Influence of the noise fluctuactions in the estimate of duration-Magnitude.

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Introduction

The aim of this report is to quantify the error related to the estimate of the Duration-Magnitude, due to the seismic noise fluctuations. We will carry on an example on data from Mt. Vesuvius.

The report is divided into the following three sections:

1. **rms noise distribution**. We find that the statistical distribution which best represent the experimental data is the log-normal one, and calculate the best fit parameters.
2. **Experimental distribution of earthquake-durations.** The function which describes the time- envelope of the seismograms at Mt. Vesuvius is well known from a study of the seismic attenuation in the area (Bianco et al. 1999). As the earthquake duration is defined as the time point at which the envelope encounters the rms-noise level, we can numerically estimate the corresponding distribution of the earthquake durations for each given Magnitude.
3. **Errors associated to the duration-Magnitude.** Finally, we deduced the errors associated to the duration-Magnitude from the corresponding distribution of the earthquake durations.

All calculations are active in the present MATHCAD worksheet, which can be easily adapted for applications different from the present example

rms noise distribution

The present section describes the procedures used to deduce the properties of the rms-noise distribution.

First we select OVO station of the seismic network - see the Open File Report by Buonocunto et al. 2001, on line at www.ov.ingv.it -. The duration Magnitude scale was calibrated (Del Pezzo and Petrosino, 2001 and references therein) for this station which was set up since 1972 with modern sensors, and was mantained in the same configuration up to the present time. It is the reference station for the calculation of the earthquake duration Magnitude at Mt. Vesuvius.

:=
D:\..\Rmsout00.txt

:=
D:\..\Rmsout99.txt

:=
D:\..\Rmsout98.txt

:=
D:\..\Rmsout97.txt

The input files containing the rms values calculated in the pre-event 10 seconds for each local earthquake recorded at OVO from 1997 to the end of 2000.

$RMS := stack(a2000, a1999, a1998, a1997)$

RMS is the vector containing the stack of all the pre-event rms values

$$RMS\_Norm := \frac{RMS}{max(RMS)}$$

RMS is normalized respect to its maximum in order to avoid any instrumental correction

$$RMS\_Hist := histogram(5000, RMS\_Norm)$$

RMS_Hist is the histogram calculated using the internal MATHCAD library. We selected 5000 intervals between the maximun and the minimum value.

The experimental probability density is calculated from the numerical frequency $\frac{RMS\_Hist^{\langle 2\rangle}}{rows(RMS)}$ divided by the bin width $\frac{1}{\left[\left(RMS\_Hist^{\langle 1\rangle}\right)_2 - \left(RMS\_Hist^{\langle 1\rangle}\right)_1\right]}$

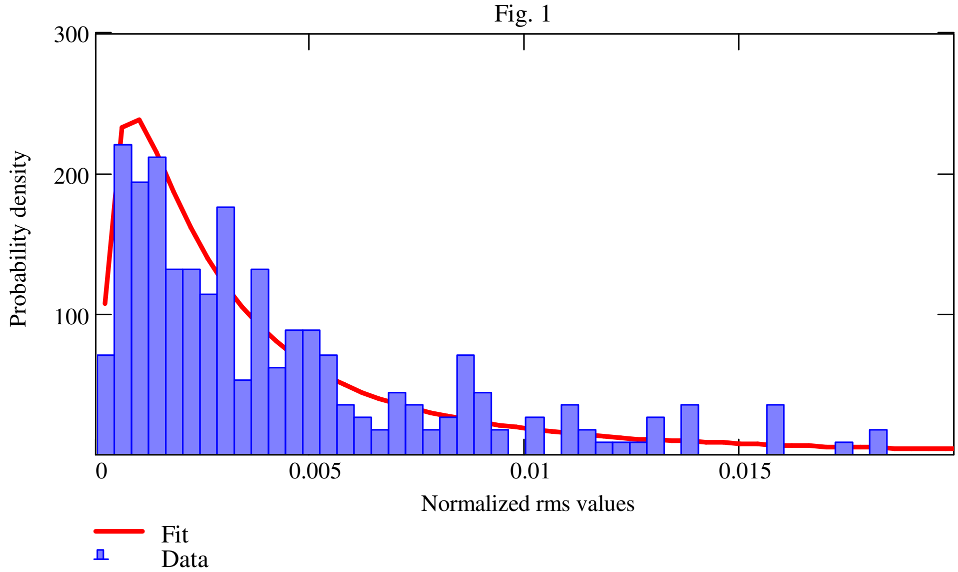
$$RMS\_Prob^{\langle 2\rangle} := \frac{RMS\_Hist^{\langle 2\rangle}}{rows(RMS)} \cdot \frac{1}{\left[\left(RMS\_Hist^{\langle 1\rangle}\right)_2 - \left(RMS\_Hist^{\langle 1\rangle}\right)_1\right]}$$

$$RMS\_Prob^{\langle 1\rangle} := RMS\_Hist^{\langle 1\rangle}$$

Using a trial and error procedure we found that the kind of distribution better approximating the experimental data set is log-normal with mean -5.9 and σ=1.1 (see the following relationship), as plotted in Fig. 1

$$PP2 := \overrightarrow{dlnorm\left(RMS\_Hist^{\langle 1\rangle}, -5.9, 1.1\right)}$$

Fig. 1



Experimental distribution of earthquake-durations

We want to define the coda-duration distribution as a function of the rms fluctuations. The coda envelope can be well approximated by the following formula (See Sato and Fehler, 1998 and Bianco et al. 1999) where A_coda is the envelope amplitude, A_0 is the amplitude at the origin and τ equals t-t0 where t0 is the origin time. Q_coda is a parameter controlling the envelope shape, and f is the frequency.

$$A\_coda := \frac{A_0}{\tau^{0.5}} \cdot \exp\left(\frac{-\pi \cdot f \cdot \tau}{Q\_coda}\right)$$

Q_coda was experimentally evaluated for Mt. Vesuvius by Bianco et al. (1999) and for f=6 Hz (that is the spectral velocity peak) assumes the following value:

$$\text{Q-coda(6 Hz)} = \frac{1}{4.9 \cdot 10^{-3}}$$

It results that

$$A\_coda := \frac{A_0}{\tau^{0.5}} \cdot \exp\left(\frac{-\pi \cdot 6 \cdot \tau}{204}\right)$$

Simplifying

$$A\_coda := \frac{A_0}{\tau^{0.5}} \cdot \exp(-0.09 \cdot \tau) \qquad (1)$$

and on the other hand

$$A_S := \frac{A_0}{\sqrt{3}} \cdot \exp(-0.09 \cdot 3) \qquad (2)$$

where we fixed at τs=3 the average travel time for S-waves for the local events at Mt. Vesuvius. A_S represents the envelope amplitude at lapse time equal to the S-wave travel time, and roughly corresponds to the S-wave maximum amplitude on the seismogram.

From relation (2) it follows that $A_0 := A_S \cdot \sqrt{3} \cdot \exp(0.27)$ and substituting in (1) it can be obtained

$$A\_coda := \frac{A_S \cdot \sqrt{3} \cdot \exp(0.27)}{\tau^{0.5}} \cdot \exp(-0.09 \cdot \tau) \qquad (3)$$

The earthquake duration, τ_d, can be deduced setting A_coda=rms-noise

$$rms\_noise := \frac{A_S \cdot \sqrt{3} \cdot \exp(0.27)}{\tau\_d^{0.5}} \cdot \exp(-0.09 \cdot \tau\_d) \qquad (4)$$

From (4) we can deduce the ratio, RSR_M between A_S and rms_noise:

$$RSR_M := \frac{\tau\_d^{0.5} \cdot \exp(0.09 \cdot \tau\_d)}{\sqrt{3} \cdot \exp(0.27)} \qquad (5)$$

From (5) τ_d can be calculated for a set of A_S values.

Distribution of the earthquake durations

A distribution of rms noise is generated using the internal rlnorm routine setting the distribution parameters equal to those experimentally calculated.

For a given As we find the earthquake duration distribution using rel. (4). For each As both τ_d and its standard deviation can be calculated, as in the following steps.

```
A_noise := rlnorm(1000, −5.9, 1.1)

τ(As) := | for n ∈ 1 .. rows(A_noise)
         |   for k ∈ 1 .. 200
         |     | t_k ← k
         |     | continue if As(t_k)^(−0.5)·exp(0.27)·√3·exp[−0.09·(t_k)] < A_noise_n
         |     | max_time_n ← t_k if As(t_k)^(−0.5)·exp(0.27)·√3·exp[−0.09·(t_k)] ≥ A_noise_n
         | max_time
         | for n ∈ 1 .. rows(max_time)
         |   max_time_n ← 0.1 if max_time_n = 0
         | max_time
```

τ(As) is a local MATHCAD routine that calculate the perturbated durations corresponding to As. τ_m is the corresponding mean value and στ(As) is its standad deviation.

$\tau\_m(As) := mean(\tau(As))$

$\sigma\tau(As) := stdev(\tau(As))$

$Mag(x) := 2.75 \cdot \log(x) - 2.35$ (6)

Duration Magnitude calibration formula for OVO station (x is the earthquake duration)

$M(As) := \overrightarrow{(2.75 \cdot \log(\tau(As)) - 2.35)}$

Vector containing the perturbed duration-Magnitude corresponding to the amplitude As

$\sigma M(As) := stdev(M(As))$

Its standard deviation.

$\tau\_m(100) = 100.452$

$\sigma\tau(100) = 11.425$

$\sigma M(100) = 0.138$

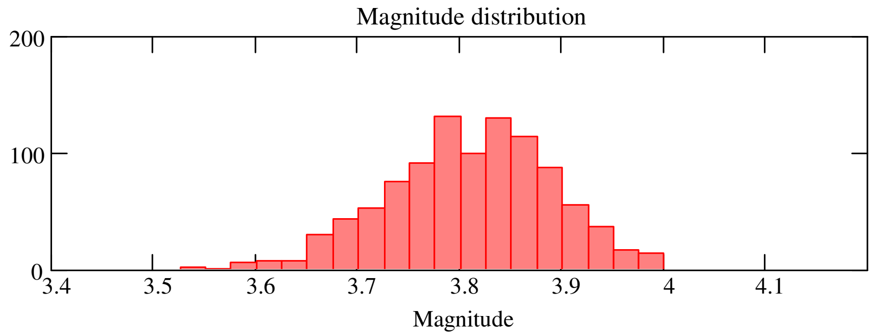
Examples obtained for As=100

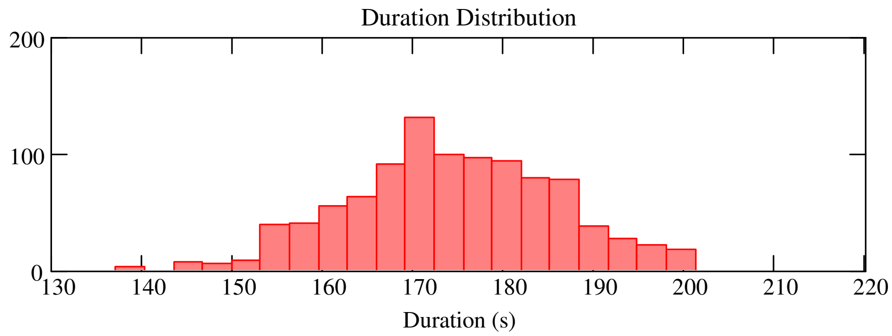
Magnitude and duration distributions

distritau:= histogram$(20, \tau(100000))$

distriM:= histogram$(40, M(100000))$

Distribution of durations and Magnitude corresponding to an amplitude of 100000



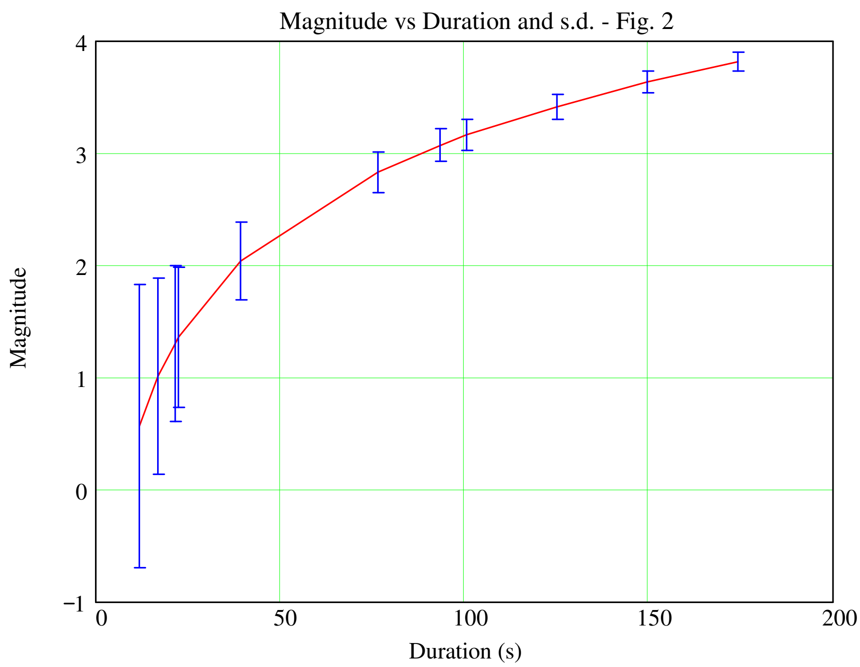


$$AAs := \begin{pmatrix} 0.01 \\ 0.02 \\ 0.035 \\ 0.04 \\ 0.25 \\ 10 \\ 50 \\ 100 \\ 1000 \\ 10000 \\ 100000 \end{pmatrix}$$

A set of S-wave amplitudes

$k := 1 .. \mathrm{rows}(AAs)$
$tau_k := \tau\_m(AAs_k)$
$Magn_k := 2.75 \cdot \log(tau_k) - 2.35$
$sigmaM_k := \sigma M(AAs_k)$

The corresponding durations. Magnitudes and their standard deviations obtained using (6)

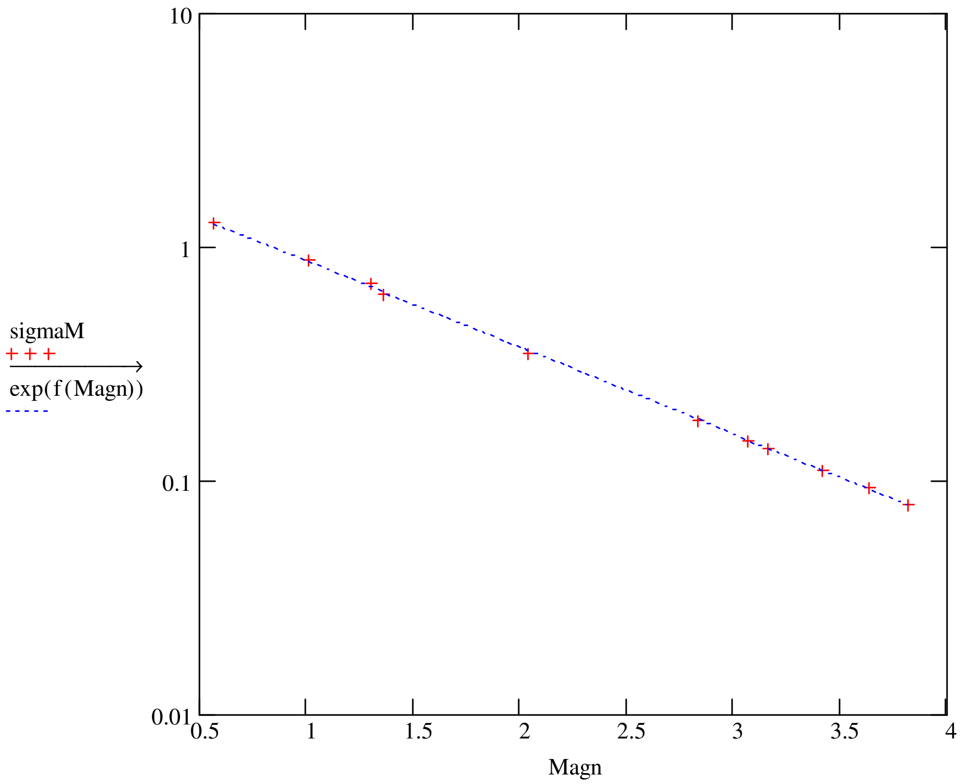


Standard deviation depends on the Magnitude, as clearly visible on the plot of Fig. 2 and this dependence can be approximated by log-linear relationship.

$y := \ln(sigmaM)$
$x := Magr$
$\mathrm{slope}(x, y) = -0.853$
$\mathrm{intercept}(x, y) = 0.71$

The dependent (y) and independent (x) variables in the linear fit

$$f(\xi) := \text{slope}(x, y) \cdot \xi + \text{intercept}(x, y) \qquad (7)$$



Concluding Remarks

We observe that the distributions of seismogram durations are approximately gaussian, for all the values of amplitude. Consequently, the distributions of the duration-Magnitude fluctuations are asymmetric, showing a longer tail at the left of the maximum with asymmetry decreasing with increasing duration-Magnitude. In a first approximation, we can consider duration-Magnitude as normally distributed and derive from the Monte-Carlo simulation the relationship between variance and duration-Magnitude, given by relationship (7)

The program implicitly contained in the present MATHCAD worksheet can be easily applied to any other seismic area. Note that the shape of the rms noise amplitude distribution is log-normal for Mt. Vesuvius. In case of application to another area the nature of the distribution must be experimentally checked

REFERENCES

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