Cold Regions Science and Technology 33 (2001) 237–246

cold regions science and technology

www.elsevier.com/locate/coldregions

A statistical validation of the snowpack model in a Montana climate

Christopher C. Lundy [a,*], Robert L. Brown [a], Edward E. Adams [a], Karl W. Birkeland [b], Michael Lehning [c]

[a] *Department of Civil Engineering, Montana State University, Bozeman, MT 59717, USA*
[b] *USDA Forest Service National Avalanche Center, P.O. Box 130, Bozeman, MT 59771, USA*
[c] *Swiss Federal Institute for Snow and Avalanche Research, CH-7260 Davos Dorf, Switzerland*

Received 1 September 2000; accepted 22 May 2001

Abstract

Recently, a computer model has been developed by the Swiss Federal Institute for Snow and Avalanche Research that simulates the evolution of a natural snow cover. Using common meteorological parameters as input, SNOWPACK predicts characteristics such as snowpack temperature and density, in addition to snow microstructure and layering. An investigation was conducted to evaluate the effectiveness of SNOWPACK in a Montana climate. A weather station was constructed in the Bridger Mountains near Bozeman, Montana, to provide the meteorological parameters necessary to run SNOWPACK. Throughout the 1999–2000 winter, weekly snow profiles were performed in undisturbed snow to provide a benchmark for the model output. Density, grain size, and crystallography were recorded on 10-cm intervals over the full snow depth, and the temperature profile was monitored with a thermocouple array. Finally, the meteorological parameters were input into SNOWPACK, and a statistical comparison was performed comparing the predicted snowpack to the observational data. Snowpack temperatures are predicted reasonably accurately by SNOWPACK. The modeled and observed densities correlated well, but the model typically underestimates snowpack settlement. Comparison of grain size and shape was problematic due to different definitions utilized by the model and observer, but still demonstrated some agreement. © 2001 Published by Elsevier Science B.V.

Keywords: Snowpack modeling; Snow metamorphism; Model validation; Snowpack evolution

1. Introduction

Recent advances in snow research, as well as the increasing availability of powerful computer systems, have led to the development of computer models (i.e. CROCUS (Brun et al., 1992), SNOWPACK (Lehning et al., 1998), and SNTHERM (Jordan, 1991)) that are becoming better at predicting the evolution of a mountain snowpack. Most of these programs use common meteorological parameters as inputs and provide as output predicted snowpack temperature, density, grain size, and crystal type. The more advanced models have already been used operationally and provide avalanche hazard forecasters and other mountain safety experts with yet another tool for evaluating the alpine snowpack.

* Corresponding author.
E-mail address: lundy@mcn.net (C.C. Lundy).

0165-232X/01/$ - see front matter © 2001 Published by Elsevier Science B.V.
PII: S0165-232X(01)00038-6

The focus of this paper is SNOWPACK, a numerical snow cover model developed by the Swiss Federal Institute for Snow and Avalanche Research. SNOWPACK is a predictive model that uses Lagrangian finite elements to solve for heat and mass transfer, stresses, and strains within the snowpack (Lehning et al., 1998). Snow is modeled as a three-phase porous media consisting of volumetric fractions of ice, water, and air. Using a microstructure-dependent viscosity, the settlement and density of the snow cover are computed. The temperature profile of the snowpack is determined based on the thermal conductivity formulation developed by Adams and Sato (1993). The model contains both equilibrium and kinetic-growth metamorphism routines that calculate the time rate of change of grain shape parameters sphericity and dendricity (Fig. 1), grain radius, and bond radius which define the microstructural characteristics of the snow. Other variables such as coordination number and bond neck length are computed from these primary quantities. Phase change of the ice and water components is taken into account, and a simple procedure for meltwater percolation through the snowpack is also utilized. Importantly, the conservation laws for mass, energy, and momentum are adhered to in all aspects of the code. Currently, wet snow metamorphism is still under development, and the models for dry snow metamorphism are being improved (Lehning et al., 1998).

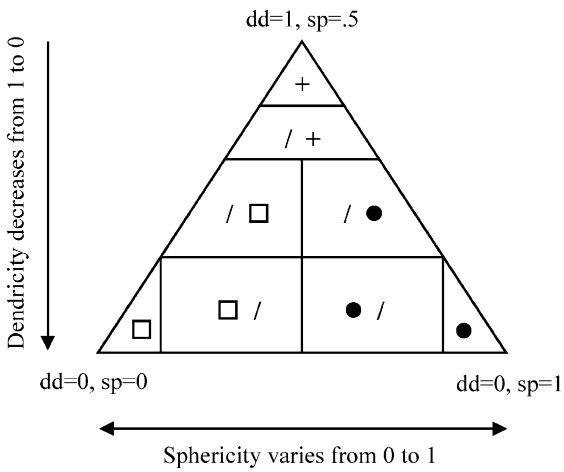

Fig. 1. Relation of the model parameters sphericity and dendricity to the ISCI symbols (Colbeck et al., 1990).

The SNOWPACK model is executed using weather and snowpack data measured at automatic weather stations. Required inputs are air temperature and relative humidity, snow surface temperature, reflected solar radiation, total snow depth, and wind speed (Lehning et al., 1999). Utilizing the reflected solar radiation allows the solarimeter to be mounted pointing downwards towards the snow surface, eliminating problems with the sensor becoming covered with new snowfall. SNOWPACK then uses an empirical estimation of the snow surface albedo to back-calculate the incoming shortwave radiation (Lehning et al., 1999). When the temperature of the surface snow is below 0 °C, the snow surface temperature is used as a Direchlet boundary condition. Otherwise, the Neumann boundary condition is utilized; in this case, the net longwave radiation is calculated from the snow surface temperature using an estimation of the atmospheric emissivity and the Stefan–Boltzmann constant (Bartelt et al., 1999).

One problem associated with efforts to model the evolution of a mountain snowpack is the lack of a meaningful comparison of the model predictions to an actual snowpack. Plots of simulated and measured snowpack parameters are abundant in the literature (Brun et al., 1992; Lehning et al., 1998), but simply comparing these graphs visually does not provide a meaningful or consistent evaluation of the similarity between observed and predicted values. Introducing statistical methods using well-established measures provides a means of quantifying the accuracy of the models

2. Purpose

To date, validation of the snowpack simulation models has not been adequately addressed using an objective and numerical approach. This will continue to limit their improvement and acceptance, since few users are willing to expend the resources necessary to operationalize a snowpack model without first having access to an extensive evaluation of the program.

To complete an objective validation of SNOWPACK, a weather station was constructed to provide the meteorological parameters necessary to run the model. During the 1999–2000 winter, regular snow profiles were conducted to provide a benchmark for the model output. The meteorological data was then input into SNOWPACK, and a "predicted" snowpack was computed. Finally, the use of statistical methods allows a thorough and objective comparison of the model output to field observations.

3. Methods

3.1. Description of field site

The field research site is located approximately 1 km north of the Bridger Bowl Ski Area near Bozeman, Montana, in an area known as Wolverine Basin. This region falls within the continental climate regime as defined by McClung and Schaerer (1993). An average annual snowfall of approximately 6.5 m is measured at the adjacent ski area. The site is a large, open, relatively flat meadow situated at an elevation of 2240 m (Fig. 2).

A weather station was constructed at the site to obtain the necessary input data for the SNOWPACK model. From 17 November 1999 to 6 April 2000, the following were collected on 30-min intervals:

- Air temperature
- Snow surface temperature
- Relative humidity
- Wind speed
- Reflected shortwave radiation
- Total snow depth.

Additionally, a temperature-measurement array was utilized to obtain a real-time temperature profile within the snowpack. It was constructed from a 3-m PVC tube fitted with thermocouples on 5-cm intervals, standing vertically with the bottom thermocouple at ground level. The PVC tube is filled with foam so that the entire unit has a low thermal conductivity.

3.2. Collection of snow profile data

On a weekly basis, snowpits were excavated in undisturbed snow near the instrument tower. Density was measured with a triangular density box of known volume, and weighed on a portable digital scale. Grain size was measured by disaggregating a sample from the layer under observation, and visually estimating the mean maximum dimension of the snow particles in the sample. A $25\times$ Pentax hand lens with built-in millimeter scale was used to facilitate this measurement. The crystal type was classified according to the ISCI system (Colbeck et al., 1990). The snowpack temperature profile was read from the thermocouple array at approximately the same time as the snow profile was performed. All observations were obtained on 10-cm intervals through the full depth of the snowpack.

Fig. 2. Aerial photo showing location of Wolverine Basin weather station.

3.3. Comparison of the predicted snowpack to the observed snowpack

While SNOWPACK includes a graphical user interface that presents a visual description of the snowpack predicted by the model, a simple visual comparison of the model results to the snowpit data is not adequate to objectively evaluate the model. A better method is to employ familiar statistical measures to evaluate the level of agreement between the modeled and observed parameters:

- Temperature
- Density
- Grain size
- Grain type.

The model itself was configured to output the snow profile data for each day at 1100 h, which corresponded with the time at which the snowpits were typically performed in the field. It is this modeled profile data that is compared to the weekly snowpit observations.

Observations from the snow profiles were taken at regular 10-cm intervals; however, the data calculated by SNOWPACK are irregularly spaced with depth. As a result, before any comparison of the predicted and observed snowpacks can be undertaken, predicted model data must be calculated at the same depths within the snowpack as the snowpit observations (i.e. on 10-cm intervals). To accomplish this task, a technique was devised to obtain model results at desired locations.

From the start, difficulty arises in that the modeled and observed snowpack heights may be different. Although the predicted snowpack height is set equal to the measured depth during periods of snowfall, the simulated depth is otherwise predicted independent of the measured height. To develop a basis for comparing the predicted and observed snowpack heights, even if the total depths differ, a normalization of depth is performed. The modeled and measured snowpack heights are each assigned a unit height, and the depths at which observed or modeled values occur are adjusted accordingly so that they are expressed as a fraction of the unit depth (Fig. 3). This can be easily expressed in mathematical terms. First, let $z_k'^{\text{mod}}$ describe the heights of the data output by the model, where m is the number of levels and k is an index ranging $0 < k < m$, and define $z_k'^{\text{mod}}$ as the predicted total snow depth. Then the normalized heights, z_k^{mod}, are given by:

$$z_k^{\text{mod}} = \frac{z_k'^{\text{mod}}}{z_m'^{\text{mod}}}. \tag{1}$$

An identical procedure is performed for the observation heights:

$$z_i^{\text{obs}} = \frac{z_i'^{\text{obs}}}{z_n'^{\text{obs}}}, \tag{2}$$

where n is the number of observations levels and i is an index that varies from $0 < i < n$.

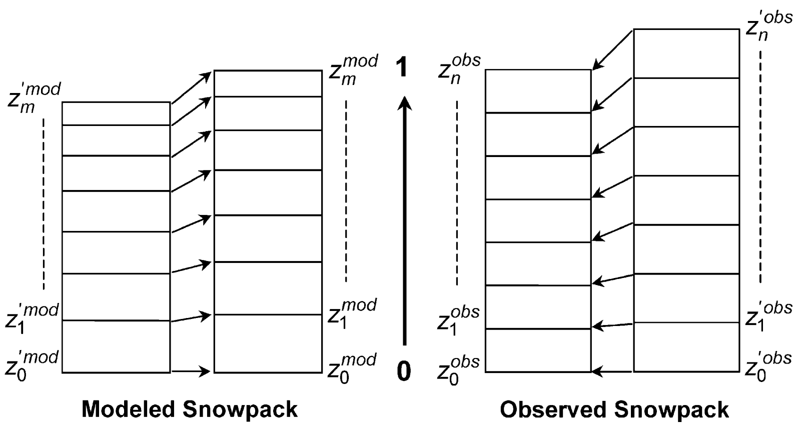

Fig. 3. Normalization of the modeled and observed snowpack depths, creating a basis for comparing the profiles even if the total depths differ.

Once a common basis for comparing the observed and modeled depths is established, linear interpolation is applied to obtain model results at depths that correspond with the observations taken on 10-cm intervals. For a series of observations such as temperature T_i^{obs}, at snow depths z_i^{obs}, a linear interpolation between the two neighboring normalized model heights, z_k^{mod} and z_{k-1}^{mod} ($z_{k-1}^{\text{mod}} < z_i^{\text{obs}} < z_k^{\text{mod}}$), yields a modeled temperature at the observed height:

$$T_i^{\text{mod}}\left(z_i^{\text{obs}}\right) = \frac{z_k^{\text{mod}} - z_i^{\text{obs}}}{z_k^{\text{mod}} - z_{k-1}^{\text{mod}}} T_{k-1}^{\text{mod}} + \frac{z_i^{\text{obs}} - z_{k-1}^{\text{mod}}}{z_k^{\text{mod}} - z_{k-1}^{\text{mod}}} T_k^{\text{mod}}. \qquad (3)$$

The equations for calculating density and grain size are identical. Since grain type is not measured on a continuous numeric scale, interpolation is not possible and the only feasible technique is to use the grain type value that occurs at the height closest to the observed location.

3.4. Statistical measures of model performance

A detailed summary of statistical descriptors that evaluate a model's ability to match an observational dataset is provided in Imam et al. (1999). These goodness-of-fit indicators fall loosely into one of two categories: residual-based and statistical association-based. By employing statistical measures from both categories, a more complete description of the model's performance is obtained.

In the residual-based category, the mean bias (B) and the root mean square error (RMSE) are commonly used for model verification (Sorooshian et al., 1983; Imam et al., 1999). These are defined as (Imam et al., 1999):

$$B = \frac{\sum_{i=1}^{y}\left(x_i^{\text{mod}} - x_i^{\text{obs}}\right)}{y} \qquad (4)$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{y}\left(x_i^{\text{mod}} - x_i^{\text{obs}}\right)^2}{y}} \qquad (5)$$

where x_i^{mod} and x_i^{obs} are a set of y predicted and measured data pairs. The mean bias indicates the direction of the expected model error, and is a useful measure of a model's tendency towards overestimation or underestimation. In contrast, the RMSE estimates the expected magnitude of error associated with a model's prediction.

Among indicators of statistical association, Pearson's correlation coefficient is perhaps the most common (Box et al., 1978):

$$r = \frac{\sum_{i=1}^{y} x_i^{\text{mod}} - x_i^{\text{obs}}}{\sqrt{\sum_{i=1}^{y}\left(x_i^{\text{mod}}\right)^2 \sum_{i=1}^{y}\left(x_i^{\text{obs}}\right)^2}}. \qquad (6)$$

The correlation coefficient has an upper bound of 1, indicating perfect positive linear correlation, and a lower bound of -1, corresponding to negative linear correlation. Although Pearson's r is familiar among many scientists and often used for validation purposes, other researchers have provided arguments against its use for model verification (Imam et al., 1999; Imam, 1994; Fox, 1981; Willmott, 1981). Another measure of association is the Nash–Suttcliffe coefficient of efficiency (Nash and Suttcliffe, 1970):

$$E = \frac{\sum_{i=1}^{y}\left(x_i^{\text{obs}} - \mu_{\text{obs}}\right)^2 - \sum_{i=1}^{y}\left(x_i^{\text{obs}} - x_i^{\text{mod}}\right)^2}{\sum_{i=1}^{y}\left(x_i^{\text{obs}} - \mu_{\text{obs}}\right)^2}, \qquad (7)$$

where μ_{obs} is the mean of the observed dataset. E is upper bounded by 1, and yields higher magnitudes with increasing model accuracy. The coefficient of efficiency can also assume negative values, which have a less intuitive interpretation. To address this, Willmott and Wicks (1980) proposed an index of agreement:

$$d = 1 - \frac{\sum_{i=1}^{y}\left(x_i^{\text{obs}} - x_i^{\text{mod}}\right)^2}{\sum_{i=1}^{y}\left(\left|x_i^{\text{mod}} - \mu_{\text{obs}}\right| + \left|x_i^{\text{obs}} - \mu_{\text{obs}}\right|\right)^2}, \qquad (8)$$

which is bounded by [0 1] so that it does not produce negative values.

The preceding statistical indicators are applicable to numeric parameters such as temperature, density, and grain size, but not to grain type which is measured on a categorical scale. Instead, Cramer's Phi, a

Chi-Square (X^2)-based statistic, is used to ascertain the degree of association between the observed and predicted grain types. Cramer's Phi is given by (Agresti, 1996):

$$V = \sqrt{\frac{X^2}{n(a-1)}}, \tag{9}$$

where n is again the number of samples in the population, and a is the number of categories present in the two populations, whichever is smaller. In this case $a = 6$, corresponding to six different grain classifications. Computation of Cramer's V is convenient as it ranges from 0 to 1 and is interpreted in the same fashion as Pearson's r. Another measure of association based on the Chi-Square statistic is Sakoda's adjusted contingency coefficient (Agresti, 1996):

$$C^* = \frac{C}{\sqrt{\frac{a-1}{a}}} = \frac{\sqrt{\frac{X^2}{X^2+y}}}{\sqrt{\frac{a-1}{a}}}, \tag{10}$$

where C is the unadjusted contingency coefficient. The adjusted contingency coefficient is also bounded by [0 1], with higher values indicating better association. Both V and C^* are computed twice; once for the majority grain type F1, and again for the minority classification F2.

The preceding discussion serves to emphasize that there is no single statistical descriptor that will effectively assess a model's ability to predict observed data. Only by combining the merits of several different measures can a complete model evaluation be obtained.

4. Results and discussion

The measured and predicted total snow depths are plotted for the research period in Fig. 4. it is apparent from this graph that during periods of increasing snow depth (snowfall), the modeled snow depth is matched to that measured at the weather station. Conversely, during settlement periods when the depth is decreasing, the model predicts depth independent of the measurement. Early in the winter, SNOWPACK predicts the settlement curves quite accurately. From mid-winter to early spring, however, the model underestimates snowpack consolidation.

Table 1 summarizes the results of the descriptive statistical analysis for the numerical parameters tem-

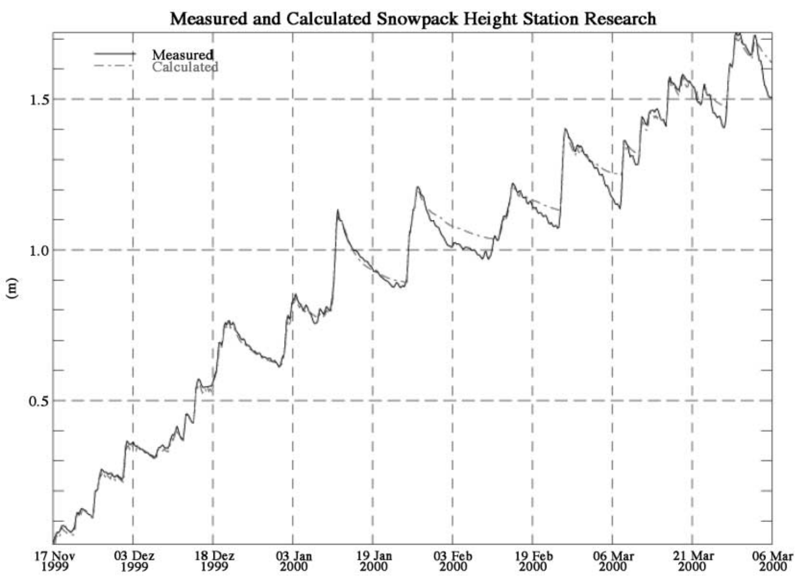

Fig. 4. Output from the SNOWPACK graphical user interface presenting modeled and measured snowpack depth over the course of the winter.

Table 1
Statistical measures comparing the predicted and observed snowpack parameters

	n	B	RMSE	r	E	d
Temperature	196	0.10 °C	0.97 °C	0.90	0.77	0.95
Density	177	−48.15 kg/m^3	69.15 kg/m^3	0.85	0.30	0.76
Grain size	179	−0.08 mm	0.42 mm	0.30	0.05	0.38

perature, density, and grain size. While there are no hard-and-fast rules for interpreting these measures, the utilization of several different statistical descriptors allows meaningful conclusions to be drawn regarding the performance of the SNOWPACK. Additionally, the analysis provides a consistent framework for comparing different versions of the model, or evaluation of the model using datasets from different seasons or geographical locations.

4.1. Temperature

By all statistical measures, the SNOWPACK model predicts the snow cover temperatures reasonably well. The RMSE is only 0.97 °C, and the mean bias B shows very little tendency toward over or underestimation. Since the temperature measurements are accurate to within ±0.5 °C, the RMSE demonstrates that the model does a good job of predicting temperature.

The measures of association r and d are nearly one and the coefficient of efficiency E is fairly high, suggesting a high degree of correlation between the observed snowpack temperatures and those predicted by the model. Referring to Fig. 5, the observed–predicted data pairs lie very close to the 1:1 line, which represents perfect agreement. It should also be noted from Fig. 5 that there are several instances where SNOWPACK predicts isothermal temperatures, but colder temperatures were measured. The graphs presented in Fig. 6 provide additional evidence that a close correlation exists between the predicted and measured temperatures, but reveal that model accuracy decreases slightly near the snowpack surface,

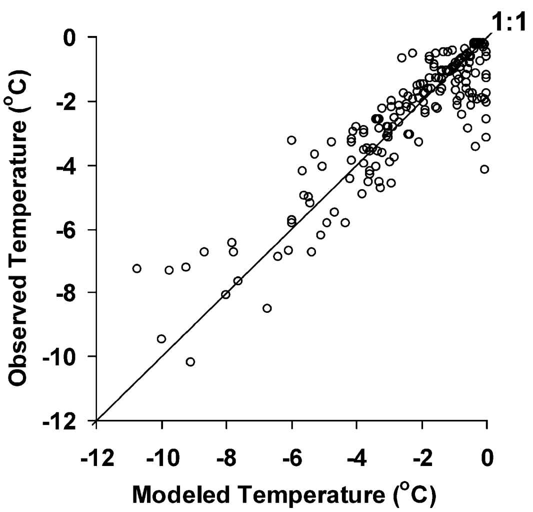

Fig. 5. Observed versus modeled temperature.

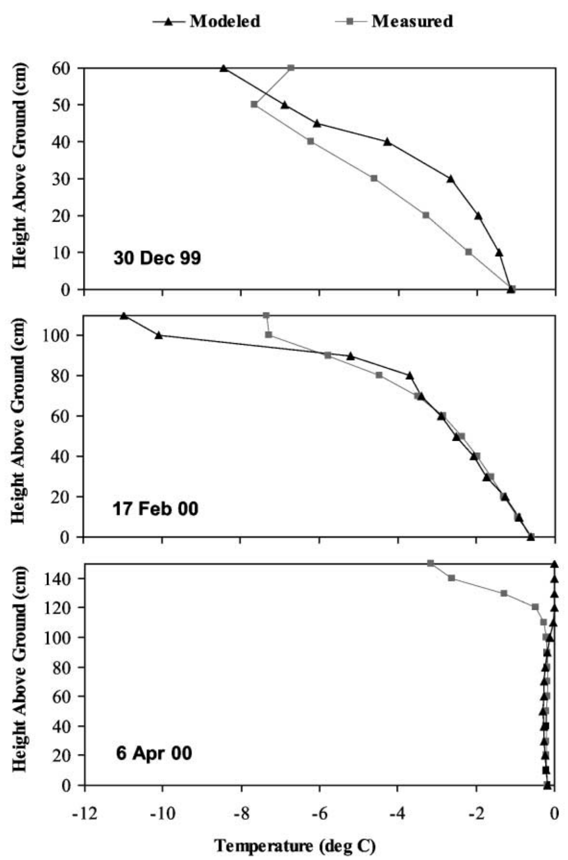

Fig. 6. Modeled and measured temperature versus snowpack depth for three different dates.

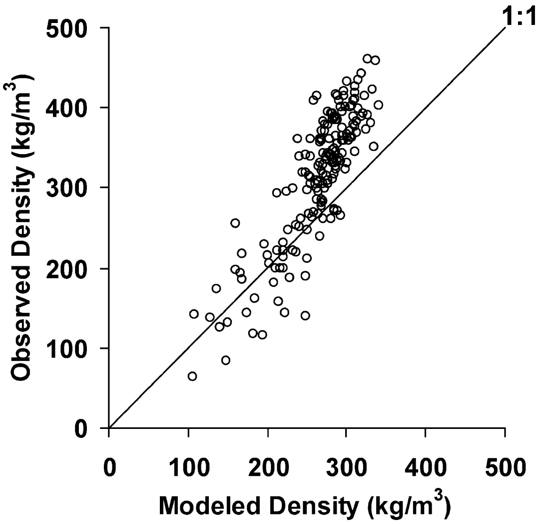

Fig. 7. Observed versus modeled density.

where often there is a greater variation of temperature over time.

4.2. Density

It is apparent from the RMSE of 69.15 kg/m^3 that SNOWPACK has some difficulty predicting the snow cover density. A mean bias B of -48.15 kg/m^3 confirms that the majority of the error present in the model's prediction is due to a consistent underestimation of the measured snowpack density. Fig. 7 illustrates that the observed–predicted data pairs are somewhat centered on the 1:1 line until the observed density approaches 250 kg/m^3. For densities greater than 250 kg/m^3, the data diverges markedly from the 1:1 line as the model increasingly underpredicts the measured density.

Despite the discrepancy in the magnitudes of the predicted and observed density, the correlation measures yielded more positive results. The coefficient of efficiency E is fairly low at 0.30, but r and d are fairly high at 0.85 and 0.76, respectively. The high r shows that there is a tendency toward a linear relationship; however, Fig. 7 reveals that the linear relationship does not follow the 1:1 line. This means that the general theory for densification may be sound, but that an adjustment of parameters may be all that is needed to improve the accuracy of density prediction.

4.3. Grain size

The comparison of predicted and observed grain size was problematic and does not supply conclusive information about the accuracy of the model. Most of the statistical measures for grain size given in Table 1 give poor results. The RMSE is large at 0.42 mm, but shows little or no bias with B equaling -0.08 mm. The statistical association scores r, E, and d are well below acceptable values.

The problem in this comparison lies partly in the differing definitions of grain size used by the model and the field observer. The model chooses 0.6 mm as the size of new snow particles, and allows only the growth of these grains. As a result, it is evident from Fig. 8 that SNOWPACK never predicts a grain diameter less than 0.6 mm, but on several occasions, grain sizes less than this were reported in the field observations. Furthermore, grain sizes greater than 1 mm are seldom predicted by the model, but were routinely observed in the field. SNOWPACK uses a grain diameter that is independent of the crystallography; essentially, all grains are treated as spheres. This is not true of the field observer, whose estimation of grain size is often tied to the shape of the snow grain, especially for faceted crystals. The esti-

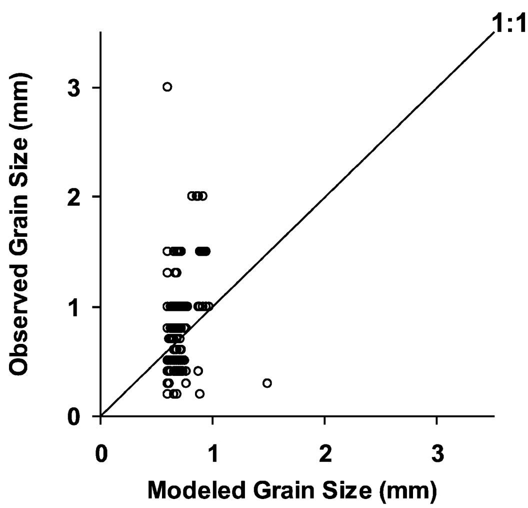

Fig. 8. Observed versus modeled grain size.

Table 2
Statistical association measures for the majority and minority grain types

	n	V	C^*
Majority (F1)	206	0.41	0.71
Minority (F2)	206	0.34	0.66

mation of grain size in the field is unavoidably a subjective measurement. Despite these contrasting definitions of grain size and the possibility of observer error, the low statistical measures suggest the need for model refinement in this area.

4.4. Grain type

Cramer's V for both F1 and F2 are both low and indicate only a weak correlation between the predicted and observed values (Table 2). The adjusted contingency coefficients C^* of 0.71 and 0.66 suggest a somewhat larger amount of association is present, but the relationship is still not very strong.

Similar to grain size, SNOWPACK uses an alternate grain classification scheme, namely, dendricity and sphericity. Since there is little basis for estimating these parameters during field observation, the model must choose a standard ISCI grain shape according to various combinations of dendricity and sphericity. A more desirable technique would be to develop a common classification system employed by both the SNOWPACK model and the observer. Another alternative is to focus less on the crystallography, and more on the microstructural parameters that actually define the physical properties of the snow. Crystal type is observed in the field primarily as an indicator of the degree of bonding and strength possessed by the snow. For instance, when rounded grains are encountered, a high degree of bonding and strength is assumed; the opposite is true when faceted crystals are observed. If a reliable method was developed for measuring bond size, and perhaps bond density, the classification of grain shape may be less important.

5. Conclusion

From 17 November 1999 to 6 April 2000, meteorological data were collected from a mountain weather station adjacent to Bridger Bowl Ski Area near Bozeman, MT. During the same period, full snowpack profiles were performed on a weekly basis within a short distance from the weather station. By running the SNOWPACK model using the collected weather data and comparing the output to the snow profiles, a thorough evaluation of the predictive capabilities of the model was possible. Statistical tests were utilized to make the comparison objective and consistent.

The statistical measures utilized in the analysis indicate that the SNOWPACK model predicts the temperature profile within the snow cover fairly accurately. However, inspection of the plot of predicted and measured temperature reveals a slightly diminishing model accuracy with colder temperatures, and additional difficulty when the modeled temperatures approach 0 °C. The ability of the model to effectively simulate snowpack temperature is crucial since most snowpack processes are strongly temperature dependent.

Though snowpack density is predicted less successfully than temperature, two of the three measures of statistical correlation give reasonable values. The data also demonstrates that the model significantly underpredicts the actual snowpack density when the densities exceeds about 250 kg/m^3. Still, our results provide information that might be useful for improving density prediction in future versions of the model.

A meaningful comparison of predicted and observed grain size is difficult due to different definitions of grain size and the subjectivity of human measurement. The statistical measures gave generally poor results and indicate little correlation between the simulated and observed values. Therefore, the results of the comparison indicate not only the need to improve the model, but also the utility of a more standardized observation technique using a definition of grain size similar to that employed by SNOWPACK.

Since snow crystallography is not measured on a continuous or numeric scale, the use of alternative statistical measures is required. The results of the statistical association measures demonstrate a weak correlation between the modeled and observed grain types. Like grain size, crystal shape is a subjective observation; therefore developing a classification scheme that can be both utilized by the model and accurately measured in the field would be advantageous.

This analysis does not evaluate the model's prediction of surface hoar since we inactivated the routine during model execution. This portion of the model is still under development (Lehning et al., 1998) and reportedly overpredicts surface hoar occurrence (Pielmeier et al., 2000). Another area of the model needing improvement is wet snow metamorphism. Currently, SNOWPACK has only rudimentary provisions for simulating wet snow metamorphism. Since this routine influences the grain size, crystal shape, and density, its effects are included in the analysis. Future work on the wet snow capabilities of SNOWPACK will be important for ablation prediction, hydrological purposes, and for applying the model to warmer, maritime climates.

In its present form, the SNOWPACK model can become a useful tool for avalanche forecasters and other practitioners who need to know the properties and structure of the snow cover, but do not always have the ability to conduct frequent snow profiles in a given location. Of course, users of the model must understand the limitations of the current version of SNOWPACK. Future model improvements and validation should increase the overall accuracy of SNOWPACK and its usefulness as a tool for snow practitioners.

References

Adams, E.E., Sato, A., 1993. Model for effective thermal conductivity of a dry snow cover composed of uniform ice spheres. Ann. Glaciol. 18, 300–304.

Agresti, A., 1996. Introduction to Categorical Data Analysis. Wiley, New York.

Bartelt, P., Lehning, M., Brown, R.L., 1999. A physical snowpack model for the Swiss National Avalanche Warning Service. Part I: Numerical model. Unpublished manuscript.

Box, G.E.P., Hunter, W.G., Hunter, J.S., 1978. Statistics for Experimenters. Wiley, New York.

Brun, E.P., David, P., Sudul, M., Brugnot, G., 1992. A numerical model to simulate snowcover stratigraphy for operation avalanche forecasting. J. Glaciol. 38 (128), 13–22.

Colbeck, S.C., Akitaya, E., Armstrong, R., Gubler, H., Lafeuille, K., Leid, D., McClung, D., Morris, E., 1990. The international classification for seasonal snow on the ground. Int. Comm. Snow Ice, 23 pp.

Fox, D.G., 1981. Judging air quality model performance: a summary of the AMS workshop on dispersion model performance. Bull., Am. Meteorol. Soc. 62, 599–609.

Imam, B., 1994. Non-linear uncertainty analysis for multiple criteria natural resource decision support system. PhD Dissertation, University of Arizona.

Imam, B., Sorooshian, S., Mayr, T., Schaap, M., Wosten, H., Scholes, B., 1999. Comparison of pedotransfer functions to compute water holding capacity using the van Genuchten model in inorganic soils. IGBP-DIS Working Paper #22.

Jordan, R., 1991. A one-dimensional temperature model for a snow cover. CRREL Report 91-16.

Lehning, M., Bartelt, P., Brown, R.L., Russi, T., Stockli, U., Zimmerli, M., 1998. A network of automatic weather and snow stations and supplementary model calculations providing snowpack information for avalanche warning. Proc. 1998 International Snow Sci. Workshop, Sun River, Oregon, pp. 225–233.

Lehning, M., Brown, R.L., Bartlet, P., Fierz, C., Satyawali, P., 1999. A physical snowpack model for the Swiss Avalanche Warning Service. Part II: Snow microstructure, meteorological boundary conditions, and applications. Unpublished manuscript.

McClung, D., Schaerer, P., 1993. The Avalanche Handbook. The Mountaineers Press, Washington.

Nash, J.E., Suttcliffe, J.V., 1970. River flow forecasting through conceptual models, Part I—A discussion of principles. J. Hydrol. 10, 282–290.

Pielmeier, C., Schneebeli, M., Stucki, T., 2000. Snow texture: a comparison of empirical versus simulated texture index for alpine snow. Ann. Glaciol. 32, in press.

Sorooshian, S., Gupta, V.K., Fulton, J.L., 1983. Evaluation of maximum likelihood parameter estimation techniques for conceptual rainfall runoff models: influence of calibration data variability and length on model credibility. Water Resour. Res. 19 (1), 251–259.

Willmott, C.J., 1981. On the validation of models. Phys. Geogr. 2, 184–194.

Willmott, C.J., Wicks, D.E., 1980. An empirical method for the spatial interpolation of monthly precipitation within California. Phys. Geogr. 1, 59–73.