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Reliability of sampling designs for spatial snow surveys

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Abstract

Spatial patterns are an inherent property of most naturally occurring materials at a large range of scales. To describe spatial patterns in the field, several observations are made according to a certain sampling design. The spatial structure can be described by the semivariogram range, and nugget and sill variances. We test how reliably seven sampling designs estimate these parameters for simulated spatial fields with predefined spatial structures using a Monte Carlo approach. Five designs have been used previously in the field for snow cover sampling, whereas two designs with semi-random sampling locations have not been used in the field. The designs include 84–159 sampling locations covering small mountain slopes typical of snow avalanche terrain. The results from the simulations show that all designs: (a) give reasonably unbiased estimates of the semivariogram parameters when averaged over many simulations, and (b) show considerable spread in the semivariogram parameter estimates, causing large uncertainty in the semivariogram estimates. Our results suggest that any comparisons of the estimated semivariogram parameters made with the sampling designs will be associated with large uncertainties. To remedy this, we suggest that optimal sampling designs for sampling slope scale snow cover parameters must include more sampling locations and a stratified randomized sampling design in the future.

Keywords: Random fields; Spatial prediction; Simulation; Geostatistics; Snow cover

1. Introduction

Spatial patterns are an inherent property of most naturally occurring materials at a large range of scales. In snow cover, such patterns are important for several processes that are driven by local, rather than average, properties (Colbeck, 1991). Understanding and modeling such processes requires an accurate description of the spatial variation found in the snow.

It has long been recognized that snow is not a spatially homogeneous material (Seligman, 1936). At the slope-scale, which is important for snow avalanche release, inhomogeneities parallel to the ground within snow layers are produced primarily by differences in snow depths, substrate, and the influence of wind (Sturm and Benson, 2004). Field surveys have been done to describe quantitatively the variation in snow at the slope scale (Conway and Abrahamson, 1984; Birkeland et al., 1995; Campbell and Jamieson, 2004; Kronholm et al., 2004; Landry et al., 2004; Logan et al., 2007). While

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recent advances in instrumentation have enabled field crews to obtain a large number of objective measurements compared to earlier studies, the destructive sampling and dynamic nature of the snow cover (snow properties can change significantly in a matter of hours) limits the number of sampling locations on a slope. The sample size has implications for the reliability of the experimental semivariogram and for the parameters of the fitted model, nugget and sill variances, and range of spatial dependence (Webster and Oliver, 1992). These three parameters describe the spatial structure of the data and must be known to predict values at unmeasured locations using kriging. To address this, previous field surveys used different nested sampling designs to optimize the reliability of the semivariogram. Optimization of the sampling design is done by changing the sample spacing, extent and support. The extent and spacing are defined by the maximum and minimum distance, respectively, between sampling sites, and the sample support is the integration volume of a sample. Optimization of sampling designs in a spatial setting has been addressed by a number of studies (e.g. Oliver and Webster, 1986).

Despite optimization of the sampling designs used in snow surveys, the accuracy of the semivariogram obtained is still a concern. In this study, we investigate how reliably different sampling designs estimate the semivariogram range, and nugget and sill variances. This information is important when deciding which sample design to use, and when comparing semivariogram estimates from different studies. First, we point out some of the general considerations specific to sampling designs for snow at the slope scale. We then use a Monte Carlo approach to investigate how reliably previously used sampling designs and two random designs resolve spatial patterns at the slope scale. Although our focus is on sampling designs used to describe spatial variability in snow cover studies, our approach is generally applicable to similar spatial studies.

2. Practical design limitations

Dry snow slab avalanches, which are the most dangerous snow avalanche type for skiers, are likely to release once a crack in a weak layer reaches 0.1–10 m in length (Schweizer, 1999). When concerned with snow stability, spatial variation within this range is, therefore, the most important to describe and spatial extent can be limited to <30 m. To reduce the avalanche hazard associated with snow stability investigations, slopes larger than 15–30 m should be avoided during unstable snow conditions. Further, measurements should be at least 0.5 m apart in a field survey to avoid sampling disturbed snow. Sample support is determined by the method used, and hence by the objective of the study. For example, if the aim is to study the spatial variation of point stability, a test method, which involves a significant volume of snow must be used. This could typically involve a method, which tests a surface area of 30×30 cm and a depth down to the critical weak layer. If the objective is to study the variation of strength of bonds between snow grains, the test method must involve a sample volume smaller than the scale of the bonds.

Reliable estimation of spatial structure from field surveys sometimes relies on re-sampling certain areas of the study site according to the results from a preliminary field survey (e.g. Oliver and Webster, 1987). The destructive nature of snow cover measurements makes returning to a site for undisturbed measurements impossible, and one set of samples must suffice to reveal the spatial structure. This makes optimal design of the sample scheme crucial.

Increased measurement density generally improves the reliability of the sample semivariogram (Webster and Oliver, 1992). For the sampling designs tested here, the investigators generally were able to make 100–150 measurements in a day. Making more measurements than this with available snow instruments is difficult, and for some instruments the number of possible measurements is considerably smaller. In slope-scale snow surveys, sample sites are much easier to locate if they are in a regular pattern and are spaced at regular intervals. Random sampling designs for field surveys are, therefore, less practical than regular grids. Finally, the sampling design must enable a description of potentially anisotropic spatial structures in two dimensions, which measurements along a single transect cannot do. In this study, we did not test sampling designs involving multiple transects (e.g. Oliver and Webster, 1987).

3. Methods

First, we decided on the spatial structure that the sampling designs had to reproduce. The structure was defined by the generating semivariogram range and nugget and sill variances. Second, one realization of this structure was simulated and the semivariogram parameters estimated from the exhaustive simulated field. If the estimated parameters differed too much (see definition below) from the generating parameters, the realization was discarded and another independent realization produced. Third, when a realization with useable parameters was found, this field was sampled with each of the seven sampling designs. Finally, from these samples we estimated the semivariogram parameters. To obtain a robust estimate of the reliability of each sampling design, a total of 5000 useable simulations were produced and used in the analysis.

3.1. Sampling designs tested

Five of the seven sampling designs tested here have been used previously for field surveys. Summary information for all seven designs tested is given in Table 1, together with relevant references and Fig. 1 shows the layout of sampling designs. For comparison with the semi-regular sampling designs used in previous field surveys, we tested two designs with semi-randomly located sampling sites within an area of $12\,\text{m} \times 12\,\text{m}$ (*Random12*) and $30\,\text{m} \times 30\,\text{m}$ (*Random30*). The locations on these designs were chosen randomly from all possible locations on a $0.5\,\text{m} \times 0.5\,\text{m}$ grid.

3.2. Simulation of the autocorrelated random field

The spatial structure of all simulations was described by a Gaussian marginal distribution around a mean value of 0 and an isotropic spherical semivariogram γ given by (Cressie, 1993):

$$\gamma(h) = \begin{cases} c_0 + c\left\{\frac{3h}{2a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right\}, & h \leqslant a, \\ c_0 + c, & h > a. \end{cases}$$

The parameter $\theta = (a, c_0, c)$ defined the semivariogram shape over the lag distances h by the range $a \geqslant 0$ the partial nugget variance $0 \leqslant c_0 \leqslant 1$, and the partial sill variance $c \geqslant 0$. In the results presented here we fixed the nugget at $c_0 = 0$, the sill at $c = 1$, and the range at $a = 5\,\text{m}$. We chose a spherical semivariogram model because it describes the variation in natural phenomena well, and is found to fit snow cover measurements better than other models in most cases (Kronholm, 2004; Kronholm et al., 2004).

Although field experiments have observed mostly isotropic structures (e.g. Kronholm et al., 2004), this is probably an over-simplification of natural conditions. Similarly, the assumption of a Gaussian marginal distribution is an over-simplification as some properties of snow layers, such as their penetration resistance, have been shown to be non-Gaussian (Kronholm et al., 2004).

For the simulations we used the circular embedding method (Dietrich and Newsam, 1993) implemented in the RandomFields package (Schlather, 2001) for R (R Development Core Team, 2005). The simulations were made on a rectangular grid with 61×61 cells, which is comparable to a $30\,\text{m} \times 30\,\text{m}$ grid with $0.5\,\text{m}$ spacing. For each simulation all 3721 cells were assigned a simulated value.

3.3. Sample semivariogram

To calculate the sample semivariogram, the simulated field was sampled to each design at the sampling locations $z(x_i)$, $i = 1,2,3,\ldots$. The classical sample semivariogram was calculated by (Webster and Oliver, 2001)

$$\hat{\gamma}(h) = \frac{1}{2m(h)} \sum_{i=1}^{m(h)} \{z(x_i) - z(x_i + h)\}^2.$$

Table 1
Summary information about sampling designs

Design	Reference	Sampling points	Minimum spacing (m)	Extent (m)
LH1	Birkeland et al. (2004a, b), Landry et al. (2004)	84	1.0	30.9
LH2	Birkeland et al. (2004a, b), Landry et al. (2004)	125	0.5	38.2
Random12		144	0.5	16.6
Random30		144	0.5	40.0
MT2004	Birkeland et al. (2004a), Logan et al. (2007)	159	0.5	18.4
MT2005		145	0.5	19.8
Swiss	Kronholm (2004), Kronholm et al. (2004)	113	0.5	19.0

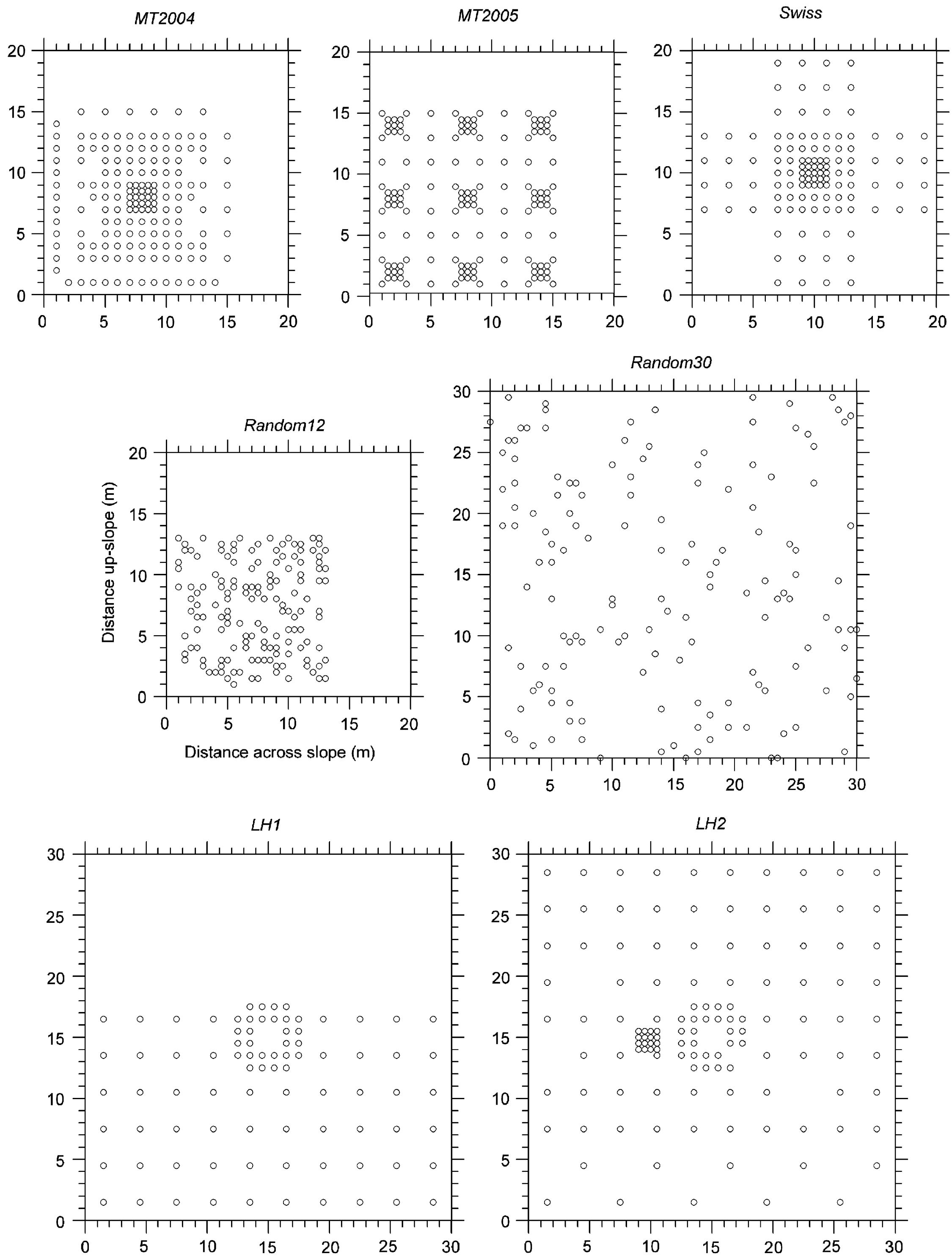

Fig. 1. Sample designs tested in this study. All designs are drawn to same scale. See Table 1 for references.

All semivariograms were computed to a lag distance of 8 m using 13 equally spaced bins with 0.2 m as the lowest bin limit and 8.0 m as the upper bin limit, and a bin width of 0.6 m. Each bin had $m(h)$ point pairs. A fixed upper limit for sample semivariograms was used despite the different extents of the sampling designs. We chose this to facilitate direct comparison of the sample semivariograms for all sample designs.

3.4. Model semivariogram

A spherical function was fitted to each sample semivariogram. The parameters for the best fitting semivariogram $\bar{\theta} = (\bar{a}, \bar{c}_0, \bar{c})$ were determined by minimizing the weighted residual sum of squares (WRSS) function given by (Cressie, 1993)

$$\mathrm{WRSS}(\bar{\theta}) = \sum_{j=1}^{K} |N(h(j))| \left\{ \frac{\hat{\gamma}(h(j))}{\bar{\gamma}(h(j); \bar{\theta})} \right\}^2 .$$

The function calculates the sum of the differences between the semivariogram estimate $\hat{\gamma}$ and the semivariogram model $\bar{\gamma}$ for each of the K bins ($j = 1, 2, \ldots, K$). This function gives more weight to bins with more paired comparisons and to bins at shorter lag distances. Calculation of the sample semivariogram and function fitting was done with the geoR package (Ribeiro and Diggle, 2001) for R (R Development Core Team, 2005). As an initial estimate for the minimization function we used the semivariogram parameters initially used for the simulations.

3.5. Monte Carlo simulations

Simulations were made using a Monte Carlo approach. For the set of generating semivariogram parameters $\theta = (a, c_0, c)$, 5000 independent simulations were made. Some realizations had a spatial structure, which differed significantly from the generating process. These realizations were identified by calculating the semivariogram range, and nugget and sill variances for the exhaustive dataset using the same method as described above. In addition, we calculated the mean for the exhaustive dataset. The realization was used for further analysis only if the model parameters estimated from the exhaustive dataset satisfied the criteria set for the spread of the mean, the sill and nugget variance and the range. The criteria are listed in Table 2.

Table 2
Generating values and criteria for spread of exhaustive simulated fields satisfied by 5000 realizations used for analysis

Parameter	Generating value	Allowed range
Mean	0	Generating mean ± 0.2
Range a	5 m	Generating range ± 0.2 m
Nugget c_0	0	Generating nugget ± 0.05
Sill c	1	Generating sill ± 0.1

4. Results

The process of calculating the sill and nugget variances and the range and mean for each exhaustive simulation and selecting only those that were within the limits shown in Table 2 resulted in 25827 simulations of which only 5000 (19%) were used in the analysis.

4.1. Semivariance in lag distance bins

The number of point pairs in each lag distance bin is shown in Fig. 2 and the spread of the 5000 observed semivariance values in each lag distance bin is shown in Fig. 3. The *LH1* design did not have any point pairs in the first lag distance bin (Fig. 2) and, therefore, only had nine bins whereas the other six sampling designs had 10 bins. The semivariance values in most bins were positively skewed and the median value was below that of the value expected from the generating value (shown with a line). The bin median value in the *Random30* design was closest to that expected from the generating model.

In Fig. 4 the standard deviation of the semivariance in each of the lag distance bins is compared for all designs. A plot of a more robust measure of the spread in each bin, such as the inter-quartile range, shows the same trends as the plot of standard deviation despite the skew in the data of each bin. The standard deviation generally increases with the lag distance for lag distances smaller than the generating range. For the *LHs1* and *2* designs the standard deviation shows peaks corresponding to the bins with relatively few point pairs (Fig. 2). For the bins at lag distances <2 m the *MT2005* and the *Random12* designs had the lowest standard deviation but for bins with lag distances >2 m the lowest standard deviation was found for the *Random30* design.

4.2. Estimated range

Histograms of the estimated range values are shown in Fig. 5, which demonstrates a considerable spread and positive skew for all designs. In a few cases, the spherical function fitted the sample

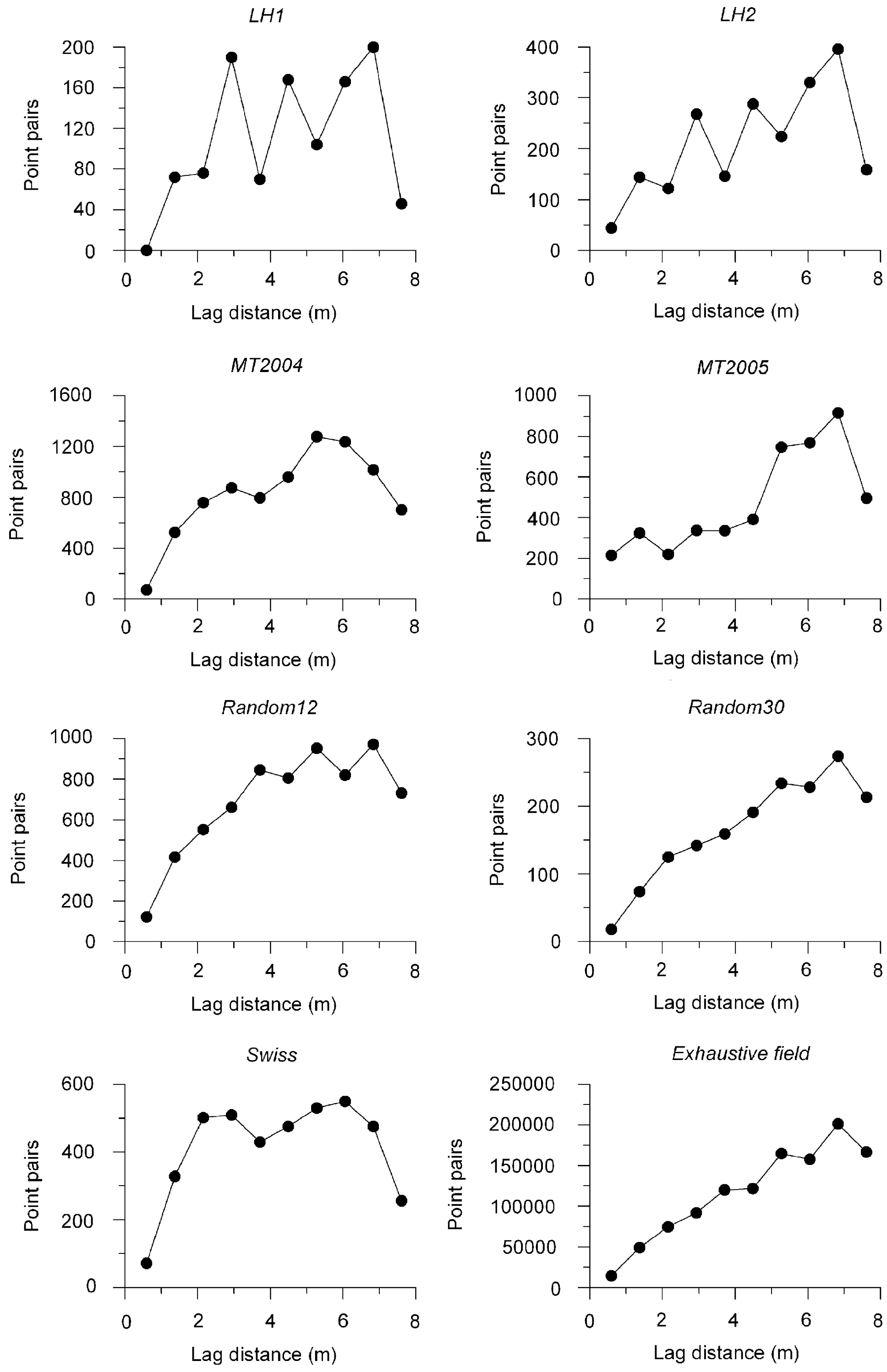

Fig. 2. Number of point pairs in each of fixed lag distance bins.

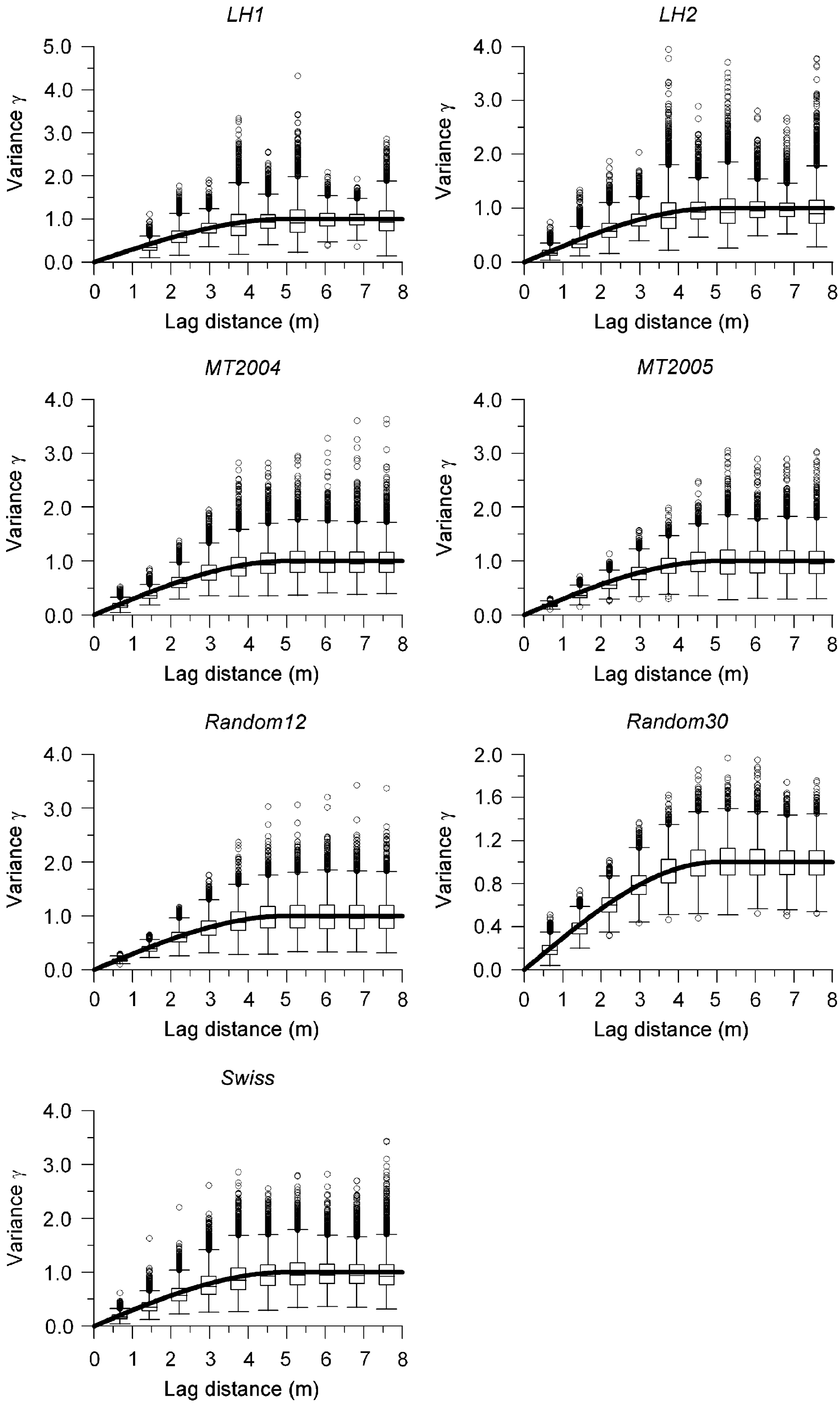

Fig. 3. Spread of 5000 observations of semivariance within each of the fixed bins.

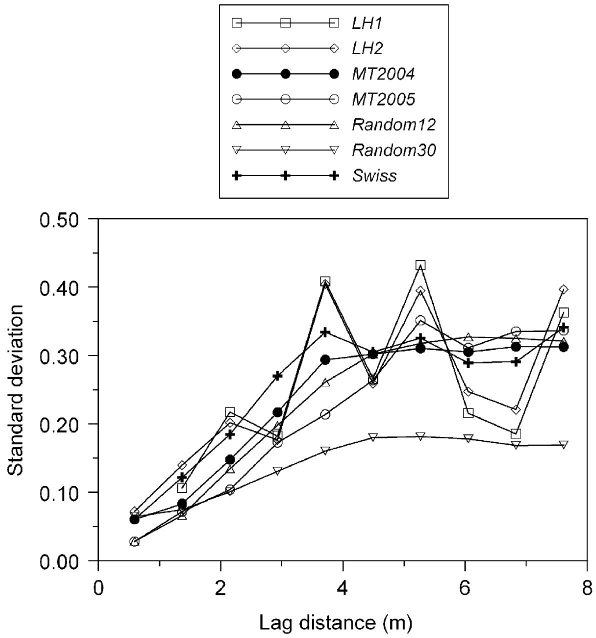

Fig. 4. Standard deviation for observed variance in each bin shown in Fig. 3.

semivariograms poorly, resulting in an unreasonably large estimated range combined with either a large nugget variance or a large sill variance. The *Swiss* design resulted in the largest number of unreasonably large estimates of the range.

Table 3 gives standard measures of the spread and central tendency for a subset of the estimated values. Because of the occasional outliers we truncated the upper and lower 5% of the distributions before calculating the measures of spread and central tendency. Robust measures of spread and central tendency show similar results. The spread of the range estimated from data sampled by the designs shows much higher variation than the spread in the ranges estimated from the exhaustive datasets, which were within $5\,\mathrm{m} \pm 0.2\,\mathrm{m}$. The *Random30* design had the smallest spread. According to the mean of the truncated estimates, all designs overestimated the range with between 13% and 5%. This may, however, be partially due to the non-normal distribution of the values, and the histograms in Fig. 5 show that the mode of the values is close to the generating range of 5 m.

4.3. Estimated sill variance

Fig. 6 shows histograms of the estimated sill variances and Table 3 shows the mean and spread of the estimates after truncation of the upper and lower 5% of the data values. The spread of the estimated values was large compared to the spread in the estimates from the exhaustive datasets, which were within 1 ± 0.1 units. The distributions from all designs had a positive skew but a mode and mean (of the truncated values) located close to the generating sill variance of 1. The *Random30* design had the lowest spread while the *Swiss* design had the largest number of unreasonably large estimates of the sill variance.

4.4. Estimated nugget variance

The estimates of the nugget variance (Fig. 7) had more spread than the estimates from the exhaustive datasets, which were within 0 ± 0.05 units. Because of the distribution of the estimates of the nugget, we did not calculate central and spread parameters as for the estimated range and sill variance. Of the seven tested designs, the *MT2005* and *Random12* designs had the largest density of estimates in the histogram closest to 0.

5. Discussion

There is good agreement between the central values for the sill and nugget variance and the range found by the sampling designs and the generating values, but the spread in the estimated parameters was relatively large (Figs. 5–7). Since the selection of the realizations used for sampling ensured that the estimates from the exhaustive dataset were close to the generating values, most of the spread in the parameters estimated from the sampled datasets must be due to the sampling design and the sample size. From a practical perspective, this spread quantifies the difficulty of taking one sample (as is done in field snow studies) and comparing that to another sample. Even if the conditions being sampled are the same, we may see sizable differences between two samples due to the sampling design alone.

Our methods may contain two possible sources of error. First, simulating spatial fields of given statistical parameters is difficult. We have minimized this issue by selecting only those simulated fields, which satisfied strict limits on the parameters on the exhaustive field. Second, the automated routine used to fit a semivariogram model to the sample semivariogram may result in more spread in the estimated semivariogram parameters than if

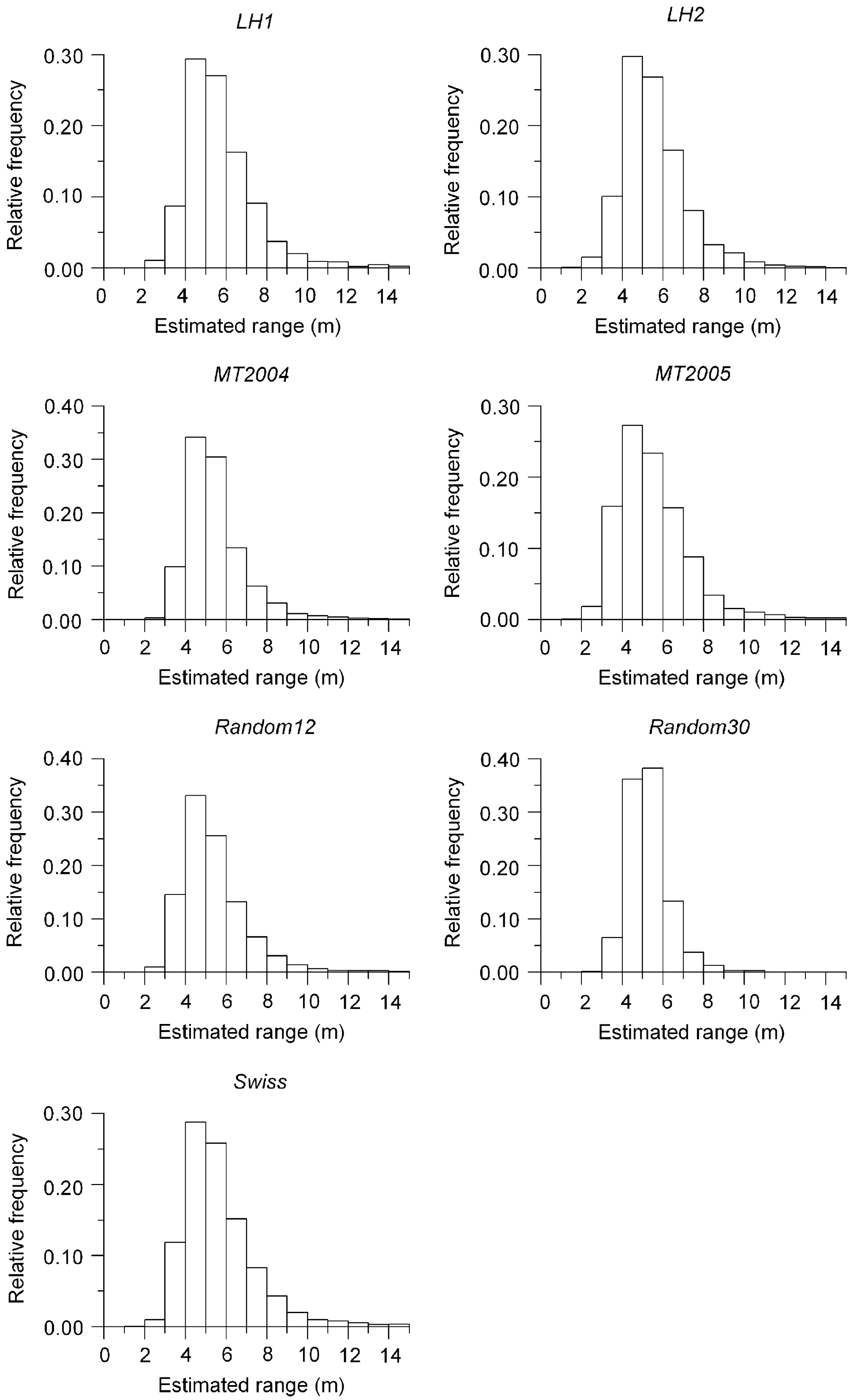

Fig. 5. Distribution of estimated range for the exhaustive dataset and for seven tested sample designs.

Table 3
Summary statistics for estimates of sample semivariogram range and sill variance

Design	Range a (m)			Sill c		
	Mean	Variance	Coefficient of variation	Mean	Variance	Coefficient of variation
LH1	5.67	1.708	0.23	1.023	0.048	0.214
LH2	5.55	1.538	0.224	0.989	0.058	0.244
MT2004	5.33	1.054	0.192	0.996	0.0645	0.255
MT2005	5.42	1.642	0.236	1.004	0.0729	0.269
Random12	5.26	1.243	0.212	1.001	0.075	0.274
Random30	5.23	0.521	0.138	0.986	0.018	0.136
Swiss	5.74	2.576	0.28	1.027	0.0891	0.291

Statistics were calculated from 5000 realization with upper and lower 5% truncated, and are therefore based on 4500 datapoints.

each semivariogram was fitted by an expert. For example, some spread in our estimates may be due to fitting a spherical semivariogram model to a sample semivariogram when another type of model, such as an exponential model, would have provided a better fit. Yet, our approach seems reasonable because we know the model type used to generate the simulated fields. Further, our technique of truncating the most extreme values of the estimated range and sill variance should remove most of the worst misfit cases from the dataset used for the statistics in Table 3. Finally, the spread of the estimates of the nugget and sill variances and the range follows what we would expect from the variance within each lag distance bin (Fig. 2). From the spread within each bin we would expect that the *Random12* and *MT2005* designs which have the smallest spread in the bins at short lag distances would be the most reliable designs for estimating the nugget, while the *Random30* design with the lowest spread within the bins at longer lag distances would be the best design for reliable estimation of the sill variance and the range. This is indeed the case (Figs. 5–7), indicating that the variance is already introduced before the fitting routine is applied.

Choosing a good sample design helps decrease the spread of the estimated variogram parameters. However, we have shown this can only be accomplished to a certain degree. In addition to choosing a good design, including more sampling locations, either by using faster sampling techniques or more instruments simultaneously, will also decrease the spread as shown by Webster and Oliver (1992). Our results suggest that the *Random30* design was generally the most reliable design and the Random12 design also gave relatively low spread in the estimated values. Yet, a random design is not practical for some field surveys, for example on snow. A possible solution is to use semi-random or stratified sampling where only a part of the design is randomized. This approach was used by the CLPX experiment to sample snow cover properties at larger scales than the ones targeted in our simulations (Cline et al., 2003), but the approach would be similar if employed at the slope-scale.

The observed spread in the semivariogram model parameters has implications for estimates of these values from field surveys with the tested sample designs. First, the spread means that an estimate of the spatial structure of a single sample (for example from one field day) can only be a rough estimate of the true spatial structure. Second, the lack of reliability in the estimates means that direct comparison of spatial structures obtained on different slopes must be made with care, even if the same sampling design is used. The estimated bias and spread presented here is useful for addressing the uncertainty in the parameter estimates when a comparison of parameters is made.

Our approach based on simulation is not exactly the same as might be applied to data from a field survey. For example, we assumed that there were no inaccuracies associated with the data values at a given sampling location, which is possible only in theory and with simulated data. We also assumed that all measurement locations in a sampling design could be used for geospatial analysis. In reality this is unlikely because typically some data might be discarded because they are considered faulty or because of instrument problems. To simulate this, a number of randomly chosen measurement locations could be left out of the simulations. In spite of the

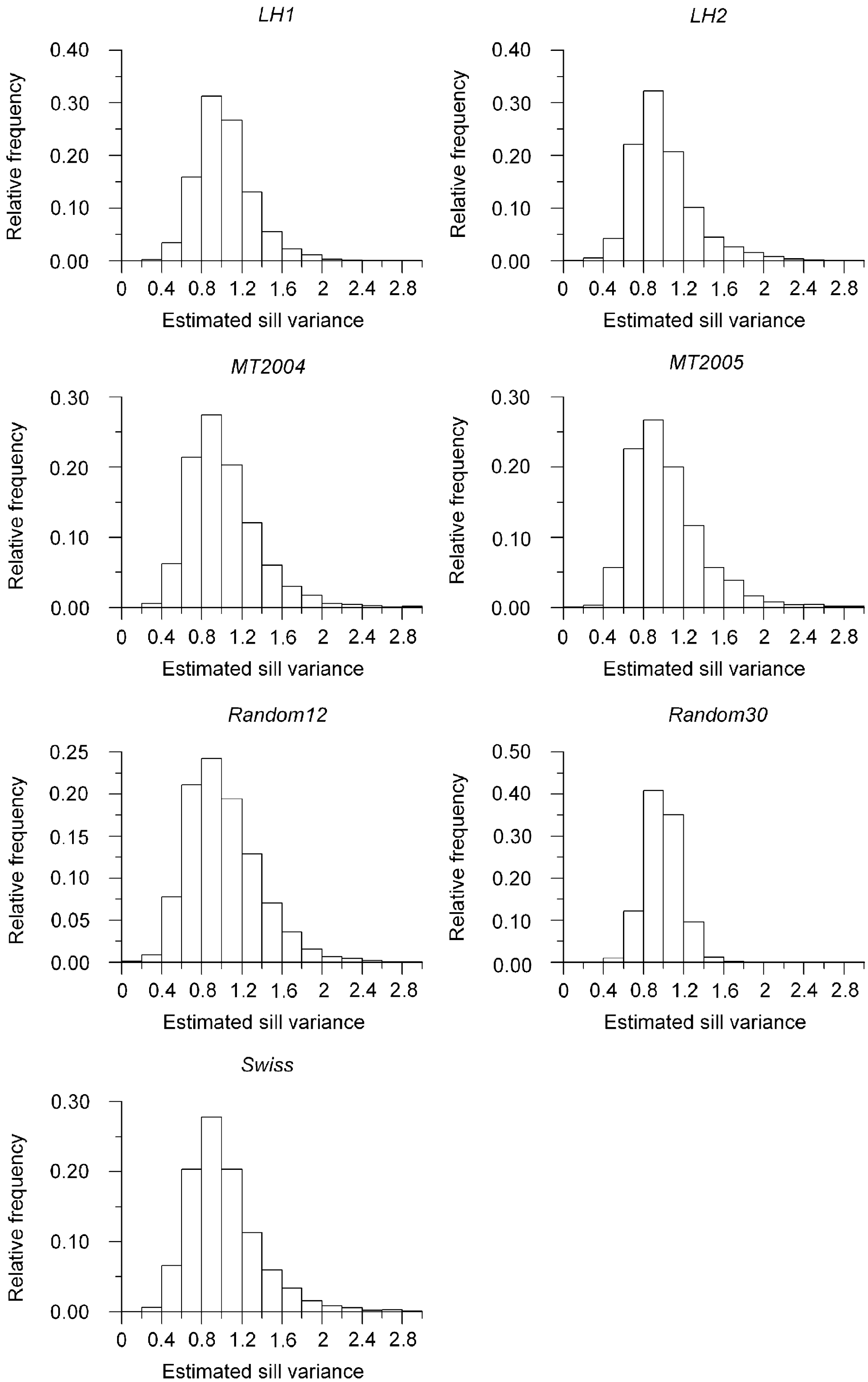

Fig. 6. Distribution of estimated sill for exhaustive dataset and for seven tested sample designs.

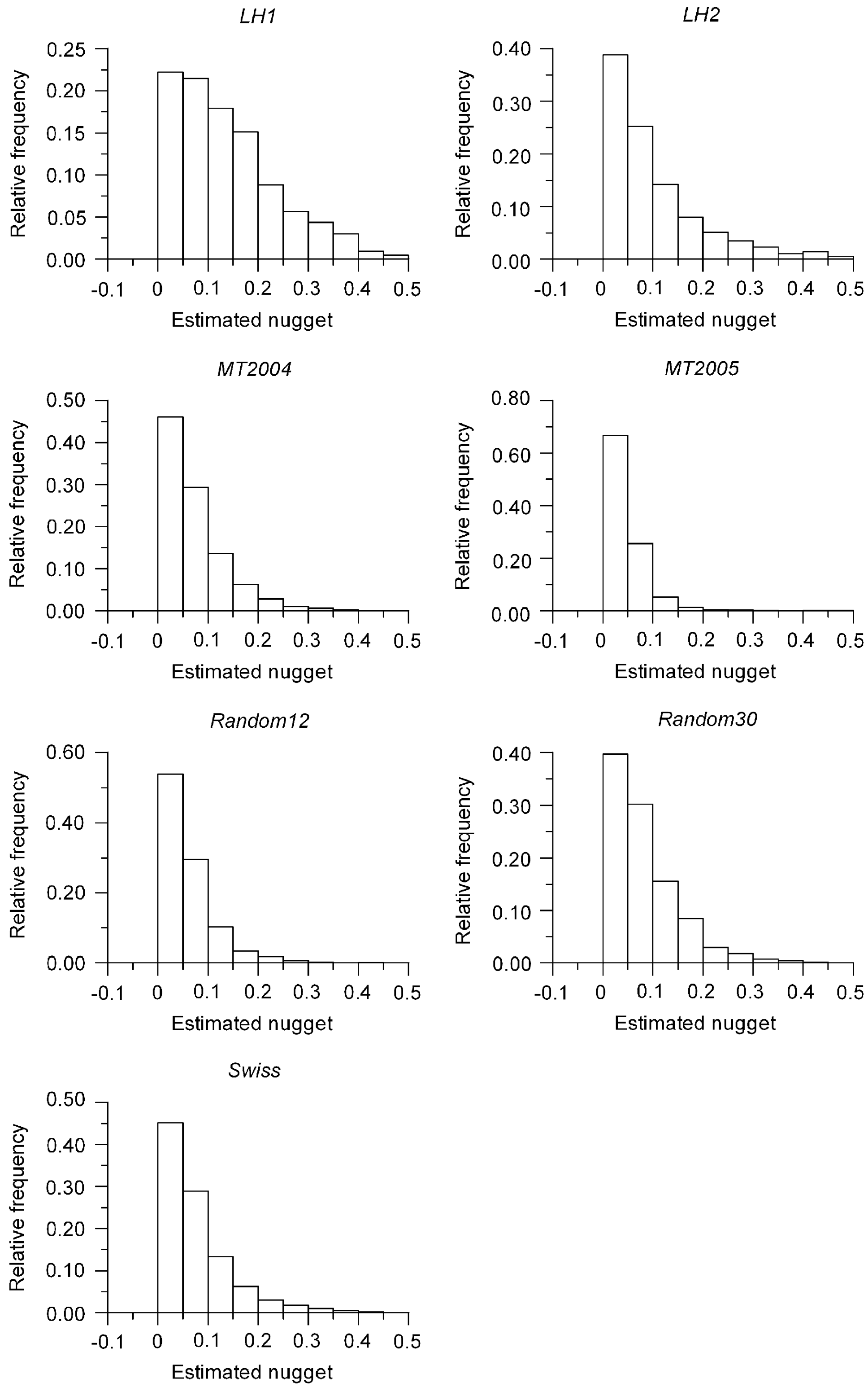

Fig. 7. Distribution of estimated nugget for exhaustive dataset and for seven tested sample designs.

differences between our use of simulated data and field surveys, we believe our analysis of sampling designs can be used when designing field surveys.

Our results suggest that the sample designs with point measurements used in the past to characterize spatial structure of snow slopes (e.g., Kronholm et al., 2004; Logan et al., 2007) might be inadequate to accurately characterize the true spatial structure of snow slopes. Instead, new methods that allow nearly continuous spatial measurements might be necessary. One promising possibility for such continuous measurements is the use of radar (Marshall et al., 2004), though the sensitivity of this instrument is still less than for some instruments used for point measurements.

6. Conclusions

Using Monte Carlo style stochastic simulations, we tested how reliably various sampling designs estimated the spatial structure of a field of data with known spatial structure. Our simulations suggest that:

- When averaged over many simulations, all the tested designs tend to have a relatively low bias with the estimated range being overestimated up to 13% compared to the generating value and the sill being slightly underestimated, but within 5% of the generating value.
- Estimates of range, nugget and sill have a relatively large spread. This clearly limits our ability to compare estimates of these values obtained on different slopes or by different studies, even when studies use the same sample design. To decrease the spread in the estimates, greater measurement density is needed, something that is not possible with snow given current instrumentation and a one-day sampling period to limit temporal changes in the snowpack. However, recent instrumentation advances, such as radar, might allow adequate sampling to characterize the true spatial structure for some snowpack parameters.
- The previously used field survey designs tested were less reliable at characterizing spatial structure than a design with randomly chosen sampling locations. Random sampling designs have not been used in the field due to the difficulty in implementing them. However, we suggest that some degree of randomization should be included in future sampling designs of slope-scale snow properties.

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