Wood Andersom Magnitude Scale for Mt. Vesuvius

- A revised ML scale for VT events at Mt. Vesuvius -

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A Mathcad-8 Professional Program

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Abstract

A Mathcad-8 program to calculate a revised magnitude scale is presented. An application to Mt. Vesuvius is included as a program test. Wood-Anderson seismograms for 131 local earthquakes recorded at station BKE (Osservatorio Vesuviano seismic network) were synthesized to estimate local magnitude from the original definition:
Ml=log Amax(Δ) - logAo(Δ)
The distance correction logAo(Δ) was empirically determined simulating a wave packet which propagates in a structure with assigned Q.
Moment magnitude (calculated both with Kanamori and Tatcher-Hanks formulas) was also determined from the displacement spectra.
Finally a relation between Wood-Anderson magnitude and duration magnitude was derived, allowing the estimate of local magnitude from the duration of the earthquake.

Theory

Local Magnitude definition

The definition of local magnitude is:

Ml=log Amax(Δ) - logAo(Δ) (1)

where Amax is the Wood Anderson maximum amplitude, and Ao is the Wood Anderson maximum amplitude for the reference earthquake. This scale uses as reference the earthquake of Magnitude 3 which in California, where the scale was set up, takes the max amplitude of 1 mm at a distance Δ of 100 km. For California the formula giving the local Magnitude as a function of distance is:

Mlcal=logAmax(Δ)+2.76log(Δ)-2.48+C (2)

where C is a correction term taking into account the deviation of the scale at the station of the network.
We normalize the scale for Mt Vesuvius in such a way that an earthquake at Δ=10 km has the same local Magnitude as in California. This means that at 10 km from the source an earthquake of a given Magnitude in California, would have the same maximum amplitude as at Mt. Vesuvius. This allows a comparison of the Magnitude values at Mt. Vesuvius with those for California. A similar normalization for a distance close to the source was proposed by Hutton and Boore (1987) for local earthquakes. In this way the above authors eliminated the strong regional attenuation anomalies for S wave propagation.
The empirical formula for the attenuation of the maximum amplitude with distance at Mt. Vesuvius was calculated using a numerical simulation. First we generate a synthetic S-wave packet, with a flat spectrum at a distance close to the source (0.1 km). A sequence of 125 random numbers between 1 and -1 with a uniform distribution simulate the wave packet. Then we multiply the sequence by a Hanning window. The signal represents the S-wave packet sampled at 1/125 sps.

Simulation of the synthetic wave packet

$k := 0 .. 100$

$R_k := \frac{k+1}{10}$ This is the distance range in km

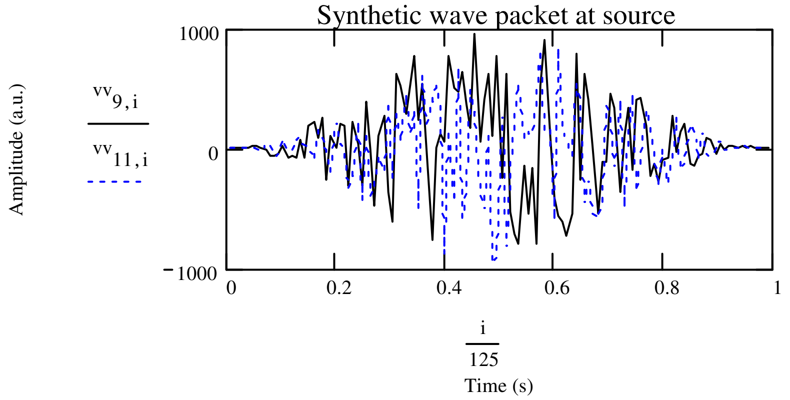
$rr_k := \log(R_k)$

$v(k) := \mathrm{runif}(125, -1, 1)$ This is the vector of 125 samples, uniformly distributed

$hn := \mathrm{hanning}(125)$ This is the hanning window

$i := 1 .. 124$

$vv_{k,i} := hn_i \cdot v(k)_i \cdot 1000$ This is the wave packet. The amplitude is arbitrary



$\beta := 2$ This is the S-wave wave velocity

$VV(k) := \mathrm{CFFT}\left[\left(vv^T\right)^{\langle k\rangle}\right]$ This is the Fourier transform of the synthetic signal

Now we apply the attenuation operator with Q=60 as measured at Mt Vesuvius

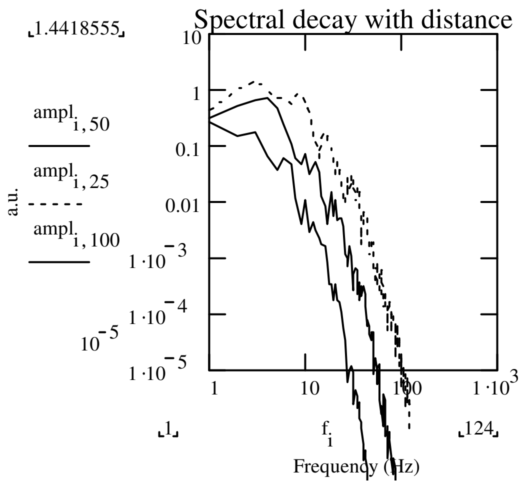
$f_i := i$

$fc := 10$ This is the corner frequency of the source spectrum

$$ampl_{i,k} := \frac{\left|VV(k)_i\right| \cdot 0.1}{R_k \cdot \left[1 + \left(\frac{f_i}{fc}\right)^2\right]} \cdot \left(\left(\exp\left(\frac{-\pi \cdot f_i \cdot R_k}{\beta \cdot 60}\right)\right)\right)$$

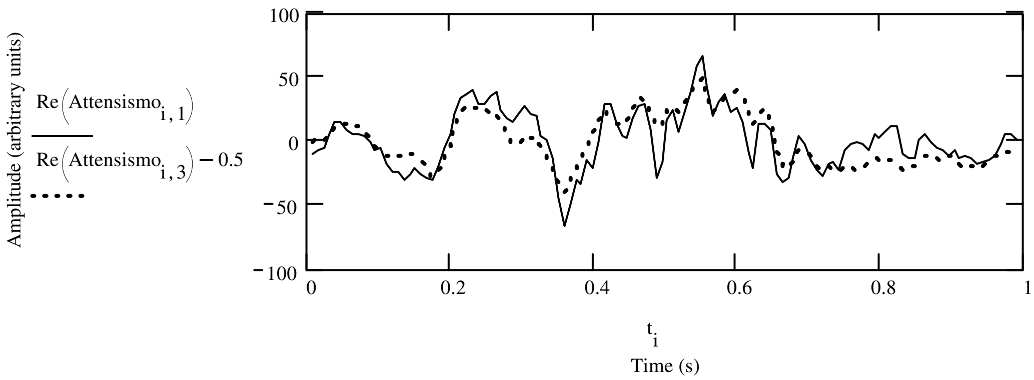
$fase_i := \arg\left(VV(k)_i\right)$

$VVatt_{i,k} := ampl_{i,k} \cdot \exp\left(i \cdot fase_i\right)$

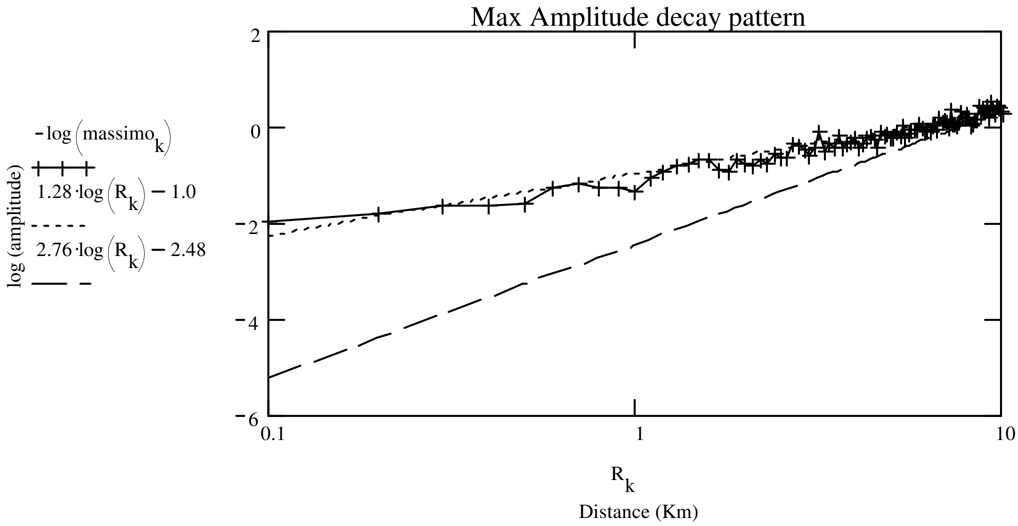


$Attensismo^{\langle k\rangle} := \mathrm{ICFFT}\left(VVatt^{\langle k\rangle}\right)$ This is the synthetic seismogram at different distances from the source. The next plot shows the seismogram recorded at 0.1 and 0.3 km distance from the source.

$t_i := \frac{i}{125}$ $R_1 = 0.2$ $R_{100} = 10.1$



$massimo_k := \max(\mathrm{Re}(Attensismo^{<k>}))$ This is the vector of the maximum amplitudes at different distances.



Best fit with a relation of the form of (2)

$y_k := -\log(massimo_k)$

$ter_k := 1$

$G^{<0>} := rr$

G is the matrix of coefficients

$G^{<1>} := ter$

$par := \left(G^T \cdot G\right)^{-1} \cdot G^T \cdot y$ This is the least square fit for the coefficients of relation (2)

$par = \begin{bmatrix} 1.3422631 \\ -1.0849864 \end{bmatrix}$ This is the solution

The relation for Mt Vesuvius is 1.28 log(Δ) + b. b has to be determined by the normalization at 10 km distance. The normalization is given by the amplitude of a Ml=3 earthquake at 10 km. For this earthquake:

Ml=log Amax +2.76 log 10 -2.48 = 3.----> log Amax = 2.72

For Mt. Vesuvius, Ml=2.72+1.28 log 10+b=3, which gives b=-1.1, then the formula is:

Ml=log Amax +1.28 log (Δ) -1.1

Application to an example

These are the input traces:

:=

A:\mag\03010947.e

:=

A:\mag\03010947.n

$trace := trac \cdot volt \cdot 10^{-3}$ The original signal is in mV. This is the correction to Volts

$trace2 := trac2 \cdot volt \cdot 10^{-3}$

$last(trace) = 2499$

$max(trace) = 0.007872 \cdot V$ Traces in Volts. Note that Mathcad automatically checks the units.

$last(trace2) = 2499$

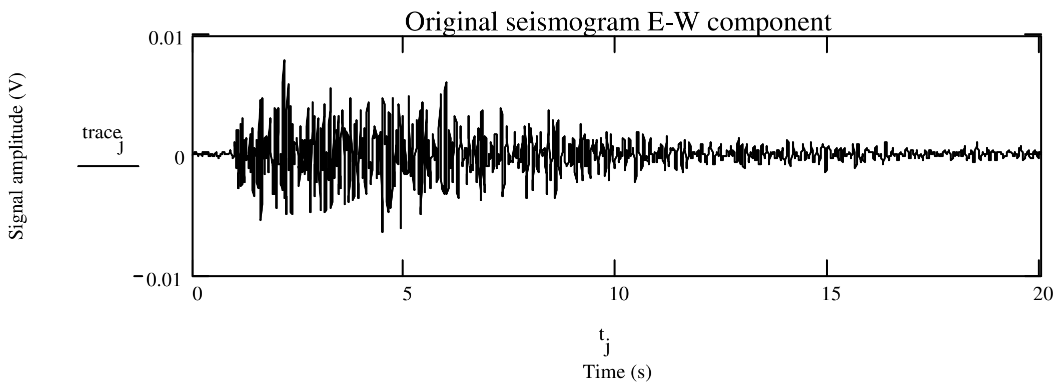
$max(trace2) = 0.00776 \cdot V$

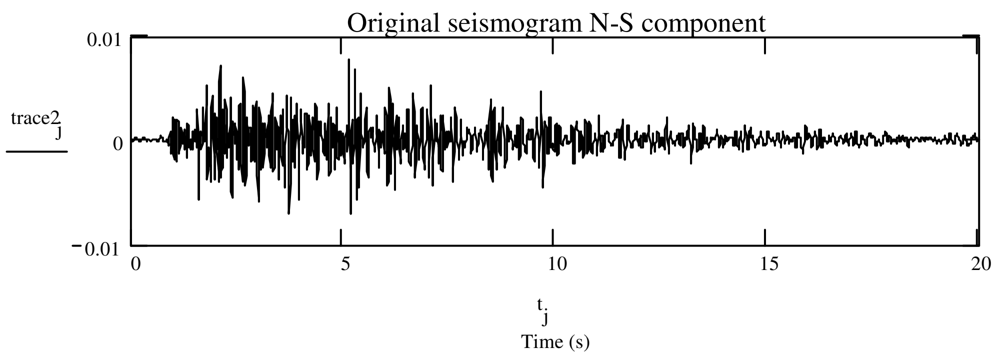
$j := 1 .. last(trace)$

$t_j := j \cdot \frac{1}{125} \cdot s$

$T := last(trace) \cdot \frac{1}{125} \cdot sec$

$T = 19.992\,s$ T is the seismogram window duration





$f0 := 1\cdot Hz$

γ is the damping coefficient of the Lennartz portable station which recorded the event shown above

$\gamma := 0.68$

$$Ga := \frac{4}{2.4\cdot 0.01}\cdot\frac{V}{\frac{m}{sec}}$$

$Ga = 166.6666667 s\cdot m^{-1}\circ V$ This is the internal damping main coil motor constant

$G := 125\cdot V\cdot s\cdot m^{-1}$ G is the motor constant at γ

$traspecvel := cfft(trace - mean(trace))$ These are the Fourier Transforms of the detrended traces

$traspecvel2 := cfft(trace2 - mean(trace2))$

$last(traspecvel) = 2499$ The number of points in the Fourier Transform

$last(traspecvel2) = 2499$

$f_j := \left(\frac{1}{125}\cdot last(trace)\right)^{-1}\cdot j\cdot Hz$ The frequency in Hz

Anti-aliasing Filter Lennartz:

$a0 := 1$ $\quad b_1 := 0.3887$

$a_1 := 1.2217$ $\quad b_2 := 0.3505$

$a_2 := 0.9686$ $\quad b_3 := 0.2756$

$a_3 := 0.5131$ $\quad fc := 25\cdot Hz$

$$filtro(f) := \frac{a0}{\left[\prod_{i=1}^{3}\left[1 + a_i\cdot\frac{f}{fc}\cdot i + b_i\cdot\left(\frac{f}{fc}\cdot i\right)^2\right]\right]}$$

The transfer function of the anti-alias filter

$$H(f) := \frac{(G\cdot(2\cdot\pi\cdot f)^2)\cdot filtro(f)}{(2\cdot\pi\cdot f0)^2 - (2\cdot\pi\cdot f)^2 + 2\cdot i\cdot(2\cdot\pi\cdot f)\cdot\gamma\cdot(2\cdot\pi\cdot f0)}$$

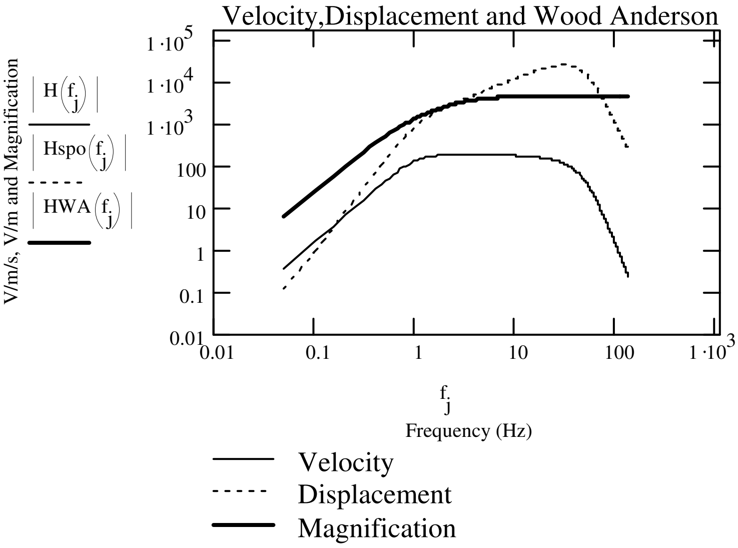
This is the velocity response curve in V/m/s

$Hspo(f) := H(f)\cdot i\cdot 2\cdot\pi\cdot f$ This is the displacement response curve, obtained multiplying for frequency

The the Wood Anderson magnification curve is:

$$HWA(f) := \frac{2800\cdot(2\cdot\pi\cdot f)^2}{(-2\cdot\pi\cdot f)^2 + \left(2\cdot\pi\cdot\frac{1}{0.8\cdot sec}\right)^2 + 2\cdot i\cdot(2\cdot\pi\cdot f)\cdot 0.8\cdot\left(2\cdot\pi\cdot\frac{1}{0.8\cdot sec}\right)}$$

$\left|Hspo\left(f_6\right)\right| = 21.2881184 m^{-1}\circ V$ Note the physical dimensions in V/m



In the plot the amplitude response for Velocity, Displacement and Wood Anderson is shown.

Transformation of the Signal in Wood Anderson Equivalent

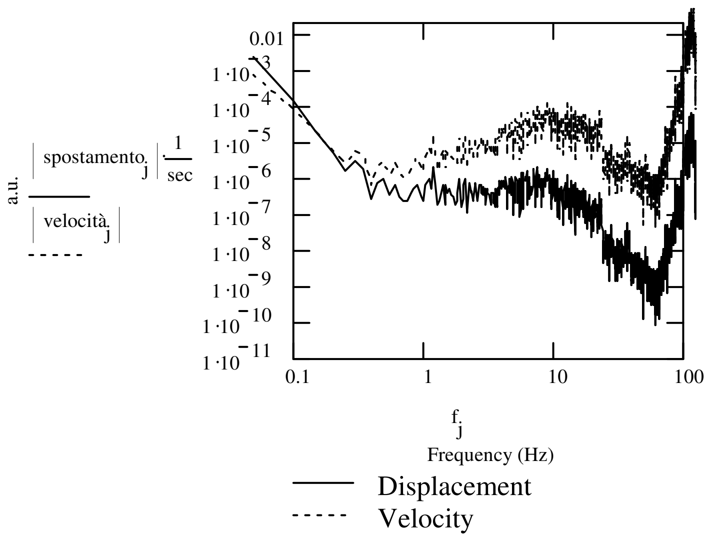
$j := 1 .. \text{last}(\text{trace})$

$\text{spostamento}_j := \dfrac{\text{traspecvel}_j}{\text{Hspo}(f_j)}$ The real ground displacement spectrum for E-W component

$\text{velocità}_j := \dfrac{\text{traspecvel}_j}{H(f_j)}$ The real ground velocity spectrum

$\text{spostamento2}_j := \dfrac{\text{traspecvel2}_j}{\text{Hspo}(f_j)}$ The same for N-S component

$\text{velocità2}_j := \dfrac{\text{traspecvel2}_j}{H(f_j)}$

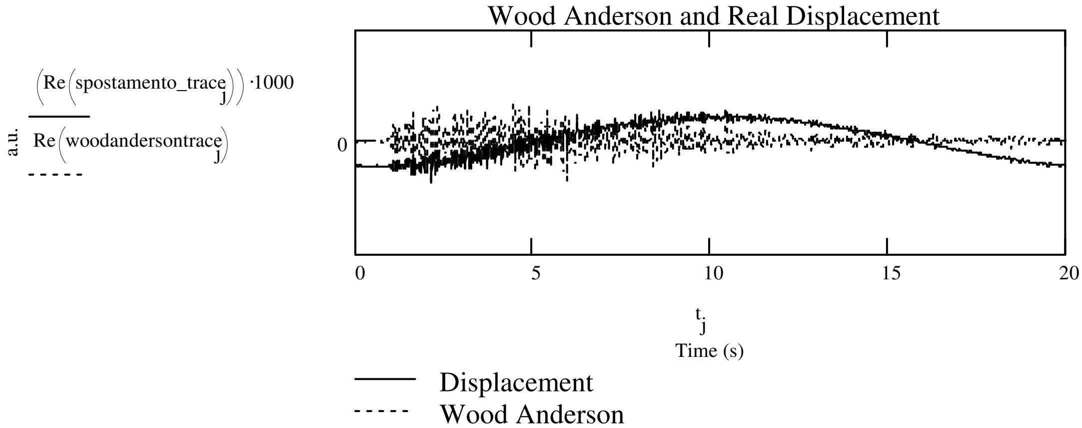


$\text{woodandersonspec}_j := \text{HWA}(f_j) \cdot \text{spostamento}_j$ Wood Anderson

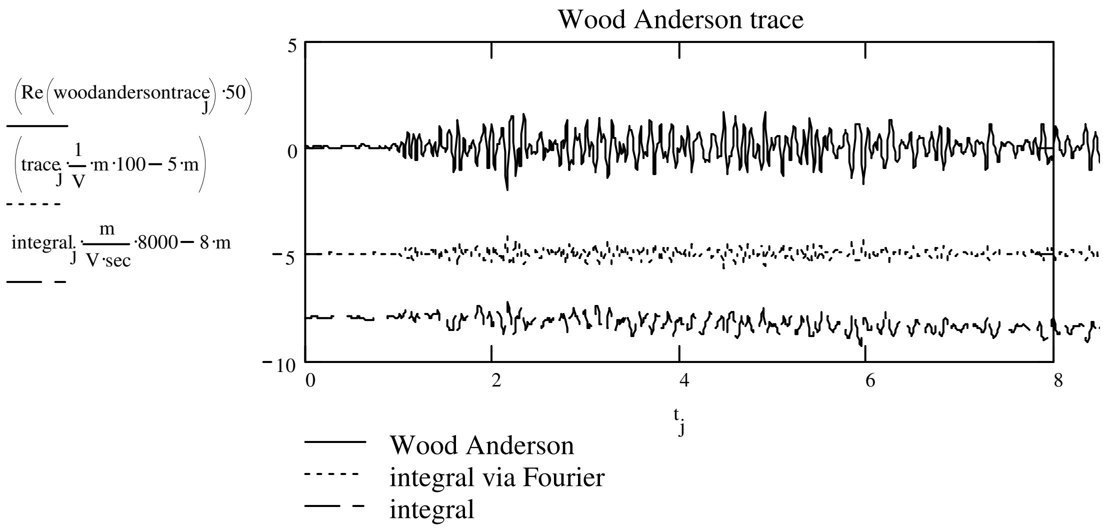
$woodandersonspec2_j := HWA(f_j) \cdot spostamento2_j$

$woodandersontrace := icfft(woodandersonspec)$ Wood Anderson converted trace for the two
$woodandersontrace2 := icfft(woodandersonspec2)$ ground motion components

$spostamento\_trace := icfft(spostamento)$
$velocità\_trace := icfft(velocità)$



$integral_0 := 0 \cdot V \cdot sec$
$integral_j := integral_{j-1} + trace_j \cdot (t_j - t_{j-1})$ The trace is integrated in time domain too, to check the results
$integral_2 = -0.0000014 s \cdot V$ chek of the units



$\max(Re(spostamento)) = 0.0000266m$

$\max(Re(woodandersontrace)) = 0.0328461m$

$\max(Re(woodandersontrace2)) = 0.0409515m$

$$AWAspe := \frac{\sqrt{\max(Re(woodandersontrace))^2 + \max(Re(woodandersontrace2))^2}}{2}$$

$$\sqrt{\max(Re(woodandersontrace))^2 + \max(Re(woodandersontrace2))^2} = 0.0524966m$$

Local Magnitude Evaluation

$Q := 60$ Quality factor measured at Mt. Vesuvius (Bianco et al.,1999). It is independent of frequency
$v := 2$ Seismic wave velocity.

$California(r, AWAspe) := \log(2 \cdot AWAspe) - 2.48 + 2.76 \cdot \log(r)$ Local Magnitude for California

$Vesuvio(r, AWAspe) := \log(2 \cdot AWAspe) + 1.28 \cdot \log(r) - 1.1$ Local Magnitude for Mt. Vesuvius

Moment Magnitude (Kanamori):

$fo := 1 \cdot Hz$

$\rho := 2.700 \cdot \frac{gm}{cm^3}$

$vs := 2 \cdot 10^5 \frac{cm}{sec}$

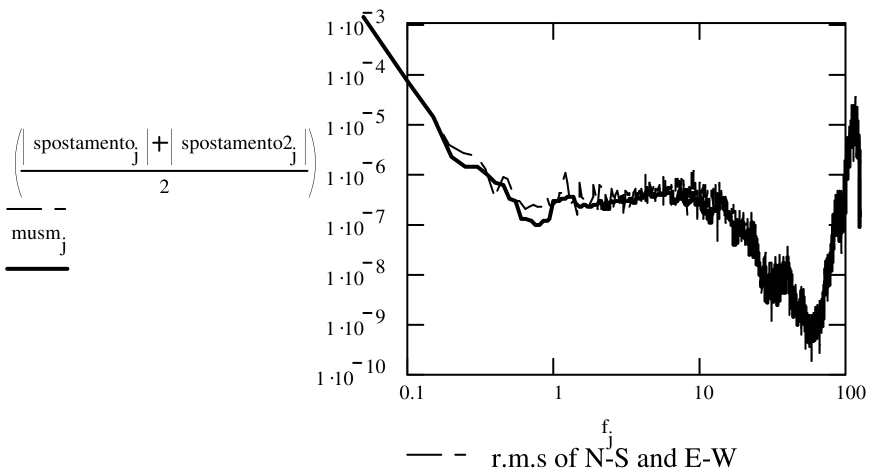
$norm := \frac{2 \cdot T}{\sqrt{last(trace)}}$

$norm = 0.79984 s$

$Mo(R, omega) := \frac{\left(omega \cdot \exp\left(\frac{\pi \cdot R \cdot fo}{vs \cdot Q} \right) \cdot 4 \cdot \pi \cdot \rho \cdot vs^3 \cdot R \right)}{0.85}$

$mum_j := \frac{\left| spostamento_j \cdot \frac{1}{m} + spostamento2_j \cdot \frac{1}{m} \right|}{2}$

$musm := medsmooth(mum, 11)$



$$\text{omega} := \frac{10^2 \cdot \text{norm} \cdot \text{cm}}{500} \cdot \sum_{k=100}^{600} \text{musm}_k$$

Authomatical evaluation of omega in cm*s:

$$\text{omega} = 0.0000104\text{s} \circ \text{cm}$$

$$\text{Mo}(364000\,\text{cm}, \text{omega}) = 1.3318806\,10^{18} \circ \text{dyne} \cdot \text{cm}$$

Moment Magnitude (Kanamori) and (Tatcher) Formulas:

$$\text{Mw} := \frac{\log\left(\text{Mo}(364000\,\text{cm}, \text{omega}) \cdot \frac{1}{\text{dyne} \cdot \text{cm}}\right)}{1.5} - 10.73$$

$$\text{Tatcher} := \frac{\log\left(\text{Mo}(364000\,\text{cm}, \text{omega}) \cdot \frac{1}{\text{dyne} \cdot \text{cm}}\right)}{1.5} - \frac{16}{1.5}$$

$\text{Mw} = 1.3529769$ **Moment Magnitude (Kanamori)**

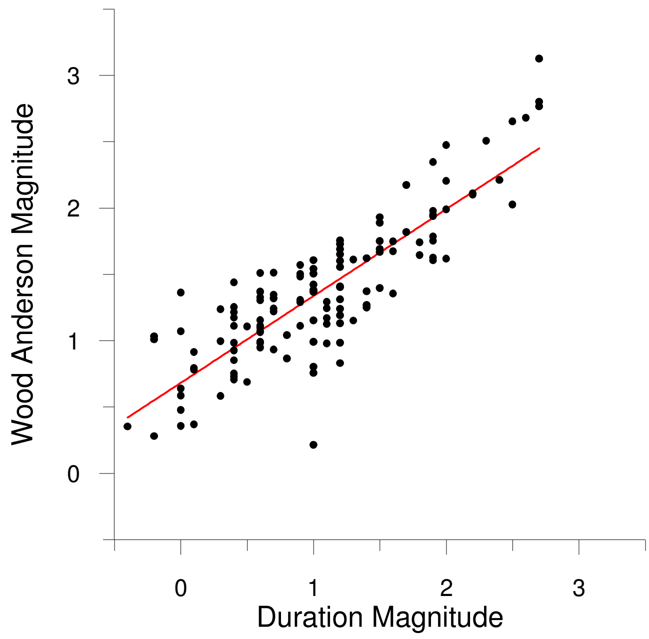
$\text{Tatcher} = 1.4163102$ **Moment Magnitude with Tatcher and Hanks Formula**

$\text{California}\left(3.64, 1000 \cdot \text{AWAspe} \frac{1}{\text{m}}\right) = 0.7887708$ **California W A Magnitude**

$\text{Vesuvio}\left(3.64, \text{AWAspe}\, 10^3 \cdot \frac{1}{\text{m}}\right) = 1.3383408$ **Vesuvius W A Magnitude**

We calculated the Vesuvius Magnitude, the Kanamori Magnitude and the Tatcher and Hanks Magnitude for 181 earthquakes recorded in 1996 by a Lennartz seismic station (BKE) of the Osservatorio Vesuviano Seismic Network. For 131 events it was possible to compare the Vesuvius Wood-Anderson Magnitude with the Duration magnitude estimated for the seismic station OVO using the empirical formula $M_D = 2.75 \log \tau - 2.35$. The Wood Anderson Magnitude can be related to the Duration Magnitude performing a linear fit (see the next plot), which provides the following relation:

$M_{WA} = 0.682 + 0.655\, M_D$



As we know the relation between the Duration Magnitude and log τ, we can combine the two formulas to obtain a final relation between the Wood Anderson Magnitude and the duration of the earthquake:

$M_{WA} = 0.682 + 0.655 * (2.75 \log \tau - 2.35) = 1.8 \log \tau - 0.9$