

RELATING SPATIAL VARIABILITY TO SNOW STABILITY USING A CELLULAR AUTOMATON MODEL INITIALIZED WITH FIELD DATA

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ABSTRACT: This research uses a two-dimensional cellular automaton model, with inputs taken from field data, to mimic snow slab avalanche release. The initial weak layer shear strength in the model follows a normal distribution with known mean and standard deviation. For each realization of the spatial distribution of weak layer shear strength, the model stresses all cells equally until the weakest cell fractures. The stress from the fractured cell is transferred to cells in the neighborhood of the fractured cell, possibly causing a propagation of the fracture and a model avalanche. The stochastic shear strength field makes the model avalanche size stochastic for statistically constant initial conditions. The standard deviation of shear strength strongly affects the proportion of model avalanches covering nearly all cells in the model, with low variability leading to a high proportion of large model avalanches, a result that supports previous conceptual models. Stress transfer properties in the model are also important for the proportion of large model avalanches, but no field data exist to constrain these values. Spatial auto-correlation of shear strength is likely to be important for the size of model avalanches, and will be built into a future version of the model.

KEYWORDS: stochastic, avalanche release, shear strength, shear stress, fracture propagation

1. INTRODUCTION

Numerous studies over the past two decades show that spatial variability in snow stability and snow structure exists at the slope scale (Conway and Abrahamson, 1984; Birkeland and others, 1995), and recent work quantifies those patterns (Kronholm and Schweizer, 2003; Logan and others, 2004; Kronholm and others, in press). However, the connection between spatial variability and the stability of a particular slope is still poorly understood, although Kronholm and Schweizer (2003) recently put forth a qualitative hypothesis. Knowledge of this relationship is critical for those assessing avalanche danger, but testing such hypotheses in the field is difficult due to the destructive sampling of slopes for spatial variability measurements and the hazards associated with sampling unstable slopes.

Numerical experiments such as cellular automaton (CA) models are one way to overcome these difficulties and tie well into the stochastic approach suggested by Schweizer (1999). CA models involve a number of discrete cells which interact according a set of rules applied iteratively until a certain condition (e.g. fracture of all cells in

the model) is met. Recently, CA models have been used to investigate avalanche release processes (Faillettaz and others, 2002; Zaiser, in press). However, these studies differ from approach presented here, since they were not initialized with field data, and they did not focus on the possible relations between snow cover spatial variability and stability.

In this paper we use a simple two-dimensional CA model to mimic dry snow slab release processes. Our goal is not to model all the details of how avalanches release, but to use this model to investigate how the size of model avalanches change with changing spatial variability of weak layer strength and changing slab properties. The model inputs are based on results from field measurements. We find that the standard deviation of the distribution plays a very significant role for the release probability of large model avalanches, with larger standard deviations leading to fewer large model avalanches. The model is also quite sensitive to the slab properties which control the stress transfer away from fractured areas of a slope.

2. THE MODEL

The two-dimensional CA model is similar to the Burridge – Knopoff model described by Ferguson and others (1998) for modeling earthquakes. For our purposes, the model can be thought of as two layers where the upper is the

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slab and the lower is the buried weak layer. The model consisted of 100 x 100 cells denoted by the coordinates (x, y) . Each cell was assigned a initial shear stress value $\tau_{x,y}^{init}$ and a shear strength value $\Sigma_{x,y}$ so that its static stability could be calculated as $S_{x,y} = \Sigma_{x,y} / \tau_{x,y}^{init}$. The initial shear strength values were stochastic and their simulation is described in detail below. In this paper we used a globally constant initial stress field, such that $\tau_{x,y}^{init}$ was constant for all (x, y) and $\tau_{x,y}^{init} = \min(\Sigma_{x,y})$. This initial setup ensured that the static stability $S_{x,y}$ of the weakest cell was 1, causing it to fracture. After fracture of a cell the stress on that cell was transferred to any non-fractured neighboring cells within a certain distance thus possibly causing other cells to fracture as described in detail below.

2.1. Stress transfer

When the static stability of a cell at (i, j) , $S_{i,j} \leq 1$, the stress on the fractured cell was transferred to its neighbors without loss, except at the model boundaries where stress could 'leak' because all cells outside the model boundaries were assumed to be non-fractured. Stress was transferred equally well in the cross-slope direction (x) as in the up/down-slope direction (y).

All non-fractured cells within a distance D of the fractured cell (called the neighborhood) received a fraction of the stress. Thus, a stress transfer distance of $D = 1$ gives 4 neighbors while $D = 1.5$ gives 8 neighbors. In this paper we investigate how the model avalanche size changes when changing D . Only non-fractured cells in the neighborhood received transferred stress. In case the fractured cell did not have any non-fractured neighbors within the neighborhood, no stress was transferred away from the fractured cell.

The fraction of stress received by a cell from a fractured neighbor was taken to be a function of the inverse distance between the cells to a power P . In this paper we investigate how P affects the proportion of large avalanches. Applications of CA models in earthquake research generally use $P = 3$ (e.g. Ferguson and others, 1998). Cells in the neighborhood of (i, j) were stressed by the transferred stress $\tau_{x,y}^{trans}$ added to the initial shear stress $\tau_{x,y}^{init}$.

The two parameters D and P represent the stiffness of the slab in the model.

2.2. Initial shear strength

The marginal statistical distribution of weak layer shear strength values in the field has been investigated most thoroughly by Jamieson and Johnston (2001). They report on 28 data sets, each with 30 to 38 shear frame tests. Depending on the goodness-of-fit test used, they found that 20 to 24 of the 28 data sets were normally distributed. They did not comment on the spatial variation of shear strength. Their values vary widely from 219 to nearly 6000 Pa, with a mean close to 2000 Pa. Föhn (1987) also found normally distributed shear frame results. Our own measurements with sets of more than 60 shear frame tests are also described reasonably well by a normal distribution (Logan and others, 2004). Based on these findings, all numerical experiments were made with a normally distributed marginal strength field with a specified mean $\bar{\Sigma} = 1500$ Pa and standard deviation σ_{Σ} (Figure 1a). In the model runs for this paper we did not introduce spatial auto-correlation in the initial shear strength field, so the shear strengths were completely randomly distributed over the model cells (Figure 1b).

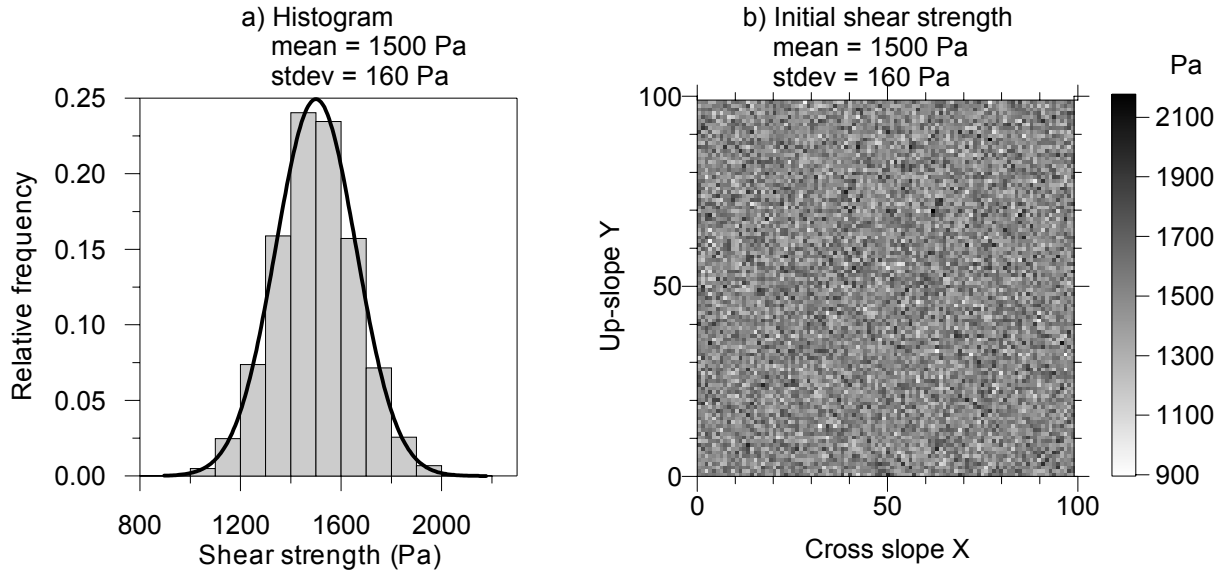


Figure 1: a) Histogram of the theoretical (line) and actual (bars) initial shear strength distribution for one model realization. b) Spatial distribution of the initial shear strength for the realization shown in a).

2.3. Numerical experiments

The model was run a number of times, each time involving a new realization of the initial strength and stress fields. Each realization resulted in one avalanche which involved from 1 (the initial fracture) to all 10^4 cells in the model. We refer to the number of cells that fractured during a realization as the model avalanche size A .

A fracture spreads by propagating away from an initial fracture through its neighbor cells. The spatial distribution of shear strength values in the model cells, and not just the marginal distribution (non-spatial) is therefore important for A . Because the spatial distribution of the initial shear strength was stochastic, so was A . In order to investigate the model avalanche size distribution (and especially the non-frequent extreme outcomes) for varying standard deviations

in the initial shear strength field, each model run consisted of 10^6 realizations of the strength and the stress fields with the same shear strength standard deviation. For each realization we recorded a) the model avalanche size A and b) the mean stability \bar{S} of all cells in the model at the time of the initial fracture.

To test the sensitivity of the model to changing some of the input parameters, we ran three different numerical experiments (Table 1). In the first experiment we changed the standard deviation of the shear strength distribution and investigated how A changed. In experiment 2 the effect of the distance D over which the stress was transferred was studied. In the last experiment we investigated the effect of changing the decay power P on A .

Table 1. Specifications for the numerical experiments. All runs in an experiment included 10^6 realizations on 100 x 100 cells. Values marked in **bold** were varied through the experiments.

Experiment	Marginal distribution		Stress transfer	
	Mean $\bar{\Sigma}$ (Pa)	Standard deviation σ_{Σ} (Pa)	Distance D (units)	Decay power P
1	1500	100 – 175	1.5	3
2	1500	110	1 – 45	3
3	1500	120	3	1 – 16

3. RESULTS

The model avalanche sizes for all model runs showed a tendency to either remain below a certain threshold A_{crit} , typically around 25 cells, or to become large, spreading to most of the 10^4 cells in the model. Only few realizations in a model run produced model avalanche sizes between the threshold A_{crit} and the full model size. Two typical runs showing this behavior are shown in Figure 2. A linear log-log relationship covered the small range of model avalanche sizes between 1 and A_{crit} . Previous studies using CA models have shown similar relationships (Faillettaz and others, 2002; Zaiser, in press), and several authors have documented linear log-log size/frequency relationships for field measurements of avalanches (Birkeland and Landry, 2002; Faillettaz and others, 2002; Rosenthal and Elder, 2003).

Changing the standard deviation in experiment 1 primarily changed the proportion of large model avalanches while the threshold A_{crit}

did not vary much. We therefore investigated the effect on the proportion of 'large' avalanches that the model runs produced at the different standard deviations (Figure 3). All model avalanches that covered > 9500 cells (95%) in the model were considered 'large'. Changing the definition of 'large' to anywhere from 85% to 99% had no effect on the results.

The proportion of large avalanches appeared very sensitive to the standard deviation of the shear strength distribution, especially for low standard deviations (Figure 3a). In the model coefficients of variation of the shear strength distribution above 10% rarely produces large avalanches. The proportion of large model avalanches is also very sensitive to the stress transfer distance D (Figure 3b). For stress transfer distances above 3, the model rarely produces large avalanches with the settings used in experiment 2.

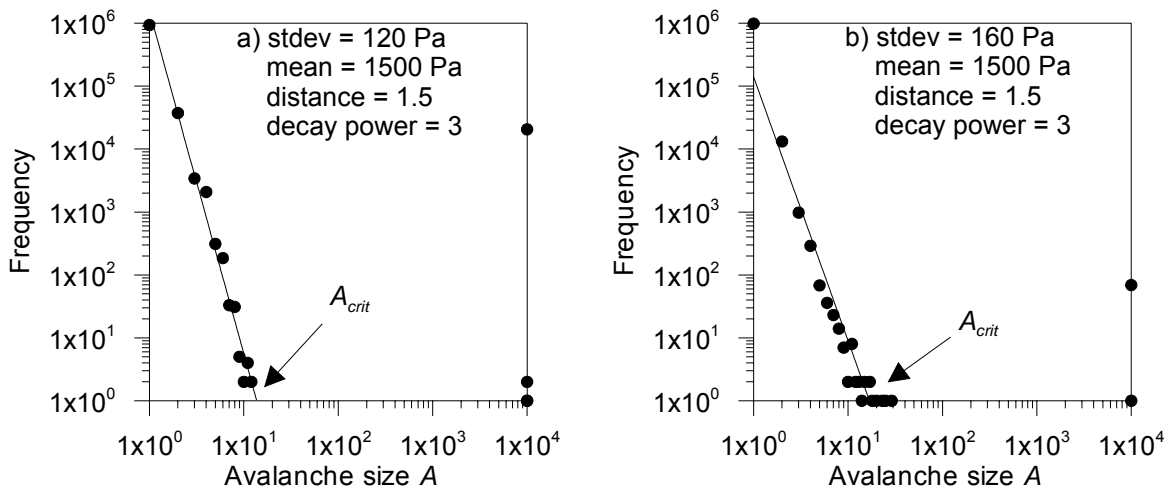


Figure 2: Log-log plots of the size distribution for model avalanches for typical runs of 10^6 realizations with a standard deviation of a) 120 and b) 160. The best fit linear log-log relationship is shown for $A \leq 25$.

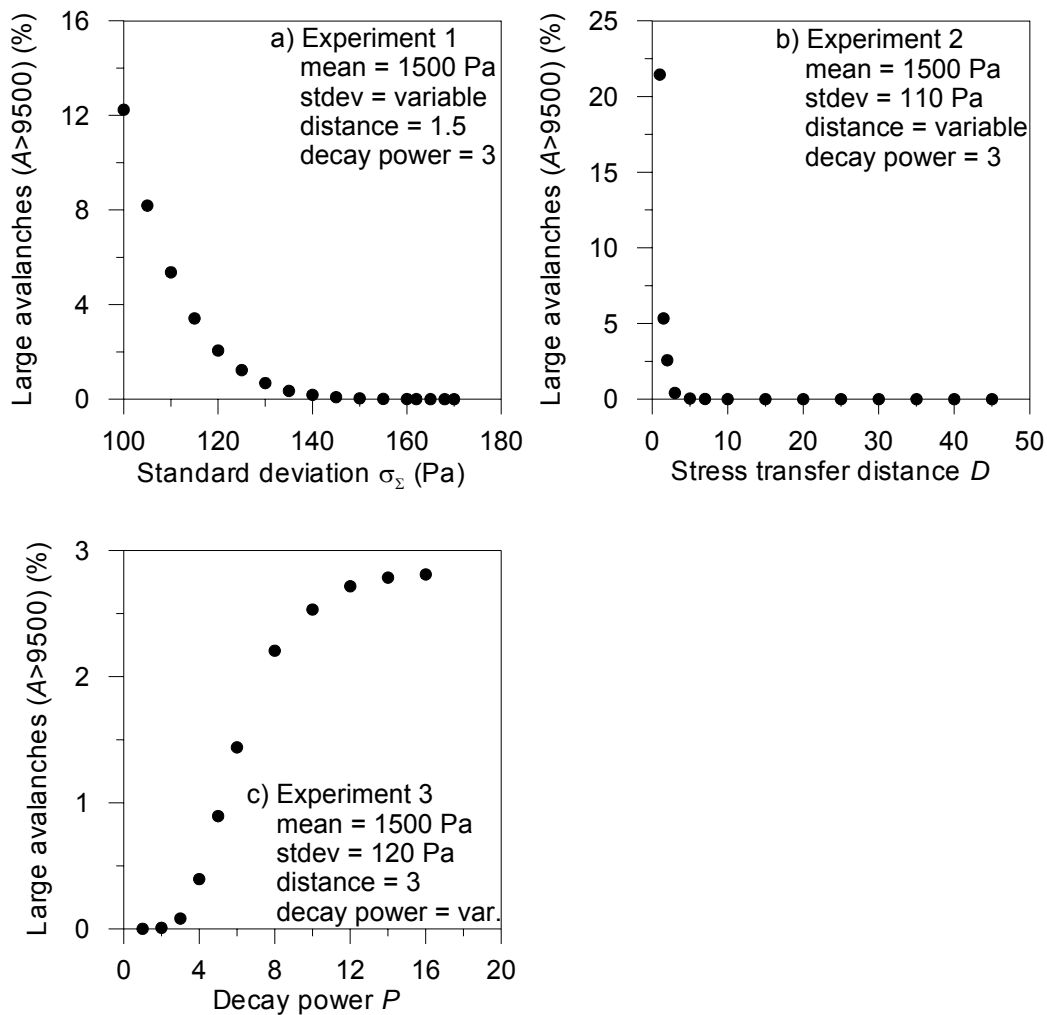


Figure 3: The proportion of large model avalanches ($> 95\%$ of the model cells) after changing a) the standard deviation of the shear strength distribution, b) the stress transfer distance and c) the decay power.

As the decay power P increases, the proportion of large model avalanches increases up to a maximum level that is approached asymptotically (Figure 3c). Increasing P causes the redistributed stress to be focused on a smaller neighborhood around a fractured cell, thus effectively decreasing the stress transfer distance D to 1 for high values of P .

A relationship exists between the mean model stability \bar{S} and the model avalanche size A (Figure 4). This reflects the distribution of shear

strength values; when the weakest shear strength in a realization is low, the initial shear stress in the model is also low. Large model avalanches generally happen when the lowest shear strength in the model is high, leading to a large loading of all cells in the model.

Figure 4 also shows that the mean model stability at the first cell fracture is always > 1 , suggesting that the mean stability of the model does not predict the stability of the model as a whole.

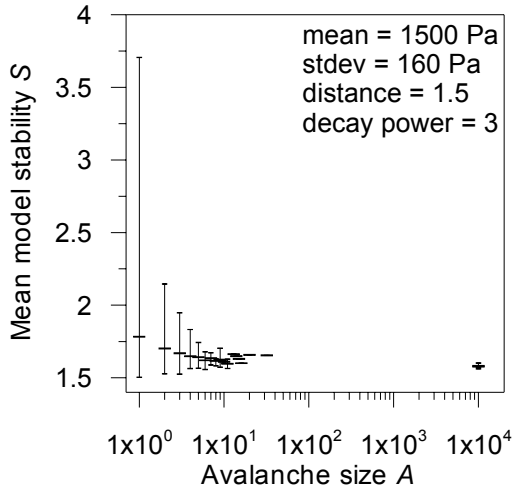


Figure 4: Mean (wide bar), maximum, and minimum (narrow bars) values of mean model stability at the first fracture for all avalanche sizes.

4. DISCUSSION

The stochastic initial shear strength values mean that the size of the model avalanches is also stochastic even for statistically constant initial conditions (Figure 2) as already noted by Faillettaz and others (2002). The stochastic strength values can cause large model avalanche to be released from weak cells although the mean stability of the model cells is well above the critical level (Figure 4), what Zaiser (in press) calls a 'knock-down' effect. This has been shown for landslides (Hergarten, 2002), and is also true for actual avalanche slopes; McClung (2002) emphasizes targeted sampling and searching for unstable conditions rather than attempting to define the mean stability of a slope when assessing stability, and our model results support this important point.

While the weakest cell in any model realization is important for the initial fracture, the strength of its neighbors and the rest of the cells (i.e. the spatial structure) control whether the initial fracture will propagate. When the standard deviation of the initial shear strength distribution is low, all cells in the model will be closer to fracture as the initial fracture takes place. The transferred stress from the initially fractured cell thus has a higher chance of causing large avalanches than for an initial distribution with a higher standard deviation, where the mean model stability will be higher (Figure 3a). Although the relationship between the model and real avalanche slopes is not straightforward, this result supports the hypothesis put forth by Kronholm and Schweizer (2003) that for a specific mean initial shear

strength the standard deviation is critically important for slope stability.

Most observed values of standard deviations or coefficients of variations (e.g. Jamieson and Johnston, 2001) are not likely to produce large avalanches if used as input in the CA model. Since large loads (e.g. avalanche researchers) cannot be added to critically unstable avalanche slopes without them failing, this supports the idea of a direct relation between the model and real avalanche slopes. However, more work is needed to investigate this relationship because the model is sensitive to many factors, including the size of the grid.

The spatial dimension of our model is scaled indirectly through the shear strength values which were measured with a 250 cm² shear frame. However, because the model scales the absolute value of the initial shear strength with an initial shear stress, only the coefficient of variation of the shear strength values is important, and not the absolute values. It is therefore not possible to relate the model size of 100 x 100 cells to a specific scale in nature.

Zaiser (in press) found that the maximum shear strength value was important for the release of large avalanches in his 1-dimensional model because the strong cells blocked a fracture from propagating past it. This is not the case in our 2-dimensional model because a spreading fracture can move around a single cell or an area of high shear strength. Such 'islands of safety' have been observed to be standing back on a slope after a slab release. To explain such observations a 2-dimensional model is necessary.

The implemented model was sensitive to the three parameters tested here: standard deviation of the initial shear strength σ_{Σ} , the stress transfer distance D and the decay power P (Figure 3). Running the model with the correct values of these parameters is therefore important. Typical values of σ_{Σ} and mean shear strength $\bar{\Sigma}$ were estimated from field measurements, but thus far only a few studies have attempted to measure the stress transfer properties of snow (e.g. Camponovo and Schweizer, 1997) and we are aware of no data to help determine the exact value of P and D . Another problem with the current implementation of the slab properties in the model is that they are spatially constant in the model, while field measurements show that slab properties as well as properties of individual layers in the slab may vary considerably over a slope (Conway and Abrahamson, 1984; Birkeland and others, in press; Kronholm and others, in press).

Spatial variations in slab properties must be accounted for in a more realistic model.

5. CONCLUSIONS

This paper presents a CA model that attempts to mimic some avalanche release processes. The model is used to explore how spatial variations in shear strength and changing slab properties might affect slope stability. The model demonstrates sensitivity to changes in the standard deviation of shear strength values and to changes in stress transfer properties of the slab. Lower shear strength standard deviations and stress transfers over shorter distances are more likely to produce large model avalanches. Though our model only approximates avalanche release processes, our results agree with previous conceptual models relating snow cover variations and slope stability. Finally, the spatial structure of the initial shear strength must be important for the spread of fractures in the model. To investigate this further, future work must simulate spatial auto-correlation in the initial shear strength values in the model.

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