DIY: Trigonometry-Basic

To review basic Trigonometric concepts, watch the following set of YouTube videos introducing concept of basic right triangle trigonometry, Pythagorean Theorem, basic trigonometric ratios, angular and linear velocity and application problems. They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

Note: If you need a review of any basic geometry topics, see the DIY Geometry Review.

1. Radians and degrees:- https://www.youtube.com/watch?v=EnwWxMZVBeg
2. Radian and degree conversion:- https://www.youtube.com/watch?v=z8vj8tUCkxY
3. Arc Length: https://www.youtube.com/watch?v=6lF1Kz6c2r4
4. Arc Length and area of a sector https://www.youtube.com/watch?v=SlfRoDI3esA
   alternate: https://www.youtube.com/watch?v=4HFkHP43sWA (continuation of video #3).
5. Linear and angular velocity https://www.youtube.com/watch?v=e78oMCdKMx0
   https://www.youtube.com/watch?v=yDHM6rd8P94 (Note: the presenter used the phrase "rate of speed" which is not quite correct. Speed is the rate of change of distance--distance traveled per unit of time. The rate of change of speed would actually be how fast the speed is changing—acceleration.)
6. Pythagorean Theorem:- https://www.youtube.com/watch?v=WqhlG3Vakw8
7. Basic Trigonometric ratios Part1 :-
   https://www.youtube.com/watch?v=Jsiy4TxgIME&list=PLD6DA74C1DBF770E7
8. Basic Trigonometric ratios Part2 :-
   https://www.youtube.com/watch?v=G-T_6hCdMQc&index=2&list=PLD6DA74C1DBF770E7
9. Unit Circle and circular functions :- https://www.youtube.com/watch?v=1m9p9iubMLU
10. Reciprocal ratios:- https://www.youtube.com/watch?v=MSoYRaSN_9g
11. Trig ratios for special triangles:-
    https://www.youtube.com/watch?annotation_id=annotation_884961&feature=iv&src_vid=cMqetVG8vRU&v=vG6Y8P7gRGM
12. Finding trig ratios of any angle using a calculator
    https://www.youtube.com/watch?v=PitZSgfquYc
13. Finding reciprocal ratios using a calculator https://www.youtube.com/watch?v=V0jlXvJy27k
    Alternate video: https://www.youtube.com/watch?v=oHH3Gcfa_ps
14. Solving right triangles: https://www.youtube.com/watch?v=a5WQlcFTXyk
15. Application of trigonometric ratios. https://www.youtube.com/watch?v=3H28-wzsF3s

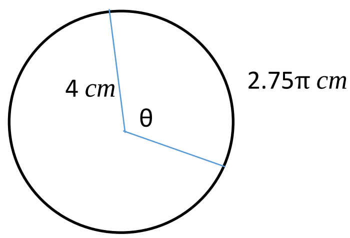
Practice problems: The following problems use the techniques demonstrated in the above videos.

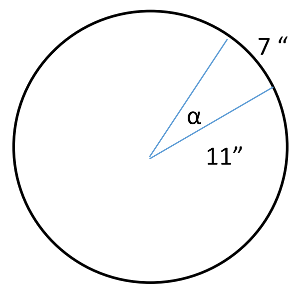
1) Convert the following degrees to radians. *(Give exact answers.)*
   a) $30°$ b) $275°$ c) $150°$

2) Convert the following radians to degrees (*round to 2 decimal places*)
   a) 1.2 b) 0.03 c) 2.569 d) Convert 2.569 to degrees-minute-seconds

3) Find the length of an arc of a circle of radius 11inches intercepted by a central angle of $\frac{\pi}{4}$.
   (Write the exact answer. Do not use calculator)

4) Find the area of the sector of a circle of radius 7m with a central angle of $\frac{\pi}{6}$. *(Write the exact answer. Do not use calculator)*

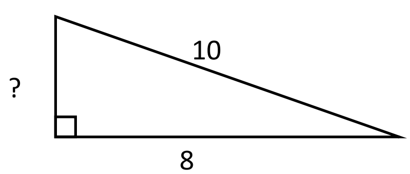
5) For the given arc length, find the central angle (in radians) and the area of the sectors below. *(Write the exact answer. Do not use calculator)*
   a)


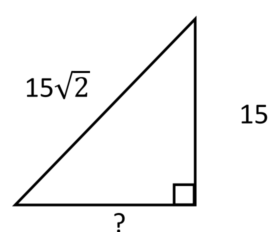
   b)


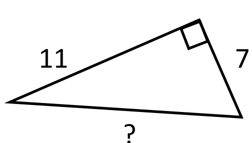
6) a) Find the linear and angular speed of a point on the outer edge of a flywheel of radius 10 cm if the flywheel is rotating at 85 times per second. *(Write the exact answer. Do not use calculator)*

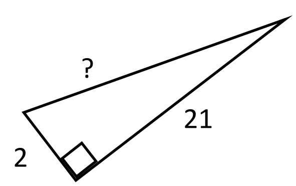
   b) Repeat for a point halfway between the center and the outer edge of the flywheel.

7) Determine if the following sides are sides of a right triangle
   a) 5, 6, 7 b) 12, 13, 18 c) 9, 12, 15

8) Find the missing side. *(Give exact answers)*
   a)


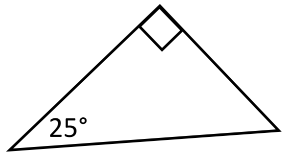
   b)


   c)


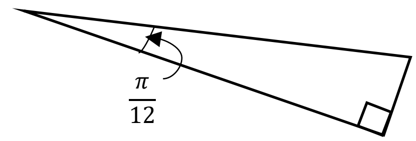
   d)


9) Find the missing angle

a)



b)



10) Fill in the blanks (assume the angle is θ)

a) $\sin\theta = \frac{\rule{2cm}{0.4pt}}{\textit{hypotenuse}}$ b) $\rule{2cm}{0.4pt} = \frac{\textit{adjacent}}{\textit{opposite}}$ c) $\csc\theta = \frac{\rule{2cm}{0.4pt}}{\textit{opposite}}$

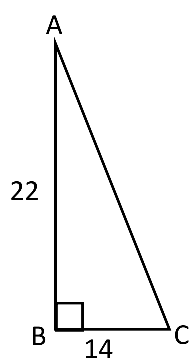
d) $\cos\theta = \frac{\textit{adjacent}}{\rule{2cm}{0.4pt}}$ e) $\cot\theta = \frac{\cos\theta}{\rule{2cm}{0.4pt}}$ f) $\sin\theta = \frac{1}{\rule{2cm}{0.4pt}}$

g) $\tan\theta = \frac{\rule{2cm}{0.4pt}}{\cot\theta}$ h) $\sec\theta = \frac{1}{\rule{2cm}{0.4pt}}$ i) $\rule{1cm}{0.4pt} = \frac{\textit{opposite}}{\textit{adjacent}}$

11) Two cars start at the same point. Car A travels due north at a speed of 80 mph and car B travels due west at a speed of 75mph. How far are the two cars from each other after traveling for 2 hours? *(Round the answer to the nearest mile)*

12) Find the indicated ratio. Give the exact answer. Do not use calculator

a)



tan C = ________

sin A=________

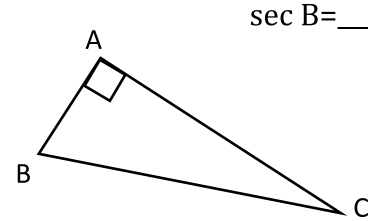
cos B=________

b) $m\angle C = \frac{\pi}{6}$

sin C = ___________

cot A=________

sec B=_________



c) $\tan\theta = \frac{5}{24}$ $(0 < \theta < 90°)$

cot θ = ____________

csc θ= _____________

cos θ= _____________

d) β = 45°

tan β = ___________

sec β=________

csc β=_________

13) Find the values of the following circular functions without using the calculator.

a) $\cos 30°$ b) $\tan\frac{\pi}{3}$ c) $\cot\frac{\pi}{4}$ d) $\csc 45^0$

e) $\csc 0°$ f) $\sin 90°$ g) $\sec\pi$ h) $\tan\pi/2$

14) Use a calculator to find the values of the following trigonometric function values: *(Round answers to 4 decimal places)*

a) $\cos 35°$ b) $\csc\,(209\frac{1}{6})°$ c) $\tan 58.23°$

d) $\cot 1.89\pi$ e) $\sin 2°\,10'$ f) $\sec 90°$

15) Anna is on the top of a building. She finds that the angle of elevation of the top another building is 32° 25' and angle of depression to the bottom of the same building is 68° 56'. What is the height in feet of the two buildings if it they are 52ft apart?

16) An isosceles triangle has a base of length 12". The angle opposite the base is 22°. Find the length of the two equal sides and the measure of the base angles. *(Round answers to nearest hundredths)*

Answers:

1)
a) $\frac{\pi}{6}$ *radians* b) $\frac{55}{36}\pi$ *radians* c) $\frac{5\pi}{6}$ *radians*

2)
a) 68.75° b) 1.72° c) 147.19° d) 147^0 11' 34"

3) $\frac{11\pi}{4}$

4) $\frac{49\pi}{12}$

5) a. 0.6875π *radians*
$A = 5.5\pi\ cm^2$
b. $\frac{7}{11}$ *radians*
$A = 38.5\ in^2$

6) a) $v = 1700\pi$ cm/sec
$\omega = 170\pi$ rad/sec
b) $v = 850\pi$ cm/sec
$\omega = 170\pi$ rad/sec

7) a. No b. No c. Yes

8) a. 6 b. 15 c. $\sqrt{170} \approx 13.04$ d. $\sqrt{445} \approx 21.095$

9) a. 65° b. $\frac{5\pi}{12}$ *radians*

10)

a) Opposite
b) cot θ
c) hypotenuse
d) hypotenuse
e) sin θ
f) csc θ
g) 1
h) cos θ
i) tan θ

11) 219 miles

12)

a) $\tan C = \frac{11}{7}$

$\sin A = \frac{7\sqrt{170}}{170}$

$\cos B = 0$

b) $\sin C = \frac{1}{2}$

$\cot A = 0$

$\sec B = 2$

c) $\cot \theta = \frac{24}{5}$

$\csc \theta = \frac{\sqrt{601}}{5}$

$\cos \theta = \frac{24\sqrt{601}}{601}$

d) $\tan \beta = 1$

$\sec \beta = \sqrt{2}$

$\csc \beta = \sqrt{2}$

13)

a) $\frac{\sqrt{3}}{2}$
b) $\sqrt{3}$
c) 1
d) $\sqrt{2}$
e) undefined
f) 1
g) -1
h) undefined

14) a.0.1892 b.-2.0519 c. 1.6128
d. -2.7776 e. 0.0378 f. 1.0001

15) 135 ft and 168 ft

16) 31.45” and 79°

Detailed Solutions

1) a) 30° we know that $1^\circ = \frac{\pi}{180}$ radian

Therefore, $30^\circ = \cancel{30} \cdot \frac{\pi}{6\cancel{180}} = \boxed{\frac{\pi}{6} \text{ radians}}$

b) 275° again, $1^\circ = \frac{\pi}{180}$ radians

Therefor, $275^\circ = \underset{55}{\cancel{275}} \cdot \frac{\pi}{\underset{36}{\cancel{180}}}$ radians

$= \boxed{\frac{55\pi}{36} \text{ radians}}$

c) 150° we know $1^\circ = \frac{\pi}{180}$ radians

$150^\circ = 5\cancel{150} \cdot \frac{\pi}{6\cancel{180}}$ radians

$= \boxed{\frac{5\pi}{6} \text{ radians}}$

2a) 1.2 radians, we know 1 rad $= \frac{180}{\pi}$ degrees

1.2 radians $= 1.2 \times \frac{180}{\pi}$ deg

$\boxed{= \left(\frac{216}{\pi}\right)^\circ \approx 68.75^\circ}$

b) 0.03 rad, 1 rad $= \left(\frac{180}{\pi}\right)$ deg

0.03 rad $= 0.03 \left(\frac{180}{\pi}\right)$ deg

$= \boxed{\frac{5.4}{\pi} \text{ deg} \approx 1.72^\circ}$

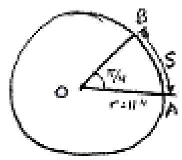
c) 2.569 rad, 1 rad $= \frac{180}{\pi}$ deg

2.569 rad $= 2.569 \left(\frac{180}{\pi}\right)$ deg

$= \boxed{\left(\frac{462.42}{\pi}\right) \text{ deg} \approx 147.19^\circ}$

d) $2.569\left(\frac{180}{\pi}\right) = 147.19285757^\circ = 147^\circ + .19285757^\circ(60 min/^\circ) = 147^\circ\ 11.57145' = 147^\circ\ 11' + .57145\left(60\frac{sec}{min}\right) = 147^\circ\ 11'\ 34"$

3)

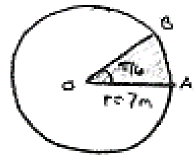


Let s = length of arc

$= r \cdot \theta$

$= (11)(\pi/4) = \boxed{\frac{11}{4}\pi \text{ in.}}$

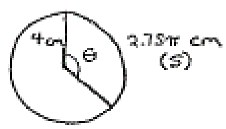
4)



Area of sector, $A = \frac{1}{2} r^2 \theta$ (θ in radians)

$A = \frac{1}{2}(7)^2\left(\frac{\pi}{6}\right) = \boxed{\frac{49}{12}\pi \text{ m}^2}$

5, a)



$s = r\theta$

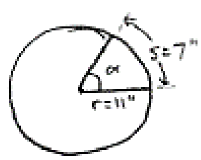
$2.75\pi = 4\theta$

$\theta = \frac{2.75}{4}\pi = \boxed{.6875\pi \text{ rad.}}$ or $\frac{11}{16}\pi$

$A_{sector} = \frac{1}{2} r^2 \theta = \frac{1}{2}(4)^2(.6875\pi)$

$= \frac{11}{2}\pi$ or $\boxed{5.5\pi \text{ cm}^2}$

b)



$s = r\alpha$

$7 = 11\alpha$

$\boxed{\alpha = \frac{7}{11} \text{ in.}}$

$A_{sector} = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(11)^2 \cdot \frac{7}{11}$

$\frac{77}{2}$ or $\boxed{38.5 \text{ in}^2}$

6. a) angular velocity $\omega = 85 \frac{\text{revolutions}}{\text{sec}} \cdot 2\pi \frac{\text{radians}}{\text{revolution}} = \boxed{170\pi \text{ rad./sec}}$

linear velocity $v = \frac{s}{t}$ or $r \cdot \omega = (10 \text{ cm})(170\pi) = \boxed{1700\pi \text{ cm/sec.}}$

or 17π m/sec.

b) For a point half-way from the center to the edge, the angular velocity will be the same, 170π rad/sec.

Linear velocity will change since the radius is changed.

$v = r \cdot \omega = 5(170\pi) = \boxed{850\pi \text{ cm/sec}}$ or 8.5π m/sec.

7 a) 5, 6, 7

If these are sides of a right triangle then they should satify the Pythagorean Theorem. Since hypotenuse is the longest side of a right triangle we can say

$5^2 + 6^2 = 7^2$

$25 + 36 = 49$

$61 = 49$ is not a true statement

Hence 5, 6, 7 are <u>not</u> the sides of a right triangle.

$\boxed{NO}$

b) 12, 13, 18.

For these side measures to be sides of a
right triangle, they have to satisfy the Pythago-
rean Theorem. Since hypotenuse is the longest
side of a right triangle we can say that

$12^2 + 13^2 = 18^2$

$144 + 169 = 324$

$213 = 324$ is a false statement

Hence 12, 13, 18 are <u>not</u> sides of a right triangle

$\boxed{NO}$

c) 9, 12, 15

The above side measures can be sides of a right triangle if and only if they satify the Pythagorean theorem. We know that the longest sides of a right triangle is the hypotenues. Therefore

$9^2 + 12^2 = 15^2$

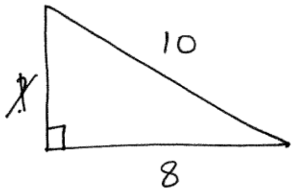
$81 + 144 = 225$

$225 = 225$ is a true statement

Hence 9, 12, 15 <u>are</u> sides of a right triangle.

$\boxed{Yes}$

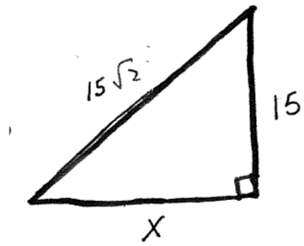
8 a)



Since this is a right triangle hence we can apply the Pythagorean Theorem. So,

$$x^2 + 8^2 = 10^2$$
$$x^2 + 64 = 100$$
$$\quad -64 \quad -64$$
$$\sqrt{x^2} = \sqrt{36}$$
$$\boxed{x = 6}$$

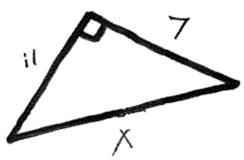
b)



Since this is a right triangle hence we can apply the Pythagorean Theorem. So,

$$15^2 + x^2 = (15\sqrt{2})^2$$
$$225 + x^2 = (225)(2)$$
$$225 + x^2 = 450$$
$$-225 \qquad -225$$
$$\sqrt{x^2} = \sqrt{225}$$
$$\boxed{x = 15}$$

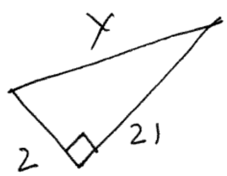
c)



Applying Pythagorean Theorem on this right triangle we get

$$11^2 + 7^2 = x^2$$
$$121 + 49 = x^2$$
$$\sqrt{170} = \sqrt{x^2}$$
$$\boxed{x = \sqrt{170}} \approx 13.04$$

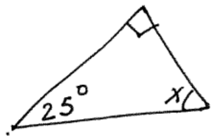
d)



Applying Pythagorean Theorem on this right triangle we get

$$2^2 + 21^2 = x^2$$
$$4 + 441 = x^2$$
$$\sqrt{445} = \sqrt{x^2}$$
$$\boxed{x = \sqrt{445}} \approx 21.095$$

9 a)



since sum of all the interior angles of a triangle is 180° we can say

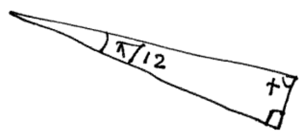
$$90° + 25° + x = 180°$$

$$\text{or} \quad 25° + x = 90°$$

$$-25 \qquad -25$$

$$\text{or} \quad \boxed{x = 65°}$$

b)



since sum of all the interior angles of a triang is 180° or π we can say

$$12\left(x + \frac{\pi}{12} + \frac{\pi}{2}\right) = (\pi)12 \quad \left[\frac{\pi}{2} = 90°\right]$$

$$12x + \pi + 6\pi = 12\pi$$

$$12x + 7\pi = 12\pi$$

$$-7\pi \qquad -7\pi$$

$$\frac{12x}{12} = \frac{5\pi}{12}$$

$$\boxed{x = \frac{5\pi}{12}}$$

10 a) $\sin\theta = \dfrac{\boxed{\text{opposite}}}{\text{hypotenuse}}$

b) $\boxed{\cot\theta} = \dfrac{\text{adjacent}}{\text{opposite}}$

c) $\csc\theta = \dfrac{\boxed{\text{hypotenuse}}}{\text{opposite}}$

d) $\cos\theta = \dfrac{\text{adjcent}}{\boxed{\text{hypotenuse}}}$

e) $\cot\theta = \dfrac{\cos\theta}{\boxed{\sin\theta}}$

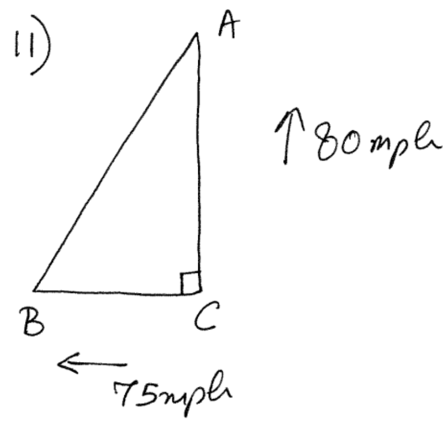
f) $\sin\theta = \dfrac{1}{\boxed{\csc\theta}}$

g) $\tan\theta = \dfrac{\boxed{1}}{\cot\theta}$

h) $\sec\theta = \dfrac{1}{\boxed{\cos\theta}}$

i) $\boxed{\tan\theta} = \dfrac{\text{opposite}}{\text{adjacent}}$

11)



$AC = 80 \times 2 = 160$ miles

$BC = 75 \times 2 = 150$ miles

since this forms a right triangle we can use the pythagorean theorem

Therefore $AB^2 = AC^2 + BC^2$

$= 160^2 + 150^2$

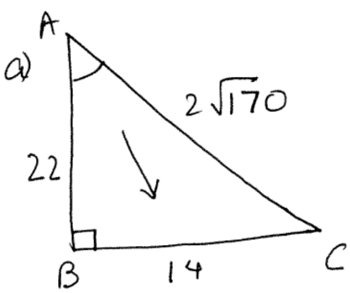
$= 25600 + 22500$

$= 48100$

$AB = \sqrt{48100}$

≈ 219 miles.

12) a)



using pythagorean theorem we get

$AC^2 = AB^2 + BC^2$

$= 22^2 + 14^2$

$= 484 + 196$

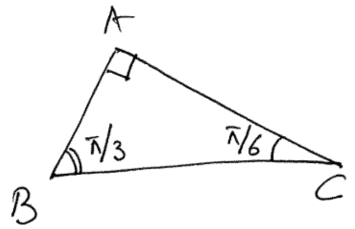
$= 680$

$AC = \sqrt{680} = \sqrt{4 \cdot 170} = 2\sqrt{170}$

$\tan C = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{\cancel{22}\,11}{\cancel{14}\,7} = \boxed{\dfrac{11}{7}}$

$\sin A = \dfrac{\text{opposite}}{\text{Hypotenuse}} = \dfrac{\cancel{14}\,7}{\cancel{2}\sqrt{170}} = \dfrac{7 \cdot \sqrt{170}}{\sqrt{170} \cdot \sqrt{170}} = \boxed{\dfrac{7\sqrt{170}}{170}}$

$\cos B = \cos 90^\circ = \boxed{0}$

12b) 

Since $\angle C = \pi/6$

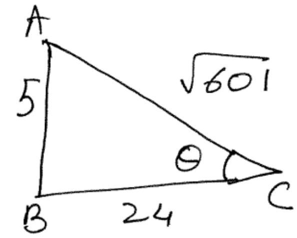
$\angle B = \frac{\pi \cdot 3}{2 \cdot 3} - \frac{\pi}{6} = \frac{3\pi - \pi}{6}$

$= \frac{\cancel{2}\pi}{\cancel{6}} = \frac{\pi}{3}$

$\sin C = \sin \frac{\pi}{6} = \boxed{\frac{1}{2}}$

$\cot A = \cot 90^\circ = \boxed{0}$

$\sec B = \sec \frac{\pi}{3} = \boxed{2}$



12c) $\tan \theta = \frac{5}{24} = \frac{\text{opposite}}{\text{adjacent}}$

using pythagorean theorem

$AC^2 = 5^2 + 24^2$

$= 25 + 576 = 601$

$AC = \sqrt{601}$

$\cot \theta = \frac{1}{\tan \theta} = \frac{24}{5}$

$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \boxed{\frac{\sqrt{601}}{5}}$

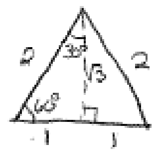
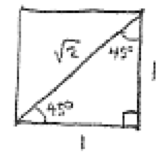
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{\sqrt{601}} \cdot \frac{\sqrt{601}}{\sqrt{601}} = \boxed{\frac{24\sqrt{601}}{601}}$

12d) $\beta = 45^\circ$

$\tan \beta = \tan 45^\circ = \boxed{1}$

$\sec \beta = \sec 45^\circ = \boxed{\sqrt{2}}$

$\csc \beta = \csc 45^\circ = \boxed{\sqrt{2}}$

13.  

a) $\cos 30° = \frac{adj}{hyp.} = \boxed{\frac{\sqrt{3}}{2}}$

b) $\tan\frac{\pi}{3} = \tan 60° = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \sqrt{3}$

c. $\cot\frac{\pi}{4} = \cot 45° = \frac{adj}{opp.} = \frac{1}{1} = 1$

d) $\csc 45° = \frac{hyp}{opp.} = \boxed{\sqrt{2}}$

e) $\sin 90° = \boxed{1}$

f. $\sec\pi = \frac{1}{\cos\pi} = \frac{1}{-1} = \boxed{-1}$

14. a) $\cos 35° = \boxed{0.8192}$

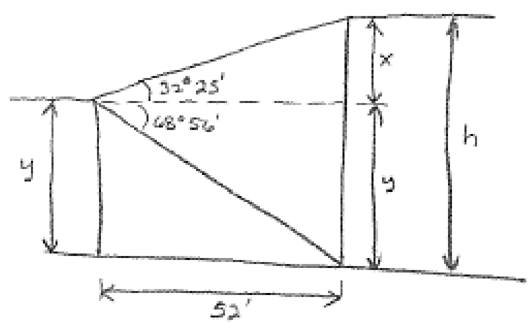
b) $\csc\left(209\frac{1}{6}\right)° = \frac{1}{\sin\left(209+\frac{1}{6}\right)} = \frac{1}{-.48735} \approx \boxed{-2.0519}$

c) $\tan(58.23°) = \boxed{1.6147}$

d) $\cot(1.89\pi) = \frac{1}{\tan(1.89\pi)} = \frac{1}{-.360022} \approx \boxed{-2.7776}$

e) $\sin(2°10') = \sin\left(2+\frac{10}{60}\right) \approx 0.0378$

f) $\sec 90° = \frac{1}{\cos 90°} = \frac{1}{0} = \boxed{\text{undefined}}$

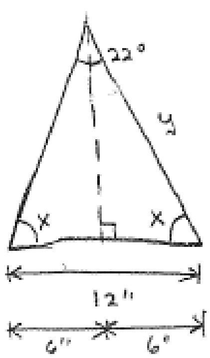
15. 

$\tan 32°25' = \frac{x}{52'} \rightarrow x = 52(.63503)$
$= 33.02$ ft.

$\tan 68°56' = \frac{y}{52} \rightarrow y = 52(2.596056)$
$= 134.99 \approx 135$ ft.

ht. of shorter building $= y = \boxed{135 \text{ ft.}}$

ht. of taller building $= h = x + y$
$= 33.02 + 134.99$
$= 168.015$ ft. $\approx \boxed{168 \text{ ft.}}$

16. 

Since all angles of a triangle add to 180°, $2x + 22 = 180$
$2x = 158$
$\boxed{x = 79°}$ = base angles.

To find y (hypotenuse), use adj. = 6" (half the 12" base.)

then, $\cos 79° = \frac{6}{y} \rightarrow y = \frac{6}{\cos 79°} = \frac{6}{.1908} = \boxed{31.45''}$

Additional Resources

Click on the links below to download worksheets under "Basics" for more practice:

1. Right triangle trigonometry
2. Angular and Linear Speed
3. Application Problems

Alternatively;

1. Go To https://www.kutasoftware.com/freeipc.html
2. Under "**Trigonometry**" click on:
   - Right triangle trigonometry
3. Go to https://studylib.net/doc/8934183/linear-and-angular-velocity-worksheet-w-answers
4. Go to https://cpb-us-e1.wpmucdn.com/cobblearning.net/dist/c/2995/files/2016/08/right_trig_word_problems-17ss2mh.pdf
5. You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets.
6. For help please contact the ***Math Assistance Area***.