Tribute to Thomas C. Spencer

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Ladies and gentlemen, colleagues and friends,
Dear Tom

When I was asked to present a laudation of my friend and mentor Tom Spencer and his scientific work it was immediately clear to me that I could not possibly decline this invitation, although I am notoriously reluctant to travel far distances.

Let me begin my remarks by describing the times of Tom's début in science! In the late sixties, one of the central problems in theoretical and mathematical physics was perceived to be whether *quantum theory, locality* and the *special theory of relativity* are compatible with one another; in other words, whether local, relativistic quantum field theories make sense mathematically. There was overwhelming evidence that *quantum electrodynamics* provides an astonishingly accurate description of processes involving electrons, positrons and electromagnetic radiation. Until the late sixties, this was really the only compelling example of a physically viable relativistic quantum field theory. But it was studied purely perturbatively, and its mathematical status remained very unclear. There was not a single model of local relativistic quantum fields known that had been shown to be mathematically consistent. One of the leading promoters of the problem to rigorously construct quantum field models was the late Arthur Wightman of Princeton University, who had some very talented followers and students. One of them was Arthur Jaffe, who, in the second half of the sixties, decided to join forces with an eminent analyst, James Glimm, who had independently begun to study models of relativistic quantum fields. Their declared goal was to construct mathematically consistent models of local relativistic quantum fields and to study their properties. Towards the end of the sixties, they had made encouraging progress in constructing quantum field models on two-dimensional Minkowski space following the Hamiltonian approach. At that time, Jim attracted a very promising PhD student who would soon play an important role in the Glimm-Jaffe endeavor.

Thomas Crawford Spencer was born on December 24, 1946, in the United States of America. He went to school in the US and in Brazil. He studied mathematics at Berkeley and then decided to return to the East Coast to take up PhD studies in mathematics at the Courant Institute. Those were difficult times for young Americans, because the war in Vietnam was raging, and many of them were drafted into the United States armed forces – not few of them to loose their future. Most fortunately, Tom succeeded to circumambulate this fate and to start working on a PhD project on constructive quantum field theory under the supervision of James Glimm. He earned his doctorate in 1972 for a thesis entitled *"Perturbations of the $P(\phi)_2$ - Quantum Field Hamiltonian"*. He then continued to collaborate with Jim at Courant until 1974. From 1974 to 1975, Tom was a postdoctoral researcher at Harvard University, in the group of Arthur Jaffe. I will never forget that, in the fall of that year, my collaboration with Tom started. From 1975 till 1977, he joined Jim at Rockefeller University in New York to serve as an associate professor, until Rockefeller abolished its mathematics department. Between 1978 and 1980, he was a professor at the math department of Rutgers University. From there he returned to the Courant Institute, where he stayed for six years. In 1986, he received an offer of a professorship in the School of Mathematics of the Institute for Advanced Study at Princeton, where he has been working ever since.
In 1983, Tom got married to Bridget Murphy. Bridget is an exceptionally generous and understanding woman and furnishes a wonderful example of how important it is for scientists to choose the right partners to share their lives with. They have a daughter and a son.
But I fear Tom may think that I have already described somewhat too many details of his biography. Thus, let's proceed to highlight some of his main scientific accomplishments.

It is characteristic of Tom's scientific efforts that he always worked on very concrete problems, but ones belonging to a *major theme in theoretical and mathematical physics*. Here is a list of such themes:

1. Constructive quantum field theory
2. Phase transitions and spontaneous symmetry breaking
3. Critical phenomena and the idea of universality
4. Disordered systems

5. Chaotic dynamical systems

Let me mention some of Tom's remarkable contributions to Theme 1. In the very early seventies, under the influence of seminal work by Symanzik and Nelson, it became customary to follow the euclidian functional integral approach – based on the Wick rotation in the time variable to the imaginary axis – rather than the Hamiltonian approach to construct relativistic quantum field models, such as the $P(\phi)_2$ - and the $\lambda\phi_3^4$ - models. This made the quantum field problems look like problems in classical statistical mechanics. Around the turn of the year from 1972 to 1973, Tom co-authored a paper with James Glimm on the problem of removing the space-time cutoff in the construction of a euclidian $P(\phi)_2$ - model in two dimensions. They devised a remarkable inductive construction to successively enlarge the space-time domain where the interaction is turned on. Their ideas gave rise to the development of cluster expansions for continuum systems, in particular quantum field models, the first of which was the Glimm-Jaffe-Spencer cluster expansion published in the celebrated volume 25 of the Springer Lecture Notes in Physics that contains the proceedings of the 1973 Erice school. Glimm, Jaffe and Spencer then went on to analyze the particle structure of two-dimensional scalar quantum field models within the euclidian approach, thus furnishing the basis for an application of Haag-Ruelle scattering theory to those models. Tom then came up with an ingenious analysis of properties of the Bethe-Salpeter kernel in $P(\phi)_2$ - models. In a joint paper with Francesco Zirilli, he used those properties to prove asymptotic completeness for two-particle scattering processes below the three-particle threshold. Their work gave rise to many subsequent studies of scattering theory in quantum field theory.
Glimm, Jaffe and Spencer were first to succeed in extending the Peierls argument and low-temperature expansions to one-component $\lambda\phi^4$ - theory in two space-time dimensions, proving the existence of a phase transition, the spontaneous breaking of the $\phi \to -\phi$ symmetry and exponential decay of connected Green functions.

This leads me to sketch some of Tom's remarkable contributions to Theme 2: Towards the end of 1975, Barry Simon, Tom Spencer and I discovered a method to analyze phase transitions and spontaneous symmetry breaking in models with *continuous symmetries* and *massless Goldstone modes*, such as the classical rotor- and Heisenberg models. The method was based on a combination of an upper bound on the connected two-point correlation function derived from the Källén-Lehmann representation of relativistic quantum field theory with a lower bound on the two-point function expressing a sum rule. Not surprisingly, our strategy was first applied to the continuum

$\lambda|\vec{\phi}|^4$ - euclidian field theory in three dimensions. Subsequently, an analogue of the Källén-Lehmann representation was discovered for lattice models satisfying "reflection positivity", leading to the method of *"infrared bounds"*, which, as of today, remains the most successful method to study phase transitions in models with continuous symmetries. This method was subsequently extended to certain quantum spin systems in work of Dyson, Lieb and Simon, and others.
In work with Oliver McBryan, Tom proved a power-law decay estimate on the spin-spin correlation in the 2D rotor model, using some clever contour deformations in a statistical integral. This method became one among several crucial technical ingredients in a subsequent study of the *Berezinskii-Kosterlitz-Thouless transition* in 2D Coulomb gases, the 2D rotor model and the 2D SOS model of interfaces. These systems can be represented as gases of arbitrarily large neutral compounds of point-vortices whose free energies are shown to tend to zero, as their diameter tends to infinity, provided the temperature is small enough. Hence these gases are dilute at low enough temperature. This work heralded the advent of a novel method called *multi-scale analysis*. It was then applied – mutatis mutandis – to the analysis of the phase transition in the one-dimensional Ising model with $1/r^2$ - ferromagnetic spin-spin interactions, which had first appeared in an analysis of the Kondo effect by Philip Anderson. Similar, simpler ideas yielded proofs of existence of the phase transition in the 3D rotor model and of the deconfinement transition in the 4D $U(1)$ - lattice gauge theory; examples of systems that can be represented as gases of vortex loops and -sheets.

Let's speed up this review a little and proceed to talking about Theme 3, i.e., *critical phenomena and the idea of universality*. Tom was involved in proving bounds on critical exponents characterizing continuous transitions passing through a critical point. Among significant findings are bounds on critical exponents for percolation and for disordered magnets and random Schrödinger operators derived together with Jennifer and Lincoln Chayes and Daniel Fisher. Most remarkable, however, is the result that, above dimension 4, the weakly self-avoiding walk behaves like simple random walk, which David Brydges and Tom proved with the help of a new analytical tool called *lace expansion* that involves an induction in the time-scale of the memory of walks interpolating between simple and self-avoiding random walk. This tool has later been successfully used in further work on self-avoiding walks and on branched polymers by Brydges, Slade and their collaborators.
In the nineties, Tom became interested in understanding critical behavior and universality in a family of 2D ferromagnetic Ising models (work with Pinson), a large class of random surface models (with Naddaf, Conlon and Brydges) and a 3D hyperbolic Sigma model (with Zirnbauer). The work on perturba-

tions of the 2D Ising model had much impact on efforts by young Italians around Gallavotti. These studies involved a mix of analytical methods that are delights for courageous analysts, in particular rigorous renomalization group methods, the Helffer-Sjöstrand (-Witten) method exploiting convexity, homogenization, the theory of singular integral operators, and new variants of central limit theorems, etc. Finally, in a very dedicated joint effort with Disertori and Zirnbauer, Tom investigated the transition in a 3D supersymmetric hyperbolic Sigma model that mimics the Anderson transition for random Schrödinger operators. In this work, the use of Ward identities derived from supersymmetry was pioneered; ideas that, in the hands of Mastropietro et al. have become an important tool. More recently, the DSZ results have been extended to the edge-reinforced random walk by Disertori, Sabot and Tarres, following suggestions by Gawedzki.

Let's turn to Theme 4, *disordered systems*. Here, Tom's most significant results concern a proof of Anderson localization for a class of discrete random Schrödinger operators in any dimension, for large disorder or in the band tails, and for some 1D quasi-periodic Schrödinger operators. These results represent subtle applications of multi-scale analysis combined with estimates that may be called "large-deviation estimates with spatial structure". The analysis of localization for 1D quasi-periodic Schrödinger operators turned out to be particularly tricky and might well be worth more than ten Martinis. Interesting results on average Green functions and smoothness of the density of states of random band matrices were contributed by Disertori, Pinson and Spencer, using a supersymmetric formalism. As far as I know, Tom was instrumental in reviving interest in random matrix theory, where, in recent years, spectacular advances have been made by various groups of researchers, including Erdös and Yau and their collaborators.
Using a coarse-graining technique, partial success was scored in understanding the phase transition in the 3D random-field Ising model, which subsequently was understood completely by Imbrie and by Bricmont and Kupiainen. Finally, there was work by Imbrie and Spencer on various lattice models of disordered systems.

The work on 1D quasi-periodic Schrödinger operators stimulated efforts to understand the chaotic properties of the Chirikov standard map. A project in this direction was courageously pursued by Lenart Carleson and Tom and led to presumably hundreds of pages of unpublished work. A little part of their program was recently brought to fruition by Mira Shamis and Tom. They have proven bounds on the Lyapunov exponent of the standard map based on the Thouless formula and estimates on the density of states of an associated random Schrödinger operator.
Clearly, these remarks do not do justice to Tom's efforts in studying chaotic

dynamical systems.

I would like to take another minute to talk about Tom's outstanding qualities as a colleague and friend. I first met Tom in the fall of 1973. Jim had invited me to visit New York and give a seminar at the Courant Institute. I thought I had proven the global Markov property for $P(\phi)_2$ - models and was looking forward to expose this exciting result. But when I arrived at Courant, Ola Bratteli pointed out a serious gap in my proof. I felt utterly discouraged and had to change the topic of my seminar. I decided to explain my proof of exponential ϕ - bounds, besides a new proof of the *local* Markov property. At the end of my talk, Tom qualified my results as interesting, and, in his paper on the Bethe-Salpeter kernel, he generously quoted my exponential bounds. This experience made it clear for me that Tom was truly a gentleman scientist!

Tom and I enjoyed a ten-year period of intense collaboration – from 1975 till 1985 – that, on all counts, was a great success. It is possible that without Barry Simon, who first proposed that the three of us collaborate on phase transitions, my collaboration with Tom might not have started. I am happy to acknowledge on today's occasion that the success Tom and I scored is most essentially due to all his wonderful and sometimes quite magical mathematical ideas that enabled us to climb some major peaks in the landscape of mathematical physics. Tom was my mountain guide in these endeavors and my mathematical mentor.

Our collaboration ended, because we live far apart from each other, geographically, and because we started to have ever heavier duties as professors and to raise families. But our friendship has lasted to this day. In fairly frequent visits at the Institute for Advanced Study, I could witness Tom's exceptional generosity towards young colleagues. It is not rare that visitors at the IAS get discouraged; for example, because success in a project is more elusive than hoped. Sooner or later, most visitors of the School of Mathematics end up in Tom's office, telling him what they are trying to accomplish and seeking his advice. One of Tom's great qualities is that he has very diverse interests and broad experience and, hence, is often able to suggest to his visitors a new perspective or some simplifying ideas. In discussions he shows admirable patience. He is genuinely modest and kind.

Dear Tom: I feel very lucky to have had you as a mentor and partner for scientific endeavors and adventures and, most importantly, as a close friend. I congratulate you on today's highly deserved recognition, and I wish you all the best for the future!

Thank you!