A tripartite array for the study of seismo-volcanic signals. A MATHCAD-11 program.

E. Del Pezzo
INGV Osservatorio Vesuviano

Abstract

The present worksheet contains a MATHCAD 11 program for the estimate of slowness vector components of a seismic signal recorded by a short period small aperture tripartite array. I use a simplified Plane-Wave-Fitting technique (PWF - Del Pezzo and Giudicepietro, Computer and Geosciences, 28 , 2002, 59-64) based on the estimate of the cross-correlation between signal pairs in time domain. An example of application to a synthetic signal shows that the estimate of the slowness vector components is almost unbiased evn in condition of low signal-to-noise ratio.

The method

Given a tripartite array of seismic stations located at points with cartesian co-ordinates x_i, y_i, I make the hypothesis that a wave packet of given slowness vector $p = (p_x, p_y)$ crosses the array with the inter-distances between the station pairs sufficiently small to permit a good degree of correlation among the traces. The travel time difference between station i and j is given by the dot product

$$t_{i,j} := p \cdot \Delta x_{i,j} \qquad (1)$$

where $\Delta x_{i,j}$ is the two element vector of the differences between the co-ordinates of station i and j: $\Delta x_{i,j} := (x_i - x_j, y_i - y_j)$ (2)

and $t_{i,j}$ is estimated through the cross-correlation function between the signal pairs $a(t)_i$ and $a(t)_j$:

$$C(\tau)_{i,j} := \int_{-\infty}^{\infty} a(t)_i \, a(t+\tau)_j \, dt \qquad (3)$$

The maximum of equation (3) occurs at a time, t_{max} which corresponds to the phase delay between signals at station i and j. t_{max} estimates for any couple i,j $t_{i,j}$ of eq. (1)

I denote with t the one column matrix given by $\begin{pmatrix} t_{\mathbf{1},\mathbf{2}} \\ t_{\mathbf{1},\mathbf{3}} \\ t_{\mathbf{2},\mathbf{3}} \end{pmatrix}$; consequently p can be estimated solving the overdetermined equation system (1) by least squares. From the estimate of p, apparent velocity v and azimuth ϕ can be estimated by:

$$v := \left[\left(p_x\right)^2 + \left(p_y\right)^2 \right]^{\frac{-1}{2}}$$

$$\phi := \operatorname{atan}\left(\frac{p_x}{p_y}\right)$$

Errors can be estimated by the covariance matrix of the data (Menke, 1984)

The users who have installed MATHCAD 11 on their PC can change the input parameters to test in different conditions. Input parameters are enlighted in red color. Set TOL=0.000001 from worksheet options.

Synthetic Test input for PWF program

Generation of the synthetic traces at an array of 3 station (tripartite array). This is the test-input for the present simplified version of the PWF program (Plane wave fitting technique)

$f2 := \mathbf{3}$

predominant frequency of the synthetic signal: f=3 Hz

$par1 := \mathbf{0.3}$ $par2 := \mathbf{0.1}$ $par3 := \mathbf{0.4}$

These parameters change the shape of the test signal

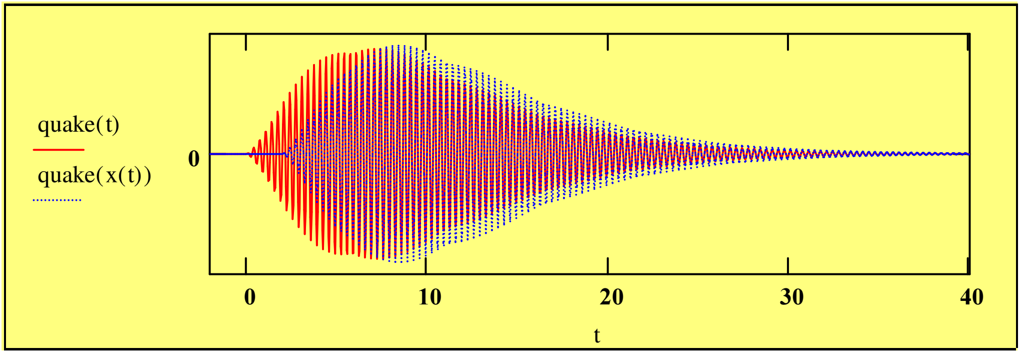
$$signal(t) := \left(\frac{t}{par1}\right)^{\mathbf{1.6}} \cdot \exp\left[\frac{-(par2 \cdot t)}{par3}\right] \sin(\mathbf{2} \cdot \pi \cdot f2 \cdot t)$$

$$quake(t) := \left(\begin{array}{|l} \mathbf{0} \quad \text{if } t \le \mathbf{0} \\ signal(t) \quad \text{if } t > \mathbf{0} \end{array} \right)$$

synthetic signal

$x(t) := t - \mathbf{2}$

time shift example



srate := **100**

Sampling rate

Npti := **2048**

Data points

Signal generation

$i := \mathbf{1} .. Npti$

$t_i := \frac{i}{srate}$

$s2_i := quake(t_i)$

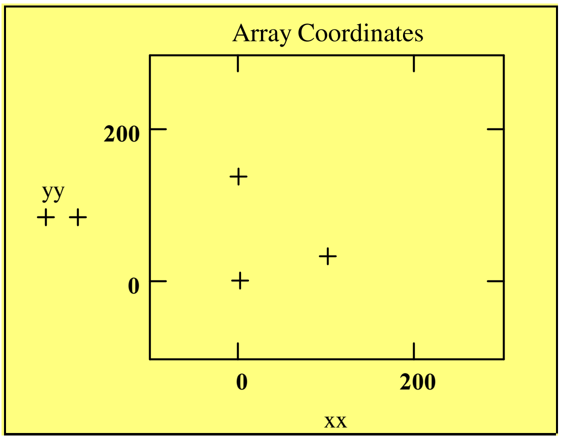
Co-ordinates of the array test

$$A := \begin{pmatrix} 2 & 3.2 \\ 0.3 & 140.2 \\ 101.3 & 35.4 \end{pmatrix}$$

Units in meters

$$xx := A^{\langle 1 \rangle}$$

$$yy := A^{\langle 2 \rangle}$$



$Nstaz := rows(A)$

$Nstaz = \mathbf{3}$

Test azimuth $az := 7$ gradi

Test v app $vapp := 1600$ metri / s

$k := \mathbf{1} .. Nstaz$

$n := \mathbf{1} .. Nstaz$

$$slox := \frac{\mathbf{1}}{vapp} \cdot \cos(az \cdot deg)$$

Slowness test in s/m. Azimuth in degrees from East to North

$$sloy := \frac{\mathbf{1}}{vapp} \cdot \sin(az \cdot deg)$$

$$\Delta t_{k,n} := \left[slox \cdot \left(xx_k - xx_n \right) + sloy \cdot \left(yy_k - yy_n \right) \right]$$

$slox = \mathbf{0.00062034}$

Co-ordinates of the slowness vector test

$sloy = \mathbf{0.00007617}$

Example of true time shift

$\Delta t_{\mathbf{1},\mathbf{2}} = -\mathbf{0.00938048}$

$x_{i,k} := t_i - \Delta t_{k,\mathbf{1}} - \mathbf{0.7}$

$noise\_amplitude := \mathbf{6}$

Addition of noise. Noiuse is gaussian (function rnorm)

$noise^{\langle k \rangle} := rnorm(Npti, \mathbf{0}, noise\_amplitude)$

$s2_{i,k} := quake\left(x_{i,k}\right) + noise_{i,k}$

Shift is a function based on the cross-correlation between the signal pairs

$$
\text{shift}(X,Y) := \left|
\begin{array}{l}
XX \leftarrow \text{correl}(X,Y) \\
XXX \leftarrow \text{recenter}(XX) \\
XXXX \leftarrow \text{submatrix}\left(XXX, \text{round}\left(\frac{\text{last}(XXX)}{\mathbf{2}} - \mathbf{50}\right), \text{round}\left(\frac{\text{last}(XXX)}{\mathbf{2}} + \mathbf{50}\right), \mathbf{1}, \mathbf{1}\right) \\
\text{massimo} \leftarrow \max(XXXX) \\
\text{for } v \in \mathbf{1} .. \text{last}(XXXX) \\
\quad \text{index} \leftarrow v \ \text{ if } \ XXXX_v \geq \text{massimo} \\
\text{alfa} \leftarrow \text{index} - \text{round}\left(\frac{\text{last}(XXXX)}{\mathbf{2}}\right) \\
\text{t\_shift} \leftarrow \frac{\text{alfa}}{\text{srate}} \\
\text{t\_shift}
\end{array}
\right.
$$

Data input

$$
i := \mathbf{1} .. \text{last}\left(s2^{\langle \mathbf{1} \rangle}\right)
$$

$$
k := \mathbf{1} .. \text{Nstaz}
$$

$t_i := \frac{i}{srate}$

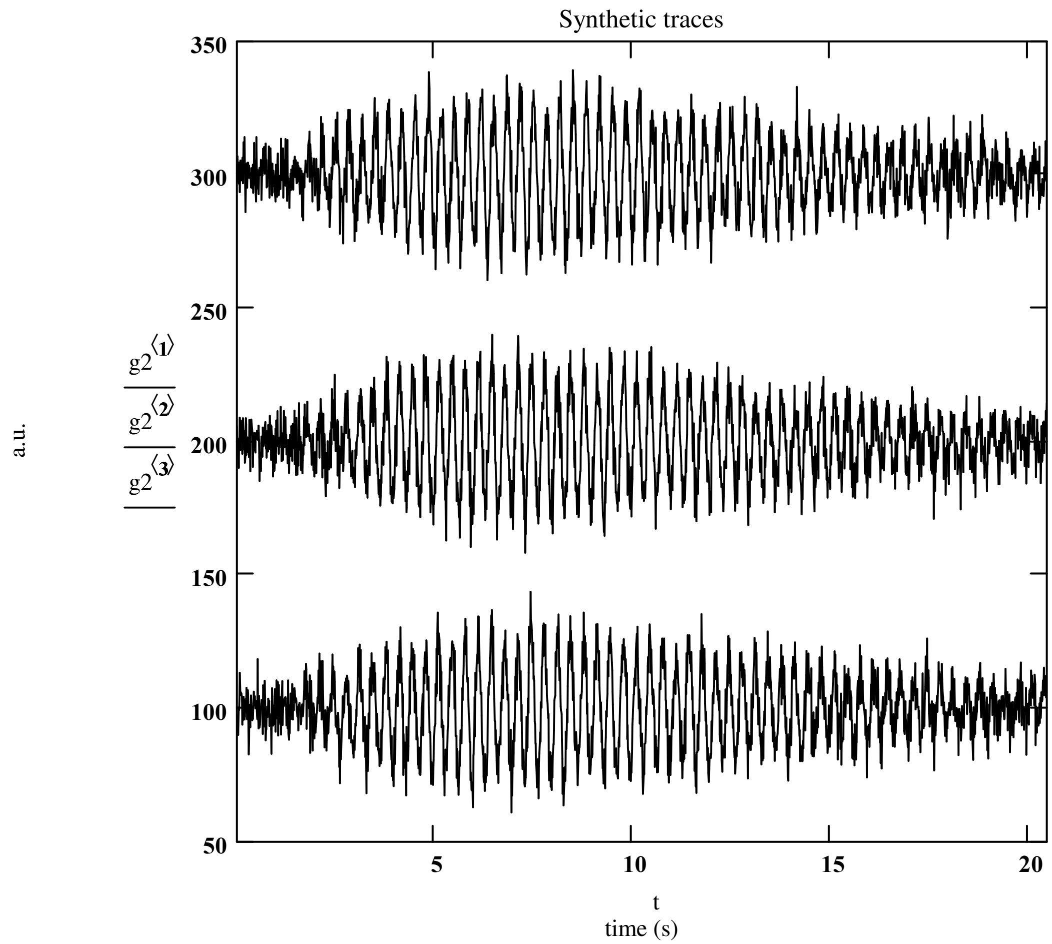
$g2 := s2$

$cols(s2) = \mathbf{3}$

$o := \mathbf{1} .. cols(s2)$

$g2^{\langle o \rangle} := g2^{\langle o \rangle} + o \cdot \mathbf{100}$

Data plot. 16 traces maximum are allowed to be plotted



Coordinate := augment(xx, yy)

$$\Delta XX := \begin{pmatrix} xx_1 - xx_2 & yy_1 - yy_2 \\ xx_1 - xx_3 & yy_1 - yy_3 \\ xx_2 - xx_3 & yy_2 - yy_3 \end{pmatrix}$$

$$xx = \begin{pmatrix} 2 \\ 0.3 \\ 101.3 \end{pmatrix} \qquad yy = \begin{pmatrix} 3.2 \\ 140.2 \\ 35.4 \end{pmatrix}$$

Data selection. Set the number of windows in which the signal is segmented

$$n\_win := \mathbf{20}$$

$$t\_w := \left| \begin{array}{l} t\_win \leftarrow \dfrac{\max(t) - \min(t)}{n\_win} \\ \text{for } n \in \mathbf{1} .. n\_win \\ \quad \left| \begin{array}{l} tin_n \leftarrow \min(t) + (n - \mathbf{1}) \cdot t\_win \\ tfin_n \leftarrow tin_n + t\_win \end{array} \right. \\ \text{augment}(tin, tfin) \end{array} \right.$$

Define the start and end time for each window

$$signal := \left| \begin{array}{l} \text{for } n \in \mathbf{1} .. n\_win \\ \quad signal_n \leftarrow \text{submatrix}\left[s2, \left(t\_w^{\langle \mathbf{1} \rangle}\right)_n \cdot srate, \left(t\_w^{\langle \mathbf{2} \rangle}\right)_n \cdot srate, \mathbf{1}, \text{cols}(s2) \right] \\ signal \end{array} \right.$$

Segmentation of the signal

$$Npti := \text{last}\left[\left(signal_{\mathbf{1}} \right)^{\langle \mathbf{1} \rangle} \right]$$

The observable is the time value corresponding to the max of cross-correlation function obtained from shift function

Generation of the data Matrix and covariance Matrix

$s := \mathbf{1} .. \text{last}(\text{signal})$

$m := \mathbf{1} .. \text{last}\left(xx^{\langle \mathbf{1} \rangle}\right)$

$n := \mathbf{1} .. \text{last}\left(xx^{\langle \mathbf{1} \rangle}\right)$

$$\text{vector} := \left| \begin{array}{l} \text{for } s \in \mathbf{1} .. \text{last}(\text{signal}) \\ \quad \left| \begin{array}{l} \text{Data}_{s,\mathbf{1}} \leftarrow \text{shift}\left[\left(\text{signal}_s\right)^{\langle \mathbf{2} \rangle}, \left(\text{signal}_s\right)^{\langle \mathbf{1} \rangle}\right] \\ \text{Data}_{s,\mathbf{2}} \leftarrow \text{shift}\left[\left(\text{signal}_s\right)^{\langle \mathbf{3} \rangle}, \left(\text{signal}_s\right)^{\langle \mathbf{1} \rangle}\right] \\ \text{Data}_{s,\mathbf{3}} \leftarrow \text{shift}\left[\left(\text{signal}_s\right)^{\langle \mathbf{3} \rangle}, \left(\text{signal}_s\right)^{\langle \mathbf{2} \rangle}\right] \end{array} \right. \\ \text{Data} \end{array} \right.$$

$$\text{Datacolumns} := \left| \begin{array}{l} \text{for } s \in \mathbf{1} .. \text{last(signal)} \\ \quad A^{\langle s \rangle} \leftarrow \left(\text{vector}^{T}\right)^{\langle s \rangle} \\ A \end{array} \right.$$

Solution of the inverse prorblem

$$P := \left| \begin{array}{l} \text{for } s \in \mathbf{1} .. \text{last(signal)} \\ \quad P^{\langle s \rangle} \leftarrow \left[-\left(\Delta XX^{T} \cdot \Delta XX\right)\right]^{-\mathbf{1}} \cdot \Delta XX^{T} \cdot \text{Datacolumns}^{\langle s \rangle} \\ P \end{array} \right.$$

$$\text{velapp} := \left| \begin{array}{l} \text{for } s \in \mathbf{1} .. \text{last(signal)} \\ \quad \text{slowp}_s \leftarrow \dfrac{\mathbf{1}}{\sqrt{\left[\left(P^{\langle s \rangle}\right)_{\mathbf{1}}\right]^{\mathbf{2}} + \left[\left(P^{\langle s \rangle}\right)_{\mathbf{2}}\right]^{\mathbf{2}}}} \\ \text{slowp} \end{array} \right.$$

$$\text{backp} := \left| \begin{array}{l} \text{for } s \in \mathbf{1} .. \text{last(signal)} \\ \quad \text{angolo}_s \leftarrow \text{angle}\left[\left(P^{\langle s \rangle}\right)_{\mathbf{1}}, \left(P^{\langle s \rangle}\right)_{\mathbf{2}}\right] \\ \text{angolo} \end{array} \right.$$

$$errdata := \left(1\cdot\frac{1}{srate}\right)^{2}$$

Estimate of the error on data

$$sigmaq := errdata\cdot identity(3)$$

$$covP := \left[\left(\Delta XX^{T}\cdot\Delta XX\right)^{-1}\cdot\Delta XX^{T}\right]\cdot sigmaq\cdot\left[\left(\Delta XX^{T}\cdot\Delta XX\right)^{-1}\cdot\Delta XX^{T}\right]^{T}$$

$$covP = \begin{pmatrix} 5.50104251\times 10^{-9} & 1.36150663\times 10^{-9} \\ 1.36150663\times 10^{-9} & 3.58489842\times 10^{-9} \end{pmatrix}$$

$$erx := \left(covP_{1,1}\right)^{0.5}$$

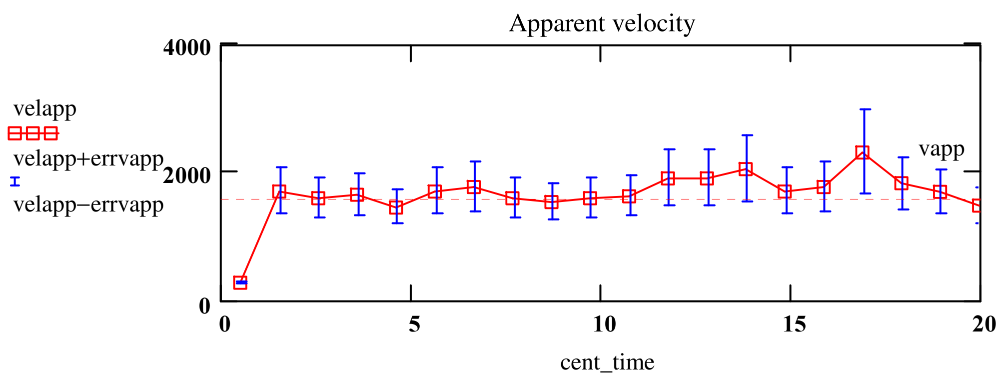
$$ery := \left(covP_{2,2}\right)^{0.5}$$

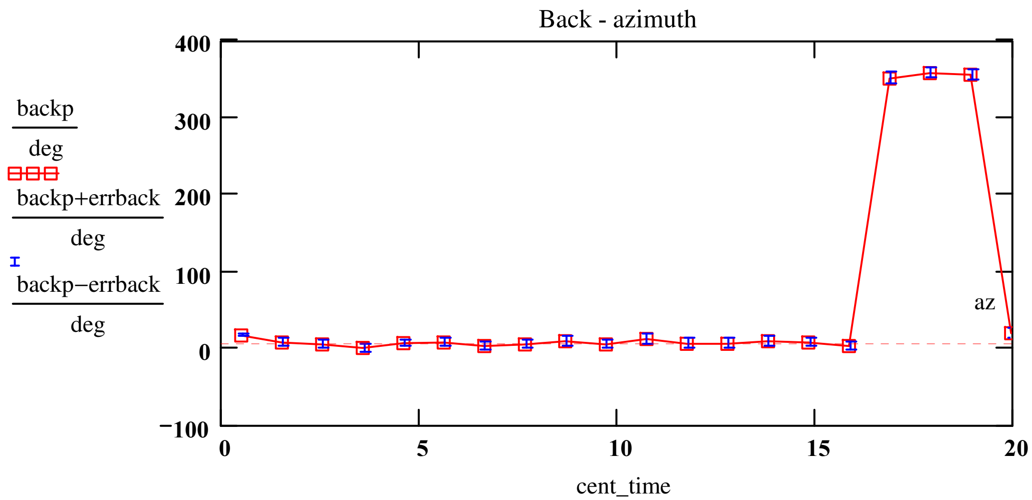
$$\text{errvapp} := \left|\begin{array}{l} \text{for } s \in 1..\text{last(signal)} \\ \quad \text{errvapp}_s \leftarrow \left[\left[\dfrac{-2\cdot\left(P^{\langle s\rangle}\right)_2}{\left[\left[\left(P^{\langle s\rangle}\right)_1\right]^2+\left[\left(P^{\langle s\rangle}\right)_2\right]^2\right]^{\frac{3}{2}}}\right]^2\cdot \text{erx}^2+\left[\dfrac{-2\cdot\left(P^{\langle s\rangle}\right)_1}{\left[\left[\left(P^{\langle s\rangle}\right)_1\right]^2+\left[\left(P^{\langle s\rangle}\right)_2\right]^2\right]^{\frac{3}{2}}}\right]^2\cdot \text{ery}^2\right]^{\frac{1}{2}} \\ \text{errvapp} \end{array}\right.$$

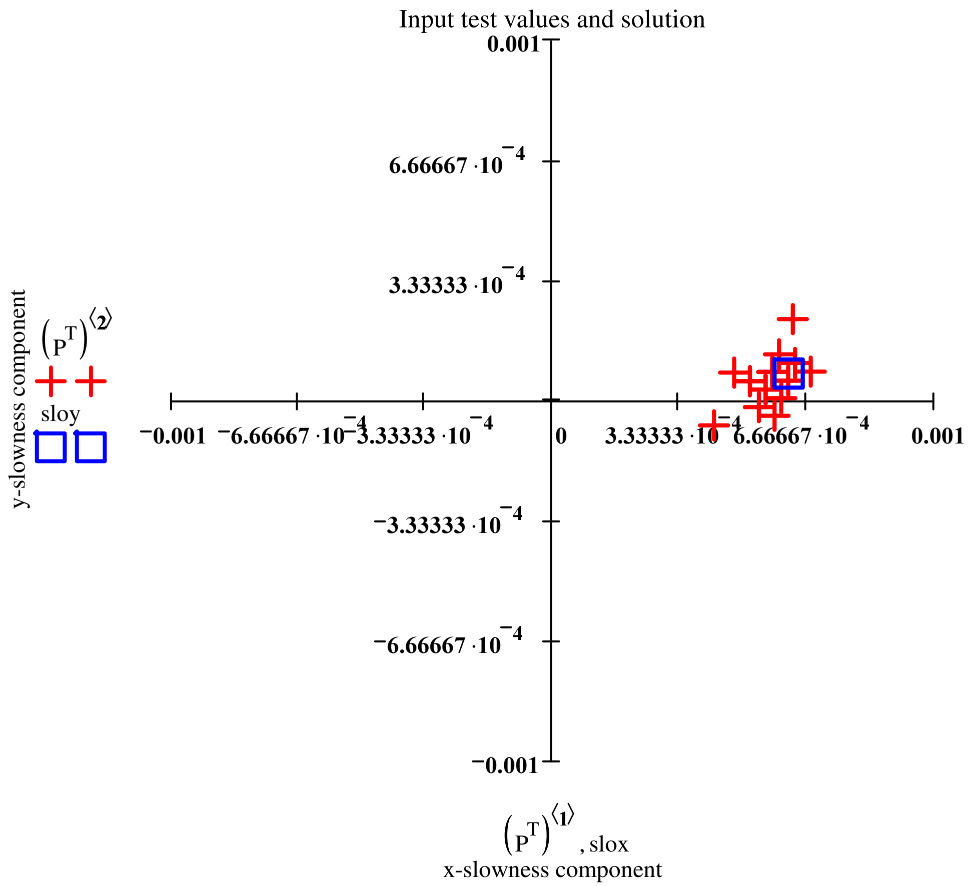
$$\text{errback} := \left|\begin{array}{l} \text{for } s \in 1..\text{last(signal)} \\ \quad \left|\begin{array}{l} \text{continue} \quad \text{on error } \dfrac{\left(P^{\langle s\rangle}\right)_1}{\left(P^{\langle s\rangle}\right)_2} \\ \text{errback}_s \leftarrow \left[\left[\dfrac{1}{\left(P^{\langle s\rangle}\right)_2\cdot\left[1+\dfrac{\left[\left(P^{\langle s\rangle}\right)_1\right]^2}{\left[\left(P^{\langle s\rangle}\right)_2\right]^2}\right]}\right]^2\cdot \text{erx}^2+\left[\dfrac{-\left(P^{\langle s\rangle}\right)_1}{\left[\left(P^{\langle s\rangle}\right)_2\right]^2\cdot\left[1+\dfrac{\left[\left(P^{\langle s\rangle}\right)_1\right]^2}{\left[\left(P^{\langle s\rangle}\right)_2\right]^2}\right]}\right]^2\cdot \text{ery}^2\right]^{\frac{1}{2}} \end{array}\right. \\ \text{errback} \end{array}\right.$$

$$cent\_time_s := \frac{\left(t\_w^{\langle 2 \rangle}\right)_s + \left(t\_w^{\langle 1 \rangle}\right)_s}{2}$$

$$k := 1 .. rows(P)$$




Back - azimuth
backp
deg
backp+errback
deg
backp−errback
deg
400
300
200
100
0
−100
0
5
10
15
20
az
cent_time



Input test values and solution
0.001
6.66667 ·10^−4
3.33333 ·10^−4
−3.33333 ·10^−4
−6.66667 ·10^−4
−0.001
−0.001
−6.66667 ·10^−4
−3.33333 ·10^−4
0
3.33333 ·10^−4
6.66667 ·10^−4
0.001
y-slowness component
(P^T)^⟨2⟩
sloy
(P^T)^⟨1⟩, slox
x-slowness component