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A study of global mean temperatures applied to available potential energy, annual variations and static stability

By C. WIIN CHRISTENSEN, *Danish Meteorological Institute, Lyngbyvej 100, 2100 Copenhagen 0, Denmark* and A. WIIN-NIELSEN, *Geophysical Institute, Haraldsgade 6, 2200 Copenhagen N, Denmark*

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ABSTRACT

This paper contains, for pedagogical purposes, a brief derivation of quasi-geostrophic, available potential energy using the parcel method and an approximate calculation of the work performed during the displacement. For the study, we have used 7 years of globally averaged temperatures. These data indicate a seasonal variation of the global mean temperature. The seasonal variation is determined from the data with respect to amplitude and phase, and a simple model is used to relate this variation to the seasonal variation of the planetary albedo. It is also demonstrated that the seasonal changes in the global mean temperature due to the ellipticity of the orbit are out of phase with the observed changes. The same data have been used to study the vertical variation of the static stability parameter entering in quasi-geostrophic models, as a function of pressure. The changes in the vertical direction are important for the determination of the vertical structure functions. In the first approximation, the parameter is inversely proportional to the square of the pressure, but deviations occur, depending on the number and position of the pressure levels. The error in the calculation of quasi-geostrophic, available potential energy due to an incorrect specification of the stability parameter is investigated using a specific example in Section 4.

1. Introduction

The present investigation started with a study of globally-averaged temperatures at a number of pressure levels for each month during the period 1982–88. The results of this study have been reported elsewhere (Christensen and Wiin-Nielsen, 1989).

It was noted that a small seasonal variation existed in the globally averaged temperatures. This variation is analyzed with respect to amplitude and phase in Section 3 of this paper, where an attempt is made to explain the observations using a simple energy balance model. While such a variation has been noted in other data samples (North and Coakley, Jr., 1979), it has been neglected in the theoretical studies of the seasonal variation of the atmospheric temperature fields which have concentrated on the much larger variation in the deviations from the global average. In addition, it seems that the seasonal variation in global mean temperatures is documented for the first time as a function of height, while other studies have limited themselves to the temperature field at the earth's surface.

The data are also well-suited to investigate the stability in the global mean temperatures. In particular, the static stability, as it appears in quasi-geostrophic theory, can be calculated from the data, and its dependence on pressure can be investigated. In this regard, we recall that the static stability can be a function of pressure only in quasi-geostrophic models, and it is therefore ideal to use globally averaged data for this purpose. Other investigations of the static stability have been carried out. Gates (1961) used data from the contiguous United States, but the purpose of that study was primarily to compare various measures of static stability and to document the horizontal variations. Jacobs and Wiin-Nielsen (1966) used a constant lapse rate atmosphere to find the pressure dependence of the static stability. The same dependence on pressure is mainly important in the determination of the vertical structure of quasi-geostrophic, baroclinic waves, because it enters in the equation for the determination of the vertical eigenmodes (Wiin-Nielsen, 1971a and 1971b, Kasahara and Tanaka, 1989). It is thus of interest to use the data to determine the static stability and to use statistical techniques to obtain empirical formulas. Such an investigation is given in Section 4.

The concept of available potential energy, originally introduced by Lorenz (1955), is defined as the surplus of total potential energy (i.e., the sum of potential and internal energy) over the minimum of this quantity. The smallest value of the total potential energy is reached if all particles on a given isentropic surface are moved adiabatically to a level, where the pressure is equal to the global mean pressure on the isentropic surface. The concept of available potential energy has been considered in further detail by Van Mieghem (1956) and Dutton and Johnson (1967).

The introduction of available potential energy in meteorological education is not particularly simple, if one wants to use the concept in its most general form. On the other hand, for many purposes it is an advantage to discuss energy relations at an early stage. It is rather straightforward to introduce the concept of potential, internal and kinetic energy, since they may be defined for a single parcel. It may thus be desirable to use simple methods to give a general understanding of the concept of available potential energy even if rather severe approximations have to be made. Section 2 of this paper contains for pedagogical purposes an introduction of available potential energy using the so-called parcel method and the principles of work and energy.

Static stability enters into the concept of available potential energy. The result of a calculation therefore depends on the assumptions made regarding the vertical variation of this parameter. The sensitivity of the calculation to various assumptions regarding the vertical dependence of the static stability is investigated in Section 4.

2. Approximate determination of available potential energy

In this section, we shall give a simple introduction to the concept of available potential energy. Any new results cannot be expected, but it seems that the way in which the calculation is made may be novel. In addition, it uses simple concepts which will make it possible to introduce available potential energy at an early point in the meteorological education.

The general idea is to start from an atmosphere at rest and in hydrostatic equilibrium. We shall furthermore make the assumption usually made in the so-called parcel method (Hess, 1959) that the pressure of a parcel displayed from its equilibrium position will adjust immediately to the pressure of the environment. Suppose then that a parcel is displayed a vertical distance from its equilibrium position, where the specific volume is $\bar{\alpha}$ and the pressure is $\bar{p}$. Using the parcel method we may calculate the force acting on the parcel:

$$F = -\alpha \frac{\partial p}{\partial z} - g = -\alpha \frac{\partial \bar{p}}{\partial z} - g$$

$$= g \frac{\alpha - \bar{\alpha}}{\bar{\alpha}} = g \frac{\theta - \bar{\theta}}{\bar{\theta}}, \qquad (2.1)$$

where θ is the potential temperature and g is gravity.

Since for a small displacement z, we may assume that the environmental potential temperature varies linearly, and that isentropic conditions apply to the parcel, we may also write (2.1) in the form:

$$F = -g \frac{\partial \ln \bar{\theta}}{\partial z} z. \qquad (2.2)$$

The work carried out during this process is approximately:

$$W = \int_0^z -g \frac{\partial \ln \bar{\theta}}{\partial z} z \, dz = -\left(\frac{g}{2}\right) \frac{\partial \ln \bar{\theta}}{\partial z} z^2. \qquad (2.3)$$

We can finally replace z by

$$z = -\frac{\theta'}{\partial \bar{\theta}/\partial z}, \qquad \theta' = \theta - \bar{\theta}, \qquad (2.4)$$

and if we then note that the energy gained is the opposite of the work carried out we may write that the contribution to the available energy is

$$a = \left(\frac{1}{2}\right) \frac{g^2}{N^2} \frac{\theta'^2}{\bar{\theta}^2}, \qquad (2.5)$$

where

$$N^2 = \frac{g}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z} \tag{2.6}$$

is the square of the Brunt-Väisälä frequency. Eq. (2.5) gives thus the contribution to the available potential energy from a unit mass. The amount of available potential energy is then obtained by integrating over the mass of the atmosphere. In the previous development, we have used height as the vertical coordinate, but the expression in (2.5) may be expressed in other vertical coordinates. Since it is our intention to use data from isobaric surfaces, we transform to pressure as the vertical coordinate. Using standard procedures and recalling (2.1), we find

$$a = \frac{1}{2\bar{\sigma}} \alpha'^2, \qquad \bar{\sigma} = -\frac{\bar{\alpha}}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial p}, \tag{2.7}$$

which is in agreement with the integrand in the approximate expression obtained by Lorenz (1955) by series expansion of the so-called exact expression derived under much more general conditions. However, under the simplified conditions applied here, we may state that the contribution to the quasi-geostrophic, available potential energy may be obtained by moving all parcels from their actual position to a point, where the height is equal to the average height of the pressure surface, from which the particle started. The energy released in the process is equal to the expression in (2.7).

One should note that the interpretation does not mean that the available potential energy is defined as a local quantity as for example the internal or the kinetic energy. The reason is that the determination of the reference height requires the topography of the whole isobaric surface. It may at first be surprising that the derivation given here agrees with the result obtained by Lorenz (1955) by a truncated series expansion of the more general integrand followed by integration over the mass of the atmosphere. However, both methods require smallness of the displacement and a constant value of the static stability parameter in the neighborhood of an isobaric surface. The smallness of the displacement is related to the slope of an isobaric surface, which should be sloping only slightly, if quasi-geostrophic theory should apply.

3. Global temperatures

A data set of globally averaged temperatures at a number of pressure levels was provided by the European Centre for Medium-Range Weather Forecasts for the years 1982–88. For each year, the monthly averages were provided for each of the 12 months. The pressure levels were: 50, 100, 150, 200, 250, 300, 400, 500, 700, 850 and 1000 hPa. From these data, we have produced the seven year averages for each month at each level. We have called each of the temperatures a global average. Strictly speaking, they are averages between the latitudes 87.5° north and 87.5° south, but the data still cover 99.8 % of the area of the earth.

The data show a small seasonal variation at

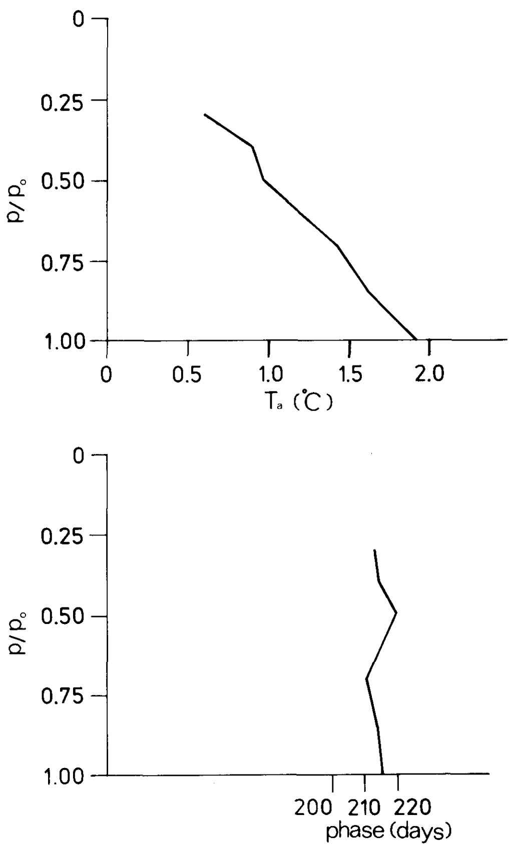

Fig. 1. The upper part shows the amplitude of the first Fourier component, T_{a1} in degrees C, as a function of normalized pressure, while the lower part gives the phase, measured in days, in the same arrangement.

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each level. At the lower levels we find the highest temperatures in June or July. This is true for all levels between 300 and 1000 hPa, or, in other words, in the major part of the troposphere. For levels between 100 and 250 hPa the maxima occur in February or March. The seasonal variation is quite small. The amplitude and phases of the first Fourier component are shown in Fig. 1 for the levels between 300 and 1000hPa. The first Fourier component is a good approximation to the seasonal variation at the low levels as can be seen from Fig. 2 showing the actual temperature variation and the first component. This is not the case at the higher levels. Fig. 3 shows the same comparison for the 400 hPa, and the deviations are now larger. At the 250 hPa level (Fig. 4) it makes little sense to use the first Fourier component only. For the levels higher than 250 hPa, the situation is similar, and it is for this reason that only the levels below 300 hPa have been included in Fig. 1.

If the distribution of continents and oceans were symmetrical around the equator, we would

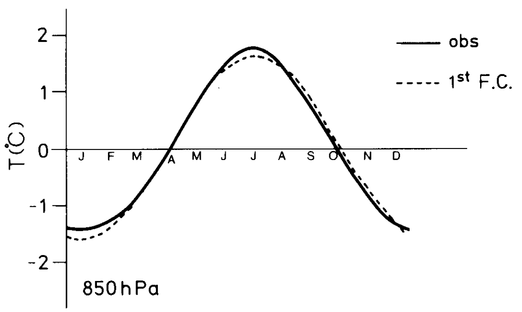

Fig. 2. A comparison between the observed seasonal variation of the deviation of temperature from the annual mean and its first Fourier component at 850 hPa.

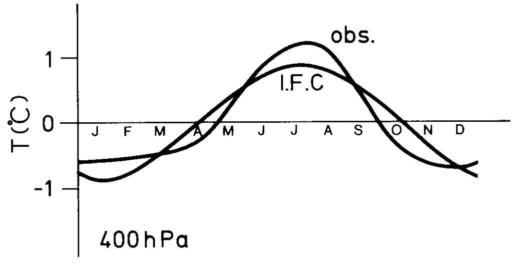

Fig. 3. As Fig. 2, but for 400 hPa.

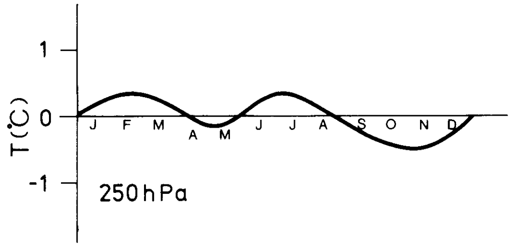

Fig. 4. The observed seasonal variation of the deviation of temperature from its annual mean at 250 hPa.

probably in the first approximation expect a constant globally averaged temperature through the year. The observed small variation may then be due to the asymmetries around the Equator. The higher temperatures at the low levels in the Northern Hemisphere summer should be due to the fact that the area of the continents is larger in this hemisphere than in the Southern Hemisphere resulting in a larger seasonal variation of the temperature in the "continental" hemisphere than in the more "maritime" Southern Hemisphere. If this is so, we would observe a seasonal variation in the global temperature. However, another process may also be of importance. The distance between the Sun and the Earth varies through the year being smallest when the Earth is in the Perihelion and largest with the planet in the Aphelion. The influence of this seasonal variation in the incoming radiation will be analysed at the end of this section.

It may be of interest to investigate, whether the seasonal variation in the global mean temperature can be obtained quantitatively from the considerations given above. In the first approximation, we may consider the surface of the Earth and adopt a simple climate energy balance model. Such models, originally designed by Budyko (1969) and Sellers (1969), have been used by North and Coakley, Jr. (1979) to describe the seasonal variation of the surface temperature using low order descriptions of the latitude dependent incident solar radiation, the albedo and the infrared outgoing flux. However, the seasonal variation in the globally averaged temperature has been neglected in these studies as small compared to the much larger variation as a function of latitude. Our problem will be to account for the small time variation of the averaged temperature.

We adopt an energy balance model applied earlier by Wiin-Nielsen (1984). The basic equation is:

$$C\frac{\mathrm{d}T}{\mathrm{d}t} = aQ - \varepsilon_{\mathrm{B}}\sigma_{\mathrm{B}}T^4, \tag{3.1}$$

in which T is the globally averaged surface temperature, t time, C the heat capacity, a the planetary co-albedo, Q the solar radiation, ε_{B} the absorption coefficient and σ_{B} the Stefan-Boltzman constant.

The steady state temperature T_0 is obtained directly from (3.1) by setting the time-derivative to zero. It is

$$T_0 = \left(\frac{a_0 Q}{\varepsilon_{\mathrm{B}}\sigma_{\mathrm{B}}}\right)^{1/4}. \tag{3.2}$$

In using (3.2), we shall adopt the values

$a_0 = 0.68, \quad Q = 344 \text{ Wm}^{-2},$

$\varepsilon_{\mathrm{B}} = 0.61, \quad \sigma_{\mathrm{B}} = 5.675 \cdot 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$

resulting in $T_0 = 287$ K. Since we are considering a small variation around this mean value, it is justified to linearize the equation. We consider therefore T' as a deviation from T_0 and write:

$$T^4 = T_0^4\left(1 + 4\frac{T'}{T_0}\right). \tag{3.3}$$

Introducing (3.2) and (3.3) in (3.1), we find

$$C\frac{\mathrm{d}T'}{\mathrm{d}t} = a'(t)\,Q - 4\varepsilon_{\mathrm{B}}\sigma_{\mathrm{B}}T_0^3 T' \tag{3.4}$$

which also may be written in the form:

$$C\frac{\mathrm{d}T'}{\mathrm{d}t} + BT' = a'(t)\,Q, \tag{3.5}$$

with $B = 4\varepsilon_{\mathrm{B}}\sigma_{\mathrm{B}}T_0^3 = 3.27 \text{ Wm}^{-2}\text{K}^{-1}$.

In the derivation of (3.5), it has been assumed that the main variation is in the co-albedo, while Q is constant. Later this situation will be reversed. The seasonal variation of the planetary albedo has been calculated by Gruber (1977). He obtains a maximum albedo in December–January and a minimum in June–July with an amplitude of about 0.022. As a first approximation we may therefore assume that the planetary co-albedo varies as

$$a'(t) = -A\cos(2\pi\tau) \tag{3.6}$$

where $A = 0.022$, and where τ is a non-dimensional time using one year as the scaling factor. Inserting (3.6) in (3.5) we obtain the following solution for T':

$$T'(t) = T_{\mathrm{A}}\cos(2\pi(\tau - \tau_*)), \tag{3.7}$$

where $T_{\mathrm{A}} = 1.98$ K and $\tau_* = 0.59$. This value corresponds to a maximum in the temperature variation occuring on day 214. Our result is thus in excellent agreement with the observed seasonal variation at the surface of the Earth as shown in Fig. 1. It should be pointed out that the value of the heat capacity in this calculation is $1.0 \times 10^7 \text{ Wm}^{-2}\text{K}^{-1}\text{s}$ corresponding to the value for the atmosphere. We have therefore made the assumption that the remaining part of the climate system plays a minor role in the short seasonal variation of the global mean temperature.

The expression (3.6) assumes that the minimum in the co-albedo is at the beginning of the year. There is a disagreement between (3.6) and the seasonal variation of the co-albedo as given by North and Coakley, Jr. (1979) based on albedo variations determined by Ellis et al. (1976), which results in a co-albedo minimum on day 56 (end of February) and a maximum on day 238 (end of August). Since the solution of (3.5) will have a maximum later than the maximum in the forcing, it is obvious that a correct result cannot be obtained with such a variation, if the present model is used.

The co-albedo variation incorporated in (3.6) is a net effect from the study of the radiation budget. According to Gruber (1977) the outgoing longwave radiation is generally in phase with the co-albedo with a maximum in June–July and a minimum in December–January. The in-phase relationship is due primarily to the influence of the cloudiness. It is interesting to note that these influences are large enough to overcome the effect of the seasonal change in Q due to the ellipticity of the Earth's orbit. This change is due to the fact the Earth is closer to the Sun in January, when it passes through the Perihelion, than in July, when it goes through the Abhelion. The annual change in Q is in the first approximation according to North and Coakley, Jr. (1979) given by

$$Q = Q_0(1 + 2e\cos(2\pi\tau - \theta_p)), \tag{3.8}$$

where e is the ellipticity ($e = 0.01674$) and θ_p is the longitude of the Perihelion measured from the winter solstice ($\theta_p = 12.73°$) just as τ is measured from this point. When we change the reference point in such a way that we measure from the beginning of the year we may with sufficient accuracy write:

$$Q = Q_0(1 + 2e\cos(2\pi\tau)). \tag{3.9}$$

The linearized equation considering the seasonal variation of Q is

$$C\frac{\mathrm{d}T'}{\mathrm{d}\tau} + \mathrm{B}T' = 2eaQ_0\cos(2\pi\tau) \tag{3.10}$$

Comparing (3.10) with the system (3.5), (3.6), we note that the only change is the sign and magnitude of the right hand side. The amplitude of the forcing function with $a = 0.7$ and $Q_0 = 344\ \mathrm{Wm}^{-2}$ is $8.06\ \mathrm{Wm}^{-2}$ in good agreement with the amplitude of the observed annual wave in the short-wave absorbed radiation ($8\ \mathrm{Wm}^{-2}$) as found by Vonder Haar and Ellis (1977). The solution of (3.10) is

$$T'(t) = T_\mathrm{B}\cos(2\pi(\tau - \tau_*{}^*)), \tag{3.11}$$

with $T_\mathrm{B} = 2.1$ K and $\tau_*{}^* = 0.087$ (corresponding to day 32).

We may thus conclude that the other parameters in the radiation budget counteract the orbital effect in such a way that the actual maximum in the globally averaged temperature occurs in August and not in early February.

The model used here describes the surface temperature only. The decrease of the amplitude with height in the troposphere and the approximate constancy of the phase in the lower troposphere have not been treated here, but they may possibly be accounted for by vertical exchange processes. The radiation budget data used in our study come partly from Gruber (1977) and partly from Vonder Haar and Ellis (1977). The two studies are not completely consistent with eachother, because the first study does not find the clear annual variation in the absorbed shortwave incoming radiation which is clearly present in the other data set.

The annual variation in the global mean surface temperature which has been discussed in this section is present in other data sets. North and Coakley, Jr. (1979) find an amplitude of 2.0 K and a phase of 209 days, where the phase is defined as the day, on which the maximum occurs. Oort (1983), using data for the period 1958–73, finds an amplitude of 1.8 K and a phase of 202 days. The three studies are thus in good agreement with each other.

4. On static stability

In this section we shall use the globally averaged temperatures at the various levels to investigate the variations in the static stability in time and with respect to a vertical coordinate, here pressure. We shall in other words use eq. (2.7) to calculate values of the stability parameter.

The interest in the static stability comes only in part from the special rôle which its pressure dependence plays in quasi-geostrophic models. These models are used only in a limited way in operational predictions, but they are still important in diagnostic data studies and in theoretical work. The changes in the static stability in the vertical direction are much more significant in the efforts to expand the spectral methods to three dimensions. So far, most models have used grid points in the vertical direction, but some efforts have been made to represent the vertical changes in terms of vertical structure functions (Wiin-Nielsen, 1971a, b, Kasahara and Tanaka, 1989, Wiin-Nielsen and Marshall, 1990). In these studies, one will note that the vertical changes of the static stability are of vital importance in determining the class of normalized, orthogonal functions, which can be used as vertical structure functions. In this regard, we refer also to the study by Jacobs and Wiin-Nielsen (1966).

It is most convenient to obtain σ at levels, which are in the middle of the layers defined by the levels, at which the temperatures are available in the data. Let the temperature and the pressure be T_2 and p_2 at the bottom of such a layer, and let T_1 and p_1 be the same quantities at the top of the layer. The expression for σ in (2.7) converted to temperature and finite differences may then be written:

$$\sigma = \frac{R}{p_\mathrm{m}}\left(\frac{R}{c_\mathrm{p}}\frac{T_\mathrm{m}}{p_\mathrm{m}} - \frac{T_2 - T_1}{p_2 - p_1}\right), \tag{4.1}$$

where $p_\mathrm{m} = (p_1 + p_2)/2$ and $T_\mathrm{m} = (T_1 + T_2)/2$.

The expression in (4.1) has been used to calculate σ for each level and each month using the global mean temperatures averaged over the seven years available here. σ has thus been obtained at 10 levels: 925, 775, 600, 450, 350, 275, 225, 175, 125 and 75 hPa.

Fig. 5 show the seasonal variation of the static stability at selected levels computed from globally averaged temperatures. Fig. 5a shows the changes at the lowest level (925 hPa), where the deviation from the annual means has been plotted. It indicates the larger stability in the months November–April and the smaller values in the remaining part of the year. Also these changes are due to the larger continentality in the Northern Hemisphere, because the atmosphere becomes more unstable due to the convection over the continental heat sources. The same type of curve (Fig. 5b) is found in the middle troposphere (450 hPa). Both curves have two maxima during the year. The second maximum in July is probably due to the larger stabilities in the Southern Hemisphere during its winter season. In the upper troposphere (Fig. 5c) we observe a smaller stability in the period May to November and a larger than average stability in the remaining part of the year. We may thus conclude that at the tropospheric levels we find the continental influence at all levels, especially in the first Fourier components. Finally, in Fig. 5d we observe that the seasonal changes of the stability in the lower stratosphere is essentially out of phase with the changes in the troposphere.

Fig. 5 shows also that the absolute values of the deviations from the annual averages increase with height. However, the static stability itself increases with height. A measure of the relative variation is the ratio of the standard variation and the mean. It is at all levels less than 10% indicating that the seasonal variation is relatively small.

It is well-known that the static stability can be a function of pressure only in a quasi-geostrophic model. For any forecast, one may naturally obtain the static stability at each pressure level where it is needed by calculating the area averaged temperatures from the initial state at the required pressure levels and proceed to calculate the static stability using eq. (4.1). It has been noted by Gates (1961) that the stability parameter σ on average appears to be inversely proportional to the square of the pressure. It would thus appear appropriate to use the present data to investigate the pressure

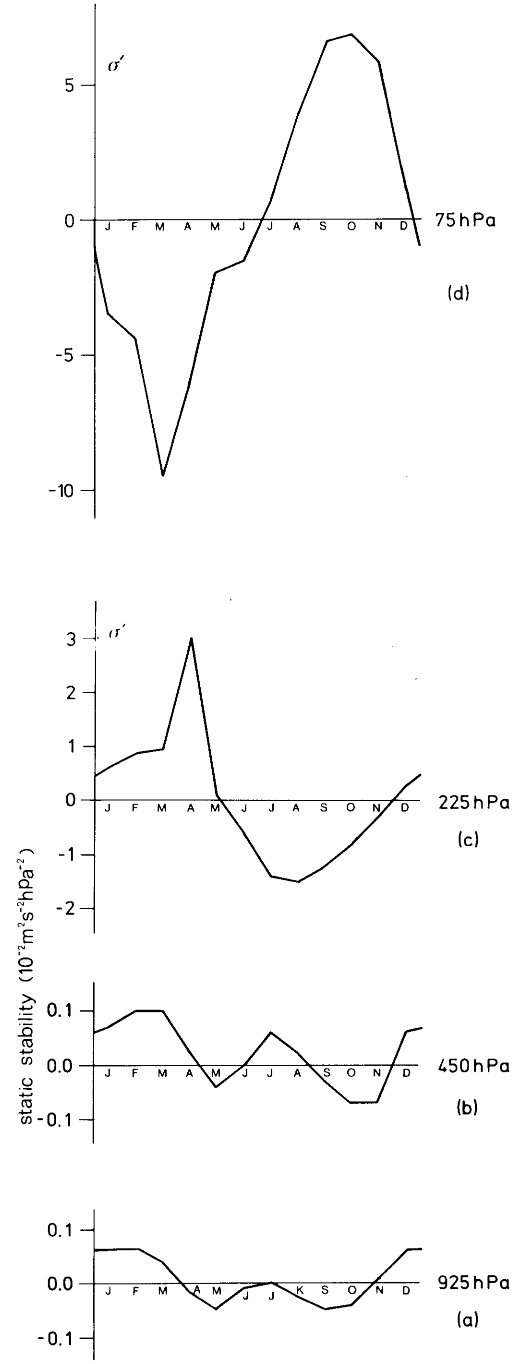

Fig. 5. The seasonal variation of the static stability, computed from globally averaged temperatures, at 925 (a), 450 (b), 225 (c) and 75 (d) hPa.

dependence. For this purpose we shall fit a functional dependence of the form

$$\sigma = \sigma_0 \left(\frac{p}{p_0}\right)^{-\delta} \tag{4.2}$$

to the data which in this case are averaged at each level over the total period. The constants σ_0 and δ are found by linear regression techniques using the equation

$$\ln \sigma = \ln \sigma_0 - \delta \ln(p/p_0) \tag{4.3}$$

Using all 10 levels ($N=10$), we find $\sigma_0 = 0.73$ and $\delta = 2.24$. Fig. 6 shows the curve (4.2) using these values of the parameters, while the dots are the original data. We note some estimates, which are too low, around 250 hPa, but the estimate

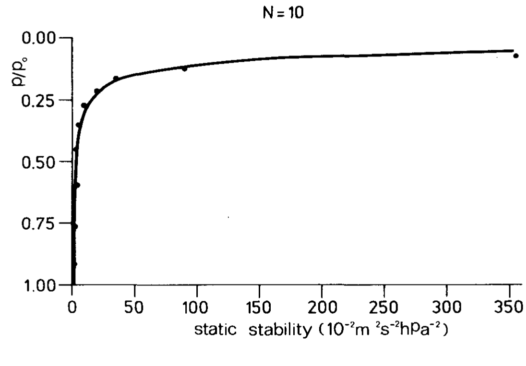

Fig. 6. The curve is the approximation in eq. (4.2) for $\sigma_0 = 0.72925$ and $\delta = 2.2365$ including all levels ($N = 10$). Dots indicate observed values.

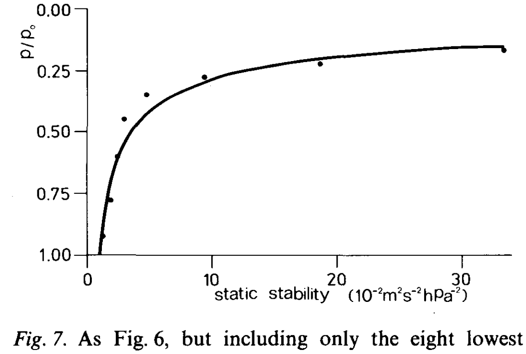

Fig. 7. As Fig. 6, but including only the eight lowest levels ($N = 8$) for $\sigma_0 = 0.9316$ and $\delta = 1.8864$.

using (2.4) are generally good. Fig. 7 is similar to Fig. 6, but the two points at the top levels have been deleted from the data ($N = 8$). They represent the very large stabilities in the stratosphere. In this case we get $\sigma_0 = 0.93$ and $\delta = 1.89$, but a similar underestimate is found in the upper troposphere.

In addition to the annual average, we have also calculated the values of the two parameters for all months of the year. These values have been used to calculate how the available potential energy depends on the vertical changes of the static stability. To construct an example we shall assume that (4.2) holds. We use then (2.7) to find how the temperature depends on pressure. Using the boundary condition $T = T_0(x, y)$ for $p = p_0$ the solution is:

$$T = \mathrm{B} p_*^{2-\delta} + (T - B)\, p_*^{\kappa},$$
$$p_* = p/p_0, \qquad \kappa = R/c_p, \tag{4.4}$$

with

$$B = \frac{\sigma_0 p_0^2}{R} \frac{1}{\delta + \kappa - 2}. \tag{4.5}$$

Eqs. (4.2) and (4.4) are used to calculate the available potential energy for an area. We find that

$$A = \frac{R^2}{g} \frac{1}{\sigma_0 p_0} \frac{1}{\delta + 2\kappa - 1} \overline{T_0^2}, \tag{4.6}$$

where the last factor is the variance of the surface temperature.

Suppose now that we use the values of σ_0 and δ corresponding to the annual average for the calculation of A in a given month, where the appropriate values are σ_0^* and δ^*. The relative error would then be

$$E = \frac{\sigma_0}{\sigma_0^*} \frac{\delta + 2\kappa - 1}{\delta^* + 2\kappa - 1} - 1. \tag{4.7}$$

The relative error has been calculated for each month of the year. The results are shown in Fig. 8 indicating that the relative error is everywhere less than 5% with positive values in the period November to April and negative errors for the remaining part of the year. We may thus conclude

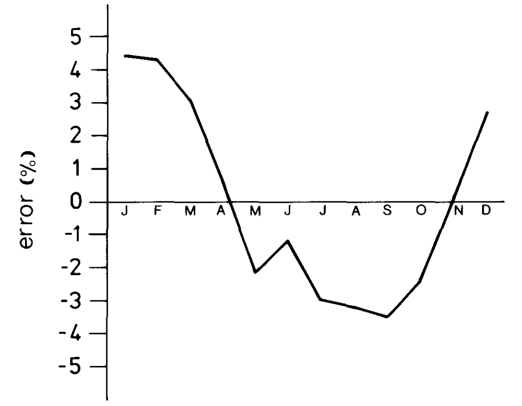

Fig. 8. The relative error in calculating the available potential energy, if the appropriate monthly values of the stability, according to eq. (4.2), are replaced by the annual mean values.

that the annual changes in the static stability are of minor importance for the calculation of the quasi-geostrophic, available, potential energy.

5. Conclusions

The investigation of globally averaged temperatures at many tropospheric and stratospheric levels has provided an explanation of the observed seasonal changes. The data set used here agrees with other observational studies showing that the global mean temperatures have a maximum in July–August and a minimum in January–February. We have shown that the observed changes are opposite to the predicted seasonal changes due to the ellipticity of the orbit of the Earth around the Sun. The observed seasonal changes can be described by the typical annual cycle in the planetrary albedo which has large values in January and small values in July. This seasonal change in the albedo is due mainly to the larger continental area in the Northern Hemisphere as compared to the other half of the Earth.

At the present time the Perihelion falls very close to the beginning of the calender year. Approximately ten thousand years from now the Perihelion will occur in the middle of the year. Assuming that the effects of ellipticity and albedo are independent of each other we should then expect that the two effects will be in phase resulting in an annual variation of the global mean temperature considerably larger than at the present time.

Agreement between predicted and observed seasonal changes in the global mean temperature is obtained only if the heat capacity used in the simple climate model is that of the atmosphere. We have thus made the assumption that the remaining parts of the climate system does not influence the changes in the global mean temperatures on a time-scale as small as a year or less. This assumption is supported by the fact that the observed changes may be described as a result of systematic seasonal changes of the planetary albedo. These changes seem in the first approximation to be determined by the influences of the clouds on the albedo.

The globally averaged temperatures have been used to study the seasonal and height changes of the static stability. As expected, we find larger than average values in January–February and smaller values half a year later in agreement with the effects of continentality on the stability. The vertical variation of static stability can with good approximation be described as inversely proportional to a power of the pressure. The value of the exponent is reasonably close to -2, but somewhat larger if both tropospheric and stratospheric levels are included and a little smaller if only the tropospheric values are included in the statistical analysis. The seasonal variations increase with height in an absolute sense, but since the static stability itself also increases with height, we should use a relative seasonal change which turns out to be quite small (less than 10%) as measured by the ratio of the root-mean-square deviation and the annual mean value.

The observed changes in stability are used to calculate the relative error in the computation of available potential energy based on a quasi-geostrophic formulation discussed in Section 2 of the paper. In fact, the relative error is calculated for each month in the case, where the annual mean value replaces the appropriate value for the month. It is found that the error in all cases is less than 5%, but with a seasonal change in agreement with the observed changes of the static stability.

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