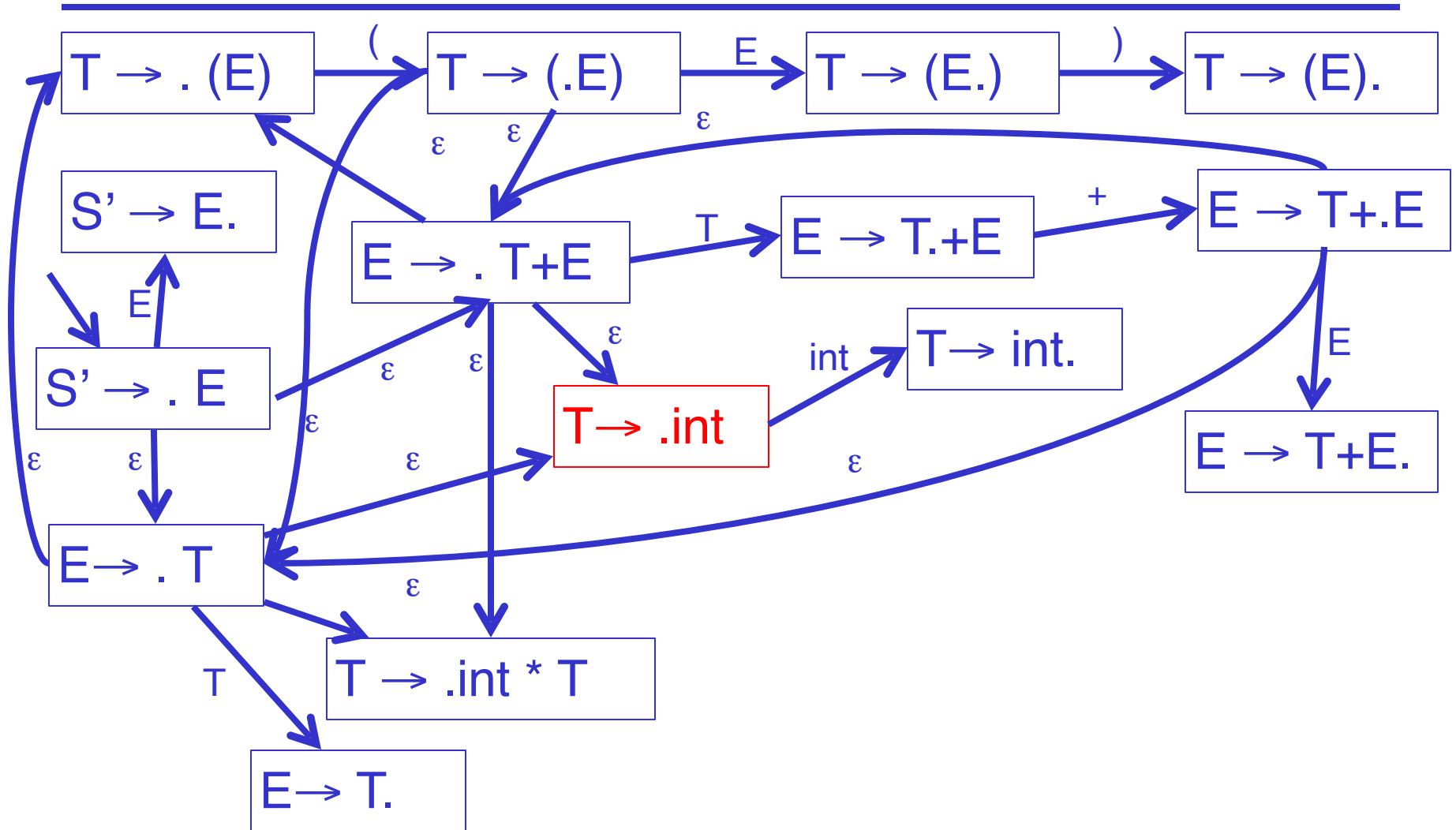
$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

# NFA for Viable Prefixes



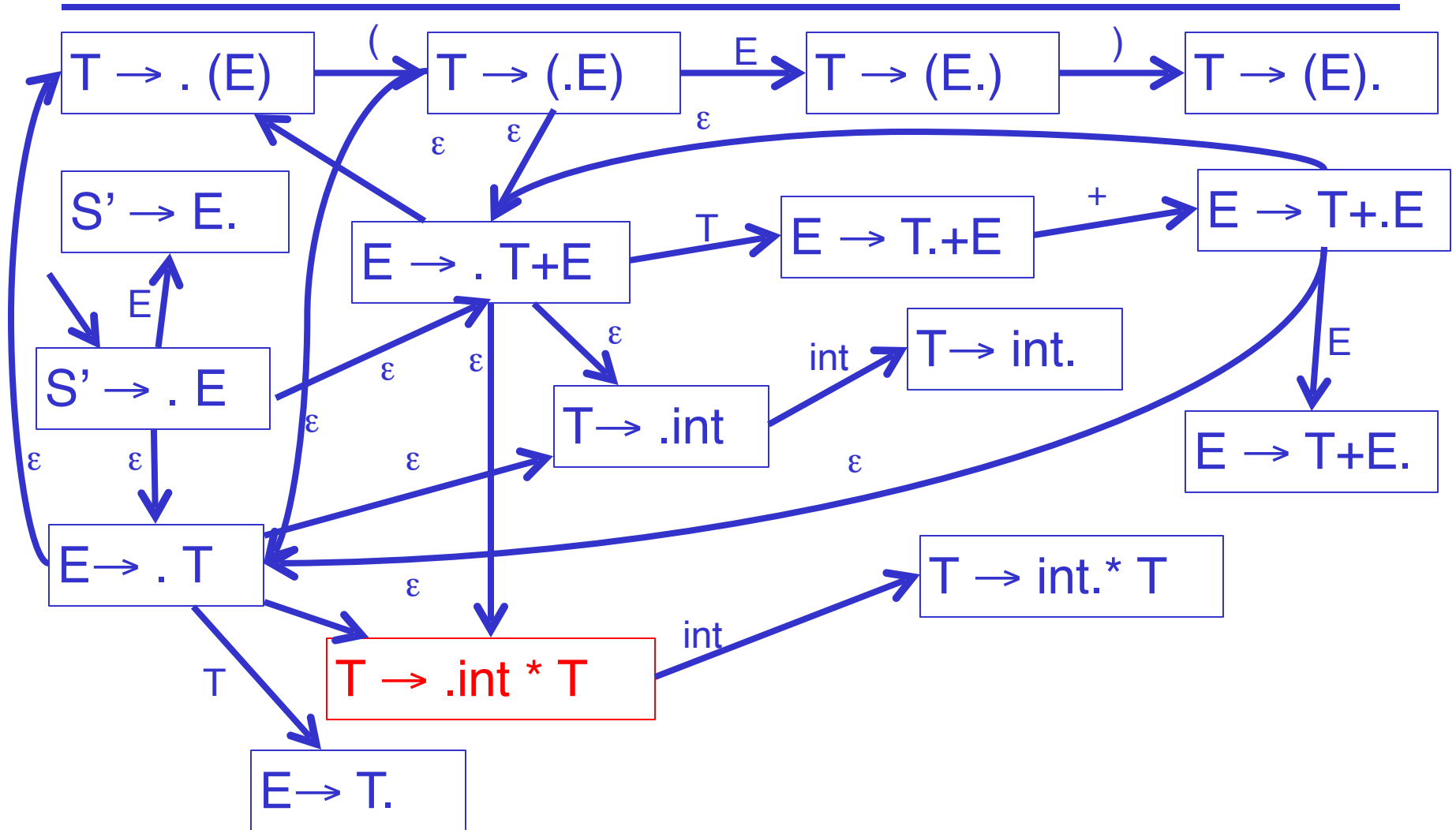
$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

# NFA for Viable Prefixes



$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

# NFA for Viable Prefixes

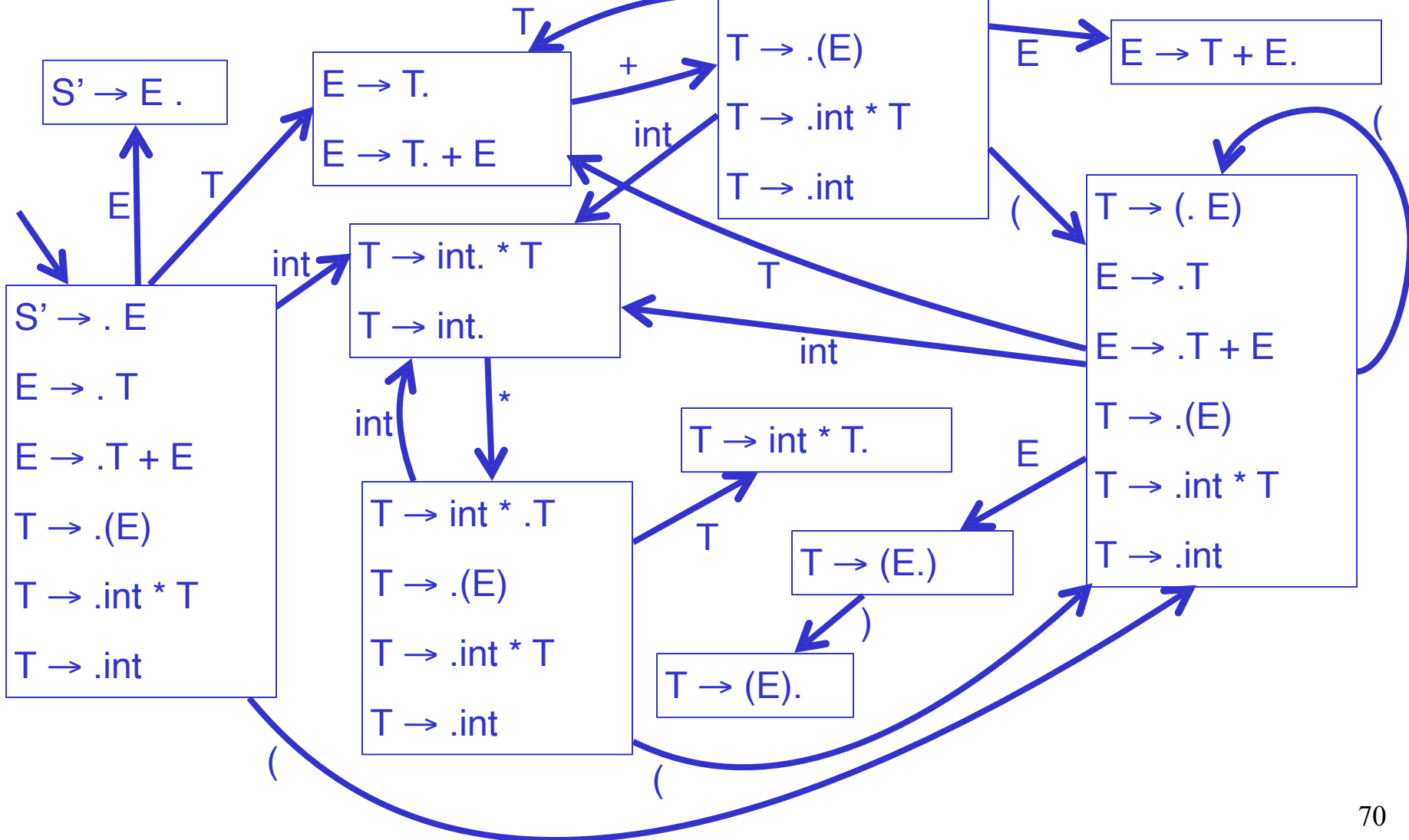






# Translation to the DFA

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



# Lingo

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The states of the DFA are

“canonical collections of items”

or

“canonical collections of LR(0) items”

The Dragon book gives another way of constructing LR(0) items

# Valid Items

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Item  $X \rightarrow \beta.\gamma$  is valid for a viable prefix  $\alpha\beta$  if

$$S' \rightarrow^* \alpha X \omega \rightarrow \alpha \beta \gamma \omega$$

by a right-most derivation

After parsing  $\alpha\beta$ , the valid items are the possible tops of the stack of items



# Items Valid for a Prefix

---

An item  $I$  is valid for a viable prefix  $\alpha$  if the DFA recognizing viable prefixes terminates on input  $\alpha$  in a state  $s$  containing  $I$

The items in  $s$  describe what the top of the item stack might be after reading input  $\alpha$

# Valid Items Example

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- An item is often valid for many prefixes
- Example: The item  $T \rightarrow (.E)$  is valid for prefixes

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...



# LR(0) Parsing

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- Idea: Assume
  - stack contains  $\alpha$
  - next input is  $t$
  - DFA on input  $\alpha$  terminates in state  $s$
- Reduce by  $X \rightarrow \beta$  if
  - $s$  contains item  $X \rightarrow \beta$ .
- Shift if
  - $s$  contains item  $X \rightarrow \beta.t\omega$
  - equivalent to saying  $s$  has a transition labeled  $t$

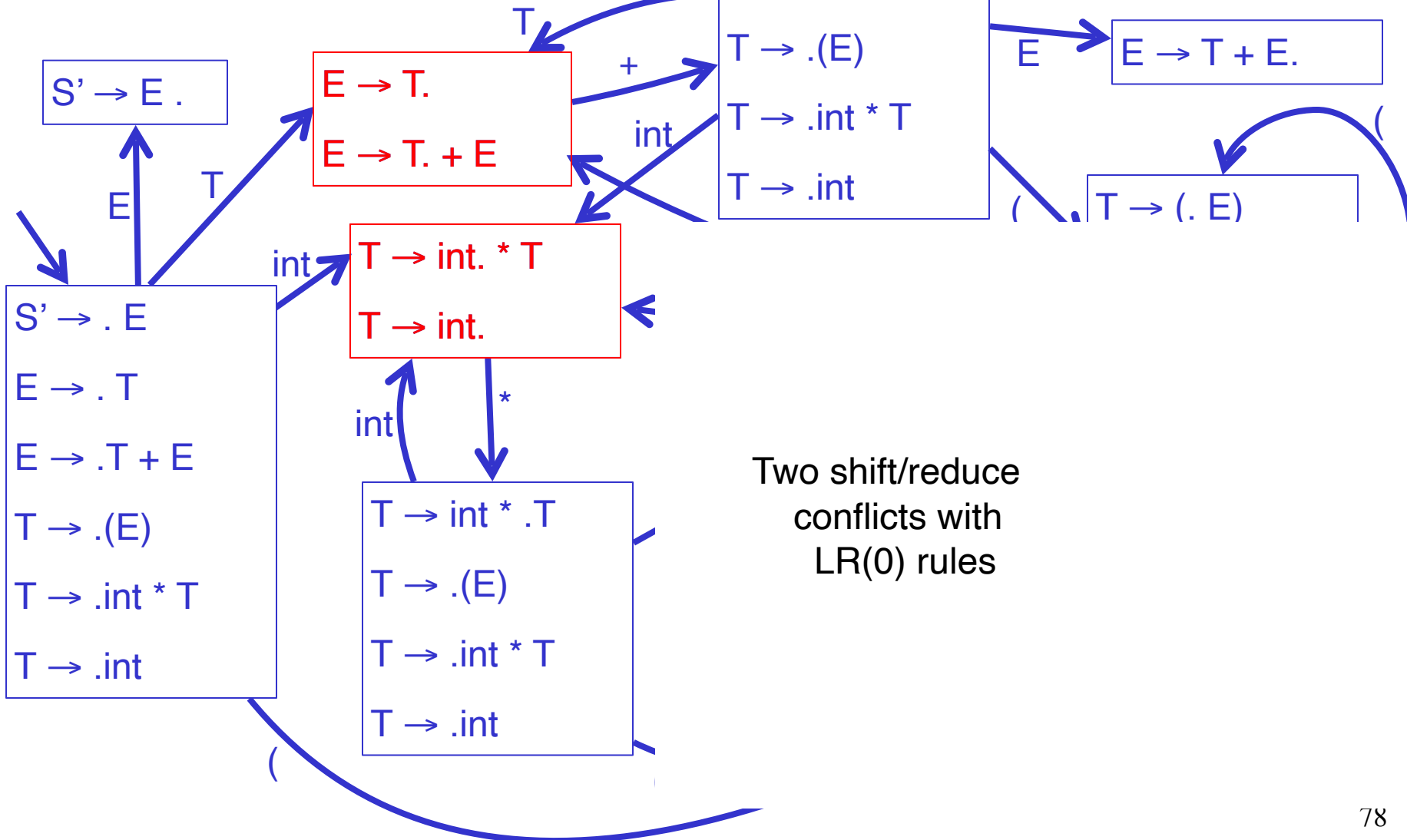
# LR(0) Conflicts

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- LR(0) has a reduce/reduce conflict if:
  - Any state has two reduce items:
    - $X \rightarrow \beta.$  and  $Y \rightarrow \omega.$
- LR(0) has a shift/reduce conflict if:
  - Any state has a reduce item and a shift item:
    - $X \rightarrow \beta.$  and  $Y \rightarrow \omega.t\delta$

# Translation to the DFA

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



Two shift/reduce conflicts with LR(0) rules


# SLR

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- LR = “Left-to-right scan”
- SLR = “Simple LR”
  
- SLR improves on LR(0) shift/reduce heuristics
  - Fewer states have conflicts

# SLR Parsing

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- Idea: Assume
  - stack contains  $\alpha$
  - next input is  $t$
  - DFA on input  $\alpha$  terminates in state  $s$
- Reduce by  $X \rightarrow \beta$  if
  - $s$  contains item  $X \rightarrow \beta$ .
  - $t \in \text{Follow}(X)$  
- Shift if
  - $s$  contains item  $X \rightarrow \beta.t\omega$



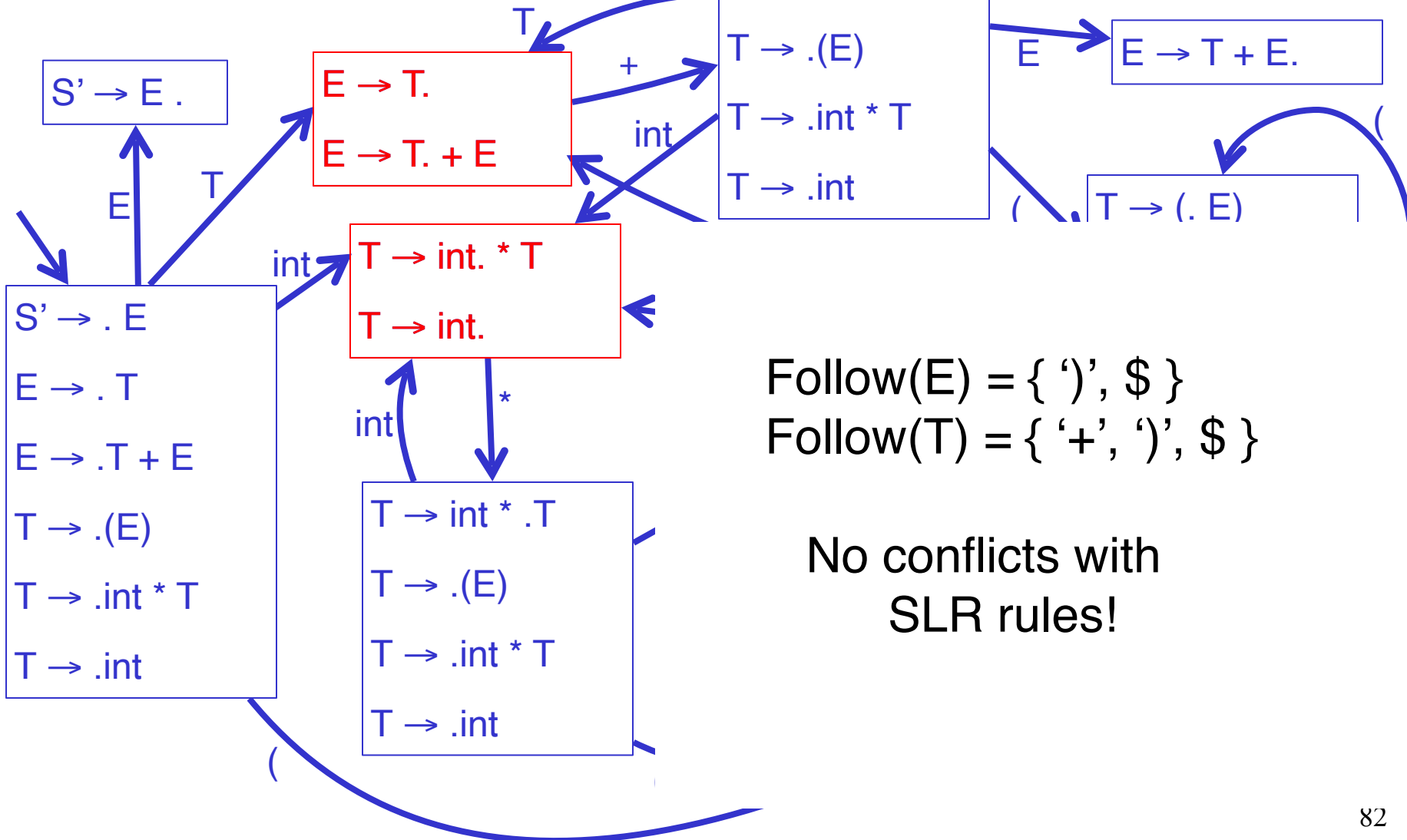
## SLR Parsing (Cont.)

---

- If there are conflicts under these rules, the grammar is not SLR
- The rules amount to a heuristic for detecting handles
  - The SLR grammars are those where the heuristics detect exactly the handles

# Translation to the DFA

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



Follow(E) = { ')', '\$ }

Follow(T) = { '+', ')', '\$ }

No conflicts with SLR rules!

# Precedence Declarations Digression

---

- Lots of grammars aren't SLR
  - including all ambiguous grammars
- We can parse more grammars by using precedence declarations
  - Instructions for resolving conflicts

## Precedence Declarations (Cont.)

---

- Consider our favorite ambiguous grammar:
  - $E \rightarrow E + E \mid E * E \mid (E) \mid \text{int}$
- The DFA for this grammar contains a state with the following items:
  - $E \rightarrow E * E \cdot$        $E \rightarrow E \cdot + E$
  - shift/reduce conflict!
- Declaring “\* has higher precedence than +” resolves this conflict in favor of reducing

## Precedence Declarations (Cont.)

---

- The term “precedence declaration” is misleading
- These declarations do not define precedence; they define conflict resolutions
  - Not quite the same thing!

# Unoptimized SLR Parsing Algorithm

---

1. Let  $M$  be DFA for viable prefixes of  $G$
2. Let  $|x_1 \dots x_n \$$  be initial configuration
3. Repeat until configuration is  $SI\$$ 
  - Let  $\alpha \omega$  be current configuration
  - Run  $M$  on current stack  $\alpha$
  - If  $M$  rejects  $\alpha$ , report parsing error
    - Stack  $\alpha$  is not a viable prefix
  - If  $M$  accepts  $\alpha$  with items  $I$ , let  $t$  be next input
    - Reduce if  $X \rightarrow \beta. \in I$  and  $t \in \text{Follow}(X)$
    - Otherwise, shift if  $X \rightarrow \beta. t \gamma \in I$
    - Report parsing error if neither applies

# Notes

---

- If there is a conflict in the last step, grammar is not SLR(k)
- k is the amount of lookahead
  - In practice  $k = 1$
- Will skip using extra start state **S'** in following example to save space on slides

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

## SLR Example

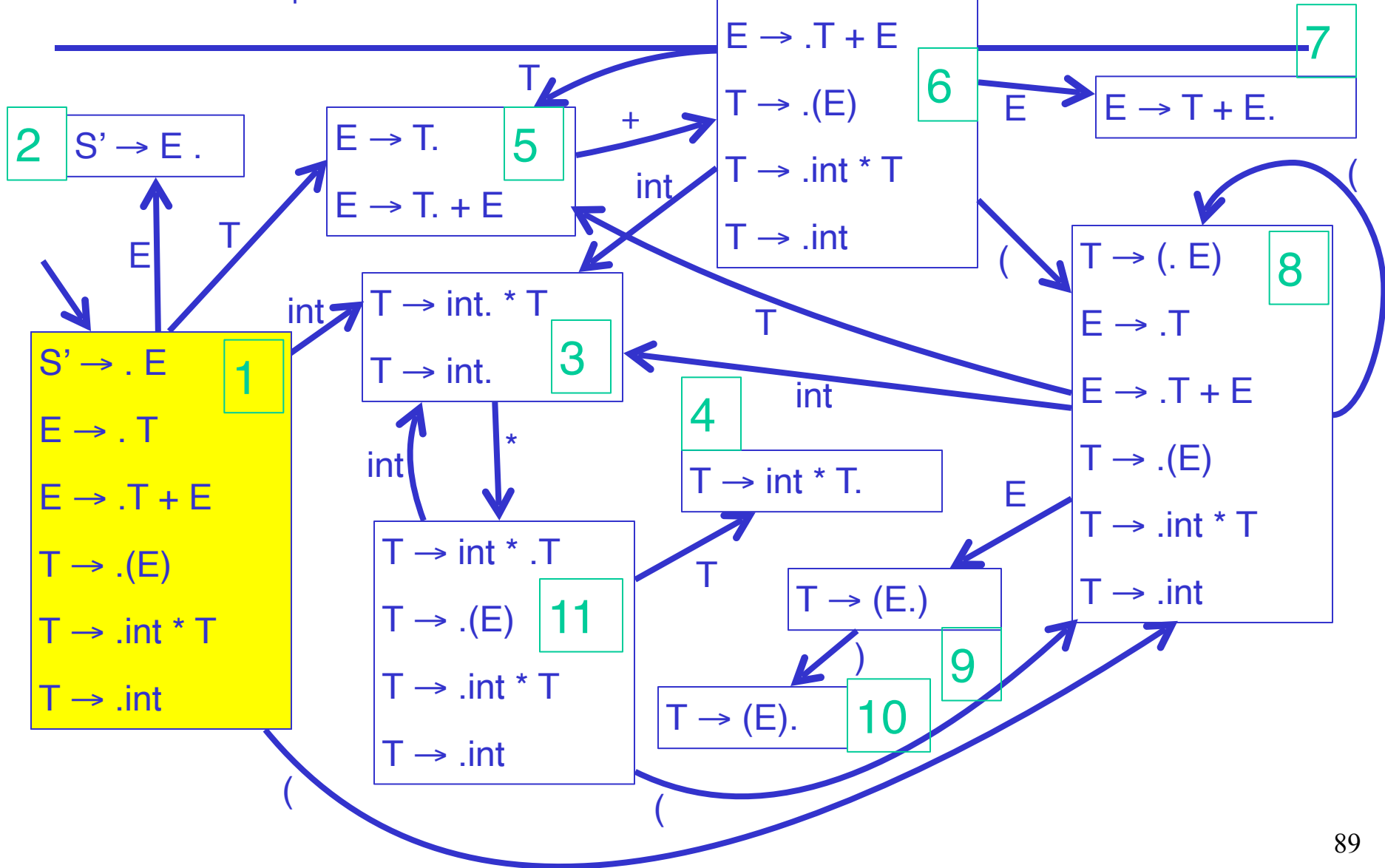
---

Configuration	DFA Halt State	Action
int * int\$	1	shift



**I int \* int\$**

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

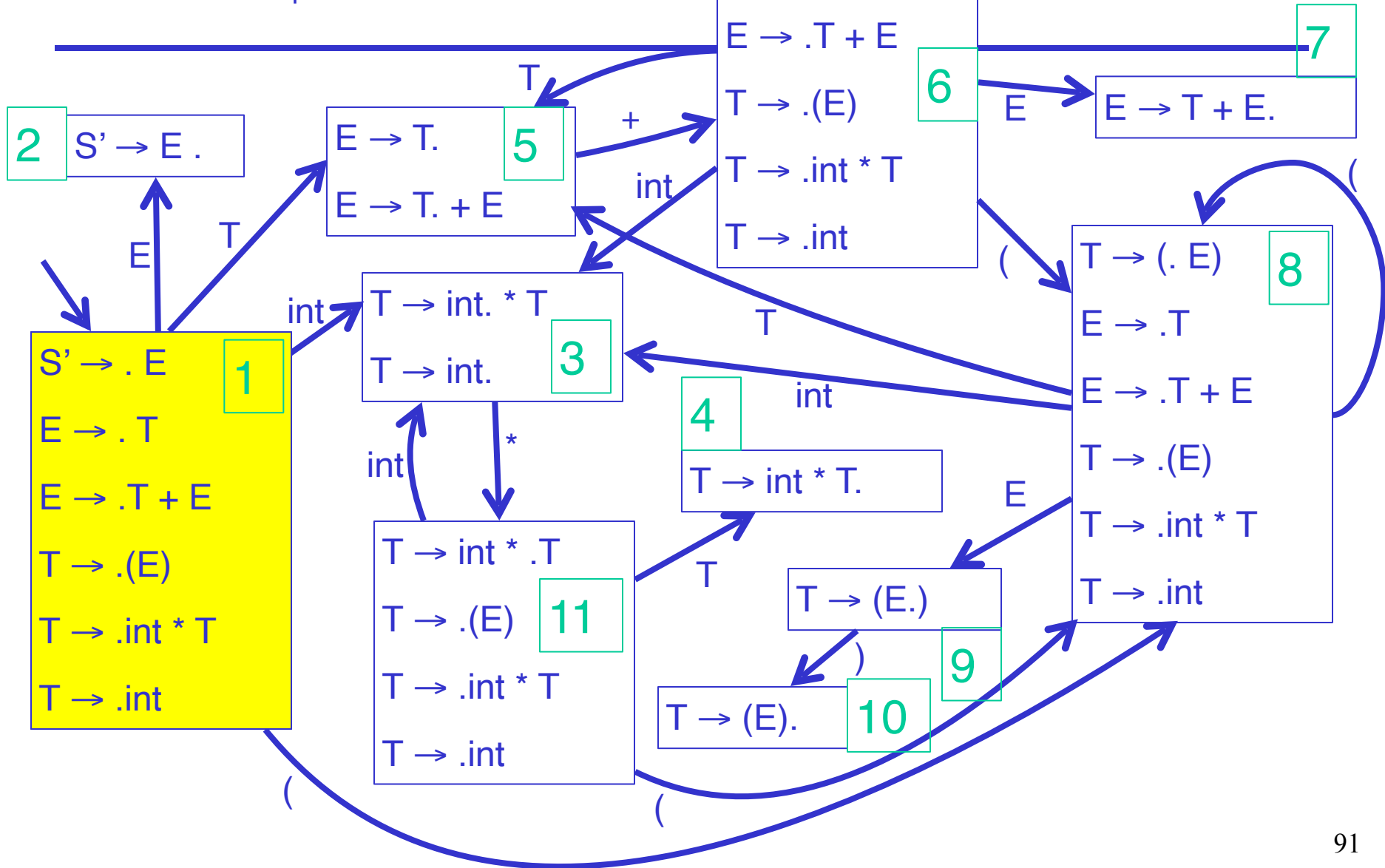
## SLR Example

---

Configuration	DFA Halt State	Action
int * int\$	1	shift
int   * int\$	3 * not in Follow(T)	shift

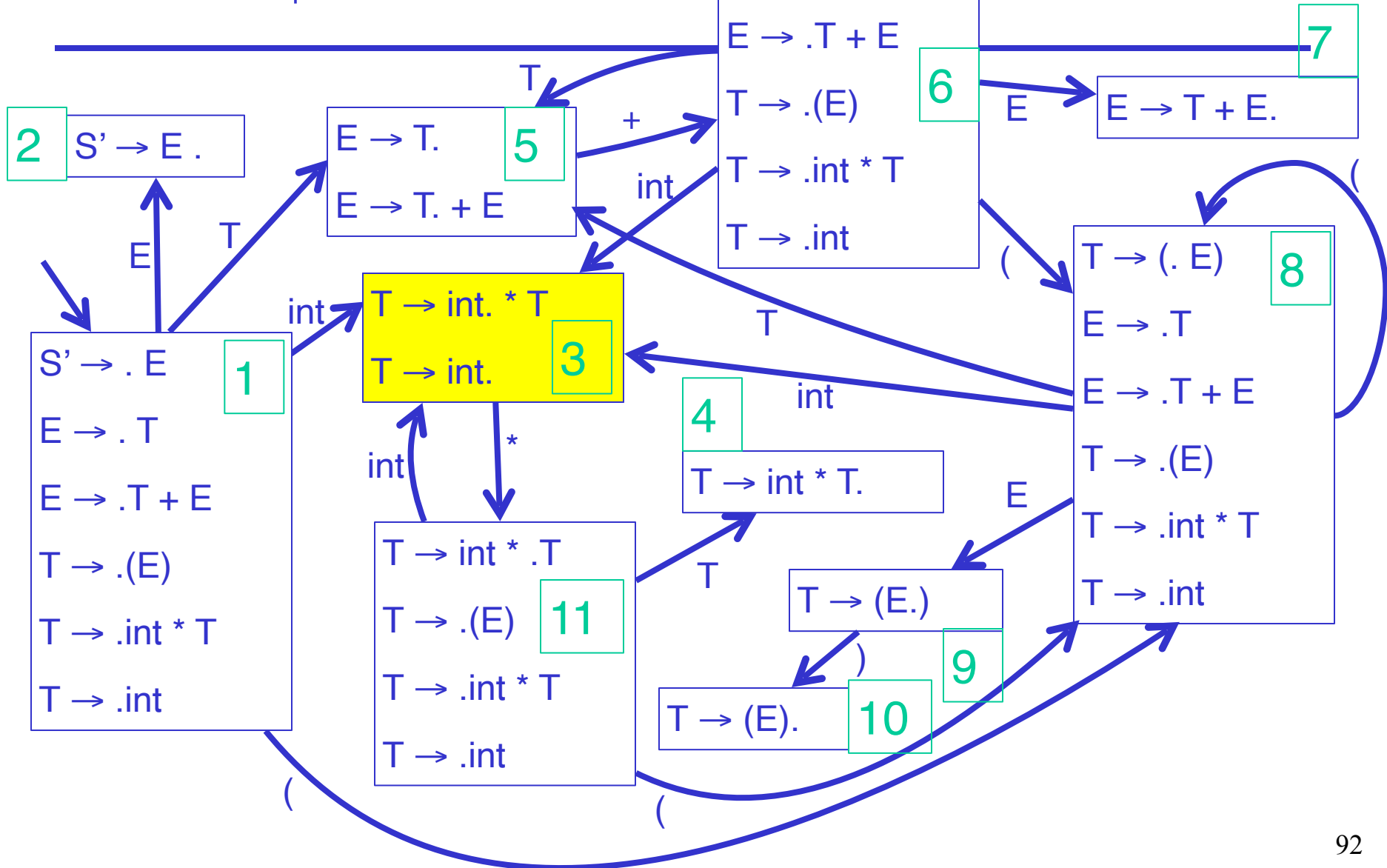
int | \* int\$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



int | \* int\$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

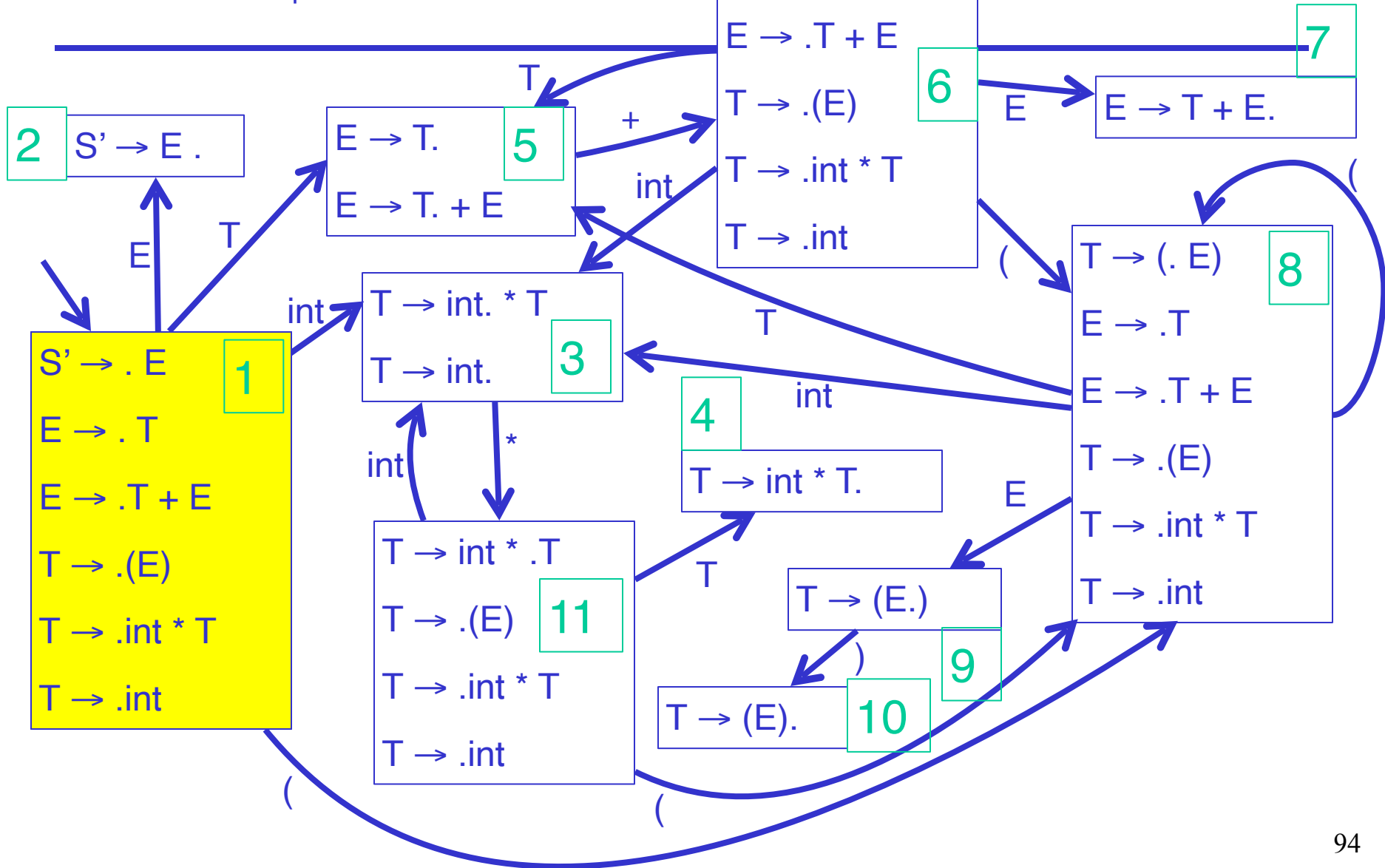
## SLR Example

---

Configuration	DFA Halt State	Action
int * int\$	1	shift
int   * int\$	3 * not in Follow(T)	shift
int *   int\$	11	shift

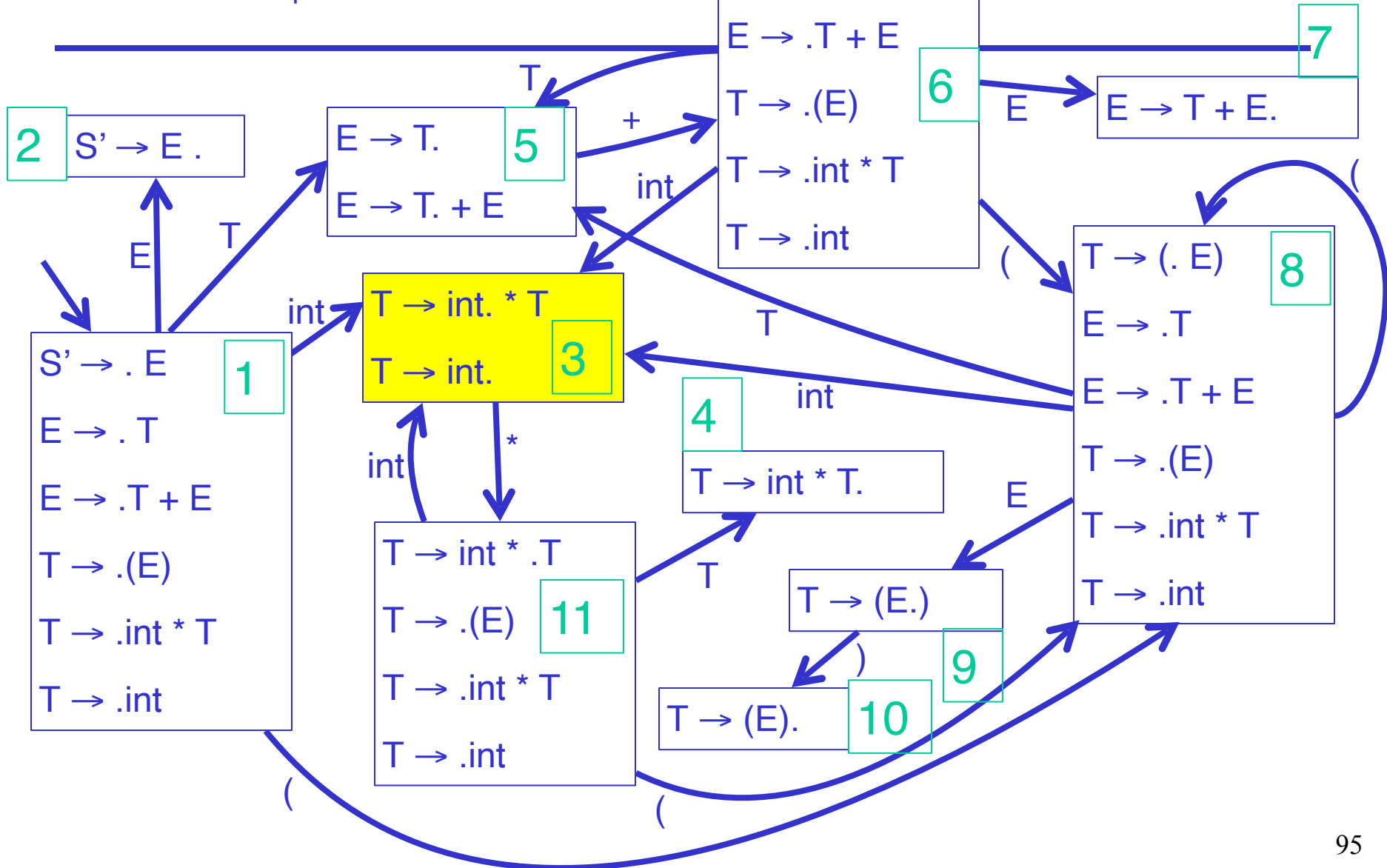
**int \* | int\$**

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



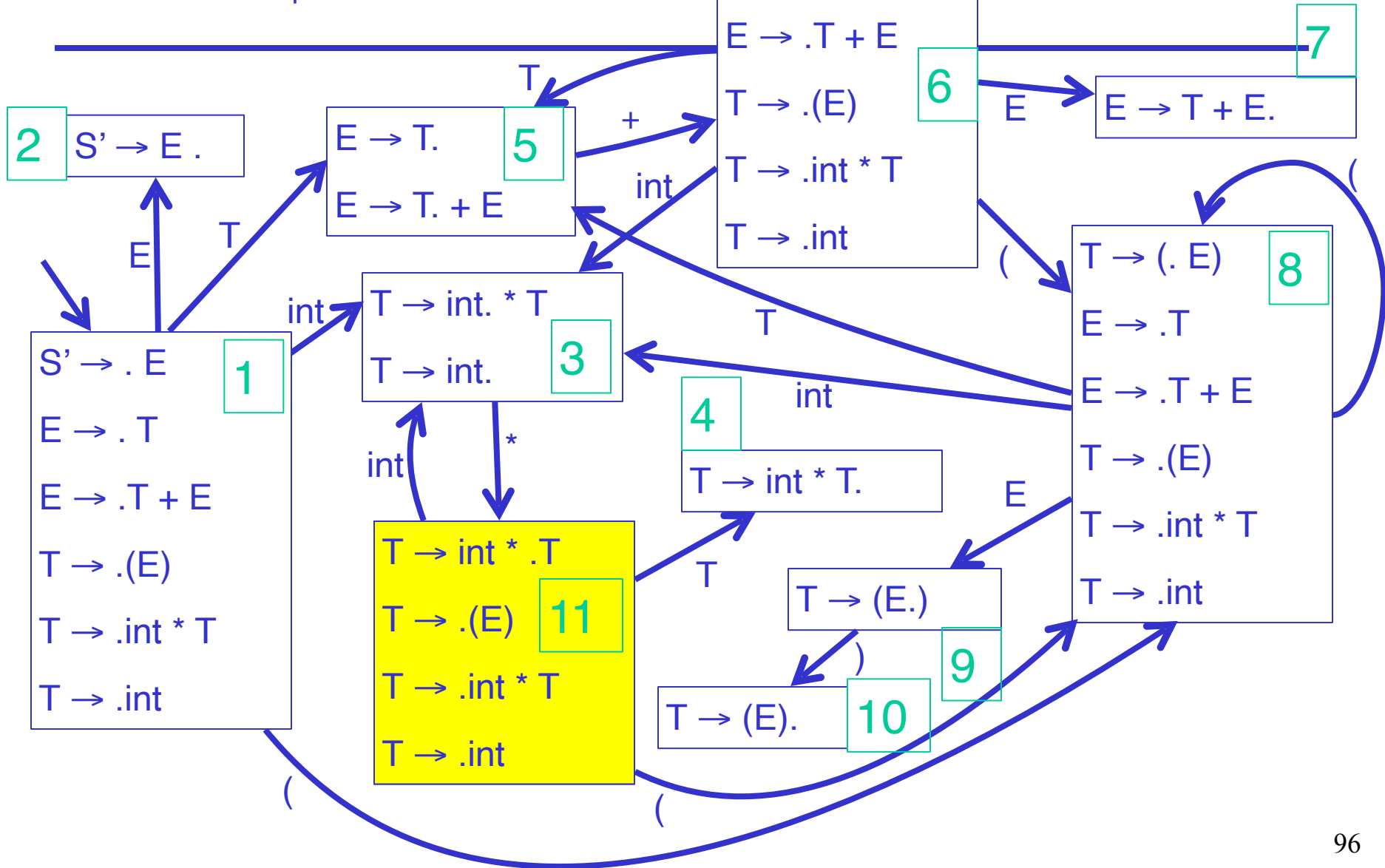
**int \* | int\$**

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



**int \* | int\$**

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$





$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

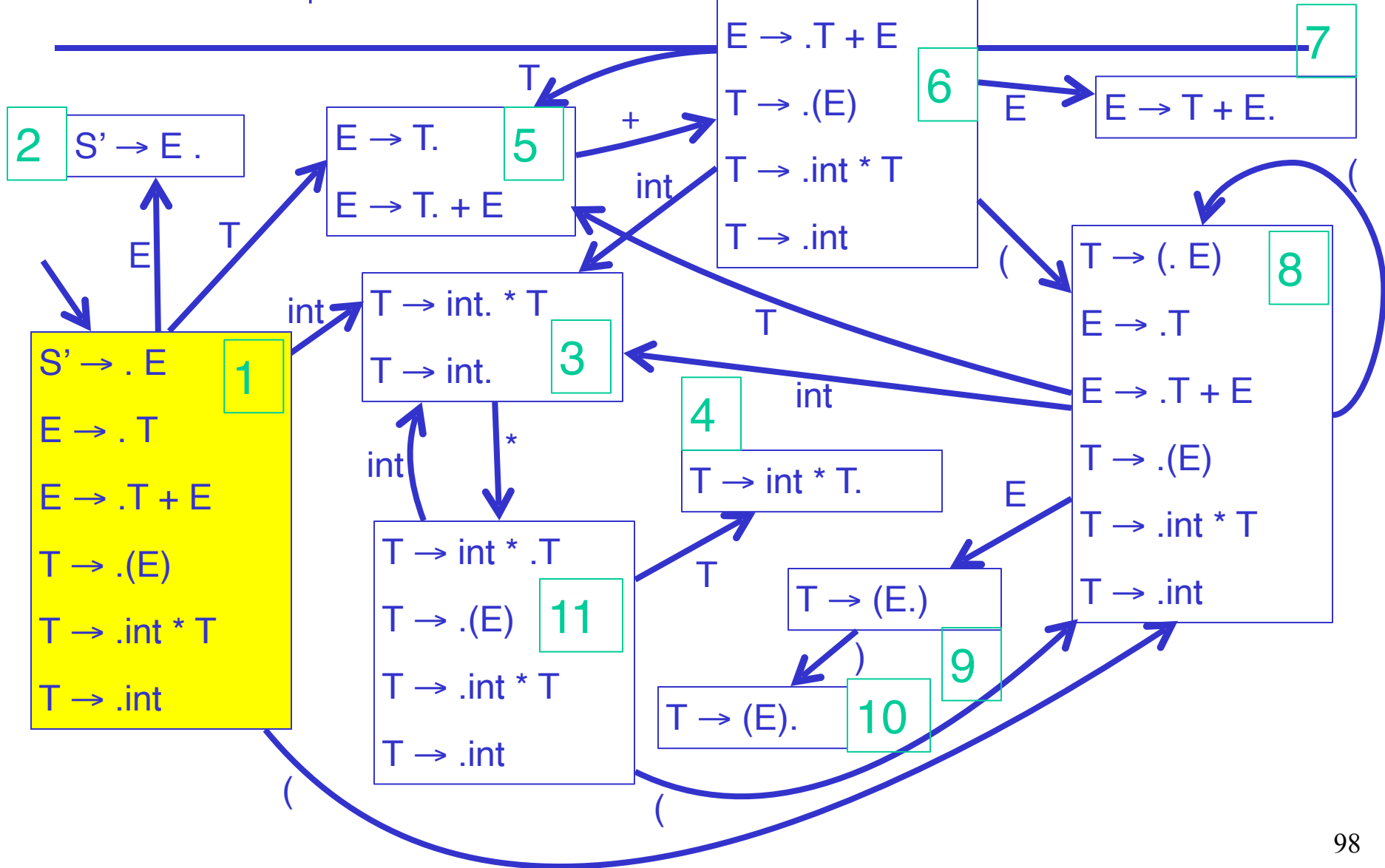
## SLR Example

---

Configuration	DFA Halt State	Action
int * int\$	1	shift
int   * int\$	3 * not in Follow(T)	shift
int *   int\$	11	shift
int * int  \$	3 \$ ∈ Follow(T)	reduce $T \rightarrow \text{int}$

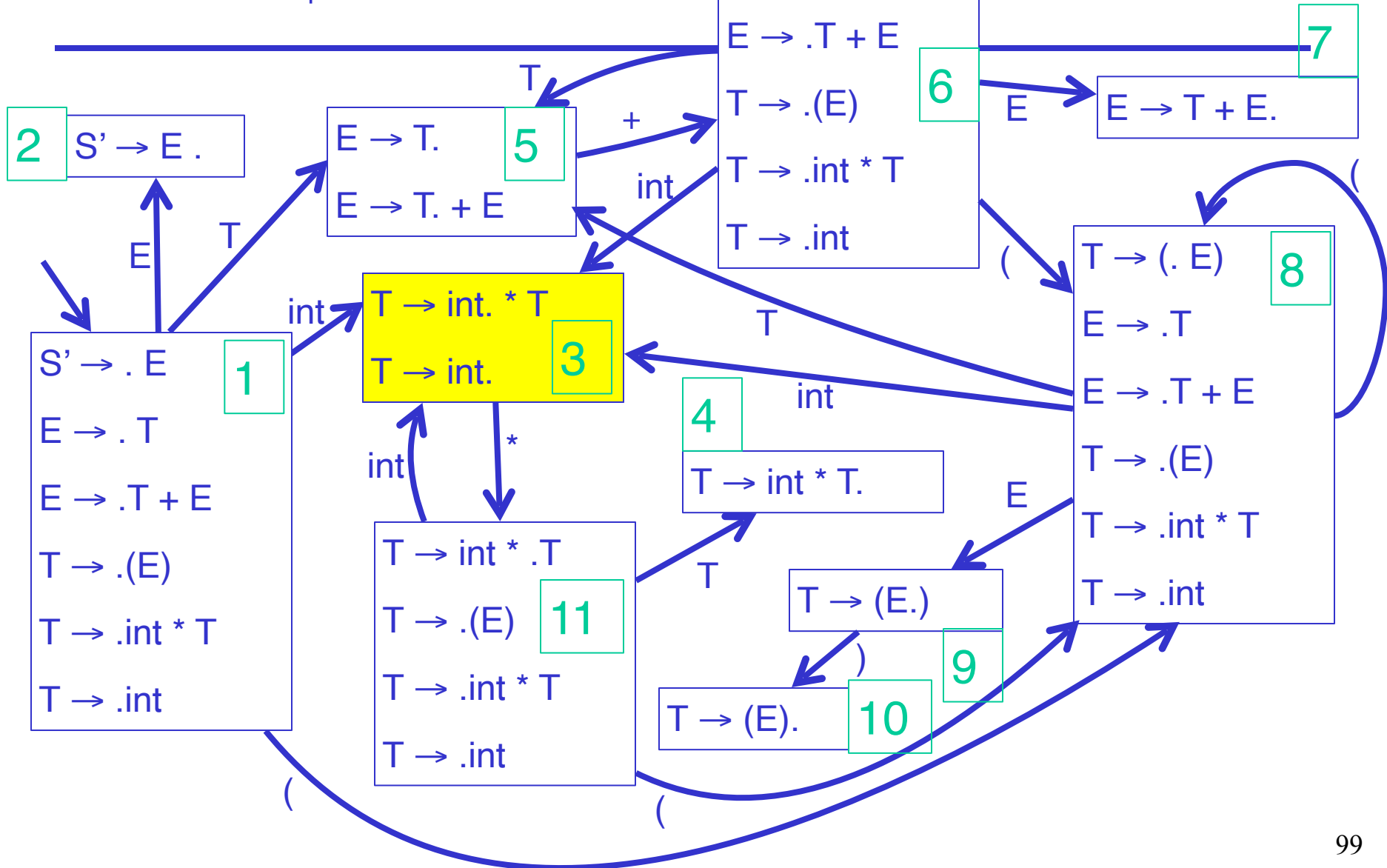
int \* int | \$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



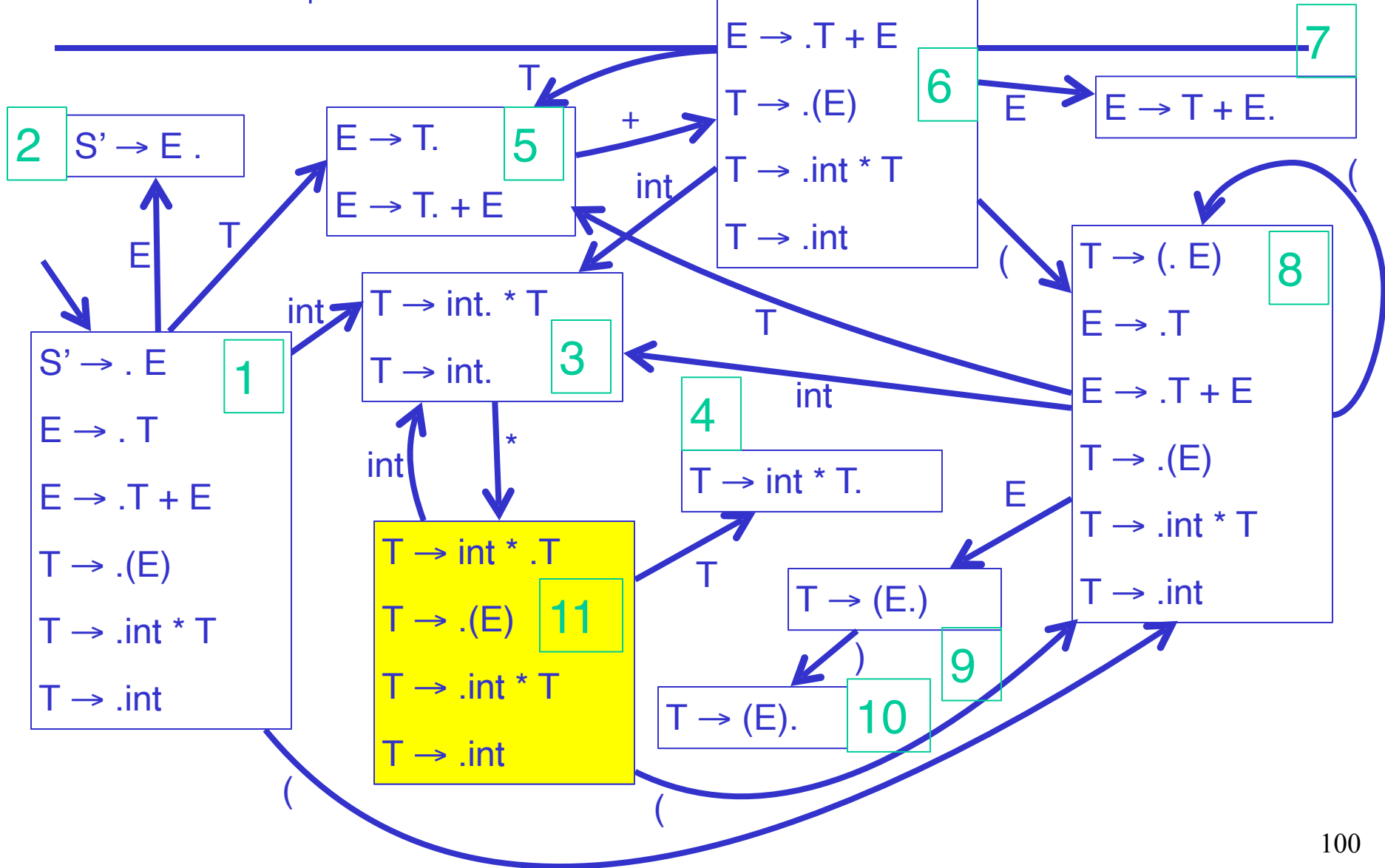
int \* int | \$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



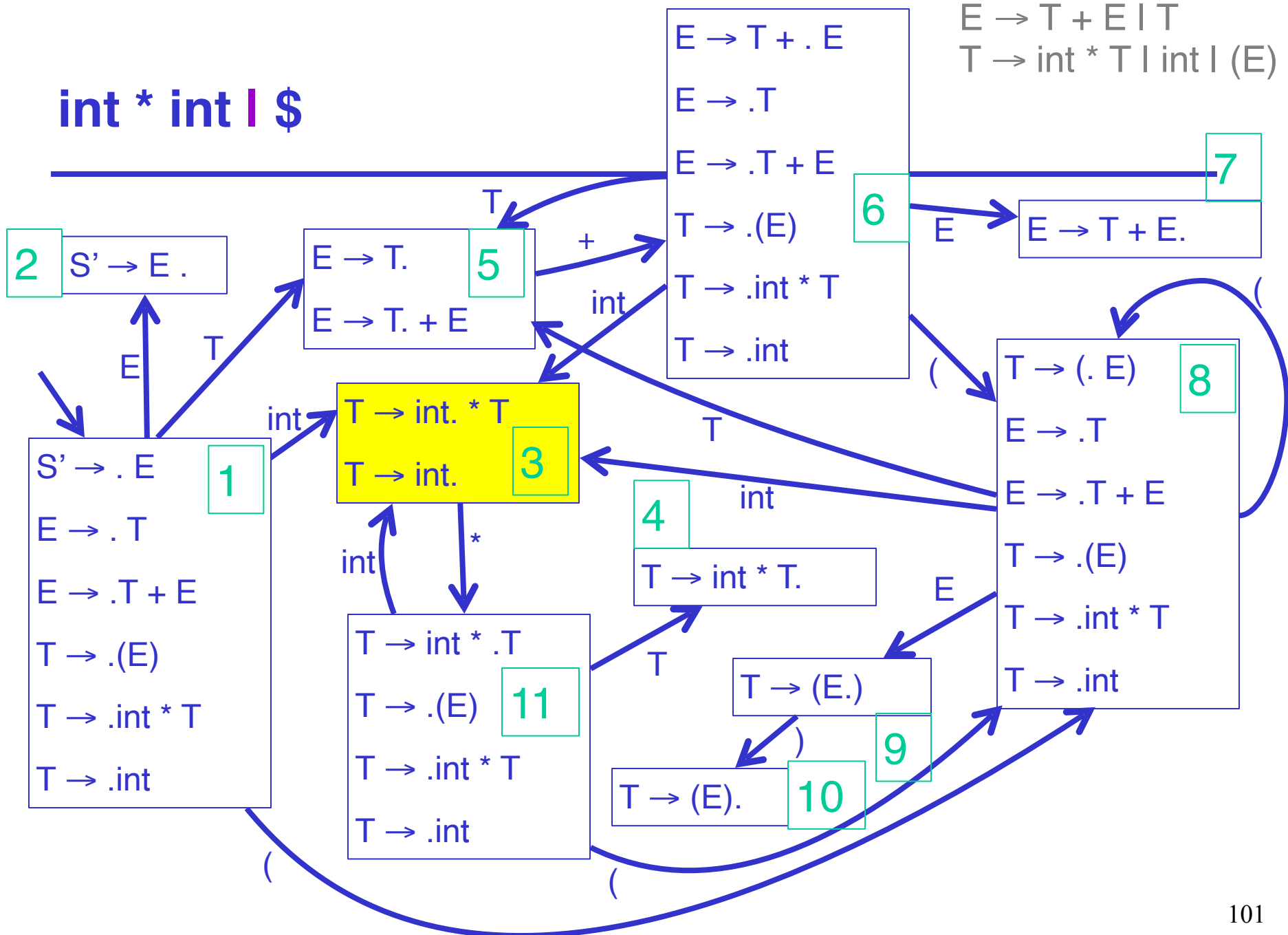
int \* int | \$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



int \* int | \$

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

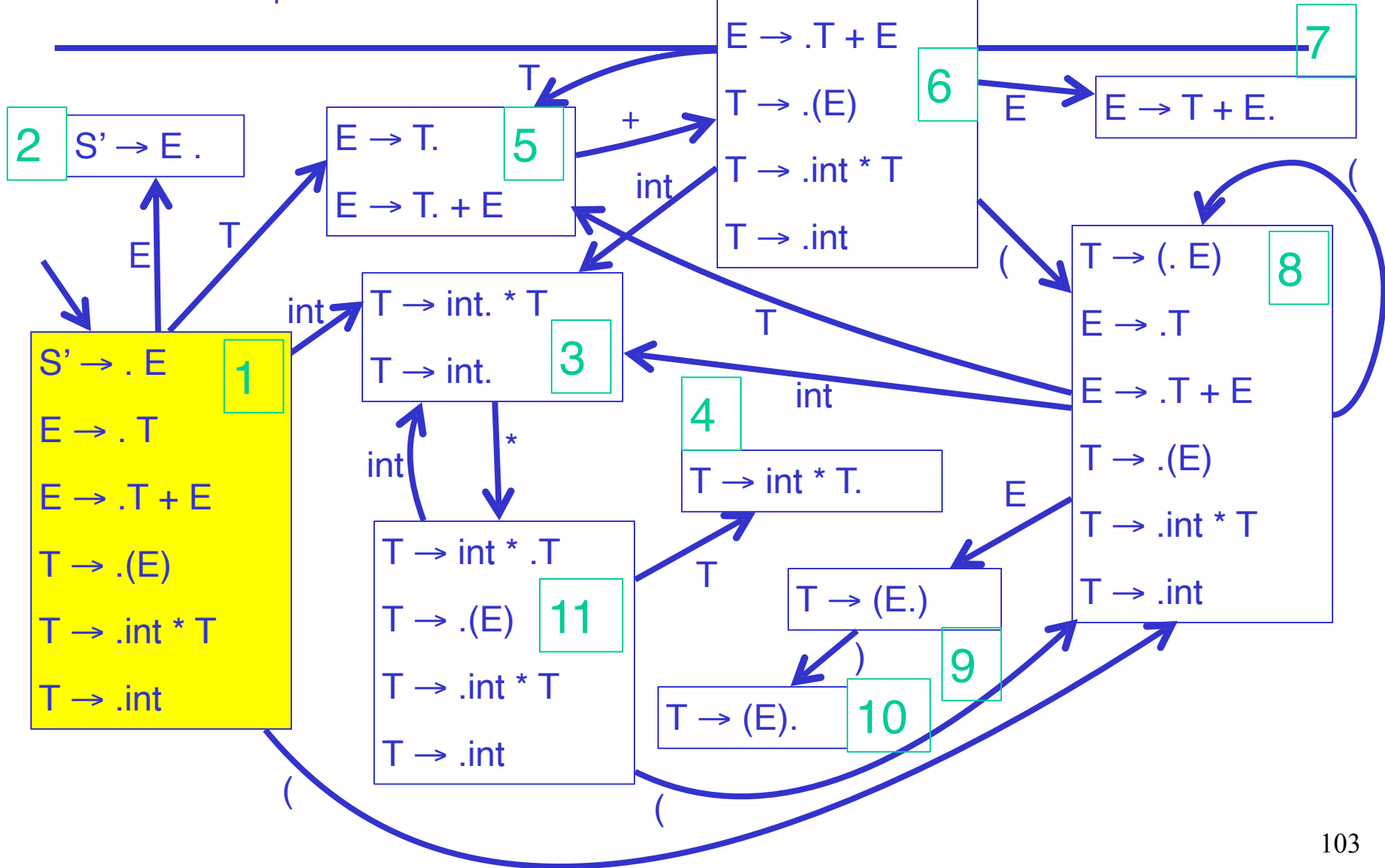
## SLR Example

---

Configuration	DFA Halt State	Action
int * int\$	1	shift
int   * int\$	3 * not in Follow(T)	shift
int *   int\$	11	shift
int * int  \$	3 \$ ∈ Follow(T)	reduce $T \rightarrow \text{int}$
int * T  \$	4 \$ ∈ Follow(T)	reduce $T \rightarrow \text{int} * T$

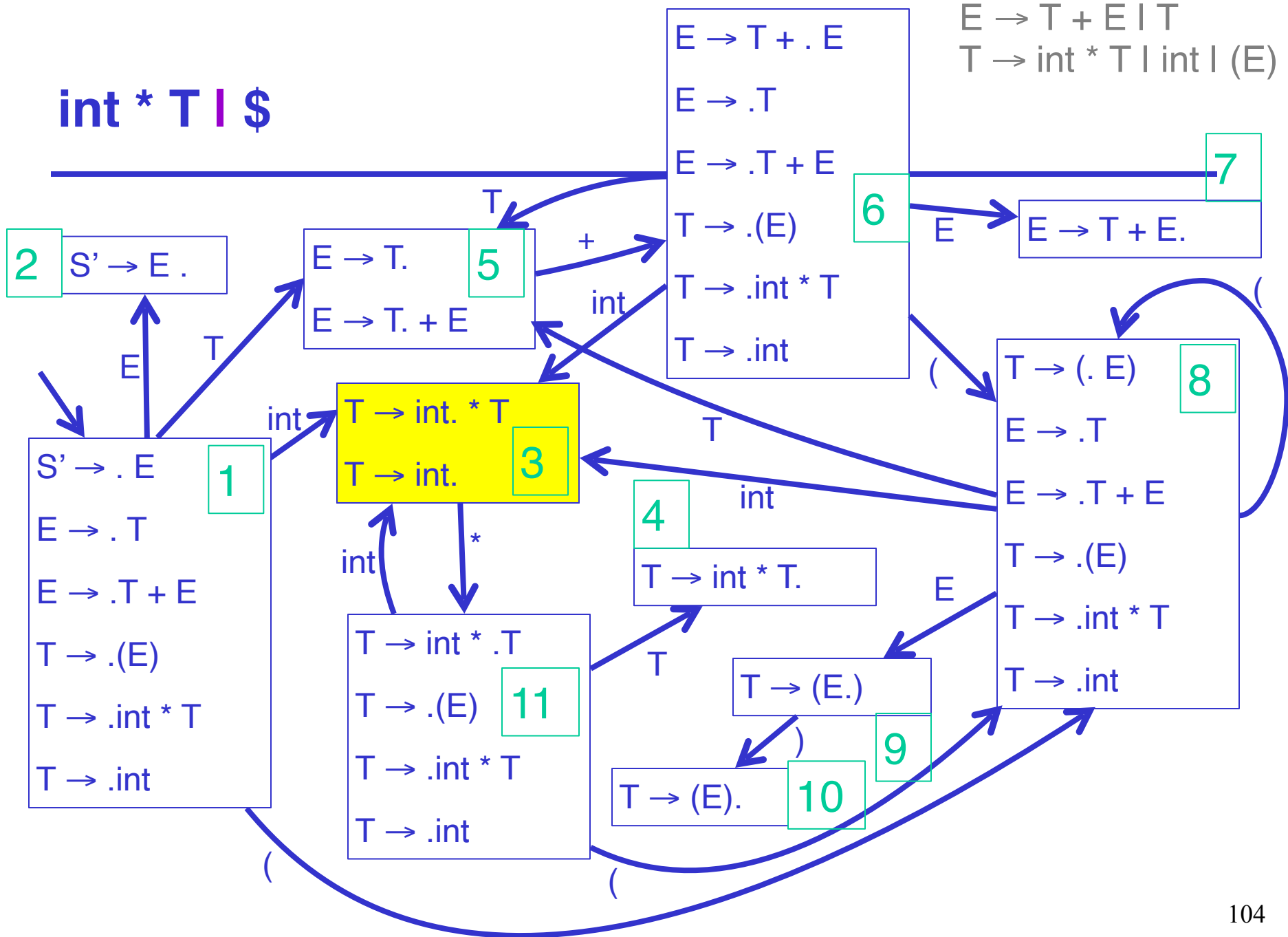
**int \* T | \$**

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



**int \* T | \$**

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

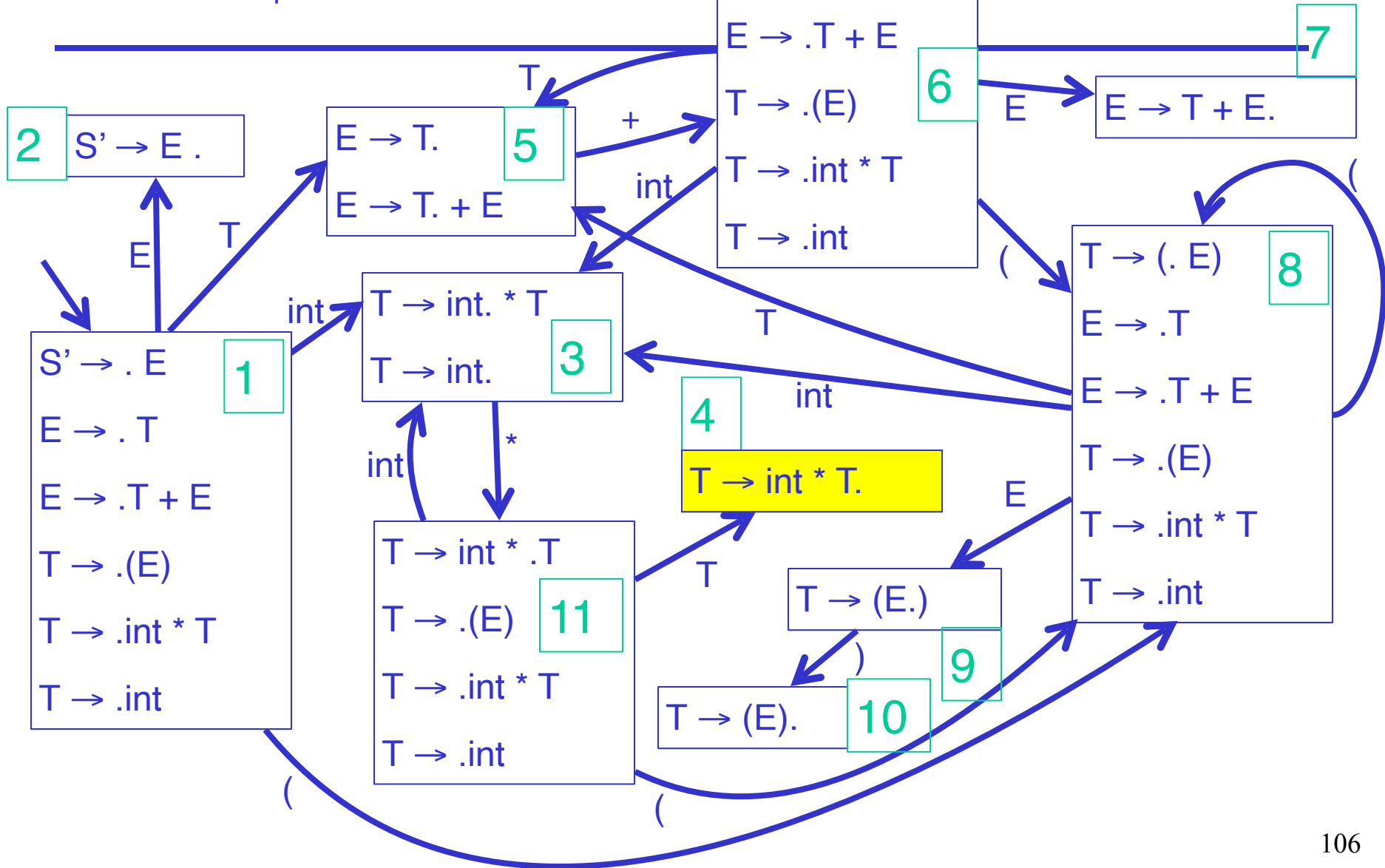






**int \* T | \$**

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

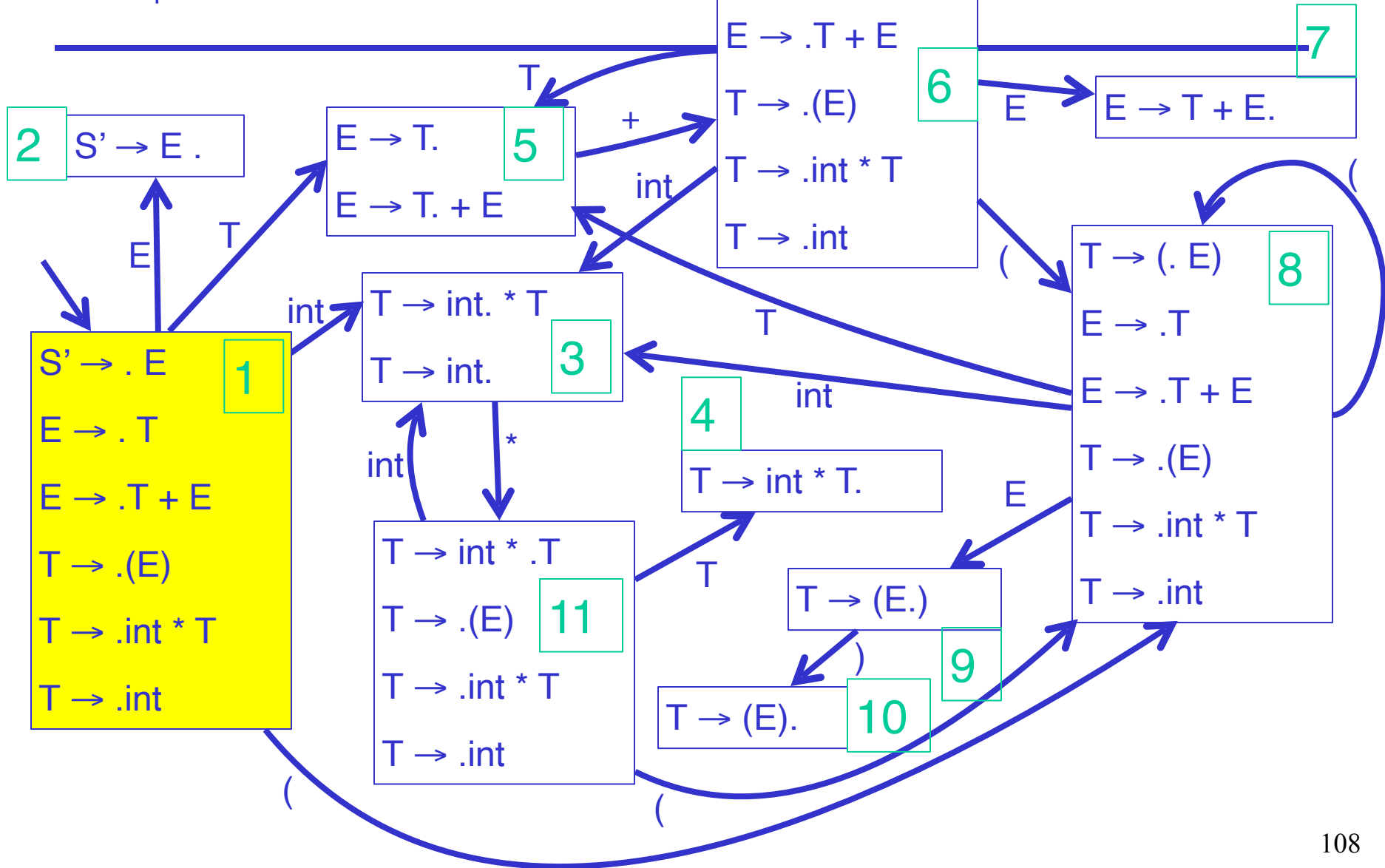
## SLR Example

---

Configuration	DFA Halt State	Action
int * int\$	1	shift
int   * int\$	3 * not in Follow(T)	shift
int *   int\$	11	shift
int * int  \$	3 \$ ∈ Follow(T)	reduce $T \rightarrow \text{int}$
int * T  \$	4 \$ ∈ Follow(T)	reduce $T \rightarrow \text{int} * T$
T  \$	5 \$ ∈ Follow(T)	reduce $E \rightarrow T$

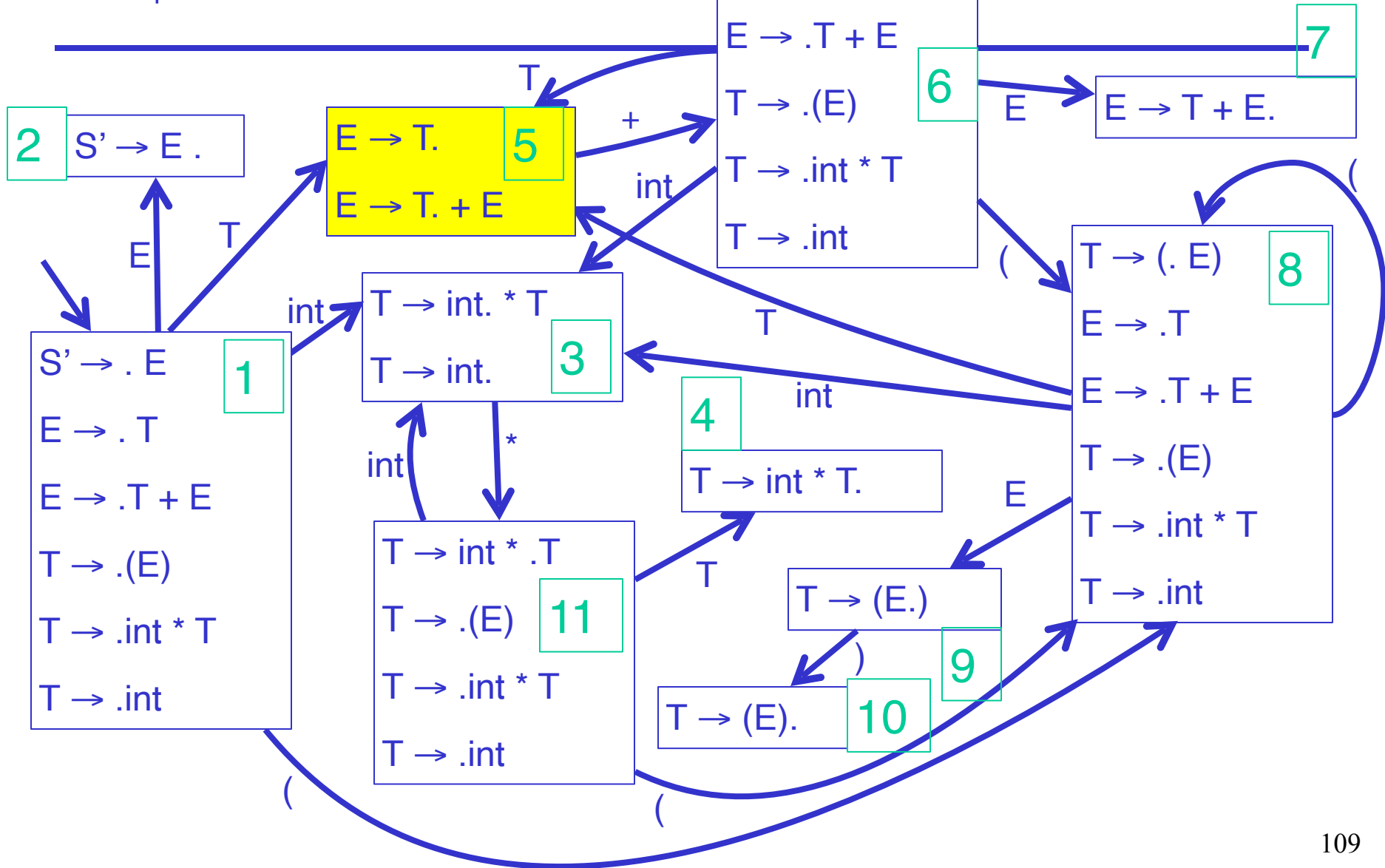
# TIS

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



# TIS

$E \rightarrow T + E \mid T$   
 $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$



$$E \rightarrow T + E \mid T$$

$$T \rightarrow \text{int} * T \mid \text{int} \mid (E)$$

## SLR Example

---

Configuration	DFA Halt State	Action
int * int\$	1	shift
int   * int\$	3 * not in Follow(T)	shift
int *   int\$	11	shift
int * int  \$	3 \$ ∈ Follow(T)	reduce $T \rightarrow \text{int}$
int * T  \$	4 \$ ∈ Follow(T)	reduce $T \rightarrow \text{int} * T$
T  \$	5 \$ ∈ Follow(T)	reduce $E \rightarrow T$
E  \$		accept

# An Improvement

---

- Rerunning the automaton at each step is wasteful
  - Most of the work is repeated
- Remember the state of the automaton on each prefix of the stack
- Change stack to contain pairs  
    ⟨ symbol, DFA state ⟩

## An Improvement (Cont.)

---

- For a stack

$\langle \text{symbol}_1, \text{state}_1 \rangle \dots \langle \text{symbol}_n, \text{state}_n \rangle$

$\text{state}_n$  is the final state of the DFA on  $\text{symbol}_1 \dots \text{symbol}_n$

- Detail: The bottom of the stack is  $\langle \text{dummy}, \text{start} \rangle$   
where
  - $\text{dummy}$  is a dummy symbol
  - $\text{start}$  is the start state of the DFA



# Goto (DFA) Table

---

- Define  $\text{goto}[i,A] = j$  if  $\text{state}_i \xrightarrow{A} \text{state}_j$
- **goto** is just the transition function of the DFA
  - One of two parsing tables

# Refined Parser Moves

---

- Shift  $x$ 
  - Push  $\langle a, x \rangle$  on the stack
  - $a$  is current input
  - $x$  is a DFA state
- Reduce  $X \rightarrow \alpha$ 
  - As before
- Accept
- Error

# Action Table

---

For each state  $s_i$  and terminal  $t$

- If  $s_i$  has item  $X \rightarrow \alpha.t\beta$  and  $\text{goto}[i,t] = k$  then  $\text{action}[i,t] = \text{shift } k$
- If  $s_i$  has item  $X \rightarrow \alpha.$  and  $t \in \text{Follow}(X)$  and  $X \neq S'$  then  $\text{action}[i,t] = \text{reduce } X \rightarrow \alpha$
- If  $s_i$  has item  $S' \rightarrow S.$  then  $\text{action}[i,\$] = \text{accept}$
- Otherwise,  $\text{action}[i,t] = \text{error}$

# SLR Parsing Algorithm

---

Let input = w\$ be initial input

Let  $j = 0$

Let DFA state 1 be the one with item  $S' \rightarrow .S$

Let stack =  $\langle \text{dummy}, 1 \rangle$  //  $\langle \text{symbol}, \text{state} \rangle$

repeat

case action[top\_state(stack), input[j]] of

shift k: push  $\langle \text{input}[j++], k \rangle$

reduce  $X \rightarrow \alpha$ :

pop  $|\alpha|$  pairs,

push  $\langle X, \text{goto}[\text{top\_state}(\text{stack}), X] \rangle$

accept: halt normally

error: halt and report error

# Notes on SLR Parsing Algorithm

---

- Note that the algorithm uses only the DFA states and the input
  - The stack symbols are never used!
- However, we still need the symbols for semantic actions

## More Notes

---

- Some common constructs are not SLR(1)
- LR(1) is more powerful
  - Build lookahead into the items
  - An LR(1) item is a pair: (LR(0) item, x lookahead)
  - $[T \rightarrow \cdot \text{int} * T, \$]$  means
    - After seeing  $T \rightarrow \text{int} * T$  reduce if lookahead is  $\$$
  - More accurate than just using follow sets
  - See Dragon Book
  - Take a look at the LR(1) automaton for your parser!