Tellus (1991), *43A*, 25–36

On a mechanism for orographic triggering of tropical cyclones in the Eastern North Pacific

By JOSEPH A. ZEHNDER and ROBERT L. GALL, *Institute of Atmospheric Physics, University of Arizona, Tucson, AZ 85721, USA*

(Manuscript received 26 September 1989; in final form 2 May 1990)

ABSTRACT

It has been observed that the subtropical Eastern Pacific Ocean, just west of the Sierra Madre Mountains in central Mexico, is a favored location for the initiation of tropical cyclones, in spite of the fact that sea surface temperatures are not anomalous in the region. Currently accepted theories of tropical cyclogenesis require the presence of a pre-existing vortex located above a suitably warm sea surface. The distribution of tropical cyclogenesis off Mexico suggests that the initial disturbances may be orographically generated. We employ a two-dimensional, adiabatic, inviscid, Boussinesq, quasi-geostrophic model of flow over topography on a β-plane to examine the generation of regions of cyclonic vorticity in the subtropics by an incident flow which may approach the mountain at a specified angle. For incident wind configurations that correspond to the climatological northeast tradewinds, the orographic disturbance consists of an evanescent mountain anticyclone. For certain southeasterly flows, corresponding to the presence of large scale standing or slowly moving waves in the region, the orographic disturbance consists of a series of cyclonic and anticyclonic vorticity centers located in the southwest quadrant of the model atmosphere. These results are consistent with the observed behavior of tropical cyclogenesis off of the coast of Mexico.

1. Introduction

Since the advent of weather satellites in 1965, the subtropical eastern North Pacific Ocean, off of the coast of Mexico, has been recognized as a prolific region for tropical cyclogenesis. The presence of this region of preferred tropical cyclogenesis has been documented in a climatological study by Renard and Bowman (1976). While the existence of this maximum has been recognized, there has been little in the way of efforts to explain it. Sea surface temperatures in the subtropical eastern North Pacific, as well as the subtropical and tropical Atlantic are favorable for the formation of tropical cyclones. However, temperatures in both regions are similar thus sea surface temperature anomalies cannot explain the maximum of tropical cyclogenesis in the Eastern Pacific.

It is widely recognized that some sort of initial cyclonic disturbance is necessary to initiate tropical cyclogenesis. This region of the eastern North Pacific is downstream (with respect to the tropical easterlies) of the mountains of central Mexico and thus the role of lee cyclogenesis in forcing the initial disturbance is suggested. This idea is reinforced by the fact that there is not a similar arrangement of mountains and warm oceans in the tropical Atlantic and tropical cyclogenesis is scattered throughout the tropical Atlantic rather than being localized in a particular region. In this paper we will examine a mechanism involving flow interacting with topography which may explain the preferred genesis of tropical cyclones in this region.

In mid-latitudes, there are a number of regions which exhibit an unusually high frequency of extra-tropical cyclogenesis. Some of this cyclogenesis may be attributed to continent-ocean temperature contrasts. However, one region with an extremely high frequency of cyclogenesis is the Gulf of Genoa, which is in the lee of the Alps (Petterssen; 1956, p. 269). There has been an extensive amount of research, both observational and theoretical, devoted to explaining mid-latitude lee cyclogenesis, especially that occuring in the Gulf of

Genoa, and it is likely that this research can be easily extended to the tropics.

One branch of theoretical research in extratropical lee cyclogenesis has focused on the interaction of large-scale baroclinic flow with topography. Merkine (1975) demonstrated that the baroclinicity of a vertically sheared flow increased in the lee of the mountain, which suggests an increased tendency for cyclogenesis. Smith (1984) associated lee cyclogenesis with the resonance of a standing Eady edge wave forced by a vertically sheared flow with a critical level at middle levels in the atmosphere. Smith examined the linear development of the wave using a quasi-geostrophic model. The nonlinear development of a topographically forced standing edge wave was examined by Bannon and Zehnder (1989). Qualitative features of the wave development obtained in that study are similar to those in Smith (1984).

We would like to investigate the formation of tropical cyclones in the eastern North Pacific by focusing on the interaction of an idealized incident flow which approximates conditions typically observed in this area of the subtropics with idealized representation of the topography of this region. An investigation of tropical lee cyclogenesis is somewhat less restrictive than the corresponding problem in mid-latitudes, since we do not need to provide a disturbance which can tap energy from the basic-state atmosphere. All we need to do is provide an initial cyclonic disturbance. The energy source is provided by the warm sea surface.

An early theory of tropical cyclone initiation, which was proposed by Charney and Eliassen (1964), involves the existence of a conditionally unstable atmosphere. Frictionally driven convergence in the center of a preexisting disturbance forces vertical motion and results in a release of convective available potential energy (CAPE). Enhanced convection results in a transverse circulation which transports more moisture into the center. This positive feedback mechanism represents a linear instability and is referred to as CISK.

An alternative mechanism, proposed by Emanuel (1986), relies on surface latent and sensible heat transfer and is referred to as the air-sea interaction instability theory (ASII). Here a preexisting disturbance is still required but the disturbance serves to increase moisture flux into the boundary layer through increased winds. In addition, as parcels flow into the low pressure center of the disturbance they are cooled adiabatically. This results in a sensible heat flux and subsequent increase of the equivalent potential temperature, θ_e, in the boundary layer. Emanuel (1986) demonstrated that tropical cyclones of realistic intensity may be maintained by the air-sea interaction alone. This theory is attractive since it does not rely on any existing conditional instability in the atmosphere. The ASII theory also requires a finite amplitude initial disturbance and thus represents a more restrictive constraint on initial disturbances that will eventually grow into tropical cyclones.

The ASII theory of tropical cyclogenesis has been investigated in numerical model simulations by Rotunno and Emanuel (1987, hereafter referred to as RE). In that paper, the development of an initial finite amplitude vortex in an atmosphere that is neutral to convection was examined. RE found that a hurricane could develop in a realistic time frame through the ASII mechanism. RE also found that the resulting development was somewhat sensitive to the size and strength of the initial vortex. A benchmark experiment was performed with an initial vortex that had maximum winds $v_m \approx 12 \text{ m s}^{-1}$ and an outer radius (radius at which wind vanishes) of $r_0 \approx 400$ km. For a "weak vortex" experiment, $v_m \approx 2 \text{ m s}^{-1}$, RE found that cool, dry downdrafts were able to decrease θ_e in the boundary layer resulting in no appreciable intensification of the vortex. If the size of the vortex was doubled ($r_0 \approx 800$ km), a similar lack of intensification of the initial vortex due to the effect of cool downdrafts was observed. The results presented in RE indicate that the initial disturbance needs to be sufficiently strong and have a length scale of a few hundred kilometers in order to develop into an intense tropical cyclone.

In this study, we will briefly discuss the climatology of tropical cyclogenesis in the subtropical eastern North Pacific Ocean. We also present a simple model which retains the important dynamical elements present in this region and provides the cyclonic disturbances necessary to initiate cyclogenesis. The model will consist of an incident wind that is independent of height (since horizontal temperature gradients in this region are small) and an idealized topography typical of central Mexico. We will examine the interaction of this incident flow with the topography using the steady-state, linear, quasi-geostrophic equations. The fact that the incident flow and the resulting disturbance are steady does not represent an unrealistic constraint, since all we need is some sort of cyclonic disturbance in the correct location (ie. over the water) with the proper length scale to initiate the tropical cyclogenesis.

In the second section of this paper, we discuss the location and temporal distribution of tropical cyclones in the subtropical eastern North Pacific during the years 1977–1987. In the third section we discuss the steady, inviscid, adiabatic, stratified, Boussinesq, quasigeostrophic model of flow over a mountain ridge. In the fourth section we present the relative vorticity fields associated with the orographic response for various incident flow configurations. The fifth and final section summarizes the important results and discusses future research.

2. The occurrence of tropical cyclones in the eastern North Pacific

In order to quantify some ideas regarding the preferred geographic location of tropical lee cyclogenesis in the eastern North Pacific, we have collected data on the initial locations of tropical cyclones here and in the North Atlantic during the years 1977–1987. Data were taken from the Mariners Weather Log for the years 1977–1983 and from Weatherwise for the years 1984–1987. The initial locations of tropical cyclones that are eventually upgraded to at least tropical storm are plotted in Fig. 1 along with the climatological mean sea surface temperatures for July.

Fig. 1 clearly demonstrates the preferred geographic location of tropical cyclogenesis downstream of the mountains of central Mexico. Of the

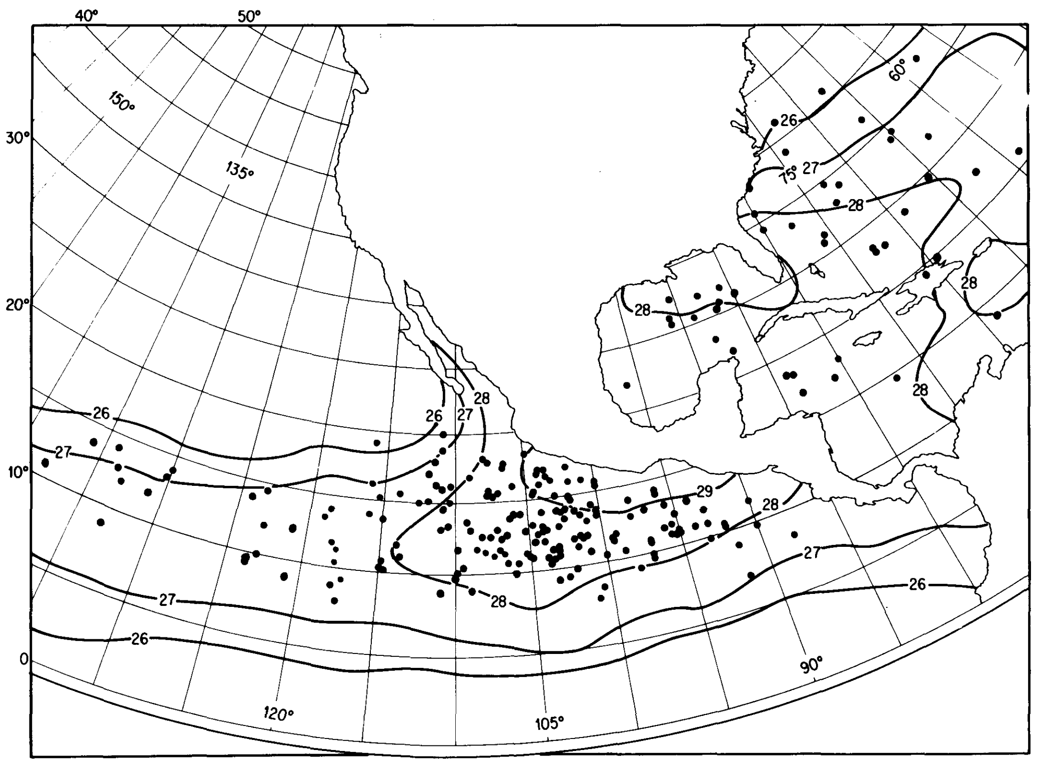

Fig. 1. Initial locations of tropical cyclones in the Eastern Pacific and Atlantic during the 1977–1987 hurricane seasons. Only cyclones that eventually are upgraded to at least tropical storm are included. Data were taken from Weatherwise and the Mariners Weather Log. Sea surface temperatures (heavy solid lines) are average values for July.

157 tropical cyclones that formed in the eastern North Pacific during the years 1977–1987, 109 were generated in the region bounded by 10° N and 20° N latitude, 110° W longitude, and the west coast of Mexico. When we compare this distribution with the initial locations of tropical cyclones in the Atlantic, we see that the Atlantic storms do not exhibit any preferred geographic location for formation. The genesis sites are scattered throughout the Carribean, the Gulf of Mexico and the subtropical Atlantic Ocean.

One may attempt to explain the large numbers of tropical cyclones in the eastern North Pacific through exceptionally high sea surface temperatures in that region. Just off the coast of Mexico the sea surface temperatures are indeed higher than in the Atlantic. However, most of the tropical cyclogenesis occurs over waters that are between 28°C and 29°C and these temperatures are about equal to those in the Carribean and Gulf of Mexico. Therefore, it is unlikely that high sea surface temperatures are responsible for the high incidence of tropical cyclones in the subtropical eastern North Pacific.

Additional information may be extracted from the data through examination of the temporal distribution of tropical cyclones in the eastern North Pacific. The dates of initiation of tropical cyclones in the eastern North Pacific for the years 1977–1987 are shown in Fig. 2a. During some years, the tropical cyclones seem to be generated nearly uniformly in time (i.e., 1983), while during other years (1978, 1982, 1985), the tropical cyclones occur in clusters, with a number of storms generated in rapid succession, followed by a period when no storms are generated. If tropical cyclogenesis were a temporally random process, we would expect to see some clustering. However, the following statistical analysis of the data in Fig. 2a shows that the observed distribution of intergenesis times is not random.

Fig. 2b shows a plot of the number of storms having a particular intergenesis time (defined as the number of days between tropical cyclone formation), along with the expected number of storms if the intergenesis times were Poisson distributed about the observed mean of 11.75 days. The Poisson distribution is the expected one if the intergenesis times are randomly distributed during the hurricane season. The expected distribution was divided into quartiles and a χ^2 test was performed.

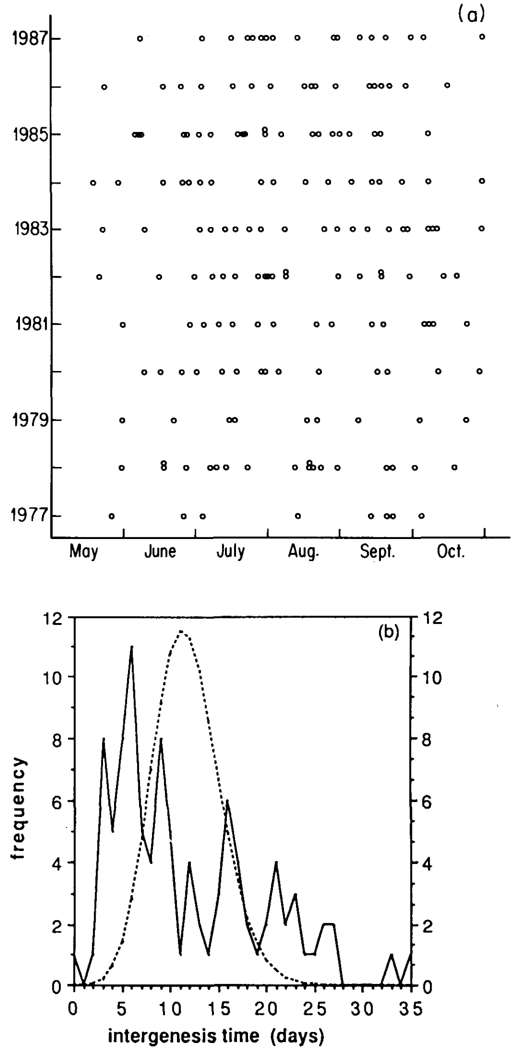

Fig. 2. (a) Dates of formation of tropical storms and hurricanes in the Eastern Pacific during the 1977–1987 hurricane seasons. Data source was the same as in Figure 1. (b) Observed (solid line) frequency of intergenesis times obtained from the data in (a). The dashed line indicates the expected distribution if the intergenesis times were distributed randomly about the observed mean of 11.75 days.

The calculated χ^2 of 66.75 is much larger than $\chi^2_{0.999}(3) = 16.3$, which indicates, with an extremely high confidence, that the observed distribution of intergenesis times is not the result of a random genesis of storms. Fig. 2b implies that there are frequent periods when tropical cyclones are generated at a rate of one every five days, while at other times the time between storms is much longer.

It is possible that both the preferred geographic location and the occasional tropical cyclone outbreaks may be due to the interaction of the incident flow with topography. The ability of mountains to disturb the airflow has been well documented and these disturbances could initiate hurricanes, but only in the vicinity of the mountains. This would explain the preferred geographic location. It is possible that at times a quasi-persistant flow configuration may exist which is favorable for the generation of the initial disturbances in the lee of the mountains while at other times the flow is not favorable for lee cyclone formation. This would help to explain the observed tropical cyclone outbreaks. In the remainder of this paper we identify one favorable incident flow configuration by developing a simple model in which regions of cyclonic relative vorticity are generated in the lee of the mountains.

3. The model

In this section, we present a simple model of flow over topography which suggests a mechanism for generation of the initial regions of cyclonic relative vorticity necessary for tropical cyclogenesis in the eastern North Pacific. Our model includes an idealized representation of the topography and characteristic wind fields in and around Mexico. The topography consists of a ridge (The Sierra Madres) extending from north of the US-Mexico border in the northwest to roughly 15°N in the southeast. This ridge has a characteristic width of about 600 km, a height of 1.5 km and is oriented at an angle of about 56° with the north-south direction. We approximate this topography by an infinite ridge of height h_0 and halfwidth a oriented at an angle to the north-south axis. Climatological surface winds are the northeast trades, with occasional departures associated with the propagation of large scale disturbances through the region. We characterize the wind field by a steady basic state wind which consists of uniform zonal and meridional components.

The model consists of a rotating, stratified, adiabatic, inviscid atmosphere described relative to a Cartesian coordinate system (x, y, z) with the positive y axis pointing toward the north and a variable Coriolis parameter, $f = f_0 + \beta_* y$. We make the Boussinesq approximation and require that the buoyancy frequency, $N_0 > 0$, of the basic state be uniform. We assume linear quasi-geostrophic dynamics and impose characteristic length, depth and velocity scales of $L = 1000$ km, $D = 10$ km and $U = 10\ \mathrm{m\ s^{-1}}$ respectively. The reduction of the primitive equations to the quasi-geostrophic form is given in Pedlosky (1979).

The vertically semi-infinite atmosphere is bounded below by a semi-infinite ridge, $z = h(x')$, where

$$h(x') = \frac{h_0}{1 + (x'/a)}, \tag{3.1}$$

$$x' = x \cos\alpha + y \sin\alpha, \tag{3.2}$$

and α is the angle between the ridge and the y axis. Positive values of α indicate a counterclockwise rotation of the ridge.

A steady non-zonal wind may be maintained in the presence of an applied forcing. If the time scale associated with the forcing is much longer than one day, the idea of an approximate geostrophic balance is still valid and the quasi-geostrophic theory may still be applied. However, the presence of an applied forcing means that the quasi-geostrophic potential vorticity is no longer a conserved quantity*. A relevant question which must be addressed regards the realism of such potential vorticity non-conserving flows.

* A reviewer suggested an alternate formulation which may be applied in the case of a non-zonal flow. If we impose a basic state zonal variation in the potential vorticity such that

$$U_0 \frac{\partial \bar{q}}{\partial x} + \beta V_0 = 0,$$

then the potential vorticity, $\bar{q}$, of the basic state would be conserved. The zonal potential vorticity gradient introduces an additional term $(\partial\bar{q}/\partial x)(\partial\phi_m/\partial y)$ on the left hand side of eq. (3.12). However, it may be readily verified that the modified vertical wavenumber resulting from the additional term in eq. (3.12) is equivalent to the one associated with an imposed zonal wind component and meridional potential vorticity gradient. This result makes sense physically since the basic state flow would be along isolines of the imposed potential vorticity distribution and changes in the relative vorticity of parcels will be in response to displacements across the isolines. The interaction of a zonal flow with topography on a β-plane is treated by Janowitz (1975), hence we consider the non-zonal, forced flow in this study.

In this paper, we examine the orographic response of imposed northeasterly and southeasterly winds. The northeasterly winds, which represent the northeast trades, are thermally forced by the intense solar heating near the equator and hence this flow will not conserve potential vorticity. The southeasterly wind represents a large scale wave that slowly propagates through the region and will conserve potential vorticity in the absence of diabatic forcing. A more realistic representation of a tropical wave would include the effect of latent heat release associated with convection. For example, consider the wave to be one of the prototype easterly Rossby waves described by Matsuno (1966), modified by latent heat release associated with the convergence in the region of the surface low and superimposed upon a uniform easterly zonal flow. In this case the wind will be southeasterly in the section of the wave that lies toward the east of the low pressure. Similar arguements can be presented for the inertia-gravity waves described by Matsuno (1966). For both types of wave, the flow is directed toward the region of maximum diabatic heating as it moves toward the north. This northward moving air experiences increasingly larger values of planetary vorticity at the same time it moves into the region of decreased static stability set up by the latent heat release. The decreased stability is what is necessary to compensate for the increasing planetary vorticity and prevent changes in the relative vorticity. This allows a steady flow with respect to the wave to be maintained. Note that since the waves are large scale with periods in excess of a week, the southeasterly flow can be maintained in the region of the topography sufficiently long for the waves described below to be set up.

The equation governing the development of the quasi-geostrophic potential vorticity is

$$\frac{\mathrm{D}}{\mathrm{D}t}\left(\nabla^2\psi + \frac{1}{S}\frac{\partial^2\psi}{\partial z^2}\right) + \beta\frac{\partial\psi}{\partial x} = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}, \tag{3.3}$$

where, ψ is the geostrophic streamfunction

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + u_{\mathrm{g}}\frac{\partial}{\partial x} + v_{\mathrm{g}}\frac{\partial}{\partial y}, \tag{3.4}$$

∇^2 is the horizontal Laplacian operator, $\beta = \beta_* L^2/U$, $S = N_0^2 D^2/f_0^2 L^2$ is the Burger number and the subscript "g" indicates horizontal wind components that are in geostrophic balance. The terms F_x and F_y represent the forcing terms that have been added to the zonal and meridional components of the momentum equations in order to maintain the steady non-zonal basic state. The zonal wind, meridional wind and potential temperature are related to the geostrophic streamfunction through the diagnostic relations

$$u_{\mathrm{g}} = -\frac{\partial\psi}{\partial y}, \qquad v_{\mathrm{g}} = \frac{\partial\psi}{\partial x}, \qquad \theta = \frac{\partial\psi}{\partial z}. \tag{3.5a,b,c}$$

The vertical component of the relative vorticity is given in terms of the geostrophic streamfunction by,

$$\zeta = \frac{\partial v_{\mathrm{g}}}{\partial x} - \frac{\partial u_{\mathrm{g}}}{\partial y} = \nabla^2\psi. \tag{3.6}$$

Conservation of potential temperature requires

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} + Sw = 0, \tag{3.7}$$

where w is the vertical velocity, which at the surface ($z=0$) is given by

$$w = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{h(x')}{\varepsilon D}\right), \tag{3.8}$$

where $\varepsilon = U/f_0 L$ is the Rossby number.

We impose a non-dimensional basic state geostrophic wind of the form

$$\boldsymbol{U}_{\mathrm{g}} = U_0\hat{\boldsymbol{x}} + V_0\hat{\boldsymbol{y}}, \tag{3.9}$$

where U_0 and V_0 are constants and $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are unit vectors in the x and y directions respectively. This basic state requires a geostrophic streamfunction of the form

$$\psi(x, y, z, t) = \frac{z}{\varepsilon F_{\mathrm{r}}} + \frac{Sz^2}{2\varepsilon} + U_0(\tau x - y) + \phi_m(x, y, z, t), \tag{3.10}$$

where $\tau = V_0/U_0$ and $F_{\mathrm{r}} = f_0^2 L^2/gD$ is the Froude number with g the acceleration due to gravity. This streamfunction also provides for a uniform potential temperature plus a uniform stratification with height. The streamfunction component $\phi_m(x, y, z, t)$ represents a perturbation to the basic state.

As was noted earlier, forcing terms must be added to the momentum equations in order to maintain the steady basic state given by eq. (3.10). Since the quasi-geostrophic flow is always close to geostrophic balance the forcing appears in the equations governing the first order ageostrophic velocity components (see Gill, 1982, p. 507). It is this first order ageostrophic velocity field that is responsible for changes in the geostrophic wind. If we require that the first order ageostrophic wind associated with the basic state be nondivergent, it may be shown that the forcing needed to maintain the steady basic state is

$$F_x = 0, \qquad F_y = V_0 \beta x. \tag{3.11a,b}$$

Using eq. (3.10) and eq. (3.11b) in eq. (3.3) and retaining only the linear terms yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\nabla^2 \phi_m + \frac{1}{S}\frac{\partial^2 \phi_m}{\partial z^2}\right) + \beta \frac{\partial \phi_m}{\partial x} = 0, \tag{3.12}$$

as the equation governing the perturbation streamfunction component, ϕ_m, where

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x} + V_0 \frac{\partial}{\partial y}. \tag{3.13}$$

A lower boundary condition on ϕ_m, obtained via the conservation of potential temperature, eq. (3.7), and the expression for the vertical velocity, eq. (3.8), is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \phi_m}{\partial z}\right) + S \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{h(x')}{\varepsilon D}\right) = 0 \quad \text{at} \quad z = 0. \tag{3.14}$$

For simplicity we restrict our discussion to steady solutions of the form

$$\phi_m(x, y, z, t) = \hat{\phi}_m(k, l) \exp i(kx + ly + mz + \gamma), \tag{3.15}$$

where k, l, m are the zonal, meridional and vertical wavenumbers and γ is a phase factor that insures that the mountain exerts a drag on the basic state flow. Using eq. (3.15) in eq. (3.12), we see that

$$m(k, l) = \pm S^{1/2}\left[\frac{\beta k}{(kU_0 + lU_0 \tau)} - (k^2 + l^2)\right]^{1/2}, \tag{3.16}$$

where the sign of the vertical wavenumber m, is determined by a radiation condition which requires that the vertical component of the group velocity be positive. The Fourier amplitude, $\hat{\phi}_m(k, l)$, is obtained via the lower boundary condition, and is given by

$$\hat{\phi}_m(k, l) = \frac{S\hat{h}(k, l)}{im(k, l)}, \tag{3.17}$$

where

$$\hat{h}(k, l) = \iint_{-\infty}^{\infty} h(x', y')\, \mathrm{e}^{-i(kx + ly)}\, \mathrm{d}x\, \mathrm{d}y$$
$$= 2\eta_0 \pi^2 a \exp(-|k'|\, a)\, \delta(l') \tag{3.18}$$

is the Fourier transform of the mountain profile where $\eta_0 = h_0/\varepsilon D$ and $k' = k \cos\alpha + l \sin\alpha$, $l' = -k \sin\alpha + l \cos\alpha$, are the wavenumbers perpendicular and parallel to the mountain. The orographically forced component, $\phi_m(x, y, z)$, is obtained by performing the inverse Fourier transform

$$\phi_m(x, y, z) = \frac{1}{2\pi} \iint_{-\infty}^{\infty} \hat{\phi}_m(k, l) \times \mathrm{e}^{i[kx + ly + m(k, l)z + \gamma]}\, \mathrm{d}k\, \mathrm{d}l. \tag{3.19}$$

The solution given by eq. (3.15), subject to (3.17) and (3.18) will consist of a vertically evanescent mountain anticyclone if m is imaginary, or vertically propagating waves if m is real. The horizontal structure of the disturbances is determined by m as well. If m is imaginary, the integrand in eq. (3.19) will be a rapidly oscillating function of k for large $|x|$, hence the integral will tend to zero in this limit. If m is real, there may be cancellations in the argument of exponential term in eq. (3.19), and the integral can be non-zero even for large values of $|x|$. Thus, the evanescent waves will be confined to the vicinity of the mountains, while the vertically propagating waves may be found at large distances from the mountain.

Since the topography used in this study is in the form of an infinite ridge, only wave modes with $l' = 0$ are forced. This requires that $l = k \tan\alpha$, and allows us to write eq. (3.16) as

$$m(k) = \pm S^{1/2}\left[\frac{\beta/U_0}{(1 + \tau \tan\alpha)} - k^2 \sec^2\alpha\right]^{1/2}. \tag{3.20}$$

If the ridge is oriented along the y-axis ($\alpha = 0$), the vertical wavenumber may be written

$$m(k) = \pm S^{1/2}[\beta/U_0 - k^2]^{1/2}. \tag{3.21}$$

In this case, westerly flows ($U_0 > 0$) generate the vertically propagating waves while easterly flows ($U_0 < 0$) result in a vertically evanescent mountain anticyclone. This case corresponds to the flow configuration and topographic orientation treated by Janowitz (1975).

If the ridge is oriented at an angle ($\alpha > 0$) with the y-axis, westerly flows still generate vertically propagating waves provided that $(1 + \tau \tan \alpha) > 0$. For easterly flows we may write eq. (3.20) as

$$m(k) = \pm S^{1/2}\left[\frac{-\beta/|U_0|}{1-(V_0/|U_0|)\tan\alpha} - k^2 \sec^2 \alpha\right]^{1/2}. \tag{3.22}$$

Here, for certain southeasterly flow configurations, we may have vertically propagating waves provided that $1-(V_0/|U_0|)\tan\alpha < 0$ or $V_0 > |U_0| \cot \alpha$. For northeasterly flows, $V_0 < 0$, and the disturbance is a vertically evanescent anticyclone. The above discussion provides information on the allowed responses for various flow configurations. We must also insure that the topographically forced waves satisfy the stationary phase requirement and that energy propagates away from the mountain.

The phase and energy propagation are determined from the dispersion relation, which may be obtained by substituting a solution of the form

$$\phi_m(x, y, z, t) = \hat{\phi}_m(k, l) \exp i(kx + ly + mz + \omega t), \tag{3.23}$$

into eq. (3.12). This substitution requires

$$\omega = -kU_0 - lU_0\tau + \frac{\beta k}{(k^2 + l^2 + m^2/S)}. \tag{3.24}$$

The phase and group velocities are given by

$$\boldsymbol{C}_\mathrm{p} = \frac{\omega}{k}\hat{\boldsymbol{x}} + \frac{\omega}{l}\hat{\boldsymbol{y}} + \frac{\omega}{m}\hat{\boldsymbol{z}}, \tag{3.25}$$

and

$$\boldsymbol{C}_\mathrm{g} = -\nabla_k \omega, \tag{3.26}$$

where

$$\nabla_k = \hat{\boldsymbol{x}}\frac{\partial}{\partial k} + \hat{\boldsymbol{y}}\frac{\partial}{\partial l} + \hat{\boldsymbol{z}}\frac{\partial}{\partial m}. \tag{3.27}$$

The restriction that the topography forces only the $l' = 0$ wave modes, along with eq. (3.25), yields a phase velocity

$$\begin{aligned}\boldsymbol{C}_\mathrm{p} = &\left[-U_0(1+\tau\tan\alpha) + \frac{\beta}{(k^2\sec^2\alpha + m^2/S)}\right]\hat{\boldsymbol{x}} \\ &+ \frac{1}{\tan\alpha}\left[-U_0(1+\tau\tan\alpha) + \frac{\beta}{(k^2\sec^2\alpha + m^2/S)}\right]\hat{\boldsymbol{y}} \\ &+ \frac{k}{m}\left[-U_0(1+\tau\tan\alpha) + \frac{\beta}{(k^2\sec^\alpha\alpha + m^2/S)}\right]\hat{\boldsymbol{z}}.\end{aligned} \tag{3.28}$$

Stationary phase requires that each component of $\boldsymbol{C}_\mathrm{p}$ vanish, or that

$$U_0(1+\tau\tan\alpha) = \frac{\beta}{(k^2\sec^2\alpha + m^2/S)}. \tag{3.29}$$

The group velocity may be written

$$\begin{aligned}\boldsymbol{C}_\mathrm{g} = &\left[U_0 - \frac{\beta}{(k^2\sec^2\alpha + m^2/S)} + \frac{2\beta k^2}{(k^2\sec^2\alpha + m^2/S)^2}\right]\hat{\boldsymbol{x}} \\ &+ \left[U_0\tau + \frac{2\beta kl}{(k^2\sec^2\alpha + m^2/S)^2}\right]\hat{\boldsymbol{y}} \\ &+ \frac{2\beta km}{(k^2\sec^2\alpha + m^2/S)^2}\hat{\boldsymbol{z}}.\end{aligned} \tag{3.30}$$

The radiation condition requires that energy propagate away from the mountain. In order to satisfy this condition, the vertical component of the group velocity must be positive. This requires

$$\mathrm{sign}(m) = \mathrm{sign}(k), \tag{3.31}$$

when choosing the sign of the vertical wavenumber in eq. (3.20). The radiation condition also requires that the zonal component of the group velocity be positive (negative) for waves in a westerly (easterly) flow. Using eqs. (3.29) and (3.30), we may write the zonal component of the group velocity as

$$C_{gx} = -V_0 \tan\alpha + \frac{2k^2 U_0^2 (1+\tau\tan\alpha)^2}{\beta}. \qquad (3.32)$$

The constraint on the direction of energy propagation imposes a restriction on the allowed zonal wave modes. For waves in an easterly flow $C_{gx} < 0$ which requires $k < k_c$, where

$$k_c = \sqrt{\frac{V_0 \beta \tan\alpha}{2U_0^2(1+\tau\tan\alpha)^2}}. \qquad (3.33)$$

The radiation condition represents a long wave cutoff of the allowed wave modes. For waves in a westerly flow, $k > k_c$, and the radiation condition imposes a short-wave cutoff of allowed waves. In addition, vertically propagating waves require real values of k. This condition is satisfied only if $V_0 > 0$. This means that there are no vertically propagating waves generated in northeasterly flows.

The orographically forced geostrophic streamfunction component is obtained by evaluating the integral in eq. (3.19). The integral over l may be performed by first transforming the delta function, $\delta(l')$. In general, a delta function of the form $\delta(f(l))$ may be written

$$\delta(f(l)) = \sum_{i=1}^{n} \frac{1}{|f'(l_i)|}\delta(l - l_i), \qquad (3.34)$$

where $f(l_i) = 0$ and $f'(l_i) \neq 0$. Since $l' = -k\sin\alpha + l\cos\alpha$, the delta function may be written

$$\delta(l') = \sec\alpha\, \delta(l - k\tan\alpha), \qquad (3.35)$$

and the integration over l is equivalent to replacing l with $k\tan\alpha$.

An additional constraint on ϕ_m is that the mountain exerts a drag on the basic state flow or equivalently that the basic state flow exert a positive (negative) force on the mountain for westerly (easterly) zonal flows. The force exerted on the mountain is

$$F = \rho_0 U f_0 L D \int_{-\infty}^{\infty} \phi(x_p, z=0) \frac{\partial h}{\partial x_p}\, dx_p, \qquad (3.36)$$

where ρ_0 is a reference value of the density and $x_p = x + y\tan\alpha$, measures distance perpendicular to the ridge. The component of the geostrophic streamfunction that consists of vertically propagating components, evaluated at the surface, may be written

$$\phi_m(x_p, 0) = \int_0^{k_*} \frac{e^{-|k| a \sec\alpha} \sin(kx_p + (k_*^2 - k^2)^{1/2} z + \gamma)}{(k_*^2 - k^2)^{1/2}}, \qquad (3.37)$$

which is an odd function of x_p. It may be shown that the necessary phase shift is $\gamma = 0$ for westerly flows and $\gamma = \pi$ for easterly flows.

We are interested in the relative vorticity associated with the orographic response. This is obtained by applying the horizontal Laplacian operator to eq. (3.19) prior to performing the integration over k. It may be shown that the relative vorticity associated with the orographic response may be written

$$\zeta_m(x_p, z) = \eta_0 S^{1/2} \sec^2\alpha \times \int_0^{k_*} \frac{\left(\begin{array}{l} k^2 e^{-|k| a \sec\alpha} \\ \times \sin[kx_p + (k_*^2 - k^2)^{1/2} z + \pi] \end{array}\right)}{(k_*^2 - k^2)^{1/2}}\, dk - \eta_0 S^{1/2}\sec^2\alpha \times \int_{k_*}^{k_c} \frac{k^2 e^{-|k| a\sec\alpha} \cos(kx_p)\, e^{-(k_*^2 - k^2)^{1/2} z}}{(k^2 - k_*^2)^{1/2}}\, dk. \qquad (3.38)$$

For waves in an easterly flow,

$$k_* = \sqrt{\frac{\beta\cos^2\alpha / U_0}{(1+\tau\tan\alpha)}}, \qquad (3.39)$$

and k_c is given by eq. (3.33). For waves in a westerly flow the constraint on group velocity provides a shortwave cutoff and the streamfunction components may be written

$$\zeta_m(x_p, z) = \eta_0 S^{1/2}\sec^2\alpha \times \int_{k_c}^{k_*} \frac{k^2 e^{-|k| a\sec\alpha} \sin(kx_p + (k_*^2 - k^2)^{1/2} z)}{(k_*^2 - k^2)^{1/2}}\, dk - \eta_0 S^{1/2}\sec^2\alpha \times \int_{k_*}^{\infty} \frac{k^2 e^{-|k| a\sec\alpha}\cos(kx_p)\, e^{-(k_*^2 - k^2)^{1/2} z}}{(k^2 - k_*^2)^{1/2}}\, dk. \qquad (3.40)$$

Eqs. (3.38) and (3.40) are evaluated numerically using a Gaussian quadrature scheme.

4. Results

The perturbation relative vorticity associated with various orientations of the incident flow are presented in this section. The topographic parameters are chosen as $h_0 = 1.5$ km, $a = 300$ km and $\alpha = 56°$. These values correspond to the topography in central Mexico. Since the mountains are centered around 17°N latitude we take the Coriolis parameter to be $f_0 = 4.27 \times 10^{-5}\ \mathrm{s}^{-1}$ and $\beta_* = 2.2 \times 10^{-11}\ \mathrm{m}^{-1}\ \mathrm{s}^{-1}$.

Fig. 3a shows the relative vorticity, in units of f_0, for an easterly zonal wind $U_0 = -10\ \mathrm{m\ s^{-1}}$ and various values of V_0. For all of the incident flow configurations the relative vorticity near the mountain is anticyclonic, which is consistent with the mountain being an isentropic surface. When the meridional wind, V_0, is negative or less than the critical value, there are weak regions of cyclonic vorticity on either side of the mountain anticyclone. When the meridional wind is greater than the critical value, there is a series of cyclonic and anticyclonic vorticity maxima on the downstream (negative x_p) side of the mountain. When the meridional wind is slightly larger than the critical value the cyclonic vorticity maximum is located near the mountain and has a width of about 300 km, which is a horizontal scale consistent with the ASII mechanism of tropical cyclone formation. As the meridional wind component increases the first maximum becomes broader, weaker, and moves further downstream. This broadening and weakening of the relative vorticity maximum implies that a limited range of angles of incidence will result in a response which provides a region of cyclonic vorticity of appropriate magnitude and horizontal scale for tropical cyclone formation.

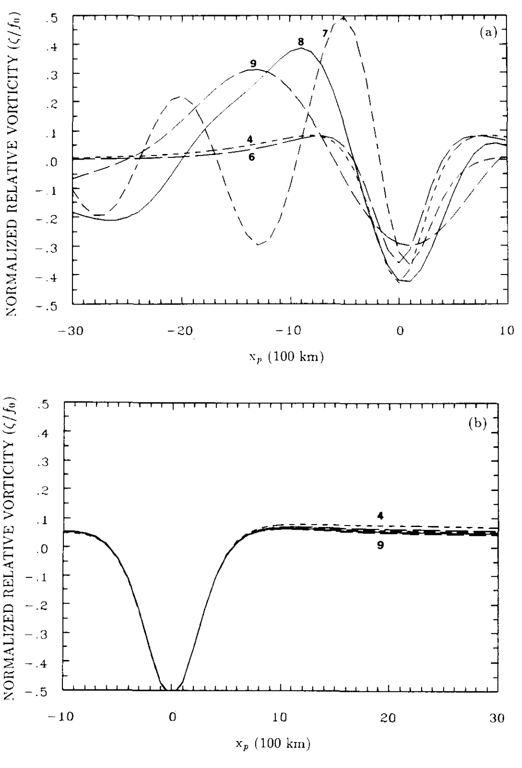

Fig. 3. Relative vorticity in units of f_0 at the surface as a function of the distance perpendicular to the ridge, x_p, for (a) an easterly zonal wind $U_0 = -10\ \mathrm{m\ s^{-1}}$ and (b) a westerly zonal wind $U_0 = 10\ \mathrm{m\ s^{-1}}$ with a variety of values of the meridional wind $V_0 > 0$. The topography has a height $h_0 = 1.5$ km a width $a = 300$ km and is centered about $x_p = 0$. Note the shift location of $x_p = 0$ between the figures.

Fig. 3b shows the relative vorticity for a westerly zonal wind $U_0 = 10\ \mathrm{m\ s^{-1}}$ and a variety of values of V_0. Here we only consider $V_0 > 0$ since northwesterly winds are not observed in this region of the subtropics during the summer. In this case the relative vorticity is anticyclonic with only slight generation of cyclonic relative vorticity on the lee side (positive x_p) of the mountain. There will be lee wave generation for this flow configuration but this occurs for topography with larger length scales than are treated here.

The differences in the characteristics of the lee waves for southeasterly and southwesterly flows may be understood by considering the behavior of the integrals in eqs. (3.38) and (3.40). The primary contribution to the integrals comes from values of k near the critical wavenumber, k_*, which is defined by eq. (3.39). For easterly zonal flows the critical wavenumber is

$$k_* = \sqrt{\frac{\beta \cos^2 \alpha / |U_0|}{V_0/|U_0| \tan \alpha - 1}}, \tag{4.1}$$

while for westerly zonal flows,

$$k_* = \sqrt{\frac{\beta \cos^2 \alpha / U_0}{1 + V_0/U_0 \tan \alpha}}. \tag{4.2}$$

For like values of $|U_0|$ and V_0, k_* will be larger for easterly flows than for westerly ones. This means that the preferred waves have shorter length scales in easterly flows than in westerlies. The relatively small scale topography in central Mexico will be more efficient in generating waves in easterly flows than in westerlies.

The orographic response generated by the flow configurations considered here is consistent with the premise that the mountains are responsible for initiating tropical cyclogenesis in the eastern North Pacific. The interaction of the climatological northeast tradewinds with the Sierra Madres results in anticyclonic disturbances. If this were not true we would expect to see a nearly continuous generation of tropical lee cyclones. In order to generate the cyclonic vorticity centers we need a departure from the climatological wind field. Such a departure may be due to the propagation of large scale easterly waves through the region. These waves then establish favorable conditions for the formation of lee cyclones which could persist for an extended period of time as the wave passes over the topography. This could explain the apparent tropical cyclone outbreaks which occur in the Eastern Pacific (see Fig. 2).

5. Conclusion

This study focuses on the generation of steady-state lee waves forced by air flow over topography on a β-plane. The topography is in the form of an infinite ridge that is oriented in the northwest-southeast direction. The incident wind is allowed to approach the ridge at a specified angle. Values of the Coriolis parameter, $f = f_0 + \beta_* y$, and topographic height and width are chosen to correspond to the subtropical region near central Mexico. The relative vorticity fields resulting from the interaction of a number of characteristic flow configurations with the topography are examined.

In easterly zonal flows the orographic disturbance consists of an anticyclone that is centered over the topography if the meridional wind is negative or below a critical value given by $V_0 = |U_0| \cot \alpha$. If the meridional wind is above the critical value, the orographic response consists of a vertically propagating wave composed of a series of cyclonic and anticyclonic relative vorticity maxima along the surface, to the west of the topography. In westerly zonal flows the orographic disturbance consists of a wave train for a wide range of meridional wind values. However, the preferred waves in westerly zonal flows are of much larger length scale than those in an easterly flow. Since the Sierra Madre Mountains have a width of about 600 km, they are much more efficient at forcing waves in an easterly flow.

If the solutions presented here are to represent realistic initial vortex centers, the incident wind must remain steady long enough for the orographic disturbances to form. An estimate of the time necessary for the disturbances to become established may be obtained from the horizontal group velocity, given by eq. (3.32). The resonant wavenumber given by eq. (4.1) is evaluated for $V_0 = 7$ m/s and $U_0 = -10$ m/s. This value of k_* is used to determine a characteristic value of the group velocity, which is $C_{gx} = 10$ m/s. It would take a disturbance about 13 h to travel the 500 km distance to the location of the first vorticity maxima shown in Fig. 3. This time is small compared to the observed periods of tropical easterly waves, which are 4 to 10 days, (see Wallace, 1971) and thus there is sufficient time during the passage of a large scale tropical wave for the wave to first alter the incident wind field over the topography and for the cyclonic vorticity maxima to set up.

It will be necessary to verify the results presented in this paper through a thorough observational study. In a future paper we will examine wind fields, both upstream and downstream of the Sierra Madres, and attempt to establish a causal connection between observed incident winds and resulting tropical cyclogenesis. It is possible that the details of the generation of cyclonic vorticity centers off Mexico in the Eastern Pacific are due to more complicated interactions than are represented in this simple model. For example, Frank (1976) has suggested that some of the initial disturbances resulting in Eastern Pacific hurricanes may begin as synoptic scale features off the coast of Africa. However, we believe that the fundamental dynamical mechanism represented in this model is important in at least some of the tropical cyclogenesis in this region.

The presence of a steady state lee trough in this model has been discounted as a possible theory of lee cyclogenesis in middle latitudes. Pierrehumbert (1986) states that the lee trough resulting from flow over topography on a β-plane does not provide an adequate representation of a developing mid-latitude lee cyclone since it is steady state. In addition, the trough has a length scale that is much larger than those associated with mid-latitude cyclones. The deficiencies noted by Pierrehumbert (1986) however, are not necessarily applicable in the tropics. In the case of tropical lee cyclogenesis, we do not need to orographically generate a disturbance that is unstable due to baroclinic or barotropic energy exchange. Even a steady state disturbance such as we have here could undergo subsequent development after they initially set up, since the instability that results in a hurricane may be due to air-sea interaction.

The steady state solution presented above, while displaying a number of realistic features, is still highly restrictive. The β-plane approximation employed here is applicable as long as the β-plane is located at a distance greater than $a_e = (c/2\beta_*)$, away from the equator. The quantity a_e is referred to as the equatorial Rossby radius and $c = (gH)^{1/2}$ where g is the acceleration due to gravity and H is an equivalent depth of the atmosphere. Typical atmospheric values yield $a_e \approx 10°$N–$12°$N of latitude (see Gill, 1982, pp. 437 and 495). The region of the Eastern Pacific that we are interested in is at about 15°N hence we are near the limit of validity of the mid-latitude β-plane. In this region, the equatorial β-plane approximation may better represent the dynamics. The topographic parameters chosen in this study are near the limit of validity of the linear quasi-geostrophic theory. There may be other types of motion (i.e., gravity waves) that may begin to become important at these length scales. However, we believe that the results presented here adequately describe the large scale features of the orographic response.

An investigation of the time dependent aspects of this problem may also prove fruitful. In a future study we will examine the interaction of easterly waves with topography on an equatorial β-plane. We also plan to investigate this process using a primitive equation model with physical parameterizations. In this way we will be able to simulate rather than infer tropical cyclogenesis and more completely investigate the dynamical processes active in this region.

6. Acknowledgments

Partial support for this work was provided by NSF grants ATM-8613514 and ATM-8912560.

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