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Power-laws and snow avalanches

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[1] This paper presents evidence of frequency-size power-laws in several groups of snow avalanche paths. Other natural hazards, such as earthquakes and forest fires, exhibit similar power-law relationships. In addition, an analysis of the response of one group of snow avalanche paths to storms through time demonstrates a power-law between the response of the system and the binned frequency of those responses. Our results, as well as our experience with these complex, non-linear systems, are consistent with self-organized criticality. The practical implication of this work is that the frequency-size relationship for small and medium sized avalanches may be useful for quantifying the risk of large snow avalanches within a group of avalanche paths.

INDEX TERMS: 1863 Hydrology: Snow and ice (1827); 3220 Mathematical Geophysics: Nonlinear dynamics; 3299 Mathematical Geophysics: General or miscellaneous

1. Introduction

[2] This research presents evidence of power-laws in several groups of snow avalanche paths. Other natural hazards, such as earthquakes [*Gutenberg and Richter*, 1956] and forest fires [*Malamud et al.*, 1998], as well as a number of computer models simulating natural hazards [e.g., *Olami et al.*, 1992; *Drossel and Schwabl*, 1992; *Hergarten and Neugebauer*, 1998], show robust power-law frequency-size distributions consistent with self-organized critical behavior. The sandpile model was the first to demonstrate self-organized criticality [*Bak et al.*, 1987, 1988]. Work on this model discussed "avalanches" through the system, and subsequent papers implied that these concepts might be applicable to "snow piles" and snow avalanches [*Bak and Chen*, 1991]. However, the release mechanism for snow avalanches that most commonly endanger humans is quite different from the granular dynamics of sandpiles, occurring when relatively cohesive slabs (or sheets) of snow fracture in shear along underlying weak layers [*McClung and Schaerer*, 1993; see *Schweizer*, 1999 for a review]. Despite this difference, our investigations indicate that groups of snow avalanche paths exhibit power laws. Though several mechanisms are known to produce such power-laws [*Sornette*, 2000], our results and observations are consistent with self-organized criticality.

[3] Snow avalanches are triggered when the seasonal snowpack, formed under the influence of complex and dynamic physical processes, is disturbed by a stress-inducing perturbation (such as a winter storm) that exceeds a time- and space-specific stability threshold. Since the natural snowpack exists near its triple point, stress-strength relationships can change rapidly through time. In the absence of disturbances the snowpack typically stabilizes into a "non-critical" state. However, when the system is sufficiently perturbed, usually with new or wind-blown snow, our experience is that it becomes "critical" and exhibits self-organized critical behavior. This "critical" state is commonly referred to as conditional stability, and is evidenced by the fact that the size of a particular disturbance to the system may be a poor predictor of the system's response. Large winter storms frequently produce extensive avalanching, but so do some small storms; small storms usually produce no avalanches, but some large storms also produce no avalanches. Further, large storms often produce large avalanches in some avalanche paths, but many nearby paths may yield no avalanches [*Mears*, 1992]. Such system responses of unpredictable (and sometimes massive) magnitude to incremental perturbations are characteristic of self-organized criticality [*Bak et al.*, 1988].

[4] Systems exhibiting self-organized critical behavior tend toward a critical state where the distribution of event sizes can be represented by a log-linear relationship. In addition, the temporal behavior of these systems is characterized by $1/f$ noise. For this research, we cannot strictly analyze $1/f$ noise because our data are not a continuous time series, but instead represent different winter seasons and irregularly spaced storms. Still, our results suggest that the response of the snow avalanche system to the perturbation of a storm can be described with a power-law.

2. Data

[5] This investigation utilizes two unique and long-term sets of observations of naturally occurring avalanches and associated weather data, as well as a third long-term avalanche dataset from a ski area that consists primarily of avalanches artificially triggered with explosives. Each dataset is from groups of closely associated avalanche paths in relatively small geographic areas (about 10 km^2). The first dataset represents 21 seasons of observations (1974/75 through 1994/95) of naturally occurring avalanches from the Gothic vicinity of the East River valley in the Elk Mountains of Colorado, with over 3,000 avalanches from 32 routinely observed avalanche paths. Our second data set is from Jackson Hole Ski Area in Wyoming and consists primarily of artificially triggered avalanches, where the timing, frequency, and magnitude of the avalanches released is strongly influenced by human decisions and triggering methods. The Jackson Hole data cover over 7,000 individual avalanches from more than 100 avalanche paths during the winters from 1969/70 to 1994/95. Like the Gothic data, our final dataset consists entirely of natural avalanche observations. This smaller dataset represents six winter seasons (1992/93 through 1997/98) in the lower Yule Creek valley of the Elk Mountain Range of the central Colorado Rocky Mountains and includes over 300 individual snow avalanche events from 29 routinely observed avalanche paths. In addition to frequency-magnitude data, the Yule Creek dataset is unique in that it describes the "response" of the system for each system "perturbation." In our case the "response" is indexed as the percentage of the maximum possible response (maximum-sized avalanches in each avalanche path within the group of paths) for a given group of closely associated avalanche events, commonly called an avalanche cycle. The system "perturbation" is a pre-defined winter storm that must have a minimum precipitation total of 12 mm snow water equivalent so that the system approaches the conditional stability threshold that we have observed to be associated with self-organized critical behavior.

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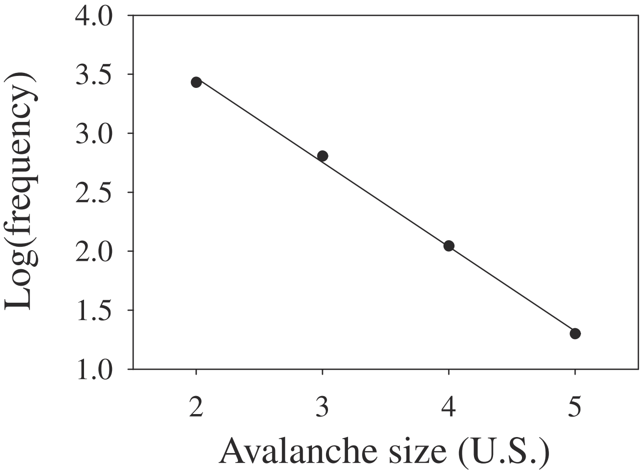

Figure 1. Log of the non-cumulative snow avalanche frequency vs size (using the U.S. classification) for 3,093 individual natural avalanche events at Gothic, Colorado. The slope of the least-squares best-fit line, α, is -0.74.

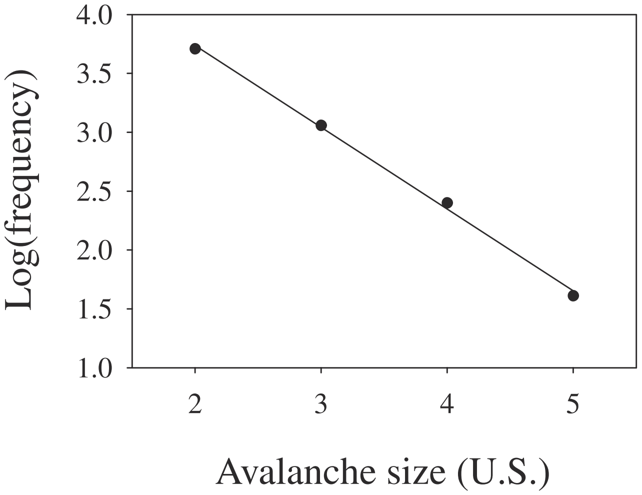

Figure 2. Log of the non-cumulative snow avalanche frequency vs size (using the U.S. classification) for 7,009 individual avalanche events at Jackson Hole Ski Area, Wyoming. Most of the avalanches in this dataset were triggered with explosives during avalanche hazard reduction work at the ski area. The slope of the least-squares best-fit line, α, is -0.70.

Finally, single observers made all the observations from Gothic and Yule Creek, helping to ensure data consistency.

[6] Our datasets classify avalanche magnitudes according to the U.S. size scale, which utilizes five categories to classify avalanches relative to the avalanche path in question [*McClung and Schaerer*, 1993]. We considered only size 2 (small relative to the path) through size 5 (maximum) avalanches, since size 1 avalanches are so small they are typically not recorded. Though the classification system yields somewhat subjective data, our observations, as well as the observations of others [e.g., *LaChapelle*, 2002; *Mears*, 2001], indicate that the U.S. classification system provides an approximate order-of-magnitude scale in terms of the mass of snow moved for a given path. While avalanche size data would ideally be in mass, calculations of avalanche mass are problematic, involve numerous variables, and no reliable method exists for converting U.S. avalanche size to mass for a particular event in a given path [*Sovilla et al.*, 2001].

3. Results and Discussion

[7] Non-cumulative frequency-size relationships for Gothic show a strong log-linear relationship, with the slope of the best-fit line, α, equal to -0.74 (Figure 1). The dataset for Jackson Hole shows a similar non-cumulative frequency-size log-linear relationship (Figure 2). Surprisingly, Jackson Hole's α of -0.70 is comparable to Gothic. Since a primary goal of avalanche hazard reduction is to pre-empt large avalanches by artificially triggering smaller ones, α should theoretically decrease (i.e., the best-fit line should be steeper) for datasets of triggered avalanches. A similar analysis of two other datasets of artificially triggered avalanches also shows log-linear frequency-size relationships, with $\alpha = -0.84$ for 27 years of data from Bridger Bowl Ski Area in Montana and $\alpha = -0.54$ for 26 years of data from Snowbird Ski Area in Utah. Further investigations are clearly necessary to discover the range of, and controls on, α for natural and triggered avalanche datasets.

[8] The Yule Creek data do not show the same clear frequency-size log-linear relationship as the other data we analyzed, perhaps because its smaller size (300 versus 3,000 to 7,000 individual avalanche events in the other datasets) did not allow for a full range of conditions to be exhibited in the data. However, a sequential graph of numbered storms (“perturbations”) and their associated avalanche index scores (“responses”) for these data (n = 104 storms) shows how some storms result in large system responses while others result in no response (Figure 3). This analysis only utilized storms capable of producing dry slab avalanches. Storms had to have at least 12 mm of snow water equivalent so the snowpack approached the critical conditional stability threshold, and we did not consider either early-season storms when the snowpack was too shallow to cover terrain anchors that would preclude avalanches or spring storms dominated by wet snow avalanche activity. Avalanche index scores are plotted against irregularly spaced storms over several seasons rather than time, as would be done for an analysis of $1/f$ noise. Still, it is noteworthy that the distribution of avalanche system responses to these storm perturbations results in a power-law between the log of the storm

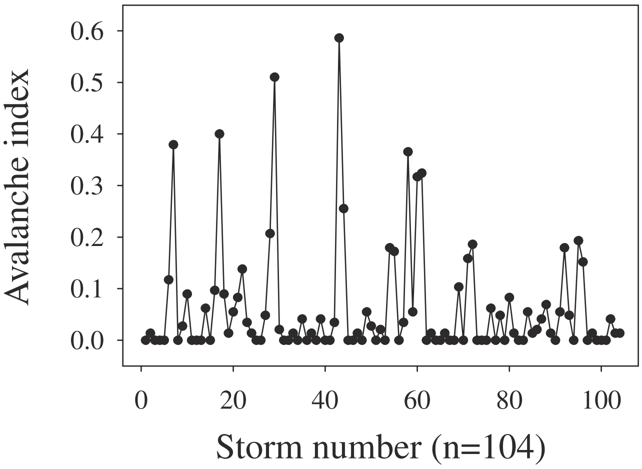

Figure 3. A sequential graph of 104 numbered storms from the 1992/93 through the 1997/98 winter seasons in Yule Creek, Colorado vs storm avalanche index scores. Each individual storm constitutes a system “perturbation,” and the avalanche index represents the system “response” to the storm as the percentage response of the system (a value of 1 would indicate that every avalanche path ran to its maximum extent).

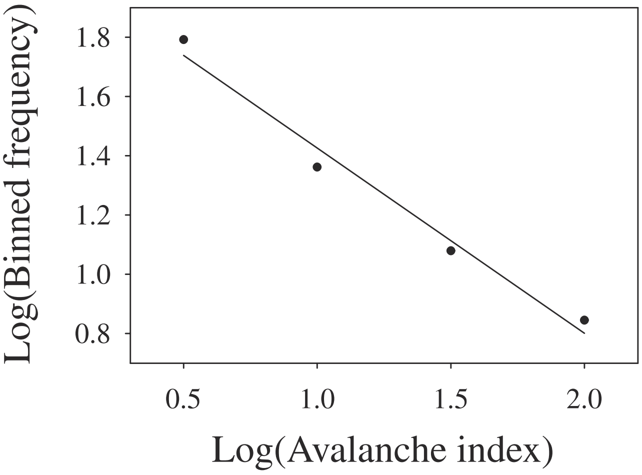

Figure 4. A power-law exists between the storm avalanche index and the log of the binned frequency related to those avalanche indices in the Yule Creek system. The slope of the least-squares best-fit line, α, is -0.63.

avalanche index and the log of the binned frequency related to those avalanche indices (Figure 4).

4. Summary

[9] Our results have several implications. First, they provide a new theoretical context for the study of snow avalanches. Despite the numerous complexities involved in snow avalanche formation and release, they show scale invariant, power-law frequency-size relationships consistent with self-organized critical behavior. In addition, at least one group of snow avalanche paths demonstrates another power-law associated with the response of the system to storm perturbations. Second, surprisingly similar power-law frequency-size relationships are exhibited both in datasets consisting entirely of natural avalanches and in datasets of avalanches triggered with explosives during avalanche hazard reduction work. However, more work is needed to better understand the controls on, and full range of, α in snow avalanches. Finally, the practical implication of our work is that the frequency-size relationship for small and medium sized avalanches may be useful for quantifying the risk of large snow avalanches within a group of avalanche paths.

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References

Bak, P., C. Tang, and K. Wiesenfeld, Self-organized criticality: An explanation of 1/*f* noise, *Phys. Rev. Lett.*, *59*, 381–384, 1987.
Bak, P., C. Tang, and K. Wiesenfeld, Self-organized criticality, *Phys. Rev. A*, *38*, 364–374, 1988.
Bak, P., and K. Chen, Self-organized criticality, *Sci. Am.*, *264*, 46–53, 1991.
Drossel, B., and F. Schwabl, Self-organized critical forest-fire model, *Phys. Rev. Lett.*, *69*, 1629–1632, 1992.
Gutenberg, B., and C. F. Richter, Earthquake magnitude, intensity, energy, and acceleration, *Bull. Seismol. Soc. Amer.*, *34*, 105–145, 1956.
Hergarten, S., and H. J. Neugebauer, Self-organized criticality in a landslide model, *Geophys. Res. Lett.*, *25*, 801–804, 1998.
LaChapelle, E. R., Personal communication. Avalanche consultant, McCarthy, Alaska, 2002.
Lay, T., and T. C. Wallace, *Modern global seismology*, Academic Press, New York, 1995.
Malamud, B. D., G. Morein, and D. L. Turcotte, Forest fires: An example of self-organized critical behavior, *Science*, *281*(5384), 1840–1842, 1998.
McClung, D., and P. Schaerer, *The avalanche handbook*, The Mountaineers, Seattle, 1993.
Mears, A. I., Snow-Avalanche Hazard Analysis for Land-Use Planning and Engineering, *Colo. Geol. Surv. Bull. 49*, Colo. Dept. of Nat. Res., Denver, Colorado, 1992.
Mears, A. I., Personal communication. Avalanche hazard consultant, Gothic, Colorado, 2001.
Olami, Z., H. J. S. Feder, and K. Christensen, Self-organized criticality in a continuous, nonconservative cellular automaton modeling earthquakes, *Phys. Rev. Lett.*, *68*, 1244–1247, 1992.
Schweizer, J., Review of dry snow slab avalanche release, *Cold Reg. Sci. and Tech.*, *30*, 43–58, 1999.
Sornette, D., *Critical phenomena in natural sciences*, Springer, Berlin, 2000.
Sovilla, B., F. Sommavilla, and A. Tomaselli, Measurements of mass balance in dense snow avalanche events, *Ann. Glac.*, *32*, 328–332, 2001.

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