## Appendix A: **Sealed-Bid Uniform-Price Auction with Synergies**.

We derive the characterization for the three regions identified in the text.

There are $(n+1) > 2$ bidders and $m = 2$ units auctioned, where $m < (n+1)$. The $(n+1)^{th}$ bidder, denoted by $h$ (the human), has a concave utility function $u(\cdot)$ with $u'(\cdot) > 0$, is normalized so that $u(0) = 0$ and $u'(0) = 1$, where $\cdot$ represents earnings net of cost of purchasing the units. $h$ demands two units valuing each at $u(V)$. Bidders $1,2,...,n$, demand only one unit valuing it at $V_1, V_2, ..., V_n$, respectively. $V_1, V_2, ..., V_n$ and $V$ are independent random variables from $F(\cdot)$ and $F_h(\cdot)$ respectively, on the common support $[0,1]$. $V_{(k)}$ denotes the $k^{th}$ order statistic of $V_1, V_2, ..., V_n$ and $F_{(k)}$ its distribution function. Let $v_1, v_2, ..., v_n, v$ be the realizations of $V_1, V_2, ..., V_n, V$ and without loss of generality, assume that $v_1 \geq v_2 \geq, ..., \geq v_n$. The good is available only in integer units. We are interested in a *sealed-bid uniform-price* (highest losing bid) auction (SBUPA). Bidders $1,...,n$, who demand a single unit have a dominant strategy to bid their value. Denote by $p$ the price per unit $h$ pays. Although the value of winning a single unit for $h$ is $u(v-p)$, there is a supper additive value for winning both units. If $h$ wins both units her utility is $u(2v + g(v) - 2p)$, i.e., she is getting an extra $g(v)$, where $g(0) = 0$ and $g'(v) > 0$. In this experiment (and derivation) we employ, $g(v) = v$. Without loss of generality assume that $b_1(v) \geq b_2(v)$ represents $h$'s two (optimal) bids.

**Lemma** *(**a**) $b_1(v) \geq v$. (**b**) $b_1(v) = b_2(v)$ if and only if $b_1(v) > v$.*

**Proof** (**a**) Suppose (**a**) does not hold. This implies that there exists $v^*$ such that $v^* > b_1(v^*) \geq b_2(v^*)$. But then, raising $b_1(v)$ from $b_1(v^*) < v^*$ to $b_1^*(v^*) = v^*$ makes $h$ better off when it matters since in such events $h$ will win one unit rather than zero with strictly positive expected utility.

(**b**) Suppose the *if* part does not hold. This implies that there exists $v^*$ such that $b_1(v^*) > v^*$ and $b_1(v^*) > b_2(v^*)$. Case 1. $b_2(v^*) \geq v^*$. In this case the pair $\{b_1^*(v^*) = b_2(v^*), b_2(v^*)\}$ dominates the alternative $\{b_1(v^*) > b_2(v^*), b_2(v^*)\}$, i.e., reducing $b_1(v^*) > b_2(v^*)$, to $b_1^*(v^*) = b_2(v^*)$ dominates. Here is the reason: If $h$ wins two or zero units, then reducing $b_1(v^*)$ does not matter. However, if $h$ wins one unit, then the price is at least $v^*$, and strictly higher with positive probability, implying that $E[u(v^* - p)] < 0$. Thus, $h$ cannot lose, and gains strictly positive expected utility by the proposed change. Case 2. $b_2(v^*) < v^*$. Using similar arguments we can show that the pair of bids $\{b_1(v^*) > v^*, b_2(v^*) < v^*\}$ is dominated by $\{b_1^*(v^*) = v^*, b_2^*(v^*) = v^*\}$ footnote .

**Part 1**: We start the analysis by assuming first that $b_1(v) = b_2(v)$ and thus, $b_1(v) = b_2(v) \geq v$. In this case $h$'s maximization problem becomes:

$$(A1) \qquad \max_{b \geq v} \left\{ \int_0^b n f(t)[F(t)]^{n-1} u(3v - 2t)\,dt + n(1 - F(b))[F(b)]^{n-1} u(v - b) \right\}.$$

The integral represents $h$'s expected utility from winning two units, an event where all $n$ rivals bid below $b$. The second part represents $h$'s expected utility from winning one unit, an event were $v_1$, the highest rivals' bid, is higher than $b$ but all other bids are below $b$. In all other events $h$ earns $u(0) = 0$. The first order condition for maximization (FOC) of $(A1)$ after rearranging is:

$$(A2) \qquad u(3v - 2b) - u(v - b) + (n - 1)u(v - b)\tfrac{1 - F(b)}{F(b)} - u'(v - b)\tfrac{1 - F(b)}{f(b)} = 0.$$

The *left hand side* (LHS) of $(A2)$ evaluated at $b = v$ is:

$$(A3) \qquad u(v) - \tfrac{1 - F(v)}{f(v)} =: H(v), \text{ and we harmlessly assume that } H'(v) > 0. \text{ footnote}$$

**Lemma** *There exists a unique value, $v = v_c$, satisfying: (**a**) $v_c = b_1(v_c) = b_2(v_c)$, that solves the FOC $(A2)$. (**b**) $\forall v > v_c$, $b_1(v) = b_2(v_c) > v$. (**c**) $\forall v < v_c$, $b_1(v) = v > b_2(v_c)$*

**Proof** (**a**) $H(0) < 0 < H(1)$ and $H'(v) > 0$. Thus, there exists a unique $v_c$, with $b_1(v_c) = b_2(v_c) = v_c$ that solves FOC $(A2)$. (**b**) Any $b = v > v_c$, implies that the LHS of $(A2)$ is strictly positive and the optimal bids are $b_1(v) = b_2(v) > v$. (**c**) Any $b = v < v_c$, implies that the LHS of $(A2)$ is strictly negative. But, Lemma 1 implies that we cannot have $b_1(v) = b_2(v) < v$, thus, $b_1(v) > b_2(v)$. By Lemma 1, $b_1(v) \geq v$, but, if $b_1(v) > v$, then $b_1(v) = b_2(v)$ is a contradiction. We conclude that when $v < v_c$, then $b_1(v) = v > b_2(v_c)$. Note that we have now also proved the *only if* part of Lemma 1. footnote

With a risk neutral (RN) $h$ and after rearranging, equation $(A2)$ becomes:

$$(A4) \qquad (v - b)[1 + (n - 1)\tfrac{1 - F(b)}{F(b)}] + v - \tfrac{1 - F(b)}{f(b)} = 0.$$

Since in our design, $F(\cdot)$ is a *uniform* distribution, $H(v_c) = 0$ implies that, $v_c = ½$. Further, with $F(\cdot)$ being a *uniform* distribution, equation $(A4)$ becomes:

$(A5)$ $\quad b^2 - \alpha(n,v)b + v = 0$, where $\alpha(n,v) =: \frac{n+(n-3)v}{n-1}$.

Differentiating the LHS of $(A5)$ with respect to $b$ yields :

$(A6)$ $\quad \frac{\partial}{\partial b}\{b^2 - \alpha(n,v)b + v\} = 2b - \alpha(n,v)$.

The second order condition for maximization (SOC) requires that $(A6)$ evaluated at the optimal $b$ $(A5)$, is negative. Thus,

$(A7)$ $\quad \frac{b^2 - v}{b} < 0$.

We write the solution to the quadratic FOC $(A5)$ as:

$(A8)$ $\quad b_{1,2} = \{\alpha(n,v) \pm [(\alpha(n,v))^2 - 4v]^{1/2}\}/2$.

Note, that once $[(\alpha(n,v))^2 - 4v] < 0$, there is no solution to equation $(A5)$. It is easy to verify that since $v \le 1$, $[(\alpha(n,v))^2 - 4v]$ is strictly decreasing in $v$ for all $n \ge 2$. Let $v_{cn}$, be that value of $v$ that solves:

$(A9)$ $\quad [(\alpha(n,v_{cn}))^2 - 4v_{cn}] = 0$.

Thus, $\forall v > v_{cn}$, $[(\alpha(n,v_{cn}))^2 - 4v_{cn}] < 0$, and the LHS of $(A5)$ is strictly positive implying that the optimal bid is $b(v) = 1$. Namely, for such (high) $v$'s, $h$ optimal strategy is "*to go for it*," bidding (at least) 1, winning two units for sure, and enjoying the synergy bonus, $v$. In what follows we restrict attention to $v$'s that satisfy $v \in (v_c, v_{cn}] = (½, v_{cn}]$.

The positive root of $(A8)$ yields, $b^2 > (\alpha(n,v))^2/2 - v = [b + \frac{v}{b}]^2/2 - v$, where the last equality is obtain by using $(A5)$. Thus, $b^2 - v > [b^2 + 2v + (v/b)^2 - 4v]/2 = [b^2 - 2v + (v/b)^2]/2 = [b - (v/b)]^2/2 > 0$, which violates $(A7)$. On the other hand, by using $(A6)$ and $(A8)$ it easy to verify that the negative root of $(A8)$ yields, $2b - \alpha(n,v) < 0$ so that the SOC is satisfied. With some additional tedious algebra one can verify that for $v \in (½, v_{cn}]$, the negative root also yields $b > v$, as required. Thus, the negative root of $(A8)$ satisfies the FOC, SOC and $b > v$, $v \in (½, v_{cn}]$ and is rewritten as:

$(A10)$ $\quad b = \{\alpha(n,v) - [(\alpha(n,v))^2 - 4v]^{1/2}\}/2$.

Although the solution proposed in $(A10)$ satisfies the FOC and the SOC, it assures only a local maximization since the objective function is not quasi-concave. Let $E[\pi(b(v))]$ denote the expected payoffs for $h$ who has $V = v \in (½, v_{cn}]$ and is using $b(v)$ as defined by $(A10)$. $E[\pi(b(v))] = \{$expected gain of winning two units$\}\{$probability of winning two units$\} + \{$expected gain of winning one unit$\}\{$probability of winning one unit$\}$. Or

$(A11)$ $\quad E[\pi(b(v))] = \{3v - 2\frac{n}{n+1}b(v)\}\{[b(v)]^n\} + \{v - b(v)\}\{n[1 - b(v)][b(v)]^{n-1}\}$

Let $E[\pi(b \ge 1)]$ denote the expected payoffs for $h$ who has $V = v(½, v_{cn}]$ and uses $b \ge 1$ on both units which assures winning both of them:

$(A12)$ $\quad E[\pi(b \ge 1, v)] = [3v - 2\frac{n}{n+1}]$,

as $3v$ is the value of winning two units and $2\frac{n}{n+1}$ is the expected payment in such a case. Let $v_n^D$, be the $v$ that equates expressions $(A11)$ and $(A12)$. It turns out that,

$(A13)$ **a**) $v_n^D \in (½, v_{cn}]$, **b**) $\forall v \in (½, v_n^D)$, the optimal bid is, $b = \{\alpha(n,v) - [(\alpha(n,v))^2 - 4v]^{1/2}\}/2$ and **c**) $\forall v \in [v_n^D, 1]$ the optimal bid is $b \ge 1$.

**Part 2**. Here, we solve for the region where $v < v_c$, implying by Lemma 1 and part (c) of Lemma 2 that, $b_1(v) = v > b_2(v)$. Simplify by concentrating on $b_2(v)$, denoting it by $b(v)$. Everything is the same as in part 1 but, we first derive the results for any $m$, $m < (n+1)$, and summarize them for our experiment where $m = 2$ at the end.

There are three regions (events) to consider here: footnote

**Region** 1: Here, $V_{(m-1)} \leq b$, thus, $E[u(v,b)] = \int_0^b u(3v-2p)dF_{(m-1)}(p)$.

**Region 2**: Here, $V_{(m)} \leq b < V_{(m-1)}$, thus, $E[u(v,b)] = u(v-b)[F_{(m)}(b) - F_{(m-1)}(b)]$.

**Region 3**: Here, $b < V_{(m)} < v$, thus, $E[u(v,b)] = \int_b^v u(v-p)dF_{(m)}(p)$.

Region 1, is the event that $h$ wins both units and earns $u(2v+v-2p)$; region 2 is the event that $h$ wins only one unit, and her bid, $b$, sets the price (which affects her gains on the unit won); region 3 is the event that $h$ wins only one unit and does not set the price. We differentiate with respect to $b$ and collect terms from the three region to obtain the following FOC for maximization:

$\partial E[u(v,b)]/\partial b = [u(3v-2b)f_{(m-1)}(b)] - \{u'(v-b)[F_{(m)}(b) - F_{(m-1)}b] +$
$u(v-b)[f_{(m)}(b) - f_{(m-1)}(b)]\} - [u(v-b)f_{(m)}(b)] = [u(2(v-b)) - u(v-b)] \times$
$f_{(m-1)}(b) - u'(v-b)[F_{(m)}(b) - F_{(m-1)}(b)]$, where $f_{(k)} \geq 0$ is the derivative of $F_{(k)}$. Finally, using $[F_{(m)}(b) - F_{(m-1)}b] = \binom{n}{n-1}[1-F(b)]^{m-1}[F(b)]^{n+1-m}$, and $f_{(m-1)}(b) = n\binom{n-1}{m-2}[1-F(b)]^{m-2}[F(b)]^{n+1-m}f(b)$, we obtain:

$$(A14) \qquad [u(3v-2b^*) - u(v-b^*)] - \frac{[u'(v-b^*)(1-F(b^*))]}{(m-1)f(b^*)} \leq 0,$$

where $b^*$, is the solution to the problem with a risk averse (RA) $h$, and where strict inequality holds only if $b^* = 0$.

**Fact 1**. When $m = 2$, as in our experimental design, when $v < v_c = ½$, $b^* < v$ so that even with synergies there is demand reduction on the second unit. To see why, consider the LHS of $(A14)$. At $b^* = v$, it is equal to, $u(b^*) - \frac{1-F(b^*)}{f(b^*)} < 0$. The strict inequality is due to fact that $[u(v_c) - \frac{1-F(v_c)}{f(v_c)}] = 0$, $b^* = v < v_c$ and $H'(v) > 0$. Note that $b^*$ is independent of the number of single unit demanders, $n$, for all (concave) $u$'s, a surprising result that is reminiscent of optimal reservation price result in single unit, IPV auctions.

For the risk neutral (RN) case $(A14)$ becomes:

$$(A15) \qquad (2v-b) - \frac{1-F(b)}{(m-1)f(b)} \leq 0,$$

with inequality *only if* the RN optimal bid is already zero, $b = 0$. It is easy to verify that a sufficient condition to assure quasi-concavity of the objective function for the RN case is:

$$(A16) \qquad (1-F(b))f'(b) - (m-2)[f(b)]^2 \leq 0.$$

We turn now to the effect of RA on bidding. Let $u(\cdot)$ be concave and assume that $v > b^* > 0$. We obtain:

$$0 = [u(3v-2b^*) - u(v-b^*)] - \frac{u'(v-b^*)(1-F(b^*))}{(m-1)f(b^*)} < [u'(v-b^*)(2v-b)] - \frac{u'(v-b^*)(1-F(b^*))}{(m-1)f(b^*)} = u'(v-b^*)[(2v-b^*) - \frac{1-F(b^*)}{(m-1)f(b^*)}]$$

We conclude that, $[(2v-b^*) - \frac{1-F(b^*)}{(m-1)f(b^*)}] > 0$.

**Fact 2**: The effect of RA is to reduce the bid of $h$ on the second unit, $b^*(v) < b(v)$, unless $b(v)$ is already zero. That is, under condition $(A16)$ (quasi-concavity of the RN case), a RA $h$ bids on the second unit no more than a RN bidder, and strictly less when $b(v) > 0$. Quasi-concavity and the fact that the FOC for a RN $h$ evaluated at $b^*$ is strictly positive is sufficient to establish fact 2. Note that for our design with a *uniform* distribution $f'(b) = 0$, so that condition $(A16)$ is satisfied for all $m \geq 2$.

In the RN case, it is convenient to rewrite the optimal $b$ (from $(A15)$ as:

$$(A17) \qquad b(v) = \left\{ \begin{array}{c} 2v - \frac{1-F(b)}{(m-1)f(b)}, \; b \in [0,v], \\ 0, \; \textit{oterwise}. \end{array} \right\},$$

which implicitly solves for (optimal) $b$. In our design $F(\cdot)$ is *uniform* on $[0,1]$, so that $(A17)$ reduces when $m > 2$ to

$$(A18) \qquad b(v) = \begin{cases} \frac{(m-1)2v-1}{m-2}, & b \in [0, v], \\ 0, & \textit{oterwise.} \end{cases}$$

When $m = 2$ and $v < v_c = 1/2$ as in our design, the LHS of $(A15)$ becomes $[2v - 1] < 0$. Thus, establishing for a RN $h$, a *uniform* distribution, and $m = 2$ (as in our design):

$$(A19) \qquad b(v) = 0, \ \forall v \in [0, ½).$$

## Appendix B: **English-Clock Auctions (ECA) with Synergies**.

Before we start note that we simplify by using $v$ for $v_h$. Also recall that the synergy bonus was modeled as earning an extra $g(v) = \gamma v$, if $h$ obtains both units. The optimal strategy for, $h$, in the ECA can be nicely described by partitioning the domain of values to three regions:

$(B1)\qquad \mathbf{A} = [0, \frac{1}{1+\gamma}), \quad \mathbf{B} = [\frac{1}{1+\gamma}, \frac{2}{2+\gamma}), \quad \mathbf{C} = [\frac{2}{2+\gamma}, 1].$

**A**. *Optimal Behavior for h when* $V = v \in \mathbf{A} = [0, \frac{1}{1+\gamma})$.

**A.1** If $v_3 \geq v$, drop the first unit at any price, $P \in [0, v_3]$. If $v_3 < v$, drop the first unit at any price, $P \in [0, min\{v, v_2\}]$.

**A.2** Drop the second unit at price, $P \in [v, max\{v, v_3\}]$.

**Proofs and observations**:

Step 1. In region A, $h$ never wants to stay IN long enough to win both units.

To win two units $h$ must stay IN with both units beyond the clock price, $P = v_2$. By dropping a unit at $P = v_2$, $h$ stops the auction, "wins one unit" (WOU) and earns, $\pi(WOU) = [v - v_2]$. Suppose that $h$ decides to stay IN with both units an extra $\varepsilon$ beyond $P = v_2$, (as long as $P + \varepsilon \leq 1$) and drop out at $P = v_2 + \varepsilon$, if $V_1 = v_1$ does not drop by then. Recall that given that $V_2 = v_2$, $V_1|V_1 \geq v_2$, is distributed *uniformly* on $[v_2, 1]$. With a probability $\frac{\varepsilon}{1-v_2}$, $v_1$ drops within the next $\varepsilon$, $h$ wins two units and earns: $(2+\gamma)v - 2E[V_1|v_2 + \varepsilon \geq V_1 \geq v_2] = [(2+\gamma)v - 2v_2 - \varepsilon]$. With a probability of $\frac{1-v_2-\varepsilon}{1-v_2}$, $v_1$ does not drop in that interval, $h$ stops the clock and wins one unit and earns $(v - (v_2 + \varepsilon))$. (Note that since we allow the possibility $P = v_2 + \varepsilon = 1$, we are also allowing the strategy that assures winning two units.) Thus, expected profits from such a strategy, "possibly winning two units" are: $\pi(WTU) = \frac{\varepsilon}{1-v_2}[(2+\gamma)v - 2v_2] + \frac{1-v_2-\varepsilon}{1-v_2}[v - v_2 - \varepsilon] = \frac{\varepsilon}{1-v_2}[(1+\gamma)v - v_2] + [v - v_2 - \varepsilon]$. However, since $v < \frac{1}{1+\gamma}$, $\{\pi(WTU) - \pi(WOU)\} = \frac{\varepsilon}{1-v_2}[(1+\gamma)v - v_2 - (1 - v_2)] = \frac{\varepsilon}{1-v_2}[(1+\gamma)v - 1] < 0$.

Step 2. Bidder $h$ never wants to win one unit at a clock price, $P > v$, as it earns negative profits rather than zero profits with no units won.

Rules A.1 and A.2 are the most general rules that implement these conclusions. (Note that $max\{v, v_3\} \geq v \geq min\{v, v_2\}$. Also note that the requirement $P \in [0, min\{v, v_2\}]$, rather than $P \in [0, v_2]$, which also assures winning no more than one unit, is to avoid staying IN with two units beyond $P = v$ when it is not desirable to win even one unit.

Note that in region A, $h$'s optimal strategy yields the same allocation and price as the strategy: "Drop unit 1 at clock price, $P = 0$ and stay IN with the second unit until the clock price reaches your value, $P = v$." Thus, theoretical predictions in region A for the allocation and price are identical to our previous work on multi-unit demand, clock auctions, with flat demand (Kagel and Levin, in press). Further, in A, bidding yields the same allocations and prices as in a sealed-bid uniform price auction (see Appendix A).

**Behavior outside equilibrium in A**: Dropping two units too early leaves no further action. Dropping the first unit early is not an error. If $h$ errs, stays with the first unit or both units too late i.e., $p > v$, and realizes it, $h$ ought to drop out right away.

***B***. *Optimal Behavior for h when* $V = v \in \mathbf{B} = [\frac{1}{1+\gamma}, \frac{2}{2+\gamma})$.

Let the clock price $P^{D} = [(2+\gamma)v - 1]$. Since $v \in [\frac{1}{1+\gamma}, \frac{2}{2+\gamma})$ in region **B**, $P^{D} - v = (1+\gamma)v - 1 \geq 0 > (2+\gamma)v - 2 = P^{D} - 1$. Thus, $v \leq P^{D} < 1$.

**B.1** If $v_2 < P^{D}$, "*Go All The Way*."(ATW)

**B.2** If $v_2 \geq P^{D}$, drop both units at clock price $P \in [P^{D}, max\{P^{D}, v_3\}]$.

**Proofs and observations**:

**B.1** For any given realization $V_2 = v_2$, this strategy yields:
$\pi(ATW) = (2+\delta)v - 2E[V_1|V_2 = v_2] = (2+\delta)v - 2\frac{1+v_2}{2} = (2+\delta)v - 1 - v_2 > (2+\delta)v - 1 - P^D = 0$. Winning one unit earns profits of, $\pi(WOU) = v - v_2$. $[\pi(ATW) - \pi(WOU)] = (1+\delta)v - 1 \geq 0$, since $v \geq \frac{1}{1+\delta}$, and strictly positive $\forall v > \frac{1}{1+\delta}$. Thus, "*Go All The Way*," dominates winning one unit or none.

**B.2** Following the strategy prescribed in B.2, yields zero units with zero profits. Winning one unit earns $v - v_2 < 0$ and staying IN beyond the clock price, $P = P^D$, to win two units earns,
$(2+\delta)v - 2E[V_1|v_2 \geq P^D] < (2+\delta)v - 2E[V_1|v_2 = P^D] = (2+\delta)v - 1 - P^D = 0$. It is easy to show that staying a $\varepsilon > 0$ beyond $P = P^D$, (as long as $P^D + \varepsilon < 1$) and dropping out only if things don't go well also yields negative expected profits.

**Behavior outside equilibrium in B**: Case **B.1**. Dropping two units too early leaves no further action. If *h* dropped the first unit too early *h* needs to stay with the second unit no longer than clock price $P = max\{v, v_3\}$. Case **B.2**. (This is the most interesting out of equilibrium behavior.) If *h* realizes that he stayed IN with both units too late i.e., although $P > max\{P^D, v_3\}$ and $v_2 > P^D$ then, if $v_2$ is still IN $(v_2 \geq P)$, *h* must drop immediately (and nothing happens relative to the optimal policy). However, if $v_2$ has dropped OUT already $(v_2 < P)$, then *h* should "*Go All The Way*," in order to win both units.

***C***. *Optimal Behavior for h when* $V = v \in \mathbf{C} = [\frac{2}{2+\delta}, 1]$.

Following the strategy prescribed in C yields two units and earns positive expected profits of: $(2+\delta)v - 2E[V_1] > 0$, since $v \geq \frac{2}{2+\delta}$ and $E[V_1] < 1$. For any given realization $V_2 = v_2$, this strategy yields profits of $\pi(ATW)$
$= (2+\delta)v - 2E[V_1|V_2 = v_2] = (2+\delta)v - (1+v_2)$. On the other hand winning one unit earns $\pi(WOU) = v - v_2$. Thus,
$\{\pi(ATW) - \pi(WOU)\} = \{[(2+\delta)v - (1+v_2)] - [v - v_2]\} = \{(1+\delta)v - 1\} \geq \{\frac{2(1+\delta)-(2+\delta)}{2+\delta}\} = \{\frac{\delta}{2+\delta}\} > 0,\ v \in [\frac{2}{2+\delta}, 1]$.
Dropping early and winning zero units earn zero profits. Thus the prescribed strategy is optimal.

**Behavior outside equilibrium in C**: If *h* dropped both units there is nothing to do. If *h* erred and dropped one unit she ought to stay IN with the second unit as long as the clock price, $P < max\{v, v_3\}$, and drop the second unit immediately when $P \geq max\{v, v_3\}$.