AVALANCHE FREQUENCY AND MAGNITUDE: USING POWER-LAW EXPONENTS TO INVESTIGATE SNOW AVALANCHE SIZE PROPORTIONS THROUGH TIME AND SPACE

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ABSTRACT: Power-laws provide a means for investigating snow avalanche frequency-magnitude relationships and their contributing factors. This research uses power laws to explore variations in avalanche size proportions through space and time, as well as investigating factors which may contribute to these variations. Data utilized for this work includes the Westwide Avalanche Network data from the western United States for regional analyses, with path-specific analyses focused on data from Utah's Little Cottonwood Canyon. Results show power-law exponents vary through space both at the regional level and between individual avalanche paths. Avalanche size proportions, with respect to space, are the product of terrain based variables at both the mountain range and the path levels, with alpha angles significantly correlated to the proportion of small to large avalanches. This research also indicates that variation in exponents through time is indicative of changes in seasonal weather and snowpack characteristics, with mean snow height also significantly correlated to the proportion of small to large avalanches. Knowledge of power-law exponents for particular avalanche paths, and their relationship to seasonal snowpack depth, may be helpful for managing avalanches along highway corridors, in ski areas, or in backcountry forecasting operations.

KEYWORDS: Power-law exponent, Snow Avalanche size proportions, Frequency and Magnitude

1. INTRODUCTION

Many natural hazards, such as earthquakes and wildfires, have been known to exhibit robust power-law behavior with respect to frequency and magnitude of events (Gutenburg and Richter 1956; Clar, Drossel, and Schwabl 1994; Malamud, Morein, and Turcotte 1998; Malamud, Millington, and Perry 2005). In brief, a power-law simply states that the size proportion of large events will always be exponentially larger than small events. Current research supports this idea with respect to snow avalanches through analysis of observed events as well as mathematical modeling (Birkeland and Landry 2002; Louchet et al. 2002; Faillettaz, Louchet, and Grasso 2004, 2006; Rosenthal and Elder 2004). As shown in McClung (2002), dry slab avalanches may not follow all characteristics of Self Organized Criticality (SOC) set forth by Bak, Tang, and Weisenfeld (1988), but the aggregate of all types has been shown to produce scale invariant behavior at some sizes (Birkeland and Landry 2002). By utilizing observations of avalanche

events per-path and season at the local scale, as well as between locations across the western United States at the regional scale, this research identifies fluctuations in the proportion of small to large avalanches, or the power-law exponent, through space and time, as well as a preliminary investigation of possible controls.

2. METHODOLOGY

2.1 Data and Study Areas

Data utilized for this study include cumulative avalanche events for many locations across the western United States, compiled by the Westwide Avalanche Network (WWAN), as well as daily observations of weather, snowpack, and avalanche occurrence at a single location along State Road 210 (SR-210) in Little Cottonwood Canyon, UT; maintained by the Utah Department Transportation (UDOT).

WWAN avalanche observations used for the regional study include 230,965 records for 131 possible station locations with various spatial and temporal extents between winter seasons 1949 and 2002. The local scale data set includes 3,340 UDOT avalanche records for ~50 active paths within 35 continuous winter seasons between 1973/74 and 2005/06, as well as meteorological

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and snowpack observations for the coincident time period.

2.1. Methods

Spatial analyses of variation in avalanche size proportion, or exponent value, includes two distinct spatial scales: between paths at the local scale, and station location at a regional scale. Temporal analysis includes observation of avalanche size proportion per-season along SR-210.

Exploration of possible controls associated with variance through time and space were chosen to best represent fundamental variables such as terrain, weather, and snowpack. This analysis includes regional scale comparisons between the derived exponent per-location as compared to regional snow climate zone and major mountain range, investigation of controls at the local scale between per-path exponents and terrain based variables inherent to each path, and seasonal exponent results in comparison to weather and snowpack variables derived for each season.

Aggregation of avalanche events by space and time is the underlying method of comparison in this research. Due to the inconsistency and lack of data recorded for size 1 observations, removal of these events from both WWAN and UDOT data sets prior to aggregation was conducted as recommended by previous research (Birkeland and Landry 2002). Once grouping of observations by space and time is complete, observations are further grouped by location at the regional scale using WWAN data, and by path and season, at the local scale using UDOT data. Simple frequency counts per-size, per-group, were conducted, and groups with insufficient counts per-size are removed. This removal of groups with insufficient data per-size discriminates poor trend line fits (R^2 < 0.5). Exponent values, or the slope of a best fit linear trend (α_e), per-group were derived by plotting the log-frequency derived per-size against the related size; defined by Perla and Martinelli (1978). An example of the exponent value (α_e = -0.55) derived for an aggregate sample of WWAN observations is shown in Figure 1.

Since avalanche sizing systems have been previously estimated to cover several orders of magnitude and be roughly logarithmic (Birkeland and Landry 2002), there is no need to derive log values for sizes 2-5 to calculate exponent results in this study. While others have argued that plotting log-frequency against log-crown height is beneficial to obtain more sample
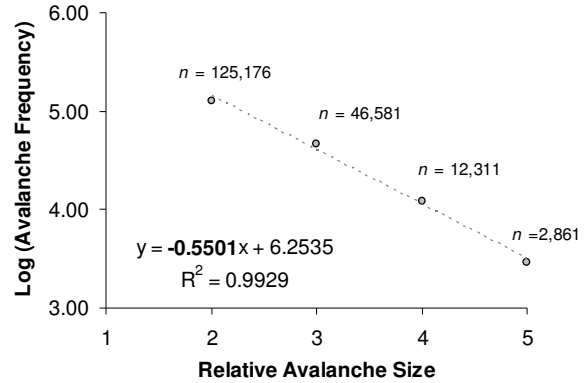

Figure 1: Power-law fit and associated exponent value for 186,929 WWAN avalanche observations greater than or equal to size R2.

points for trend fit (Faillettaz, Louchet, and Grasso 2004, 2006; Rosenthal and Elder 2004), the methods used in this study utilize standard avalanche size classifications to preserve several orders of magnitude for analysis. The drawback to using conventional sizing of 2-5 is the lack of sample points used to generate the exponent and unreliable R^2 values associated, thus R^2 will be used as a measure of fit only, and not as a measure of variance associated with each derived exponent.

3. RESULTS AND DISCUSSION

After aggregation and filtering of both data sets, the regional analysis of spatial variation per-location included 185,915 of the original 230,965 WWAN records, and 79 of 131 possible station locations. The UDOT data set, used for spatial variation at the local scale, included 3,193 of 3,340 possible records, and 39 of the original 50 avalanche paths for analysis. Seasonal analysis of this data set utilized 23 of the original 35 seasons.

3.1 Exponent variation across space

3.1.1 Regional scale variation per-location

For 79 station locations across the western U.S., the average number of records found per-location was n = 2,354 and mean R^2 value of 0.94. Analysis produced considerable variation in exponent values, ranging from α_e = -0.09 for grouped paths at Pine Creek Mill, CA to -1.09 along U.S. Highway 6, Loveland Pass, CO. This very low value for Pine Creek may be associated with the lack of records for that location (n = 30). The median exponent value for all 79
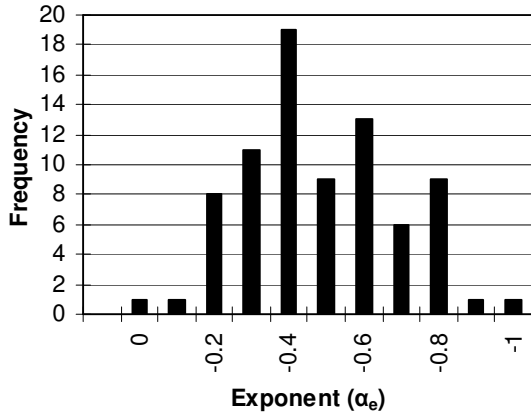
Figure 2: Frequency of exponent values for 79 WWAN station locations across the western U.S.

locations was found to be α_e = -0.50 with a standard deviation of 0.20. Descriptive statistics and histogram results indicate slightly skewed distribution (skewness = -0.31) with the majority of exponent values between -0.20 and -0.80 (Figure 2). This distribution shows greater variation in exponent values less than the mean (-0.50). This suggests the occurrence of a single large event has great influence on the exponent value, or a flattening of frequency-magnitude trend fits at a regional scale. Upon visual inspection, variation is apparent between locations within similar geographic areas, as well as across the western region as a whole (Figure 3). Exponent values in Colorado locations seemed to show the greatest variation; this may in part be due to the density of observations associated with this area as compared to other areas in the western region. Exponents in the northern states of Washington and Idaho showed greater occurrence of small avalanches to large, and California showed a lower proportion of small to large.

3.1.2 Local scale variation per-path

Results for the per-path analysis were derived from 39 avalanche paths associated with SR-210, in Little Cottonwood Canyon. The mean exponent value was α_e = -0.48, ranging from -0.23 for the Grizzly Gulch path to -0.69 for the West Hellgate path. The median value was -0.49 with a standard deviation of 0.11. Histogram results show minor characteristics of both a normal and/or bimodal distribution, and descriptive statistics indicate a positive skewness of 0.11. This skew is contradictory to that found in the per-location analysis, but also has a shorter range in values. This may be the product of significantly less
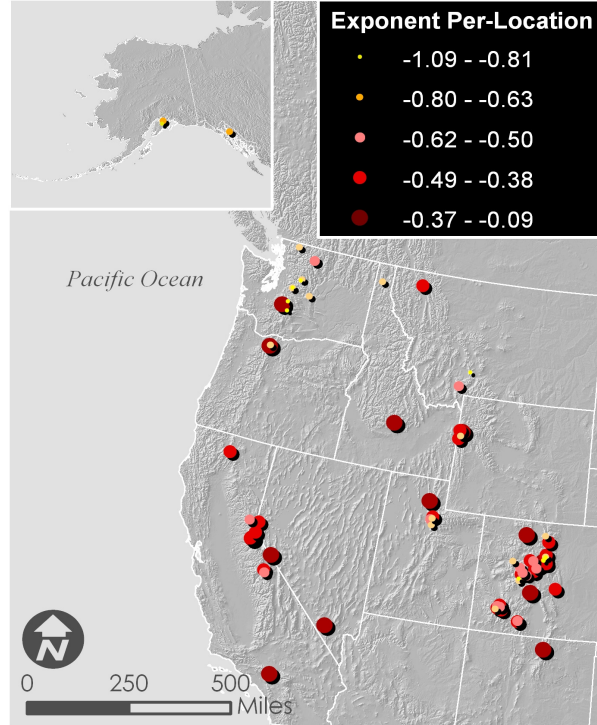

Figure 3: Spatial variation of exponent values per-WWAN station location across the western United States at the regional scale.

sample observations in the UDOT data set in comparison with the larger WWAN data set.

Spatial distribution of exponents along the SR-210 corridor shows a slight decreasing trend, or lesser proportion of small to large avalanche events moving west to east in the canyon (Figure 4). Further ANOVA testing between the upper (n = 19) and mid-canyon (n = 20) paths showed significant difference between the two groups at the 95% confidence interval. The primary difference between upper (east side) and mid-canyon paths is primarily associated with terrain-based variables such as steeper, longer, and more confined paths in the mid-canyon, and increased mean elevation in the upper paths. These variables may have significant influence on avalanche size proportions at the path-level and are investigated later in the study.

3.2 Exponent variation through time

After removal of size 1 data and seasons with insufficient counts per-size, the per-season analyses were derived from 23 of the 35 seasons within the UDOT data set between the winters of1974/75 and 2005/06. The mean number of records per-group was 97, and the median
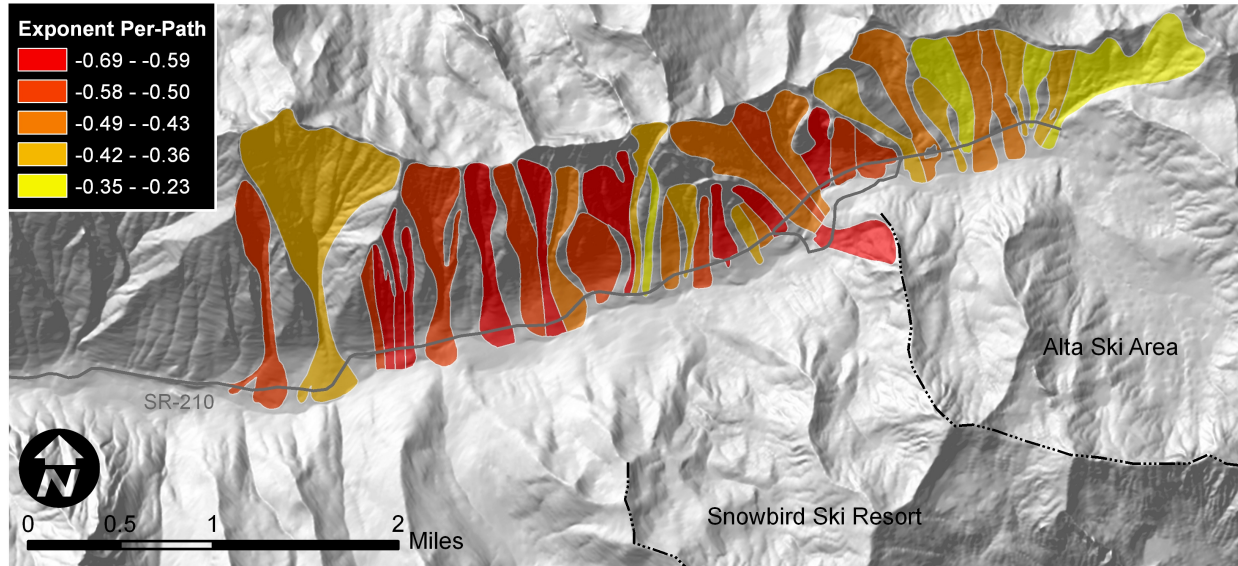
Figure 4: Spatial variation of exponent values per-avalanche path at the local scale associated with SR-210 in Lt. Cottonwood Canyon, UT.

exponent value was found to be α_e = -0.63; ranging from -0.26 for the 1980/81 season to -0.83 for the 2005/06 season. Standard deviation was 0.13 and the total range in exponent values was 0.57. Despite gaps in per-season data, there appears to be a gradual decrease (-0.01) in exponent values through time with a fit of R^2 = 0.35 (Figure 5). This suggests the proportion of small avalanches to large is increasing with time. ANOVA testing of per-season exponent results grouped into 5 year periods show a significant difference at the 95% confidence interval, and
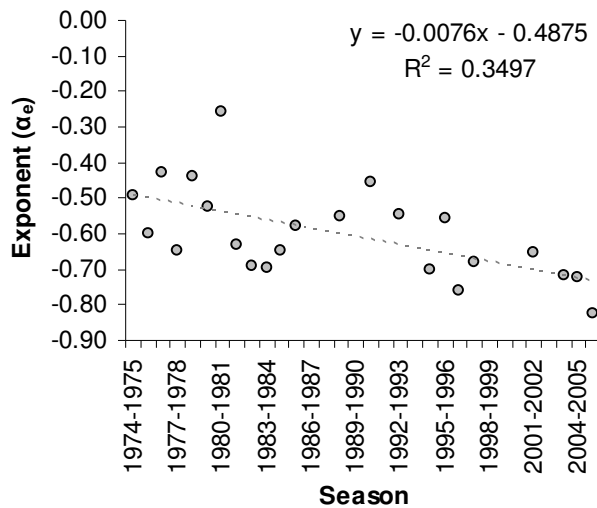

Figure 5: Temporal variation in exponent values per-season for all avalanche paths associated to SR-210, UT.

grouping of seasons into 2 parts, before and after 1991, produced significance at the 90% confidence interval.

Reasons for this decreasing trend may be associated with such variables as changes in local/regional climate (i.e., snowfall, water content, intensity of storms, etc.), or may simply be due to differences in classification from one forecaster to the next during the time span and/or more stringent recording of all sizes in recent years. Increased frequency of control activities attempting to decrease the average event size during large avalanche cycles may also be a human-based influence on size proportion change through time at this study area.

3.2 Exploring exponent controls

The final objective of this study includes exploration of possible controls influencing spatial and temporal variation in avalanche size proportions. This includes core variables associated with terrain, snowpack, and weather.

3.2.1 Regional scale spatial controls

Utilizing exponent values derived from the WWAN data set at the regional scale, the first analysis explored differences between location exponents per-Snow Climate Zone (as defined by Roch (1949), LaChapelle (1966), Dexter (1981), Armstrong and Armstrong (1987), Mock

(1995),and Mock and Birkeland (2000)), as well as by major mountain range in the western U.S.

There appeared to be little difference in mean exponent values upon the initial visual inspection of each of the 4 climate zones (Figure 6). The exception to this included locations within the Coastal Transition Climate zone which exhibited a mean exponent of α_e =-0.73, deviating significantly from the 3 other zones, which only range from α_e = -0.52 to -0.53. This deviation of Coastal Transitional may be a contributed to the significantly low number of locations representing this zone: 3 versus 13, 34, and 17. ANOVA test results concluded the difference between mean exponent values per-climate is not significant at either the 90 or 95% confidence interval, despite inclusion/exclusion of Coastal Transition climates. These tests indicate the possibility that snow climate, or snowpack and weather regimes may not be a significant contributing factor to exponent variation between spatial locations at the regional scale.

Mean exponent values for 5 major mountain ranges were also investigated. Mountain range areas were delineated visually using a hill-shade image derived from a 90m digital elevation model (DEM) for the western U.S. Resulting mean exponent values ranged from α_e = -0.68 to -0.39 and showed a slight variation by latitude upon visual inspection with the exception of the Colorado Rockies (Figure 7). The deviation shown in the Colorado Rockies may again be a product of the number of records or locations used: 38 versus 12, 5, 9, and 10. ANOVA results indicated significant differences between mean exponent values per-range at the 95% confidence interval
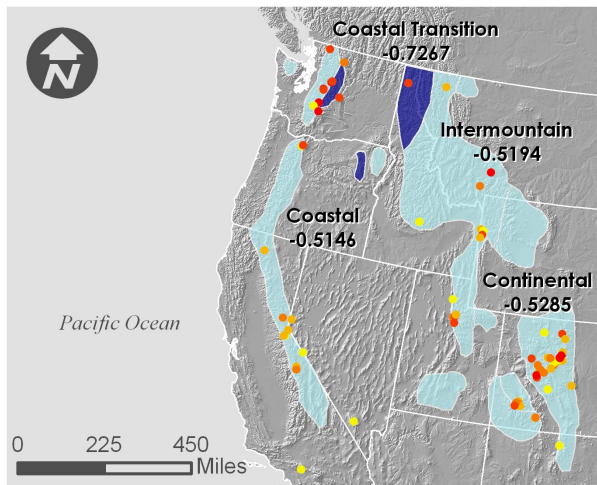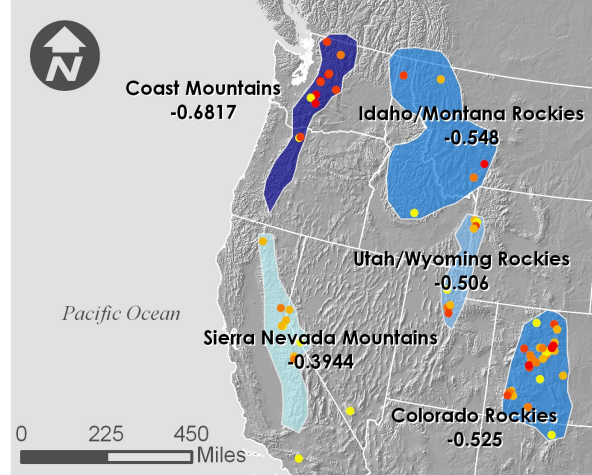

Figure 7: Difference in mean location exponent values per-mountain range in the western U.S.

with and without inclusion of the Colorado Rockies. While this test shows some significance in mean exponent values per-range, the results are somewhat subjective in that delineation of mountain ranges are not defined by peer reviewed literature. If, with further analysis by other means, this difference per-major mountain range proves to be significant, it may support terrain-based variables as important determinants of exponent variation as suspected by the previous per-path analysis.

3.2.1 Local scale spatial controls

Exponent values derived from the per-path analysis along SR-210 were compared to several terrain-based variables derived from digital GIS data for the Little Cottonwood and SR-210. These variables included confinement ratio, mean elevation, maximum displacement, mean displacement, maximum slope, and path alpha-angle.

The results from this analysis indicate insignificant R^2 values for 4 of the 6 variables. Exponent correlations associated with maximum slope (0.27) and alpha-angle (0.49) (Figure 8), showed much stronger relationships with exponent values than other independent terrain-based variables used in this analysis. Further regression testing of these relationships found significant correlation at the 95% confidence interval. These relationships make sense from a practical standpoint in that steeper paths produce a larger proportion of small slides to large and vice versa. With respect to observations along SR-210, steeper paths, such as Hellgate and the White

Figure 6: Difference in mean location exponent values per-Snow Climate Zone in the western U.S.
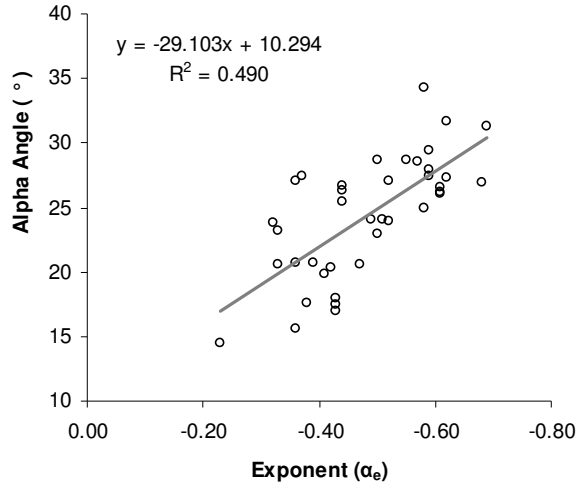
Figure 8: Derived exponent values for 39 avalanche paths along SR-210 in comparison with alpha angle per-path.

Pine Chutes, are indicator paths for natural activity and become rapidly unstable with sudden bursts of Precipitation Intensity (PI), and/or with sustained strong winds of the appropriate direction. Paths similar to Grizzly gulch are generally less sensitive to these types of events, but are more likely to produce large events associated with deep slab instabilities.

3.2.1 Local scale temporal controls

Avalanche size proportions derived per-season for SR-210 were compared with weather and snowpack variables collected at the Alta Guard study plot for the coincident time period. These variables include the total snow accumulated during each season, total water, mean height of snowpack (HS), maximum HS, mean high temperature and mean low temperature.

Comparing exponent values derived per-season with weather/snowpack variables, only 2 of the 6 independent variables showed promise. Maximum and mean HS showed the strongest correlations with R^2 = 0.47 and 0.66, respectively. Visually, the fit associated to exponent vs. mean HS seemed influenced by 1 outlier season (Figure 9). Upon further investigation, this point was found to be associated a record low season at the Alta Guard study plot: 1976/77 as the lowest with a mean HS of 66 cm and a total snowfall of 8 meters; average HS and total snow for the entire data set was found to be 194 cm and 14 m . Upon removal of this season from the analysis, a strong fit (R^2 = 0.71) was found (Figure 10). Further

regression supported the significance of both mean and max HS variables at the 95% confidence interval. These results may in part be due to the small number of samples used for this analysis, thus further analysis with a more complete temporal dataset may be required to strengthen this study.

These findings suggest increased HS may result in increased proportions of small to large events. A practical argument for this relationship is that as a snowpack becomes deeper it may also become more stable, thereby producing a greater proportion of small avalanches. Conversely, a shallow snowpack is more susceptible to deep instabilities through development of large temperature gradients and advanced faceting at the ground. This type of snowpack would likely
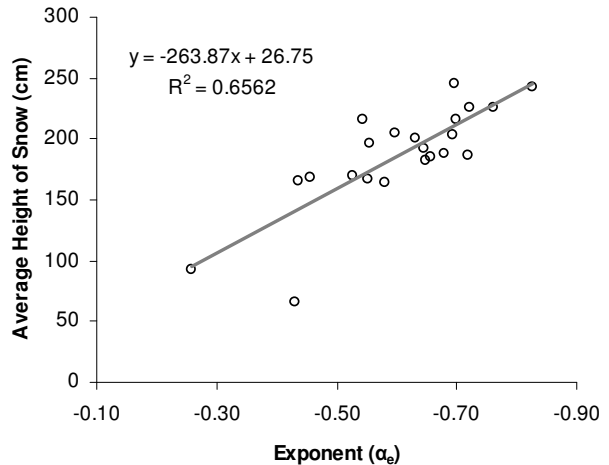

Figure 9: Derived exponent values along SR-210 in comparison with mean HS per-season.
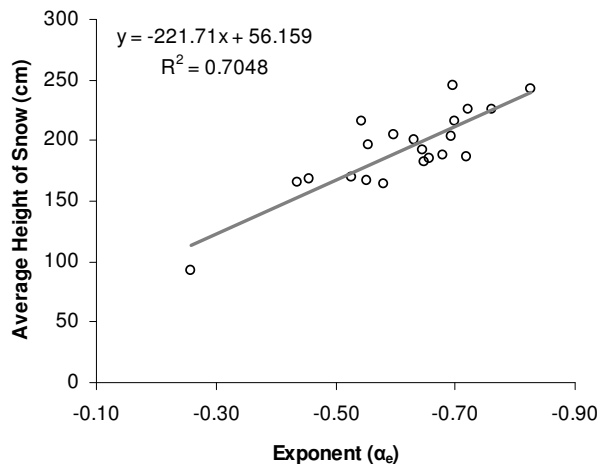

Figure 10: Derived exponent values per-season in comparison with mean HS after removal of 1976/77.

produce a greater proportion of large avalanches to small.

4. CONCLUSIONS:

Though research conducted simultaneous to this work casts some doubt on whether or not avalanches fit power laws (Bair et al. 2008), power laws still provide a reasonable approximation of magnitude/frequency relationships and power law exponents provide a way to compare those relationships over time and space. The findings of this research suggest the proportion of small to large avalanche events varies across space and time at multiple scales. Potential influences on these findings at the regional scale include discontinuous and inconsistent temporal extents within the WWAN data set, clustering of locations used for analysis, and user-defined delineations of major mountain ranges. Possible influences associated with the UDOT data set, at the local scale, include: observations made by multiple parties throughout the time span, the effects of observations in a controlled environment, dominant south facing paths associated to SR-210, and errors inherent to GIS based data used to derive per-path terrain-based variables.

This research, as well as the research of others (Birkeland and Landry 2002); Louchet et al. 2002; Faillettaz, Louchet, and Grasso 2004, 2006; Rosenthal and Elder 2004), supports the idea of robust power-law relationships and scale invariance when looking at the frequency-magnitude relationship associated to snow avalanches over certain size scales. These studies further investigate this idea by showing significant variation in avalanche size proportions through measurements of exponent values across space and time. Overall, investigations of avalanche size proportions, exponent variation, and the exploration of controlling variables should be expanded to other locations across the western U.S., Alaska, and other countries to fully understand and/or strengthen the results of this study.

Although new technologies that record events independent of human influence, and/or provide more precise and quantitative measurements of avalanche size, extent, and mass, would have the greatest influence on these analyses, further investigation of fluctuations in avalanche size proportions using different methods and/or different data is needed to support these hypotheses. Additional analysis of contributing variables is needed to fully understand the relationships between size proportions, spatially and temporally. A long term, large data set of avalanche occurrence for individual avalanche paths, new methods of measuring terrain based variables, as well as robust weather and snowpack data per-season are needed to address this problem by region, location, path, and season.

While these findings may not only provide insight into the vast behavior inherent to snow avalanche phenomena, it may also assist planners when developing new ski areas, highway corridors, or inhabited structures within avalanche prone areas, as well as provide clues for forecasters when evaluating avalanche potential for certain paths, during certain seasons.

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