# Package 'Matrix'

January 23, 2025

VersionNote do also bump src/version.h, inst/include/Matrix/version.h

Version 1.7-2

```
Date 2025-01-20
Priority recommended
Title Sparse and Dense Matrix Classes and Methods
Description A rich hierarchy of sparse and dense matrix classes,
      including general, symmetric, triangular, and diagonal matrices
      with numeric, logical, or pattern entries. Efficient methods for
      operating on such matrices, often wrapping the 'BLAS', 'LAPACK',
      and 'SuiteSparse' libraries.
License GPL (>= 2) | file LICENCE
URL https://Matrix.R-forge.R-project.org
BugReports https://R-forge.R-project.org/tracker/?atid=294&group_id=61
Contact Matrix-authors@R-project.org
Depends R (>= 4.4), methods
Imports grDevices, graphics, grid, lattice, stats, utils
Suggests MASS, datasets, sfsmisc, tools
Enhances SparseM, graph
LazyData no
LazyDataNote not possible, since we use data/*.R and our S4 classes
BuildResaveData no
Encoding UTF-8
NeedsCompilation yes
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2 Contents

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**Repository** CRAN

**Date/Publication** 2025-01-23 16:40:11 UTC

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abIndex-class

Class "abIndex" of Abstract Index Vectors

### Description

The "abIndex" class, short for "Abstract Index Vector", is used for dealing with large index vectors more efficiently, than using integer (or numeric) vectors of the kind 2:1000000 or c(0:1e5, 1000:1e6).

Note that the current implementation details are subject to change, and if you consider working with these classes, please contact the package maintainers (packageDescription("Matrix")\$Maintainer).

### **Objects from the Class**

Objects can be created by calls of the form new("abIndex", ...), but more easily and typically either by as(x, "abIndex") where x is an integer (valued) vector, or directly by abIseq() and combination c(...) of such.

#### Slots

kind: a character string, one of ("int32", "double", "rleDiff"), denoting the internal structure of the abIndex object.

x: Object of class "numLike"; is used (i.e., not of length 0) only iff the object is *not* compressed, i.e., currently exactly when kind != "rleDiff".

rleD: object of class "rleDiff", used for compression via rle.

#### Methods

```
as.numeric, as.integer, as.vector signature(x = "abIndex"): ...
[ signature(x = "abIndex", i = "index", j = "ANY", drop = "ANY"): ...
coerce signature(from = "numeric", to = "abIndex"): ...
coerce signature(from = "abIndex", to = "numeric"): ...
coerce signature(from = "abIndex", to = "integer"): ...
length signature(x = "abIndex"): ...
Ops signature(e1 = "numeric", e2 = "abIndex"): These and the following arithmetic and logic operations are not yet implemented; see Ops for a list of these (S4) group methods.
Ops signature(e1 = "abIndex", e2 = "abIndex"): ...
Ops signature(e1 = "abIndex", e2 = "numeric"): ...
Summary signature(x = "abIndex"): ...
show ("abIndex"): simple show method, building on show(<rleDiff>).
is.na ("abIndex"): works analogously to regular vectors.
is.finite, is.infinite ("abIndex"): ditto.
```

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#### Note

This is currently experimental and not yet used for our own code. Please contact us (packageDescription("Matrix")\$Main if you plan to make use of this class.

Partly builds on ideas and code from Jens Oehlschlaegel, as implemented (around 2008, in the GPL'ed part of) package **ff**.

#### See Also

```
rle (base) which is used here; numeric
```

### **Examples**

abIseq

Sequence Generation of "abIndex", Abstract Index Vectors

### **Description**

```
Generation of abstract index vectors, i.e., objects of class "abIndex".

abIseq() is designed to work entirely like seq, but producing "abIndex" vectors.

abIseq1() is its basic building block, where abIseq1(n,m) corresponds to n:m.

c(x, ...) will return an "abIndex" vector, when x is one.
```

#### Usage

### **Arguments**

```
from, to the starting and (maximal) end value of the sequence.

by number: increment of the sequence.

length.out desired length of the sequence. A non-negative number, which for seq and seq.int will be rounded up if fractional.

along.with take the length from the length of this argument.

in general an arbitrary number of R objects; here, when the first is an "abIndex" vector, these arguments will be concatenated to a new "abIndex" object.
```

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### Value

An abstract index vector, i.e., object of class "abIndex".

### See Also

the class abIndex documentation; rep2abI() for another constructor; rle (base).

#### **Examples**

all.equal-methods

Matrix Package Methods for Function all.equal()

### **Description**

Methods for function all.equal() (from R package base) are defined for all Matrix classes.

# Methods

```
target = "Matrix", current = "Matrix" \
target = "ANY", current = "Matrix" \
target = "Matrix", current = "ANY" these three methods are simply using all.equal.numeric
directly and work via as.vector().
```

There are more methods, notably also for "sparseVector"'s, see showMethods("all.equal").

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asUniqueT	Standardize a Sparse	Matrix in Triplet Format
asuniquei	Sianaaraize a Sparse	mairix in Tripiei Formai

**Description** 

Detect or standardize a TsparseMatrix with unsorted or duplicated (i, j) pairs.

# Usage

```
anyDuplicatedT(x, ...)
isUniqueT(x, byrow = FALSE, isT = is(x, "TsparseMatrix"))
asUniqueT(x, byrow = FALSE, isT = is(x, "TsparseMatrix"))
aggregateT(x)
```

### **Arguments**

Χ	an R object. anyDuplicatedT and aggregateT require x inheriting from TsparseMatrix.
	asUniqueT requires x inheriting from Matrix and coerces x to TsparseMatrix
	if necessary.
	optional arguments passed to the default method for generic function anyDuplicated.
byrow	a logical indicating if x should be sorted by row then by column.
isT	a logical indicating if x inherits from virtual class TsparseMatrix.

# Value

anyDuplicatedT(x) returns the index of the first duplicated (i, j) pair in x (0 if there are no duplicated pairs).

isUniqueT(x) returns TRUE if x is a TsparseMatrix with sorted, nonduplicated (i,j) pairs and FALSE otherwise.

asUniqueT(x) returns the unique TsparseMatrix representation of x with sorted, nonduplicated (i,j) pairs. Values corresponding to identical (i,j) pairs are aggregated by addition, where in the logical case "addition" refers to logical OR.

aggregateT(x) aggregates without sorting.

# See Also

Virtual class TsparseMatrix.

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```
anyDuplicatedT(T2) == 2,
           anyDuplicatedT(T2u) == 0,
           length(T2 @x) == 5,
           length(T2u@x) == 3)
isUniqueT(T2 ) # FALSE
isUniqueT(T2u) # TRUE
T3 <- T2u
T3[1, c(1,3)] \leftarrow 10; T3[2, c(1,5)] \leftarrow 20
T3u <- asUniqueT(T3)
str(T3u) # sorted in 'j', and within j, sorted in i
stopifnot(isUniqueT(T3u))
## Logical 1.TMatrix and n.TMatrix :
(L2 < - T2 > 0)
validObject(L2u <- asUniqueT(L2))</pre>
(N2 <- as(L2, "nMatrix"))
validObject(N2u <- asUniqueT(N2))</pre>
stopifnot(N2u@i == L2u@i, L2u@i == T2u@i, N2@i == L2@i, L2@i == T2@i,
          N2u@j == L2u@j, L2u@j == T2u@j, N2@j == L2@j, L2@j == T2@j)
# now with a nasty NA [partly failed in Matrix 1.1-5]:
L.0N <- L.1N <- L2
L.0N@x[1:2] \leftarrow c(FALSE, NA)
L.1N@x[1:2] \leftarrow c(TRUE, NA)
validObject(L.0N)
validObject(L.1N)
(m.0N \leftarrow as.matrix(L.0N))
(m.1N \leftarrow as.matrix(L.1N))
stopifnot(identical(10L, which(is.na(m.0N))), !anyNA(m.1N))
symnum(m.0N)
symnum(m.1N)
```

band-methods

Extract bands of a matrix

# **Description**

Return the matrix obtained by setting to zero elements below a diagonal (triu), above a diagonal (tril), or outside of a general band (band).

# Usage

```
band(x, k1, k2, ...)

triu(x, k = 0L, ...)

tril(x, k = 0L, ...)
```

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### Arguments

X	a matrix-like object
k, k1, k2	integers specifying the diagonals that are not set to zero, $k1 \le k2$ . These are interpreted relative to the main diagonal, which is $k = 0$ . Positive and negative values of k indicate diagonals above and below the main diagonal, respectively.
	optional arguments passed to methods, currently unused by package Matrix.

#### **Details**

```
triu(x, k) is equivalent to band(x, k, dim(x)[2]). Similarly, tril(x, k) is equivalent to band(x, -dim(x)[1], k).
```

### Value

An object of a suitable matrix class, inheriting from triangularMatrix where appropriate. It inherits from sparseMatrix if and only if x does.

#### Methods

```
    x = "CsparseMatrix" method for compressed, sparse, column-oriented matrices.
    x = "RsparseMatrix" method for compressed, sparse, row-oriented matrices.
    x = "TsparseMatrix" method for sparse matrices in triplet format.
    x = "diagonalMatrix" method for diagonal matrices.
    x = "denseMatrix" method for dense matrices in packed or unpacked format.
    x = "matrix" method for traditional matrices of implicit class matrix.
```

#### See Also

bandSparse for the construction of a banded sparse matrix directly from its non-zero diagonals.

```
## A random sparse matrix :
set.seed(7)
m \leftarrow matrix(0, 5, 5)
m[sample(length(m), size = 14)] \leftarrow rep(1:9, length=14)
(mm <- as(m, "CsparseMatrix"))</pre>
tril(mm)
              # lower triangle
tril(mm, -1) # strict lower triangle
triu(mm, 1) # strict upper triangle
band(mm, -1, 2) # general band
(m5 \leftarrow Matrix(rnorm(25), ncol = 5))
tril(m5) # lower triangle
tril(m5, -1) # strict lower triangle
triu(m5, 1) # strict upper triangle
band(m5, -1, 2) # general band
(m65 \leftarrow Matrix(rnorm(30), ncol = 5)) # not square
```

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```
triu(m65)  # result not "dtrMatrix" unless square
(sm5 <- crossprod(m65)) # symmetric
  band(sm5, -1, 1)# "dsyMatrix": symmetric band preserves symmetry property
as(band(sm5, -1, 1), "sparseMatrix")# often preferable
(sm <- round(crossprod(triu(mm/2)))) # sparse symmetric ("dsC*")
band(sm, -1,1) # remains "dsC", *however*
band(sm, -2,1) # -> "dgC"
```

bandSparse

Construct Sparse Banded Matrix from (Sup-/Super-) Diagonals

### **Description**

Construct a sparse banded matrix by specifying its non-zero sup- and super-diagonals.

### Usage

### **Arguments**

the matrix dimension (n, m) = (nrow, ncol). n, m integer vector of "diagonal numbers", with identical meaning as in band(\*, k), i.e., relative to the main diagonal, which is k=0. diagonals optional list of sub-/super- diagonals; if missing, the result will be a pattern matrix, i.e., inheriting from class nMatrix. diagonals can also be  $n' \times d$  matrix, where d <- length(k) and n' >= min(n, m). In that case, the sub-/super- diagonals are taken from the columns of diagonals, where only the first several rows will be used (typically) for off-diagonals. symmetric logical; if true the result will be symmetric (inheriting from class symmetricMatrix) and only the upper or lower triangle must be specified (via k and diagonals). character string, one of "C", "T", or "R", specifying the sparse representation repr to be used for the result, i.e., one from the super classes CsparseMatrix, TsparseMatrix, or RsparseMatrix. giveCsparse (deprecated, replaced with repr): logical indicating if the result should be a CsparseMatrix or a TsparseMatrix, where the default was TRUE, and now is determined from repr; very often Csparse matrices are more efficient subsequently, but not always.

#### Value

a sparse matrix (of class CsparseMatrix) of dimension  $n \times m$  with diagonal "bands" as specified.

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### See Also

band, for extraction of matrix bands; bdiag, diag, sparseMatrix, Matrix.

### **Examples**

```
diags <- list(1:30, 10*(1:20), 100*(1:20))
s1 <- bandSparse(13, k = -c(0:2, 6), diag = c(diags, diags[2]), symm=TRUE)
s1
s2 \leftarrow bandSparse(13, k = c(0:2, 6), diag = c(diags, diags[2]), symm=TRUE)
stopifnot(identical(s1, t(s2)), is(s1, "dsCMatrix"))
## a pattern Matrix of *full* (sub-)diagonals:
bk < -c(0:4, 7,9)
(s3 <- bandSparse(30, k = bk, symm = TRUE))
## If you want a pattern matrix, but with "sparse"-diagonals,
## you currently need to go via logical sparse:
lLis <- lapply(list(rpois(20, 2), rpois(20, 1), rpois(20, 3))[c(1:3, 2:3, 3:2)],</pre>
               as.logical)
(s4 <- bandSparse(20, k = bk, symm = TRUE, diag = lLis))
(s4. <- as(drop0(s4), "nsparseMatrix"))</pre>
n <- 1e4
bk < -c(0:5, 7,11)
bMat <- matrix(1:8, n, 8, byrow=TRUE)
bLis <- as.data.frame(bMat)</pre>
B <- bandSparse(n, k = bk, diag = bLis)</pre>
Bs <- bandSparse(n, k = bk, diag = bLis, symmetric=TRUE)</pre>
B [1:15, 1:30]
Bs[1:15, 1:30]
## can use a list *or* a matrix for specifying the diagonals:
stopifnot(identical(B, bandSparse(n, k = bk, diag = bMat)),
  identical(Bs, bandSparse(n, k = bk, diag = bMat, symmetric=TRUE))
          , inherits(B, "dtCMatrix") # triangular!
)
```

bdiag

Construct a Block Diagonal Matrix

# Description

Build a block diagonal matrix given several building block matrices.

# Usage

```
bdiag(...)
.bdiag(lst)
```

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### Arguments

```
individual matrices or a list of matrices.non-empty list of matrices.
```

#### **Details**

For non-trivial argument list, bdiag() calls .bdiag(). The latter maybe useful to programmers.

#### Value

A sparse matrix obtained by combining the arguments into a block diagonal matrix.

The value of bdiag() inherits from class CsparseMatrix, whereas .bdiag() returns a TsparseMatrix.

#### Note

This function has been written and is efficient for the case of relatively few block matrices which are typically sparse themselves.

It is currently *inefficient* for the case of many small dense block matrices. For the case of *many* dense  $k \times k$  matrices, the bdiag\_m() function in the 'Examples' is an order of magnitude faster.

### Author(s)

Martin Maechler, built on a version posted by Berton Gunter to R-help; earlier versions have been posted by other authors, notably Scott Chasalow to S-news. Doug Bates's faster implementation builds on TsparseMatrix objects.

#### See Also

Diagonal for constructing matrices of class diagonalMatrix, or kronecker which also works for "Matrix" inheriting matrices.

bandSparse constructs a banded sparse matrix from its non-zero sub-/super - diagonals.

Note that other CRAN R packages have own versions of bdiag() which return traditional matrices.

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```
## random (diagonal-)block-triangular matrices:
rblockTri <- function(nb, max.ni, lambda = 3) {</pre>
   .bdiag(replicate(nb, {
         n <- sample.int(max.ni, 1)</pre>
         tril(Matrix(rpois(n * n, lambda = lambda), n, n)) }))
}
(T4 <- rblockTri(4, 10, lambda = 1))
image(T1 <- rblockTri(12, 20))</pre>
##' Fast version of Matrix :: .bdiag() -- for the case of *many* (k x k) matrices:
##' @param lmat list(<mat1>, <mat2>, ....., <mat_N>) where each mat_j is a k x k 'matrix'
##' @return a sparse (N*k x N*k) matrix of class \code{"\linkS4class{dgCMatrix}"}.
bdiag_m <- function(lmat) {</pre>
    ## Copyright (C) 2016 Martin Maechler, ETH Zurich
    if(!length(lmat)) return(new("dgCMatrix"))
    stopifnot(is.list(lmat), is.matrix(lmat[[1]]),
               (k \leftarrow (d \leftarrow dim(lmat[[1]]))[1]) == d[2], \# k \times k
              all(vapply(lmat, dim, integer(2)) == k)) # all of them
    N <- length(lmat)
    if(N * k > .Machine$integer.max)
        stop("resulting matrix too large; would be M x M, with M=", N*k)
    M <- as.integer(N * k)</pre>
    ## result: an M x M matrix
    new("dgCMatrix", Dim = c(M,M),
        ## 'i :' maybe there's a faster way (w/o matrix indexing), but elegant?
        i = as.vector(matrix(0L:(M-1L), nrow=k)[, rep(seq_len(N), each=k)]),
        p = k * 0L:M,
        x = as.double(unlist(lmat, recursive=FALSE, use.names=FALSE)))
}
112 <- replicate(12, matrix(rpois(16, lambda = 6.4), 4, 4),
                  simplify=FALSE)
dim(T12 \leftarrow bdiag_m(112)) # 48 x 48
T12[1:20, 1:20]
```

boolmatmult-methods Boolean Arithmetic Matrix Products: %&% and Methods

# **Description**

For boolean or "pattern" matrices, i.e., R objects of class nMatrix, it is natural to allow matrix products using boolean instead of numerical arithmetic.

In package **Matrix**, we use the binary operator %&% (aka "infix") function) for this and provide methods for all our matrices and the traditional R matrices (see matrix).

### Value

a pattern matrix, i.e., inheriting from "nMatrix", or an "ldiMatrix" in case of a diagonal matrix.

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#### Methods

We provide methods for both the "traditional" (R base) matrices and numeric vectors and conceptually all matrices and sparseVectors in package **Matrix**.

```
signature(x = "ANY", y = "ANY")
signature(x = "ANY", y = "Matrix")
signature(x = "Matrix", y = "ANY")
signature(x = "nMatrix", y = "nMatrix")
signature(x = "nMatrix", y = "nsparseMatrix")
signature(x = "nsparseMatrix", y = "nMatrix")
signature(x = "nsparseMatrix", y = "nsparseMatrix")
signature(x = "sparseVector", y = "sparseVector")
```

#### Note

These boolean arithmetic matrix products had been newly introduced for **Matrix** 1.2.0 (March 2015). Its implementation has still not been tested extensively.

Originally, it was left unspecified how non-structural zeros, i.e., 0's as part of the M@x slot should be treated for numeric ("dMatrix") and logical ("lMatrix") sparse matrices. We now specify that boolean matrix products should behave as if applied to drop0(M), i.e., as if dropping such zeros from the matrix before using it.

Equivalently, for all matrices M, boolean arithmetic should work as if applied to M != 0 (or M != FALSE).

The current implementation ends up coercing both x and y to (virtual) class nsparseMatrix which may be quite inefficient for dense matrices. A future implementation may well return a matrix with **different** class, but the "same" content, i.e., the same matrix entries  $m_i j$ .

#### See Also

%\*%, crossprod(), or tcrossprod(), for (regular) matrix product methods.

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```
identical(crossprod(L), crossprod(N , boolArith=FALSE)))
crossprod(D, boolArith = TRUE) # pattern: "nsCMatrix"
crossprod(L, boolArith = TRUE) # ditto
crossprod(L, boolArith = FALSE) # numeric: "dsCMatrix"
```

BunchKaufman-class

Dense Bunch-Kaufman Factorizations

#### **Description**

Classes BunchKaufman and pBunchKaufman represent Bunch-Kaufman factorizations of  $n \times n$  real, symmetric matrices A, having the general form

$$A = UD_UU' = LD_LL'$$

where  $D_U$  and  $D_L$  are symmetric, block diagonal matrices composed of  $b_U$  and  $b_L$   $1 \times 1$  or  $2 \times 2$  diagonal blocks;  $U = \prod_{k=1}^{b_U} P_k U_k$  is the product of  $b_U$  row-permuted unit upper triangular matrices, each having nonzero entries above the diagonal in 1 or 2 columns; and  $L = \prod_{k=1}^{b_L} P_k L_k$  is the product of  $b_L$  row-permuted unit lower triangular matrices, each having nonzero entries below the diagonal in 1 or 2 columns.

These classes store the nonzero entries of the  $2b_U + 1$  or  $2b_L + 1$  factors, which are individually sparse, in a dense format as a vector of length nn (BunchKaufman) or n(n+1)/2 (pBunchKaufman), the latter giving the "packed" representation.

### **Slots**

Dim, Dimnames inherited from virtual class MatrixFactorization.

uplo a string, either "U" or "L", indicating which triangle (upper or lower) of the factorized symmetric matrix was used to compute the factorization and in turn how the x slot is partitioned.

x a numeric vector of length n\*n (BunchKaufman) or n\*(n+1)/2 (pBunchKaufman), where n=Dim[1]. The details of the representation are specified by the manual for LAPACK routines dsytrf and dsptrf.

perm an integer vector of length n=Dim[1] specifying row and column interchanges as described in the manual for LAPACK routines dsytrf and dsptrf.

### **Extends**

Class BunchKaufmanFactorization, directly. Class MatrixFactorization, by class BunchKaufmanFactorization, distance 2.

# Instantiation

Objects can be generated directly by calls of the form new("BunchKaufman", ...) or new("pBunchKaufman", ...), but they are more typically obtained as the value of BunchKaufman(x) for x inheriting from dsyMatrix or dspMatrix.

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#### Methods

```
coerce signature(from = "BunchKaufman", to = "dtrMatrix"): returns a dtrMatrix, useful
    for inspecting the internal representation of the factorization; see 'Note'.

coerce signature(from = "pBunchKaufman", to = "dtpMatrix"): returns a dtpMatrix, useful
    for inspecting the internal representation of the factorization; see 'Note'.

determinant signature(from = "p?BunchKaufman", logarithm = "logical"): computes the
    determinant of the factorized matrix A or its logarithm.

expand1 signature(x = "p?BunchKaufman"): see expand1-methods.

expand2 signature(x = "p?BunchKaufman"): see expand2-methods.

solve signature(a = "p?BunchKaufman", b = .): see solve-methods.
```

#### Note

In  ${\bf Matrix}$  < 1.6-0, class BunchKaufman extended  ${\tt dtrMatrix}$  and class pBunchKaufman extended  ${\tt dtpMatrix}$ , reflecting the fact that the internal representation of the factorization is fundamentally triangular: there are n(n+1)/2 "parameters", and these can be arranged systematically to form an  $n \times n$  triangular matrix.  ${\bf Matrix}$  1.6-0 removed these extensions so that methods would no longer be inherited from  ${\tt dtrMatrix}$  and  ${\tt dtpMatrix}$ . The availability of such methods gave the wrong impression that BunchKaufman and pBunchKaufman represent a (singular) matrix, when in fact they represent an ordered set of matrix factors.

The coercions as(., "dtrMatrix") and as(., "dtpMatrix") are provided for users who understand the caveats.

#### References

The LAPACK source code, including documentation; see https://netlib.org/lapack/double/dsytrf.f and https://netlib.org/lapack/double/dsptrf.f.

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Class dsyMatrix and its packed counterpart.

Generic functions BunchKaufman, expand1, and expand2.

```
showClass("BunchKaufman")
set.seed(1)

n <- 6L
(A <- forceSymmetric(Matrix(rnorm(n * n), n, n)))

## With dimnames, to see that they are propagated :
dimnames(A) <- rep.int(list(paste0("x", seq_len(n))), 2L)
(bk.A <- BunchKaufman(A))</pre>
```

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```
str(e.bk.A <- expand2(bk.A, complete = FALSE), max.level = 2L)</pre>
str(E.bk.A <- expand2(bk.A, complete = TRUE), max.level = 2L)</pre>
## Underlying LAPACK representation
(m.bk.A <- as(bk.A, "dtrMatrix"))</pre>
stopifnot(identical(as(m.bk.A, "matrix"), `dim<-`(bk.A@x, bk.A@Dim)))</pre>
## Number of factors is 2*b+1, b \le n, which can be nontrivial ...
(b <- (length(E.bk.A) - 1L) %/% 2L)
ae1 <- function(a, b, ...) all.equal(as(a, "matrix"), as(b, "matrix"), ...)</pre>
ae2 <- function(a, b, ...) ae1(unname(a), unname(b), ...)</pre>
## A ~ U DU U', U := prod(Pk Uk) in floating point
stopifnot(exprs = {
    identical(names(e.bk.A), c("U", "DU", "U."))
    identical(e.bk.A[["U" ]], Reduce(`%*%`, E.bk.A[seq_len(b)]))
    identical(e.bk.A[["U."]], t(e.bk.A[["U"]]))
    ae1(A, with(e.bk.A, U %*% DU %*% U.))
})
## Factorization handled as factorized matrix
b <- rnorm(n)</pre>
stopifnot(identical(det(A), det(bk.A)),
          identical(solve(A, b), solve(bk.A, b)))
```

BunchKaufman-methods Methods for Bunch-Kaufman Factorization

# **Description**

Computes the Bunch-Kaufman factorization of an  $n \times n$  real, symmetric matrix A, which has the general form

$$A = UD_UU' = LD_LL'$$

where  $D_U$  and  $D_L$  are symmetric, block diagonal matrices composed of  $b_U$  and  $b_L$   $1 \times 1$  or  $2 \times 2$  diagonal blocks;  $U = \prod_{k=1}^{b_U} P_k U_k$  is the product of  $b_U$  row-permuted unit upper triangular matrices, each having nonzero entries above the diagonal in 1 or 2 columns; and  $L = \prod_{k=1}^{b_L} P_k L_k$  is the product of  $b_L$  row-permuted unit lower triangular matrices, each having nonzero entries below the diagonal in 1 or 2 columns.

Methods are built on LAPACK routines dsytrf and dsptrf.

# Usage

```
BunchKaufman(x, ...)
## S4 method for signature 'dsyMatrix'
BunchKaufman(x, warnSing = TRUE, ...)
## S4 method for signature 'dspMatrix'
BunchKaufman(x, warnSing = TRUE, ...)
```

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```
## S4 method for signature 'matrix'
BunchKaufman(x, uplo = "U", ...)
```

# Arguments

X	a finite symmetric matrix or Matrix to be factorized. If x is square but not symmetric, then it will be <i>treated</i> as symmetric; see uplo.
warnSing	a logical indicating if a warning should be signaled for singular x.
uplo	a string, either "U" or "L", indicating which triangle of x should be used to compute the factorization.
	further arguments passed to or from methods.

### Value

An object representing the factorization, inheriting from virtual class BunchKaufmanFactorization. The specific class is BunchKaufman unless x inherits from virtual class packedMatrix, in which case it is pBunchKaufman.

### References

```
The LAPACK source code, including documentation; see https://netlib.org/lapack/double/dsytrf.f and https://netlib.org/lapack/double/dsptrf.f.
```

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

# See Also

Classes BunchKaufman and pBunchKaufman and their methods.

Classes dsyMatrix and dspMatrix.

Generic functions expand1 and expand2, for constructing matrix factors from the result.

Generic functions Cholesky, Schur, lu, and qr, for computing other factorizations.

```
showMethods("BunchKaufman", inherited = FALSE)
set.seed(0)

data(CAex, package = "Matrix")
class(CAex) # dgCMatrix
isSymmetric(CAex) # symmetric, but not formally

A <- as(CAex, "symmetricMatrix")
class(A) # dsCMatrix

## Have methods for denseMatrix (unpacked and packed),
## but not yet sparseMatrix ...
## Not run:
(bk.A <- BunchKaufman(A))</pre>
```

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```
## End(Not run)
(bk.A <- BunchKaufman(as(A, "unpackedMatrix")))
## A ~ U DU U' in floating point
str(e.bk.A <- expand2(bk.A), max.level = 2L)
stopifnot(all.equal(as(A, "matrix"), as(Reduce(`%*%`, e.bk.A), "matrix"))))</pre>
```

CAex

Albers' example Matrix with "Difficult" Eigen Factorization

# **Description**

An example of a sparse matrix for which eigen() seemed to be difficult, an unscaled version of this has been posted to the web, accompanying an E-mail to R-help (https://stat.ethz.ch/mailman/listinfo/r-help), by Casper J Albers, Open University, UK.

### Usage

```
data(CAex)
```

### **Format**

This is a  $72 \times 72$  symmetric matrix with 216 non-zero entries in five bands, stored as sparse matrix of class dgCMatrix.

#### **Details**

Historical note (2006-03-30): In earlier versions of R, eigen(CAex) fell into an infinite loop whereas eigen(CAex, EISPACK=TRUE) had been okay.

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cbind2-methods

'cbind()' and 'rbind()' recursively built on cbind2/rbind2

#### **Description**

The base functions cbind and rbind are defined for an arbitrary number of arguments and hence have the first formal argument . . . . Now, when S4 objects are found among the arguments, base cbind() and rbind() internally "dispatch" recursively, calling cbind2 or rbind2 respectively, where these have methods defined and so should dispatch appropriately.

cbind2() and rbind2() are from the **methods** package, i.e., standard R, and have been provided for binding together *two* matrices, where in **Matrix**, we have defined methods for these and the 'Matrix' matrices.

#### Usage

```
## cbind(..., deparse.level = 1)
## rbind(..., deparse.level = 1)

## S4 method for signature 'Matrix, Matrix'
cbind2(x, y, ...)
## S4 method for signature 'Matrix, Matrix'
rbind2(x, y, ...)
```

### **Arguments**

```
for [cr]bind, vector- or matrix-like R objects to be bound together; for [cr]bind2, further arguments passed to or from methods; see cbind and cbind2.

deparse.level integer controlling the construction of labels in the case of non-matrix-like arguments; see cbind.

x, y vector- or matrix-like R objects to be bound together.
```

#### Value

typically a 'matrix-like' object of a similar class as the first argument in . . . .

Note that sometimes by default, the result is a sparseMatrix if one of the arguments is (even in the case where this is not efficient). In other cases, the result is chosen to be sparse when there are more zero entries is than non-zero ones (as the default sparse in Matrix()).

### Author(s)

Martin Maechler

#### See Also

```
cbind, cbind2.
```

```
Our class definition help pages mentioning cbind2() and rbind2() methods: "denseMatrix", "diagonalMatrix", "indMatrix".
```

### **Examples**

CHMfactor-class

Sparse Cholesky Factorizations

# Description

CHMfactor is the virtual class of sparse Cholesky factorizations of  $n \times n$  real, symmetric matrices A, having the general form

$$P_1AP_1' = L_1DL_1' \stackrel{D_{jj} \ge 0}{=} LL'$$

or (equivalently)

$$A = P_1' L_1 D L_1' P_1 \stackrel{D_{jj} \ge 0}{=} P_1' L L' P_1$$

where  $P_1$  is a permutation matrix,  $L_1$  is a unit lower triangular matrix, D is a diagonal matrix, and  $L = L_1 \sqrt{D}$ . The second equalities hold only for positive semidefinite A, for which the diagonal entries of D are non-negative and  $\sqrt{D}$  is well-defined.

The implementation of class CHMfactor is based on CHOLMOD's C-level cholmod\_factor\_struct. Virtual subclasses CHMsimpl and CHMsuper separate the simplicial and supernodal variants. These have nonvirtual subclasses [dn]CHMsimpl and [dn]CHMsuper, where prefix 'd' and prefix 'n' are reserved for numeric and symbolic factorizations, respectively.

### Usage

isLDL(x)

### **Arguments**

Χ

an object inheriting from virtual class CHMfactor, almost always the result of a call to generic function Cholesky.

#### Value

isLDL(x) returns TRUE or FALSE: TRUE if x stores the lower triangular entries of  $L_1 - I + D$ , FALSE if x stores the lower triangular entries of L.

#### **Slots**

#### Of CHMfactor:

Dim, Dimnames inherited from virtual class MatrixFactorization.

colcount an integer vector of length Dim[1] giving an *estimate* of the number of nonzero entries in each column of the lower triangular Cholesky factor. If symbolic analysis was performed prior to factorization, then the estimate is exact.

- perm a 0-based integer vector of length Dim[1] specifying the permutation applied to the rows and columns of the factorized matrix. perm of length 0 is valid and equivalent to the identity permutation, implying no pivoting.
- type an integer vector of length 6 specifying details of the factorization. The elements correspond to members ordering, is\_ll, is\_super, is\_monotonic, maxcsize, and maxesize of the original cholmod\_factor\_struct. Simplicial and supernodal factorizations are distinguished by is\_super. Simplicial factorizations do not use maxcsize or maxesize. Supernodal factorizations do not use is\_ll or is\_monotonic.

### Of CHMsimpl (all unused by nCHMsimpl):

- nz an integer vector of length Dim[1] giving the number of nonzero entries in each column of the lower triangular Cholesky factor. There is at least one nonzero entry in each column, because the diagonal elements of the factor are stored explicitly.
- p an integer vector of length Dim[1]+1. Row indices of nonzero entries in column j of the lower triangular Cholesky factor are obtained as i[p[j]+seq\_len(nz[j])]+1.
- i an integer vector of length greater than or equal to sum(nz) containing the row indices of nonzero entries in the lower triangular Cholesky factor. These are grouped by column and sorted within columns, but the columns themselves need not be ordered monotonically. Columns may be overallocated, i.e., the number of elements of i reserved for column j may exceed nz[j].
- prv, nxt integer vectors of length Dim[1]+2 indicating the order in which the columns of the lower triangular Cholesky factor are stored in i and x. Starting from j <- Dim[1]+2, the recursion j <- nxt[j+1]+1 traverses the columns in forward order and terminates when nxt[j+1] = -1. Starting from j <- Dim[1]+1, the recursion j <- prv[j+1]+1 traverses the columns in backward order and terminates when prv[j+1] = -1.</li>

### Of dCHMsimpl:

x a numeric vector parallel to i containing the corresponding nonzero entries of the lower triangular Cholesky factor L or (if and only if type[2] is 0) of the lower triangular matrix  $L_1 - I + D$ .

### Of CHMsuper:

- super, pi, px integer vectors of length nsuper+1, where nsuper is the number of supernodes.
  super[j]+1 is the index of the leftmost column of supernode j. The row indices of supernode
  j are obtained as s[pi[j]+seq\_len(pi[j+1]-pi[j])]+1. The numeric entries of supernode
  j are obtained as x[px[j]+seq\_len(px[j+1]-px[j])]+1 (if slot x is available).
- s an integer vector of length greater than or equal to Dim[1] containing the row indices of the supernodes. s may contain duplicates, but not within a supernode, where the row indices must be increasing.

### Of dCHMsuper:

x a numeric vector of length less than or equal to prod(Dim) containing the numeric entries of the supernodes.

#### **Extends**

Class MatrixFactorization, directly.

#### Instantiation

Objects can be generated directly by calls of the form new("dCHMsimpl", ...), etc., but dCHMsimpl and dCHMsuper are more typically obtained as the value of Cholesky(x, ...) for x inheriting from sparseMatrix (often dsCMatrix).

There is currently no API outside of calls to new for generating nCHMsimpl and nCHMsuper. These classes are vestigial and may be formally deprecated in a future version of **Matrix**.

#### Methods

```
coerce signature(from = "CHMsimpl", to = "dtCMatrix"): returns a dtCMatrix representing the lower triangular Cholesky factor L or the lower triangular matrix L_1 - I + D, the latter if and only if from@type[2] is 0.
```

coerce signature(from = "CHMsuper", to = "dgCMatrix"): returns a dgCMatrix representing the lower triangular Cholesky factor L. Note that, for supernodes spanning two or more columns, the supernodal algorithm by design stores non-structural zeros above the main diagonal, hence dgCMatrix is indeed more appropriate than dtCMatrix as a coercion target.

determinant signature(from = "CHMfactor", logarithm = "logical"): behaves according to an optional argument sqrt. If sqrt = FALSE, then this method computes the determinant of the factorized matrix A or its logarithm. If sqrt = TRUE, then this method computes the determinant of the factor  $L = L_1 sqrt(D)$  or its logarithm, giving NaN for the modulus when D has negative diagonal elements. For backwards compatibility, the default value of sqrt is TRUE, but that can be expected change in a future version of  $\mathbf{Matrix}$ , hence defensive code will always set sqrt (to TRUE, if the code must remain backwards compatible with  $\mathbf{Matrix} < 1.6$ -0). Calls to this method not setting sqrt may warn about the pending change. The warnings can be disabled with options(Matrix.warnSqrtDefault = 0).

diag signature(x = "CHMfactor"): returns a numeric vector of length n containing the diagonal elements of D, which (if they are all non-negative) are the squared diagonal elements of L.

```
expand signature(x = "CHMfactor"): see expand-methods.
expand1 signature(x = "CHMsimpl"): see expand1-methods.
expand1 signature(x = "CHMsuper"): see expand1-methods.
expand2 signature(x = "CHMsimpl"): see expand2-methods.
expand2 signature(x = "CHMsuper"): see expand2-methods.
image signature(x = "CHMfactor"): see image-methods.
nnzero signature(x = "CHMfactor"): see nnzero-methods.
solve signature(a = "CHMfactor", b = .): see solve-methods.
```

update signature(object = "CHMfactor"): returns a copy of object with the same nonzero pattern but with numeric entries updated according to additional arguments parent and mult, where parent is (coercible to) a dsCMatrix or a dgCMatrix and mult is a numeric vector of positive length.

The numeric entries are updated with those of the Cholesky factor of F(parent) + mult[1] \* I, i.e., F(parent) plus mult[1] times the identity matrix, where F = identity for symmetric parent and F = tcrossprod for other parent. The nonzero pattern of F(parent) must match that of S if object = Cholesky(S, ...).

updown signature(update = ., C = ., object = "CHMfactor"): see updown-methods.

#### References

The CHOLMOD source code; see <a href="https://github.com/DrTimothyAldenDavis/SuiteSparse">https://github.com/DrTimothyAldenDavis/SuiteSparse</a>, notably the header file 'CHOLMOD/Include/cholmod.h' defining cholmod\_factor\_struct.

Chen, Y., Davis, T. A., Hager, W. W., & Rajamanickam, S. (2008). Algorithm 887: CHOLMOD, supernodal sparse Cholesky factorization and update/downdate. *ACM Transactions on Mathematical Software*, *35*(3), Article 22, 1-14. doi:10.1145/1391989.1391995

Amestoy, P. R., Davis, T. A., & Duff, I. S. (2004). Algorithm 837: AMD, an approximate minimum degree ordering algorithm. *ACM Transactions on Mathematical Software*, *17*(4), 886-905. doi:10.1145/1024074.1024081

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Class dsCMatrix.

Generic functions Cholesky, updown, expand1 and expand2.

```
showClass("dCHMsimpl")
showClass("dCHMsuper")
set.seed(2)

m <- 1000L
n <- 200L
M <- rsparsematrix(m, n, 0.01)
A <- crossprod(M)

## With dimnames, to see that they are propagated :
dimnames(A) <- dn <- rep.int(list(paste0("x", seq_len(n))), 2L)

(ch.A <- Cholesky(A)) # pivoted, by default
str(e.ch.A <- expand2(ch.A, LDL = TRUE), max.level = 2L)
str(E.ch.A <- expand2(ch.A, LDL = FALSE), max.level = 2L)

ae1 <- function(a, b, ...) all.equal(as(a, "matrix"), as(b, "matrix"), ...)
ae2 <- function(a, b, ...) ae1(unname(a), unname(b), ...)</pre>
```

```
## A ~ P1' L1 D L1' P1 ~ P1' L L' P1 in floating point
stopifnot(exprs = {
    identical(names(e.ch.A), c("P1.", "L1", "D", "L1.", "P1"))
    identical(names(E.ch.A), c("P1.", "L" ,
                                                "L." , "P1"))
    identical(e.ch.A[["P1"]],
             new("pMatrix", Dim = c(n, n), Dimnames = c(list(NULL), dn[2L]),
                  margin = 2L, perm = invertPerm(ch.A@perm, 0L, 1L)))
    identical(e.ch.A[["P1."]], t(e.ch.A[["P1"]]))
    identical(e.ch.A[["L1."]], t(e.ch.A[["L1"]]))
    identical(E.ch.A[["L."]], t(E.ch.A[["L"]]))
    identical(e.ch.A[["D"]], Diagonal(x = diag(ch.A)))
   all.equal(E.ch.A[["L"]], with(e.ch.A, L1 %*% sqrt(D)))
   ae1(A, with(e.ch.A, P1. %*% L1 %*% D %*% L1. %*% P1))
   ae1(A, with(E.ch.A, P1. %*% L %*%
                                               L. %*% P1))
    ae2(A[ch.A@perm + 1L, ch.A@perm + 1L], with(e.ch.A, L1 %*% D %*% L1.))
    ae2(A[ch.A@perm + 1L, ch.A@perm + 1L], with(E.ch.A, L %*%
})
## Factorization handled as factorized matrix
## (in some cases only optionally, depending on arguments)
b <- rnorm(n)
stopifnot(identical(det(A), det(ch.A, sqrt = FALSE)),
          identical(solve(A, b), solve(ch.A, b, system = "A")))
u1 \leftarrow update(ch.A, A, mult = sqrt(2))
u2 <- update(ch.A, t(M), mult = sqrt(2)) # updating with crossprod(M), not M
stopifnot(all.equal(u1, u2, tolerance = 1e-14))
```

chol-methods

Compute the Cholesky Factor of a Matrix

### **Description**

Computes the upper triangular Cholesky factor of an  $n \times n$  real, symmetric, positive semidefinite matrix A, optionally after pivoting. That is the factor L' in

$$P_1AP_1' = LL'$$

or (equivalently)

$$A = P_1' L L' P_1$$

where  $P_1$  is a permutation matrix.

Methods for denseMatrix are built on LAPACK routines dpstrf, dpotrf, and dpptrf, The latter two do not permute rows or columns, so that  $P_1$  is an identity matrix.

Methods for sparseMatrix are built on CHOLMOD routines cholmod\_analyze and cholmod\_factorize\_p.

### Usage

```
chol(x, ...)
## S4 method for signature 'dsyMatrix'
chol(x, pivot = FALSE, tol = -1, ...)
## S4 method for signature 'dspMatrix'
chol(x, ...)
## S4 method for signature 'dsCMatrix'
chol(x, pivot = FALSE, ...)
## S4 method for signature 'ddiMatrix'
chol(x, ...)
## S4 method for signature 'generalMatrix'
chol(x, uplo = "U", ...)
## S4 method for signature 'triangularMatrix'
chol(x, uplo = "U", ...)
```

#### **Arguments**

X	a finite, symmetric, positive semidefinite matrix or Matrix to be factorized. If x is square but not symmetric, then it will be <i>treated</i> as symmetric; see uplo. Methods for dense x require positive definiteness when pivot = FALSE. Methods for sparse (but not diagonal) x require positive definiteness unconditionally.
pivot	a logical indicating if the rows and columns of $x$ should be pivoted. Methods for sparse x employ the approximate minimum degree (AMD) algorithm in order to reduce fill-in, i.e., without regard for numerical stability.
tol	a finite numeric tolerance, used only if pivot = TRUE. The factorization algorithm stops if the pivot is less than or equal to tol. Negative tol is equivalent to $nrow(x) * .Machine$double.eps * max(diag(x)).$
uplo	a string, either "U" or "L", indicating which triangle of $x$ should be used to compute the factorization. The default is "U", even for lower triangular $x$ , to be consistent with chol from <b>base</b> .
	further arguments passed to or from methods.

# **Details**

For x inheriting from diagonalMatrix, the diagonal result is computed directly and without pivoting, i.e., bypassing CHOLMOD.

For all other x, chol(x, pivot = value) calls Cholesky(x, perm = value, ...) under the hood. If you must know the permutation  $P_1$  in addition to the Cholesky factor L', then call Cholesky directly, as the result of chol(x, pivot = TRUE) specifies L' but not  $P_1$ .

# Value

A matrix, triangularMatrix, or diagonalMatrix representing the upper triangular Cholesky factor L'. The result is a traditional matrix if x is a traditional matrix, dense if x is dense, and sparse if x is sparse.

#### References

The LAPACK source code, including documentation; see https://netlib.org/lapack/double/dpstrf.f, https://netlib.org/lapack/double/dpotrf.f, and https://netlib.org/lapack/double/dpptrf.f.

The CHOLMOD source code; see <a href="https://github.com/DrTimothyAldenDavis/SuiteSparse">https://github.com/DrTimothyAldenDavis/SuiteSparse</a>, notably the header file 'CHOLMOD/Include/cholmod.h' defining cholmod\_factor\_struct.

Chen, Y., Davis, T. A., Hager, W. W., & Rajamanickam, S. (2008). Algorithm 887: CHOLMOD, supernodal sparse Cholesky factorization and update/downdate. *ACM Transactions on Mathematical Software*, *35*(3), Article 22, 1-14. doi:10.1145/1391989.1391995

Amestoy, P. R., Davis, T. A., & Duff, I. S. (2004). Algorithm 837: AMD, an approximate minimum degree ordering algorithm. *ACM Transactions on Mathematical Software*, *17*(4), 886-905. doi:10.1145/1024074.1024081

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

The default method from **base**, chol, called for traditional matrices x.

Generic function Cholesky, for more flexibility notably when computing the Cholesky factorization and not only the factor L'.

```
showMethods("chol", inherited = FALSE)
set.seed(0)
## --- Dense -----
## chol(x, pivot = value) wrapping Cholesky(x, perm = value)
selectMethod("chol", "dsyMatrix")
## Except in packed cases where pivoting is not yet available
selectMethod("chol", "dspMatrix")
## .... Positive definite ......
(A1 \leftarrow new("dsyMatrix", Dim = c(2L, 2L), x = c(1, 2, 2, 5)))
(R1.nopivot <- chol(A1))
(R1 <- chol(A1, pivot = TRUE))
## In 2-by-2 cases, we know that the permutation is 1:2 or 2:1,
## even if in general 'chol' does not say ...
stopifnot(exprs = {
                       , as(crossprod(R1.nopivot), "dsyMatrix"))
  all.equal( A1
  all.equal(t(A1[2:1, 2:1]), as(crossprod(R1
                                         ), "dsyMatrix"))
  identical(Cholesky(A1)@perm, 2:1) # because 5 > 1
})
```

```
## .... Positive semidefinite but not positive definite .....
(A2 \leftarrow new("dpoMatrix", Dim = c(2L, 2L), x = c(1, 2, 2, 4)))
try(R2.nopivot <- chol(A2)) # fails as not positive definite</pre>
(R2 <- chol(A2, pivot = TRUE)) # returns, with a warning and ...
stopifnot(exprs = {
  all.equal(t(A2[2:1, 2:1]), as(crossprod(R2), "dsyMatrix"))
  identical(Cholesky(A2)@perm, 2:1) # because 4 > 1
})
## .... Not positive semidefinite .....
(A3 \leftarrow new("dsyMatrix", Dim = c(2L, 2L), x = c(1, 2, 2, 3)))
try(R3.nopivot <- chol(A3)) # fails as not positive definite</pre>
(R3 <- chol(A3, pivot = TRUE)) # returns, with a warning and ...
## _Not_ equal: see details and examples in help("Cholesky")
all.equal(t(A3[2:1, 2:1]), as(crossprod(R3), "dsyMatrix"))
## ---- Sparse -------
## chol(x, pivot = value) wrapping
## Cholesky(x, perm = value, LDL = FALSE, super = FALSE)
selectMethod("chol", "dsCMatrix")
## Except in diagonal cases which are handled "directly"
selectMethod("chol", "ddiMatrix")
(A4 <- toeplitz(as(c(10, 0, 1, 0, 3), "sparseVector")))
(ch.A4.nopivot <- Cholesky(A4, perm = FALSE, LDL = FALSE, super = FALSE))</pre>
(ch.A4 <- Cholesky(A4, perm = TRUE, LDL = FALSE, super = FALSE))</pre>
(R4.nopivot <- chol(A4))
(R4 <- chol(A4, pivot = TRUE))
det4 <- det(A4)
b4 <- rnorm(5L)
x4 \leftarrow solve(A4, b4)
stopifnot(exprs = {
    identical(R4.nopivot, expand1(ch.A4.nopivot, "L."))
   identical(R4, expand1(ch.A4, "L."))
   all.equal(A4, crossprod(R4.nopivot))
   all.equal(A4[ch.A4@perm + 1L, ch.A4@perm + 1L], crossprod(R4))
   all.equal(diag(R4.nopivot), sqrt(diag(ch.A4.nopivot)))
   all.equal(diag(R4), sqrt(diag(ch.A4)))
   all.equal(sqrt(det4), det(R4.nopivot))
   all.equal(sqrt(det4), det(R4))
   all.equal(det4, det(ch.A4.nopivot, sqrt = FALSE))
   all.equal(det4, det(ch.A4, sqrt = FALSE))
   all.equal(x4, solve(R4.nopivot, solve(t(R4.nopivot), b4)))
   all.equal(x4, solve(ch.A4.nopivot, b4))
   all.equal(x4, solve(ch.A4, b4))
```

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})

chol2inv-methods

Inverse from Cholesky Factor

### Description

Given formally upper and lower triangular matrices U and L, compute  $(U'U)^{-1}$  and  $(LL')^{-1}$ , respectively.

This function can be seen as way to compute the inverse of a symmetric positive definite matrix given its Cholesky factor. Equivalently, it can be seen as a way to compute  $(X'X)^{-1}$  given the R part of the QR factorization of X, if R is constrained to have positive diagonal entries.

### Usage

```
chol2inv(x, ...)
## S4 method for signature 'dtrMatrix'
chol2inv(x, ...)
## S4 method for signature 'dtCMatrix'
chol2inv(x, ...)
## S4 method for signature 'generalMatrix'
chol2inv(x, uplo = "U", ...)
```

# **Arguments**

Х	a square matrix or Matrix, typically the result of a call to chol. If x is square but not (formally) triangular, then only the upper or lower triangle is considered, depending on optional argument uplo if x is a Matrix.
uplo	a string, either "U" or "L", indicating which triangle of x contains the Cholesky factor. The default is "U", to be consistent with $chol2inv$ from <b>base</b> .
	further arguments passed to or from methods.

#### Value

A matrix, symmetricMatrix, or diagonalMatrix representing the inverse of the positive definite matrix whose Cholesky factor is x. The result is a traditional matrix if x is a traditional matrix, dense if x is dense, and sparse if x is sparse.

#### See Also

The default method from base, chol2inv, called for traditional matrices x.

Generic function cho1, for computing the upper triangular Cholesky factor  $L^\prime$  of a symmetric positive semidefinite matrix.

Generic function solve, for solving linear systems and (as a corollary) for computing inverses more generally.

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### **Examples**

```
(A <- Matrix(cbind(c(1, 1, 1), c(1, 2, 4), c(1, 4, 16))))
(R <- chol(A))
(L <- t(R))
(R2i <- chol2inv(R))
(L2i <- chol2inv(R))
stopifnot(exprs = {
    all.equal(R2i, tcrossprod(solve(R)))
    all.equal(L2i, crossprod(solve(L)))
    all.equal(as(R2i %*% A, "matrix"), diag(3L)) # the identity
    all.equal(as(L2i %*% A, "matrix"), diag(3L)) # ditto
})</pre>
```

Cholesky-class

Dense Cholesky Factorizations

### **Description**

Classes Cholesky and pCholesky represent dense, pivoted Cholesky factorizations of  $n \times n$  real, symmetric, positive semidefinite matrices A, having the general form

$$P_1AP_1' = L_1DL_1' = LL'$$

or (equivalently)

$$A = P_1' L_1 D L_1' P_1 = P_1' L L' P_1$$

where  $P_1$  is a permutation matrix,  $L_1$  is a unit lower triangular matrix, D is a non-negative diagonal matrix, and  $L = L_1 \sqrt{D}$ .

These classes store the entries of the Cholesky factor L or its transpose L' in a dense format as a vector of length nn (Cholesky) or n(n+1)/2 (pCholesky), the latter giving the "packed" representation.

#### Slots

Dim, Dimnames inherited from virtual class MatrixFactorization.

uplo a string, either "U" or "L", indicating which triangle (upper or lower) of the factorized symmetric matrix was used to compute the factorization and in turn whether x stores L' or L.

x a numeric vector of length n\*n (Cholesky) or n\*(n+1)/2 (pCholesky), where n=Dim[1], listing the entries of the Cholesky factor L or its transpose L' in column-major order.

perm a 1-based integer vector of length Dim[1] specifying the permutation applied to the rows and columns of the factorized matrix. perm of length 0 is valid and equivalent to the identity permutation, implying no pivoting.

#### **Extends**

Class CholeskyFactorization, directly. Class MatrixFactorization, by class CholeskyFactorization, distance 2.

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#### Instantiation

Objects can be generated directly by calls of the form new("Cholesky", ...) or new("pCholesky", ...), but they are more typically obtained as the value of Cholesky(x) for x inheriting from dsyMatrix or dspMatrix (often the subclasses of those reserved for positive semidefinite matrices, namely dpoMatrix and dppMatrix).

#### Methods

```
coerce signature(from = "Cholesky", to = "dtrMatrix"): returns a dtrMatrix representing the Cholesky factor L or its transpose L'; see 'Note'. coerce signature(from = "pCholesky", to = "dtpMatrix"): returns a dtpMatrix representing the Cholesky factor L or its transpose L'; see 'Note'. determinant signature(from = "p?Cholesky", logarithm = "logical"): computes the determinant of the factorized matrix A or its logarithm.
```

diag signature(x = "p?Cholesky"): returns a numeric vector of length n containing the diagonal elements of D, which are the squared diagonal elements of L.

```
expand1 signature(x = "p?Cholesky"): see expand1-methods.
expand2 signature(x = "p?Cholesky"): see expand2-methods.
solve signature(a = "p?Cholesky", b = .): see solve-methods.
```

#### Note

In Matrix < 1.6-0, class Cholesky extended dtrMatrix and class pCholesky extended dtpMatrix, reflecting the fact that the factor L is indeed a triangular matrix. Matrix = 1.6-0 removed these extensions so that methods would no longer be inherited from dtrMatrix and dtpMatrix. The availability of such methods gave the wrong impression that Cholesky and pCholesky represent a (singular) matrix, when in fact they represent an ordered set of matrix factors.

The coercions as(., "dtrMatrix") and as(., "dtpMatrix") are provided for users who understand the caveats.

#### References

The LAPACK source code, including documentation; see https://netlib.org/lapack/double/dpstrf.f, https://netlib.org/lapack/double/dpotrf.f, and https://netlib.org/lapack/double/dpptrf.f.

Lucas, C. (2004). *LAPACK-style codes for level 2 and 3 pivoted Cholesky factorizations*. LAPACK Working Note, Number 161. https://www.netlib.org/lapack/lawnspdf/lawn161.pdf

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Class CHMfactor for sparse Cholesky factorizations.

Classes dpoMatrix and dppMatrix.

Generic functions Cholesky, expand1 and expand2.

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```
showClass("Cholesky")
set.seed(1)
m <- 30L
n <- 6L
(A <- crossprod(Matrix(rnorm(m * n), m, n)))</pre>
## With dimnames, to see that they are propagated :
dimnames(A) <- dn <- rep.int(list(paste0("x", seq_len(n))), 2L)</pre>
(ch.A <- Cholesky(A)) # pivoted, by default
str(e.ch.A <- expand2(ch.A, LDL = TRUE), max.level = 2L)</pre>
str(E.ch.A <- expand2(ch.A, LDL = FALSE), max.level = 2L)</pre>
## Underlying LAPACK representation
(m.ch.A <- as(ch.A, "dtrMatrix")) # which is L', not L, because
A@uplo == "U"
stopifnot(identical(as(m.ch.A, "matrix"), `dim<-`(ch.A@x, ch.A@Dim)))</pre>
ae1 <- function(a, b, ...) all.equal(as(a, "matrix"), as(b, "matrix"), ...)</pre>
ae2 <- function(a, b, ...) ae1(unname(a), unname(b), ...)</pre>
## A ~ P1' L1 D L1' P1 ~ P1' L L' P1 in floating point
stopifnot(exprs = {
    identical(names(e.ch.A), c("P1.", "L1", "D", "L1.", "P1"))
    identical(names(E.ch.A), c("P1.", "L" , "L." , "P1"))
    identical(e.ch.A[["P1"]],
              new("pMatrix", Dim = c(n, n), Dimnames = c(list(NULL), dn[2L]),
                  margin = 2L, perm = invertPerm(ch.A@perm)))
    identical(e.ch.A[["P1."]], t(e.ch.A[["P1"]]))
    identical(e.ch.A[["L1."]], t(e.ch.A[["L1"]]))
    identical(E.ch.A[["L."]], t(E.ch.A[["L"]]))
    identical(e.ch.A[["D"]], Diagonal(x = diag(ch.A)))
    all.equal(E.ch.A[["L"]], with(e.ch.A, L1 %*% sqrt(D)))
    ae1(A, with(e.ch.A, P1. %*% L1 %*% D %*% L1. %*% P1))
    ae1(A, with(E.ch.A, P1. %*% L %*% L. %*% P1))
    ae2(A[ch.A@perm, ch.A@perm], with(e.ch.A, L1 %*% D %*% L1.))
    ae2(A[ch.A@perm, ch.A@perm], with(E.ch.A, L %*%
})
## Factorization handled as factorized matrix
b <- rnorm(n)
all.equal(det(A), det(ch.A), tolerance = 0)
all.equal(solve(A, b), solve(ch.A, b), tolerance = 0)
## For identical results, we need the _unpivoted_ factorization
## computed by det(A) and solve(A, b)
(ch.A.nopivot <- Cholesky(A, perm = FALSE))</pre>
stopifnot(identical(det(A), det(ch.A.nopivot)),
          identical(solve(A, b), solve(ch.A.nopivot, b)))
```

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Cholesky-methods

Methods for Cholesky Factorization

### Description

Computes the pivoted Cholesky factorization of an  $n \times n$  real, symmetric matrix A, which has the general form

$$P_1AP_1' = L_1DL_1' \stackrel{D_{jj} \ge 0}{=} LL'$$

or (equivalently)

$$A = P_1' L_1 D L_1' P_1 \stackrel{D_{jj} \ge 0}{=} P_1' L L' P_1$$

where  $P_1$  is a permutation matrix,  $L_1$  is a unit lower triangular matrix, D is a diagonal matrix, and  $L = L_1 \sqrt{D}$ . The second equalities hold only for positive semidefinite A, for which the diagonal entries of D are non-negative and  $\sqrt{D}$  is well-defined.

Methods for denseMatrix are built on LAPACK routines dpstrf, dpotrf, and dpptrf. The latter two do not permute rows or columns, so that  $P_1$  is an identity matrix.

Methods for sparseMatrix are built on CHOLMOD routines cholmod\_analyze and cholmod\_factorize\_p.

### Usage

```
Cholesky(A, ...)
## S4 method for signature 'dsyMatrix'
Cholesky(A, perm = TRUE, tol = -1, ...)
## S4 method for signature 'dspMatrix'
Cholesky(A, ...)
## S4 method for signature 'dsCMatrix'
Cholesky(A, perm = TRUE, LDL = !super, super = FALSE,
    Imult = 0, \ldots)
## S4 method for signature 'ddiMatrix'
Cholesky(A, ...)
## S4 method for signature 'generalMatrix'
Cholesky(A, uplo = "U", ...)
## S4 method for signature 'triangularMatrix'
Cholesky(A, uplo = "U", ...)
## S4 method for signature 'matrix'
Cholesky(A, uplo = "U", ...)
```

# Arguments

Α

a finite, symmetric matrix or Matrix to be factorized. If A is square but not symmetric, then it will be *treated* as symmetric; see uplo. Methods for dense A require positive definiteness when perm = FALSE and positive semidefiniteness when perm = TRUE. Methods for sparse A require positive definiteness when LDL = TRUE and nonzero leading principal minors (after pivoting) when LDL = FALSE. Methods for sparse, *diagonal* A are an exception, requiring positive semidefiniteness unconditionally.

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perm	a logical indicating if the rows and columns of $A$ should be pivoted. Methods for sparse A employ the approximate minimum degree (AMD) algorithm in order to reduce fill-in, i.e., without regard for numerical stability. Pivoting for sparsity may introduce nonpositive leading principal minors, causing the factorization to fail, in which case it may be necessary to set perm = FALSE.
tol	a finite numeric tolerance, used only if perm = TRUE. The factorization algorithm stops if the pivot is less than or equal to tol. Negative tol is equivalent to nrow(A) * .Machine\$double.eps * max(diag(A)).
LDL	a logical indicating if the simplicial factorization should be computed as $P_1'L_1DL_1'P_1$ , such that the result stores the lower triangular entries of $L_1-I+D$ . The alternative is $P_1'LL'P_1$ , such that the result stores the lower triangular entries of $L=L_1\sqrt{D}$ . This argument is ignored if super = TRUE (or if super = NA and the supernodal algorithm is chosen), as the supernodal code does not yet support the LDL = TRUE variant.
super	a logical indicating if the factorization should use the supernodal algorithm. The alternative is the simplicial algorithm. Setting super = NA leaves the choice to a CHOLMOD-internal heuristic.
Imult	a finite number. The matrix that is factorized is A + Imult * diag(nrow(A)), i.e., A plus Imult times the identity matrix. This argument is useful for symmetric, indefinite A, as Imult > max(rowSums(abs(A)) - diag(abs(A))) ensures that A + Imult * diag(nrow(A)) is diagonally dominant. (Symmetric, diagonally dominant matrices are positive definite.)
uplo	a string, either "U" or "L", indicating which triangle of A should be used to compute the factorization. The default is "U", even for lower triangular A, to be consistent with chol from <b>base</b> .
	further arguments passed to or from methods.

### **Details**

Note that the result of a call to Cholesky inherits from CholeskyFactorization but not Matrix. Users who just want a matrix should consider using chol, whose methods are simple wrappers around Cholesky returning just the upper triangular Cholesky factor L', typically as a triangularMatrix. However, a more principled approach would be to construct factors as needed from the CholeskyFactorization object, e.g., with expand1(x, "L"), if x is the object.

The behaviour of Cholesky (A, perm = TRUE) for dense A is somewhat exceptional, in that it expects without checking that A is positive semidefinite. By construction, if A is positive semidefinite and the exact algorithm encounters a zero pivot, then the unfactorized trailing submatrix is the zero matrix, and there is nothing left to do. Hence when the finite precision algorithm encounters a pivot less than tol, it signals a warning instead of an error and zeros the trailing submatrix in order to guarantee that P'LL'P is positive semidefinite even if A is not. It follows that one way to test for positive semidefiniteness of A in the event of a warning is to analyze the error

$$\frac{\|A - P'LL'P\|}{\|A\|}.$$

See the examples and LAPACK Working Note ("LAWN") 161 for details.

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#### Value

An object representing the factorization, inheriting from virtual class CholeskyFactorization. For a traditional matrix A, the specific class is Cholesky. For A inheriting from unpackedMatrix, packedMatrix, and sparseMatrix, the specific class is Cholesky, pCholesky, and dCHMsimpl or dCHMsuper, respectively.

#### References

The LAPACK source code, including documentation; see https://netlib.org/lapack/double/dpstrf.f, https://netlib.org/lapack/double/dpotrf.f, and https://netlib.org/lapack/double/dpptrf.f.

The CHOLMOD source code; see <a href="https://github.com/DrTimothyAldenDavis/SuiteSparse">https://github.com/DrTimothyAldenDavis/SuiteSparse</a>, notably the header file 'CHOLMOD/Include/cholmod.h' defining cholmod\_factor\_struct.

Lucas, C. (2004). *LAPACK-style codes for level 2 and 3 pivoted Cholesky factorizations*. LAPACK Working Note, Number 161. https://www.netlib.org/lapack/lawnspdf/lawn161.pdf

Chen, Y., Davis, T. A., Hager, W. W., & Rajamanickam, S. (2008). Algorithm 887: CHOLMOD, supernodal sparse Cholesky factorization and update/downdate. *ACM Transactions on Mathematical Software*, *35*(3), Article 22, 1-14. doi:10.1145/1391989.1391995

Amestoy, P. R., Davis, T. A., & Duff, I. S. (2004). Algorithm 837: AMD, an approximate minimum degree ordering algorithm. *ACM Transactions on Mathematical Software*, *17*(4), 886-905. doi:10.1145/1024074.1024081

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Classes Cholesky, pCholesky, dCHMsimpl and dCHMsuper and their methods.

Classes dpoMatrix, dppMatrix, and dsCMatrix.

Generic function chol, for obtaining the upper triangular Cholesky factor L' as a matrix or Matrix.

Generic functions expand1 and expand2, for constructing matrix factors from the result.

Generic functions BunchKaufman, Schur, 1u, and qr, for computing other factorizations.

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```
isPerm(ch.A1@perm)
    is.unsorted(ch.A1@perm) # typically not the identity permutation
    length(ch.A1.nopivot@perm) == 0L
})
## A ~ P1' L D L' P1 ~ P1' L L' P1 in floating point
str(e.ch.A1 <- expand2(ch.A1, LDL = TRUE), max.level = 2L)</pre>
str(E.ch.A1 <- expand2(ch.A1, LDL = FALSE), max.level = 2L)</pre>
stopifnot(exprs = {
    all.equal(as(A1, "matrix"), as(Reduce(`%*%`, e.ch.A1), "matrix"))
    all.equal(as(A1, "matrix"), as(Reduce(`%*%`, E.ch.A1), "matrix"))
})
## .... Positive semidefinite but not positive definite .....
A2 <- A1
A2[1L, ] \leftarrow A2[, 1L] \leftarrow 0
A2
try(Cholesky(A2, perm = FALSE)) # fails as not positive definite
ch.A2 <- Cholesky(A2) # returns, with a warning and ...
A2.hat <- Reduce(`%*%`, expand2(ch.A2, LDL = FALSE))
norm(A2 - A2.hat, "2") / norm(A2, "2") # 7.670858e-17
## .... Not positive semidefinite .....
A3 <- A1
A3[1L, ] \leftarrow A3[, 1L] \leftarrow -1
try(Cholesky(A3, perm = FALSE)) # fails as not positive definite
ch.A3 <- Cholesky(A3) # returns, with a warning and ...</pre>
A3.hat <- Reduce(`%*%`, expand2(ch.A3, LDL = FALSE))
norm(A3 - A3.hat, "2") / norm(A3, "2") # 1.781568
## Indeed, 'A3' is not positive semidefinite, but 'A3.hat' _is_
ch.A3.hat <- Cholesky(A3.hat)</pre>
A3.hat.hat <- Reduce(`%*%`, expand2(ch.A3.hat, LDL = FALSE))
norm(A3.hat - A3.hat.hat, "2") / norm(A3.hat, "2") # 1.777944e-16
## --- Sparse -----
## Really just three cases modulo permutation :
##
##
                         factorization minors of P1 A P1'
             type
    1 simplicial P1 A P1' = L1 D L1'
##
                                                 nonzero
##
    2 simplicial P1 A P1' = L L'
                                                positive
    3 supernodal P1 A P2' = L L'
                                                positive
data(KNex, package = "Matrix")
A4 <- crossprod(KNex[["mm"]])
ch.A4 <-
list(pivoted =
     list(simpl1 = Cholesky(A4, perm = TRUE, super = FALSE, LDL = TRUE),
```

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```
simpl0 = Cholesky(A4, perm = TRUE, super = FALSE, LDL = FALSE),
          super0 = Cholesky(A4, perm = TRUE, super = TRUE
     unpivoted =
     list(simpl1 = Cholesky(A4, perm = FALSE, super = FALSE, LDL = TRUE),
          simpl0 = Cholesky(A4, perm = FALSE, super = FALSE, LDL = FALSE),
          super0 = Cholesky(A4, perm = FALSE, super = TRUE
ch.A4
s <- simplify2array
rapply2 <- function(object, f, ...) rapply(object, f, , , how = "list", ...)</pre>
s(rapply2(ch.A4, isLDL))
s(m.ch.A4 \leftarrow rapply2(ch.A4, expand1, "L")) # giving L = L1 sqrt(D)
## By design, the pivoted and simplicial factorizations
## are more sparse than the unpivoted and supernodal ones ...
s(rapply2(m.ch.A4, object.size))
## Which is nicely visualized by lattice-based methods for 'image'
inm <- c("pivoted", "unpivoted")</pre>
jnm <- c("simpl1", "simpl0", "super0")</pre>
for(i in 1:2)
 for(j in 1:3)
    print(image(m.ch.A4[[c(i, j)]], main = paste(inm[i], jnm[j])),
          split = c(j, i, 3L, 2L), more = i * j < 6L)
simpl1 <- ch.A4[[c("pivoted", "simpl1")]]</pre>
stopifnot(exprs = {
    length(simpl1@perm) == ncol(A4)
    isPerm(simpl1@perm, 0L)
    is.unsorted(simpl1@perm) # typically not the identity permutation
})
## One can expand with and without D regardless of isLDL(.),
## but "without" requires L = L1 sqrt(D), which is conditional
## on min(diag(D)) >= 0, hence "with" is the default
isLDL(simpl1)
stopifnot(min(diag(simpl1)) >= 0)
str(e.ch.A4 <- expand2(simpl1, LDL = TRUE), max.level = 2L) # default</pre>
str(E.ch.A4 <- expand2(simpl1, LDL = FALSE), max.level = 2L)</pre>
stopifnot(exprs = {
    all.equal(E.ch.A4[["L" ]], e.ch.A4[["L1" ]] %*% sqrt(e.ch.A4[["D"]]))
    all.equal(E.ch.A4[["L."]], sqrt(e.ch.A4[["D"]]) %*% e.ch.A4[["L1."]])
    all.equal(A4, as(Reduce(`%*%`, e.ch.A4), "symmetricMatrix"))
    all.equal(A4, as(Reduce(`%*%`, E.ch.A4), "symmetricMatrix"))
})
## The "same" permutation matrix with "alternate" representation
## [i, perm[i]] {margin=1} <-> [invertPerm(perm)[j], j] {margin=2}
alt <- function(P) {</pre>
   P@margin <- 1L + !(P@margin - 1L) # 1 <-> 2
   P@perm <- invertPerm(P@perm)</pre>
```

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```
}
## Expansions are elegant but inefficient (transposes are redundant)
## hence programmers should consider methods for 'expand1' and 'diag'
stopifnot(exprs = {
    identical(expand1(simpl1, "P1"), alt(e.ch.A4[["P1"]]))
    identical(expand1(simpl1, "L"), E.ch.A4[["L"]])
    identical(Diagonal(x = diag(simpl1)), e.ch.A4[["D"]])
})
## chol(A, pivot = value) is a simple wrapper around
## Cholesky(A, perm = value, LDL = FALSE, super = FALSE),
## returning L' = sqrt(D) L1' _but_ giving no information
## about the permutation P1
selectMethod("chol", "dsCMatrix")
stopifnot(all.equal(chol(A4, pivot = TRUE), E.ch.A4[["L."]]))
## Now a symmetric matrix with positive _and_ negative eigenvalues,
## hence _not_ positive semidefinite
A5 <- new("dsCMatrix",
          Dim = c(7L, 7L),
          p = c(0:1, 3L, 6:7, 10:11, 15L),
          i = c(0L, 0:1, 0:3, 2:5, 3:6),
          x = c(1, 6, 38, 10, 60, 103, -4, 6, -32, -247, -2, -16, -128, -2, -67))
(ev <- eigen(A5, only.values = TRUE)$values)</pre>
(t.ev <- table(factor(sign(ev), -1:1))) # the matrix "inertia"</pre>
ch.A5 <- Cholesky(A5)
isLDL(ch.A5)
(d.A5 <- diag(ch.A5)) # diag(D) is partly negative
## Sylvester's law of inertia holds here, but not in general
## in finite precision arithmetic
stopifnot(identical(table(factor(sign(d.A5), -1:1)), t.ev))
try(expand1(ch.A5, "L"))
                                 # unable to compute L = L1 sqrt(D)
try(expand2(ch.A5, LDL = FALSE)) # ditto
try(chol(A5, pivot = TRUE))
                                 # ditto
## The default expansion is "square root free" and still works here
str(e.ch.A5 <- expand2(ch.A5, LDL = TRUE), max.level = 2L)</pre>
stopifnot(all.equal(A5, as(Reduce(`%*%`, e.ch.A5), "symmetricMatrix")))
## Version of the SuiteSparse library, which includes CHOLMOD
Mv <- Matrix.Version()</pre>
Mv[["suitesparse"]]
```

## **Description**

Since 2005, package **Matrix** has supported coercions to and from class graph from package **graph**. Since 2013, this functionality has been exposed via functions T2graph and graph2T, which, unlike methods for as(from, "<Class>"), support optional arguments.

## Usage

```
graph2T(from, use.weights = )
T2graph(from, need.uniq = !isUniqueT(from), edgemode = NULL)
```

#### **Arguments**

from for graph2T(), an R object of class "graph";
for T2graph(), a sparse matrix inheriting from "TsparseMatrix".

use.weights logical indicating if weights should be used, i.e., equivalently the result will be numeric, i.e. of class dgTMatrix; otherwise the result will be ngTMatrix or nsTMatrix, the latter if the graph is undirected. The default looks if there are weights in the graph, and if any differ from 1, weights are used.

need.uniq a logical indicating if from may need to be internally "uniqified"; do not set this and hence rather use the default, unless you know what you are doing!

edgemode one of NULL, "directed", or "undirected". The default NULL looks if the matrix is symmetric and assumes "undirected" in that case.

#### Value

```
For graph2T(), a sparse matrix inheriting from "TsparseMatrix". For T2graph() an R object of class "graph".
```

#### See Also

Package **igraph**, which provides similar coercions to and from its class igraph via functions graph\_from\_adjacency\_matrix and as\_adjacency\_matrix.

```
if(requireNamespace("graph")) {
  n4 <- LETTERS[1:4];  dns <- list(n4,n4)
  show(a1 <- sparseMatrix(i= c(1:4),  j=c(2:4,1),  x = 2,  dimnames=dns))
  show(g1 <- as(a1, "graph")) # directed
  unlist(graph::edgeWeights(g1)) # all '2'

  show(a2 <- sparseMatrix(i= c(1:4,4), j=c(2:4,1:2), x = TRUE, dimnames=dns))
  show(g2 <- as(a2, "graph")) # directed
  # now if you want it undirected:
  show(g3 <- T2graph(as(a2, "TsparseMatrix"), edgemode="undirected"))
  show(m3 <- as(g3, "Matrix"))
  show( graph2T(g3) ) # a "pattern Matrix" (nsTMatrix)

  a. <- sparseMatrix(i=4:1, j=1:4, dimnames=list(n4, n4), repr="T") # no 'x'</pre>
```

```
show(a.) # "ngTMatrix"
show(g. <- as(a., "graph"))
}</pre>
```

coerce-methods-SparseM

Sparse Matrix Coercion from and to those from package SparseM

## **Description**

Methods for coercion from and to sparse matrices from package **SparseM** are provided here, for ease of porting functionality to the **Matrix** package, and comparing functionality of the two packages. All these work via the usual as(., "<class>") coercion,

```
as(from, Class)
```

#### Methods

```
from = "matrix.csr", to = "dgRMatrix" ...
from = "matrix.csc", to = "dgCMatrix" ...
from = "matrix.coo", to = "dgTMatrix" ...
from = "dgRMatrix", to = "matrix.csr" ...
from = "dgCMatrix", to = "matrix.csc" ...
from = "dgTMatrix", to = "matrix.csc" ...
from = "Matrix", to = "matrix.csr" ...
from = "matrix.csr", to = "dgCMatrix" ...
from = "matrix.csr", to = "dgCMatrix" ...
from = "matrix.csr", to = "dgCMatrix" ...
from = "matrix.csc", to = "Matrix" ...
from = "matrix.csc", to = "Matrix" ...
from = "matrix.cso", to = "Matrix" ...
```

### See Also

The documentation in CRAN package **SparseM**, such as SparseM.ontology, and one important class, matrix.csr.

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colSums-methods

Form Row and Column Sums and Means

### **Description**

Form row and column sums and means for objects, for sparseMatrix the result may optionally be sparse (sparseVector), too. Row or column names are kept respectively as for **base** matrices and colSums methods, when the result is numeric vector.

#### Usage

```
colSums(x, na.rm = FALSE, dims = 1L, ...)
rowSums(x, na.rm = FALSE, dims = 1L, ...)
colMeans(x, na.rm = FALSE, dims = 1L, ...)
rowMeans(x, na.rm = FALSE, dims = 1L, ...)
## S4 method for signature 'CsparseMatrix'
colSums(x, na.rm = FALSE, dims = 1L,
         sparseResult = FALSE, ...)
## S4 method for signature 'CsparseMatrix'
rowSums(x, na.rm = FALSE, dims = 1L,
         sparseResult = FALSE, ...)
## S4 method for signature 'CsparseMatrix'
colMeans(x, na.rm = FALSE, dims = 1L,
         sparseResult = FALSE, ...)
## S4 method for signature 'CsparseMatrix'
rowMeans(x, na.rm = FALSE, dims = 1L,
         sparseResult = FALSE, ...)
```

#### **Arguments**

Χ	a Matrix, i.e., inheriting from Matrix.
na.rm	logical. Should missing values (including NaN) be omitted from the calculations?
dims	completely ignored by the Matrix methods.
• • •	potentially further arguments, for method <-> generic compatibility.
sparseResult	logical indicating if the result should be sparse, i.e., inheriting from class sparseVector.
	Only applicable when x is inheriting from a sparseMatrix class.

#### Value

returns a numeric vector if sparseResult is FALSE as per default. Otherwise, returns a sparseVector. dimnames(x) are only kept (as names(v)) when the resulting v is numeric, since sparseVectors do not have names.

## See Also

colSums and the sparseVector classes.

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### **Examples**

```
(M <- bdiag(Diagonal(2), matrix(1:3, 3,4), diag(3:2))) # 7 x 8
d \leftarrow Diagonal(10, c(0,0,10,0,2,rep(0,5)))
MM <- kronecker(d, M)
dim(MM) # 70 80
length(MM@x) # 160, but many are '0'; drop those:
MM <- drop@(MM)
length(MM@x) # 32
  cm <- colSums(MM)</pre>
(scm <- colSums(MM, sparseResult = TRUE))</pre>
stopifnot(is(scm, "sparseVector"),
          identical(cm, as.numeric(scm)))
rowSums (MM, sparseResult = TRUE) # 14 of 70 are not zero
colMeans(MM, sparseResult = TRUE) # 16 of 80 are not zero
## Since we have no 'NA's, these two are equivalent :
stopifnot(identical(rowMeans(MM, sparseResult = TRUE),
                    rowMeans(MM, sparseResult = TRUE, na.rm = TRUE)),
  rowMeans(Diagonal(16)) == 1/16,
  colSums(Diagonal(7)) == 1)
## dimnames(x) --> names( <value> ) :
dimnames(M) \leftarrow list(paste0("r", 1:7), paste0("V",1:8))
colSums(M)
rowMeans(M)
## Assertions :
stopifnot(exprs = {
    all.equal(colSums(M),
              structure(c(1,1,6,6,6,6,3,2), names = colnames(M)))
    all.equal(rowMeans(M),
              structure(c(1,1,4,8,12,3,2)/8, names = paste0("r", 1:7)))
})
```

condest

Compute Approximate CONDition number and 1-Norm of (Large) Matrices

### **Description**

"Estimate", i.e. compute approximately the CONDition number of a (potentially large, often sparse) matrix A. It works by apply a fast *randomized* approximation of the 1-norm, norm(A, "1"), through onenormest(.).

### Usage

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### **Arguments**

A	a square matrix, optional for onenormest(), where instead of A, A.x and At.x can be specified, see there.
t	number of columns to use in the iterations.
normA	number; (an estimate of) the 1-norm of A, by default $norm(A, "1")$ ; may be replaced by an estimate.
silent	logical indicating if warning and (by default) convergence messages should be displayed.
quiet	logical indicating if convergence messages should be displayed.
A.x, At.x	when A is missing, these two must be given as functions which compute A $\%$ x, or t(A) $\%$ x, respectively.
n	== nrow(A), only needed when A is not specified.
iter.max	maximal number of iterations for the 1-norm estimator.
eps	the relative change that is deemed irrelevant.

#### **Details**

condest() calls lu(A), and subsequently onenormest(A.x = , At.x = ) to compute an approximate norm of the *inverse* of A,  $A^{-1}$ , in a way which keeps using sparse matrices efficiently when A is sparse.

Note that onenormest() uses random vectors and hence *both* functions' results are random, i.e., depend on the random seed, see, e.g., set.seed().

## Value

Both functions return a list; condest() with components,

```
est a number > 0, the estimated (1-norm) condition number \hat{\kappa}; when r := r \text{cond}(A), 1/\hat{\kappa} \approx r. v the maximal Ax column, scaled to norm(v) = 1. Consequently, norm(Av) = norm(A)/est; when est is large, v is an approximate null vector.
```

The function onenormest() returns a list with components,

```
est a number > 0, the estimated norm(A, "1").

v 0-1 integer vector length n, with an 1 at the index j with maximal column A[,j] in A.

w numeric vector, the largest Ax found.

iter the number of iterations used.
```

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## Author(s)

This is based on octave's condest() and onenormest() implementations with original author Jason Riedy, U Berkeley; translation to R and adaption by Martin Maechler.

#### References

Nicholas J. Higham and Françoise Tisseur (2000). A Block Algorithm for Matrix 1-Norm Estimation, with an Application to 1-Norm Pseudospectra. *SIAM J. Matrix Anal. Appl.* **21**, 4, 1185–1201.

William W. Hager (1984). Condition Estimates. SIAM J. Sci. Stat. Comput. 5, 311-316.

#### See Also

norm, rcond.

#### **Examples**

 ${\it Csparse Matrix-class} \qquad {\it Class \ "Csparse Matrix" \ of \ Sparse \ Matrices \ in \ Column-compressed} \\ Form$ 

## **Description**

The "CsparseMatrix" class is the virtual class of all sparse matrices coded in sorted compressed column-oriented form. Since it is a virtual class, no objects may be created from it. See showClass("CsparseMatrix") for its subclasses.

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#### Slots

- i: Object of class "integer" of length nnzero (number of non-zero elements). These are the *0-based* row numbers for each non-zero element in the matrix, i.e., i must be in 0: (nrow(.)-1).
- p: integer vector for providing pointers, one for each column, to the initial (zero-based) index of
   elements in the column. .@p is of length ncol(.) + 1, with p[1] == 0 and p[length(p)] ==
   nnzero, such that in fact, diff(.@p) are the number of non-zero elements for each column.
   In other words, m@p[1:ncol(m)] contains the indices of those elements in m@x that are the
   first elements in the respective column of m.

Dim, Dimnames: inherited from the superclass, see the sparseMatrix class.

#### **Extends**

Class "sparseMatrix", directly. Class "Matrix", by class "sparseMatrix".

#### Methods

matrix products %\*%, crossprod() and tcrossprod(), several solve methods, and other matrix methods available:

```
signature(e1 = "CsparseMatrix", e2 = "numeric"): ...
AAithth signature(e1 = "numeric", e2 = "CsparseMatrix"): ...
Math signature(x = "CsparseMatrix"): ...
band signature(x = "CsparseMatrix"): ...
- signature(e1 = "CsparseMatrix", e2 = "numeric"): ...
- signature(e1 = "numeric", e2 = "CsparseMatrix"): ...
+ signature(e1 = "CsparseMatrix", e2 = "numeric"): ...
+ signature(e1 = "numeric", e2 = "CsparseMatrix"): ...
coerce signature(from = "CsparseMatrix", to = "TsparseMatrix"): ...
coerce signature(from = "CsparseMatrix", to = "denseMatrix"): ...
coerce signature(from = "CsparseMatrix", to = "matrix"): ...
coerce signature(from = "TsparseMatrix", to = "CsparseMatrix"): ...
coerce signature(from = "denseMatrix", to = "CsparseMatrix"): ...
diag signature(x = "CsparseMatrix"): ...
gamma signature(x = "CsparseMatrix"): ...
lgamma signature(x = "CsparseMatrix"): ...
log signature(x = "CsparseMatrix"): ...
t signature(x = "CsparseMatrix"): ...
tril signature(x = "CsparseMatrix"): ...
triu signature(x = "CsparseMatrix"): ...
```

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#### Note

All classes extending CsparseMatrix have a common validity (see validObject) check function. That function additionally checks the i slot for each column to contain increasing row numbers. In earlier versions of **Matrix** (<= 0.999375-16), validObject automatically re-sorted the entries when necessary, and hence new() calls with somewhat permuted i and x slots worked, as new(...) (with slot arguments) automatically checks the validity.

Now, you have to use sparseMatrix to achieve the same functionality or know how to use .validateCsparse() to do so.

#### See Also

colSums, kronecker, and other such methods with own help pages.

Further, the super class of CsparseMatrix, sparseMatrix, and, e.g., class dgCMatrix for the links to other classes.

# Examples

```
getClass("CsparseMatrix")
## The common validity check function (based on C code):
getValidity(getClass("CsparseMatrix"))
```

ddenseMatrix-class

Virtual Class "ddenseMatrix" of Numeric Dense Matrices

## **Description**

This is the virtual class of all dense numeric (i.e., **d**ouble, hence "*ddense*") S4 matrices. Its most important subclass is the dgeMatrix class.

### **Extends**

Class "dMatrix" directly; class "Matrix", by the above.

## Slots

the same slots at its subclass dgeMatrix, see there.

#### Methods

Most methods are implemented via as(\*, "generalMatrix") and are mainly used as "fallbacks" when the subclass doesn't need its own specialized method.

Use showMethods(class = "ddenseMatrix", where = "package: Matrix") for an overview.

## See Also

The virtual classes Matrix, dMatrix, and dsparseMatrix.

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## **Examples**

```
showClass("ddenseMatrix")
showMethods(class = "ddenseMatrix", where = "package:Matrix")
```

ddiMatrix-class

Class "ddiMatrix" of Diagonal Numeric Matrices

## Description

The class "ddiMatrix" of numerical diagonal matrices. Note that diagonal matrices now extend sparseMatrix, whereas they did extend dense matrices earlier.

## **Objects from the Class**

Objects can be created by calls of the form new("ddiMatrix", ...) but typically rather via Diagonal.

#### Slots

x: numeric vector. For an  $n \times n$  matrix, the x slot is of length n or 0, depending on the diag slot: diag: "character" string, either "U" or "N" where "U" denotes unit-diagonal, i.e., identity matrices.

Dim, Dimnames: matrix dimension and dimnames, see the Matrix class description.

### **Extends**

```
Class "diagonalMatrix", directly. Class "dMatrix", directly. Class "sparseMatrix", indirectly, see showClass("ddiMatrix").
```

#### Methods

```
%*% signature(x = "ddiMatrix", y = "ddiMatrix"): ...
```

### See Also

Class diagonal Matrix and function Diagonal.

```
(d2 <- Diagonal(x = c(10,1)))
str(d2)
## slightly larger in internal size:
str(as(d2, "sparseMatrix"))

M <- Matrix(cbind(1,2:4))
M %*% d2 #> `fast' multiplication
chol(d2) # trivial
```

denseLU-class 49

denseLU-class

Dense LU Factorizations

## **Description**

denseLU is the class of dense, row-pivoted LU factorizations of  $m \times n$  real matrices A, having the general form

$$P_1A = LU$$

or (equivalently)

$$A = P_1'LU$$

where  $P_1$  is an  $m \times m$  permutation matrix, L is an  $m \times \min(m, n)$  unit lower trapezoidal matrix, and U is a  $\min(m, n) \times n$  upper trapezoidal matrix. If m = n, then the factors L and U are triangular.

#### Slots

Dim, Dimnames inherited from virtual class MatrixFactorization.

x a numeric vector of length prod(Dim) storing the triangular L and U factors together in a packed format. The details of the representation are specified by the manual for LAPACK routine dgetrf.

perm an integer vector of length min(Dim) specifying the permutation  $P_1$  as a product of transpositions. The corresponding permutation vector can be obtained as asPerm(perm).

## Extends

Class LU, directly. Class MatrixFactorization, by class LU, distance 2.

#### Instantiation

Objects can be generated directly by calls of the form new("denseLU", ...), but they are more typically obtained as the value of lu(x) for x inheriting from denseMatrix (often dgeMatrix).

## Methods

coerce signature(from = "denseLU", to = "dgeMatrix"): returns a dgeMatrix with the dimensions of the factorized matrix A, equal to L below the diagonal and equal to U on and above the diagonal.

determinant signature(from = "denseLU", logarithm = "logical"): computes the determinant of the factorized matrix A or its logarithm.

```
expand signature(x = "denseLU"): see expand-methods.
expand1 signature(x = "denseLU"): see expand1-methods.
expand2 signature(x = "denseLU"): see expand2-methods.
solve signature(a = "denseLU", b = "missing"): see solve-methods.
```

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#### References

The LAPACK source code, including documentation; see https://netlib.org/lapack/double/dgetrf.f.

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Class sparseLU for sparse LU factorizations.

Class dgeMatrix.

Generic functions 1u, expand1 and expand2.

```
showClass("denseLU")
set.seed(1)
n <- 3L
(A <- Matrix(round(rnorm(n * n), 2L), n, n))</pre>
## With dimnames, to see that they are propagated :
dimnames(A) <- dn <- list(paste0("r", seq_len(n)),</pre>
                           paste0("c", seq_len(n)))
(lu.A \leftarrow lu(A))
str(e.lu.A <- expand2(lu.A), max.level = 2L)</pre>
## Underlying LAPACK representation
(m.lu.A <- as(lu.A, "dgeMatrix")) # which is L and U interlaced
stopifnot(identical(as(m.lu.A, "matrix"), `dim<-`(lu.A@x, lu.A@Dim)))</pre>
ae1 <- function(a, b, ...) all.equal(as(a, "matrix"), as(b, "matrix"), ...)</pre>
ae2 <- function(a, b, ...) ae1(unname(a), unname(b), ...)</pre>
## A ~ P1' L U in floating point
stopifnot(exprs = {
    identical(names(e.lu.A), c("P1.", "L", "U"))
    identical(e.lu.A[["P1."]],
              new( "pMatrix", Dim = c(n, n), Dimnames = c(dn[1L], list(NULL)),
                  margin = 1L, perm = invertPerm(asPerm(lu.A@perm))))
    identical(e.lu.A[["L"]],
              new("dtrMatrix", Dim = c(n, n), Dimnames = list(NULL, NULL),
                  uplo = "L", diag = "U", x = lu.A@x))
    identical(e.lu.A[["U"]],
              new("dtrMatrix", Dim = c(n, n), Dimnames = c(list(NULL), dn[2L]),
                  uplo = "U", diag = "N", x = lu.A@x))
    ae1(A, with(e.lu.A, P1. %*% L %*% U))
    ae2(A[asPerm(lu.A@perm), ], with(e.lu.A, L %*% U))
})
```

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denseMatrix-class

Virtual Class "denseMatrix" of All Dense Matrices

# Description

This is the virtual class of all dense (S4) matrices. It partitions into two subclasses packedMatrix and unpackedMatrix. Alternatively into the (currently) three subclasses ddenseMatrix, ldenseMatrix, and ndenseMatrix.

denseMatrix is (hence) the direct superclass of these (2+3=5) classes.

#### **Extends**

```
class "Matrix" directly.
```

# Slots

```
exactly those of its superclass "Matrix", i.e., "Dim" and "Dimnames".
```

## Methods

```
Use showMethods(class = "denseMatrix", where = "package:Matrix") for an overview of methods.
```

```
Extraction ("[") methods, see [-methods.
```

### See Also

colSums, kronecker, and other such methods with own help pages.

Its superclass Matrix, and main subclasses, ddenseMatrix and sparseMatrix.

```
showClass("denseMatrix")
```

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dgCMatrix-class

Compressed, sparse, column-oriented numeric matrices

## **Description**

The dgCMatrix class is a class of sparse numeric matrices in the compressed, sparse, column-oriented format. In this implementation the non-zero elements in the columns are sorted into increasing row order. dgCMatrix is the "standard" class for sparse numeric matrices in the **Matrix** package.

# **Objects from the Class**

Objects can be created by calls of the form new("dgCMatrix", ...), more typically via as(\*, "CsparseMatrix") or similar. Often however, more easily via Matrix(\*, sparse = TRUE), or most efficiently via sparseMatrix().

#### **Slots**

```
x: Object of class "numeric" - the non-zero elements of the matrix.... all other slots are inherited from the superclass "CsparseMatrix".
```

### Methods

```
Matrix products (e.g., crossprod-methods), and (among other)

coerce signature(from = "matrix", to = "dgCMatrix")

diag signature(x = "dgCMatrix"): returns the diagonal of x

dim signature(x = "dgCMatrix"): returns the dimensions of x

image signature(x = "dgCMatrix"): plots an image of x using the levelplot function

solve signature(a = "dgCMatrix", b = "..."): see solve-methods, notably the extra argument sparse.

lu signature(x = "dgCMatrix"): computes the LU decomposition of a square dgCMatrix object
```

## See Also

```
Classes dsCMatrix, dtCMatrix, lu
```

```
(m <- Matrix(c(0,0,2:0), 3,5))
str(m)
m[,1]</pre>
```

dgeMatrix-class 53

dgeMatrix-class

Class "dgeMatrix" of Dense Numeric (S4 Class) Matrices

## **Description**

A general numeric dense matrix in the S4 Matrix representation. dgeMatrix is the "standard" class for dense numeric matrices in the **Matrix** package.

## **Objects from the Class**

Objects can be created by calls of the form new("dgeMatrix", ...) or, more commonly, by coercion from the Matrix class (see Matrix) or by Matrix(..).

#### Slots

```
x: Object of class "numeric" - the numeric values contained in the matrix, in column-major order.
```

Dim: Object of class "integer" - the dimensions of the matrix - must be an integer vector with exactly two non-negative values.

Dimnames: a list of length two - inherited from class Matrix.

factors: Object of class "list" - a list of factorizations of the matrix.

#### Methods

```
The are group methods (see, e.g., Arith)
Arith signature(e1 = "dgeMatrix", e2 = "dgeMatrix"): ...
Arith signature(e1 = "dgeMatrix", e2 = "numeric"): ...
Arith signature(e1 = "numeric", e2 = "dgeMatrix"): ...
Math signature(x = "dgeMatrix"): ...
Math2 signature(x = "dgeMatrix", digits = "numeric"): ...
matrix products %*%, crossprod() and tcrossprod(), several solve methods, and other matrix
methods available:
Schur signature(x = "dgeMatrix", vectors = "logical"): ...
Schur signature(x = "dgeMatrix", vectors = "missing"): ...
chol signature(x = "dgeMatrix"): see chol.
colMeans signature(x = "dgeMatrix"): columnwise means (averages)
colSums signature(x = "dgeMatrix"): columnwise sums
diag signature(x = "dgeMatrix"): ...
dim signature(x = "dgeMatrix"): ...
dimnames signature(x = "dgeMatrix"): ...
eigen signature(x = "dgeMatrix", only.values= "logical"): ...
```

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```
eigen signature(x = "dgeMatrix", only.values= "missing"): ...
norm signature(x = "dgeMatrix", type = "character"): ...
norm signature(x = "dgeMatrix", type = "missing"): ...
rcond signature(x = "dgeMatrix", norm = "character") or norm = "missing": the reciprocal condition number, rcond().
rowMeans signature(x = "dgeMatrix"): rowwise means (averages)
rowSums signature(x = "dgeMatrix"): rowwise sums
t signature(x = "dgeMatrix"): matrix transpose
```

#### See Also

Classes Matrix, dtrMatrix, and dsyMatrix.

dgRMatrix-class

Sparse Compressed, Row-oriented Numeric Matrices

### Description

The dgRMatrix class is a class of sparse numeric matrices in the compressed, sparse, row-oriented format. In this implementation the non-zero elements in the rows are sorted into increasing column order.

**Note:** The column-oriented sparse classes, e.g., dgCMatrix, are preferred and better supported in the **Matrix** package.

### **Objects from the Class**

Objects can be created by calls of the form new("dgRMatrix", ...).

## **Slots**

- j: Object of class "integer" of length nnzero (number of non-zero elements). These are the column numbers for each non-zero element in the matrix.
- p: Object of class "integer" of pointers, one for each row, to the initial (zero-based) index of elements in the row.
- x: Object of class "numeric" the non-zero elements of the matrix.
- Dim: Object of class "integer" the dimensions of the matrix.

### Methods

```
diag signature(x = "dgRMatrix"): returns the diagonal of x
dim signature(x = "dgRMatrix"): returns the dimensions of x
image signature(x = "dgRMatrix"): plots an image of x using the levelplot function
```

### See Also

the RsparseMatrix class, the virtual class of all sparse compressed **r**ow-oriented matrices, with its methods. The dgCMatrix class (**c**olumn compressed sparse) is really preferred.

dgTMatrix-class 55

dgTMatrix-class

Sparse matrices in triplet form

## **Description**

The "dgTMatrix" class is the class of sparse matrices stored as (possibly redundant) triplets. The internal representation is not at all unique, contrary to the one for class dgCMatrix.

## **Objects from the Class**

Objects can be created by calls of the form new("dgTMatrix", ...), but more typically via spMatrix() or sparseMatrix(\*, repr = "T").

### Slots

- i: integer row indices of non-zero entries in 0-base, i.e., must be in 0: (nrow(.)-1).
- j: integer column indices of non-zero entries. Must be the same length as slot i and *0-based* as well, i.e., in 0: (ncol(.)-1).
- x: numeric vector the (non-zero) entry at position (i, j). Must be the same length as slot i. If an index pair occurs more than once, the corresponding values of slot x are added to form the element of the matrix.

Dim: Object of class "integer" of length 2 - the dimensions of the matrix.

### Methods

```
+ signature(e1 = "dgTMatrix", e2 = "dgTMatrix")
image signature(x = "dgTMatrix"): plots an image of x using the levelplot function
t signature(x = "dgTMatrix"): returns the transpose of x
```

#### Note

Triplet matrices are a convenient form in which to construct sparse matrices after which they can be coerced to dgCMatrix objects.

Note that both new(.) and spMatrix constructors for "dgTMatrix" (and other "TsparseMatrix" classes) implicitly add  $x_k$ 's that belong to identical  $(i_k, j_k)$  pairs.

However this means that a matrix typically can be stored in more than one possible "TsparseMatrix" representations. Use asUniqueT() in order to ensure uniqueness of the internal representation of such a matrix.

#### See Also

Class dgCMatrix or the superclasses dsparseMatrix and TsparseMatrix; asUniqueT.

Diagonal Diagonal

### **Examples**

```
m <- Matrix(0+1:28, nrow = 4)</pre>
m[-3,c(2,4:5,7)] \leftarrow m[3,1:4] \leftarrow m[1:3,6] \leftarrow 0
(mT <- as(m, "TsparseMatrix"))</pre>
str(mT)
mT[1,]
mT[4, drop = FALSE]
stopifnot(identical(mT[lower.tri(mT)],
                     m [lower.tri(m) ]))
mT[lower.tri(mT,diag=TRUE)] <- 0</pre>
## Triplet representation with repeated (i,j) entries
## *adds* the corresponding x's:
T2 <- new("dgTMatrix",
          i = as.integer(c(1,1,0,3,3)),
          j = as.integer(c(2,2,4,0,0)), x=10*1:5, Dim=4:5)
str(T2) # contains (i,j,x) slots exactly as above, but
T2 ## has only three non-zero entries, as for repeated (i,j)'s,
   ## the corresponding x's are "implicitly" added
stopifnot(nnzero(T2) == 3)
```

Diagonal

Construct a Diagonal Matrix

## **Description**

Construct a formally diagonal Matrix, i.e., an object inheriting from virtual class diagonal Matrix (or, if desired, a *mathematically* diagonal CsparseMatrix).

### Usage

```
Diagonal(n, x = NULL, names = FALSE)
.sparseDiagonal(n, x = NULL, uplo = "U", shape = "t", unitri = TRUE, kind, cols)
   .trDiagonal(n, x = NULL, uplo = "U", unitri = TRUE, kind)
   .symDiagonal(n, x = NULL, uplo = "U", kind)
```

### **Arguments**

n	integer indicating the dimension of the (square) matrix. If missing, then $length(x)$ is used.
Х	numeric or logical vector listing values for the diagonal entries, to be recycled as necessary. If NULL (the default), then the result is a unit diagonal matrixsparseDiagonal() and friends ignore non-NULL x when kind = "n".
names	either logical TRUE or FALSE or then a character vector of length n. If true <i>and</i> names(x) is not NULL, use that as both row and column names for the resulting matrix. When a character vector, use it for both dimnames.

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uplo	one of $c("U","L")$ , specifying the uplo slot of the result if the result is formally triangular of symmetric.
shape	one of c("t", "s", "g"), indicating if the result should be formally triangular, symmetric, or "general". The result will inherit from virtual class triangularMatrix, symmetricMatrix, or generalMatrix, respectively.
unitri	logical indicating if a formally triangular result with ones on the diagonal should be formally <i>unit</i> triangular, i.e., with diag slot equal to "U" rather than "N".
kind	one of c("d","1","n"), indicating the "mode" of the result: numeric, logical, or pattern. The result will inherit from virtual class dsparseMatrix, lsparseMatrix, or nsparseMatrix, respectively. Values other than "n" are ignored when x is non-NULL; in that case the mode is determined by typeof(x).
cols	optional integer vector with values in $\emptyset$ : (n-1), indexing columns of the specified diagonal matrix. If specified, then the result is (mathematically) D[, cols+1] rather than D, where D = Diagonal(n, x), and it is always "general" (i.e., shape is ignored).

#### Value

Diagonal() returns an object inheriting from virtual class diagonalMatrix.

```
.sparseDiagonal() returns a CsparseMatrix representation of Diagonal(n, x) or, if cols is given, of Diagonal(n, x)[, cols+1]. The precise class of the result depends on shape and kind.
```

.trDiagonal() and .symDiagonal() are simple wrappers, for .sparseDiagonal(shape = "t")
and .sparseDiagonal(shape = "s"), respectively.

. sparseDiagonal() exists primarily to leverage efficient C-level methods available for CsparseMatrix.

## Author(s)

Martin Maechler

#### See Also

the generic function diag for extraction of the diagonal from a matrix works for all "Matrices".

bandSparse constructs a *banded* sparse matrix from its non-zero sub-/super - diagonals. band(A) returns a band matrix containing some sub-/super - diagonals of A.

Matrix for general matrix construction; further, class diagonal Matrix.

```
Diagonal(3)
Diagonal(x = 10^(3:1))
Diagonal(x = (1:4) >= 2)#-> "ldiMatrix"

## Use Diagonal() + kronecker() for "repeated-block" matrices:
M1 <- Matrix(0+0:5, 2,3)
(M <- kronecker(Diagonal(3), M1))

(S <- crossprod(Matrix(rbinom(60, size=1, prob=0.1), 10,6)))</pre>
```

diagonalMatrix-class

```
(SI <- S + 10*.symDiagonal(6)) # sparse symmetric still
stopifnot(is(SI, "dsCMatrix"))
(I4 <- .sparseDiagonal(4, shape="t"))# now (2012-10) unitriangular
stopifnot(I4@diag == "U", all(I4 == diag(4)))</pre>
```

diagonalMatrix-class Class "diagonalMatrix" of Diagonal Matrices

## **Description**

Class "diagonalMatrix" is the virtual class of all diagonal matrices.

## **Objects from the Class**

A virtual Class: No objects may be created from it.

### **Slots**

```
diag: character string, either "U" or "N", where "U" means 'unit-diagonal'.
Dim: matrix dimension, and
Dimnames: the dimnames, a list, see the Matrix class description. Typically list(NULL, NULL) for diagonal matrices.
```

#### **Extends**

```
Class "sparseMatrix", directly.
```

#### Methods

These are just a subset of the signature for which defined methods. Currently, there are (too) many explicit methods defined in order to ensure efficient methods for diagonal matrices.

```
coerce signature(from = "matrix", to = "diagonalMatrix"): ...
coerce signature(from = "Matrix", to = "diagonalMatrix"): ...
coerce signature(from = "diagonalMatrix", to = "generalMatrix"): ...
coerce signature(from = "diagonalMatrix", to = "triangularMatrix"): ...
coerce signature(from = "diagonalMatrix", to = "nMatrix"): ...
coerce signature(from = "diagonalMatrix", to = "matrix"): ...
coerce signature(from = "diagonalMatrix", to = "sparseVector"): ...
t signature(x = "diagonalMatrix"): ...
    and many more methods
solve signature(a = "diagonalMatrix", b, ...): is trivially implemented, of course; see also solve-methods.
which signature(x = "nMatrix"), semantically equivalent to base function which(x, arr.ind).
```

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"Math" signature(x = "diagonalMatrix"): all these group methods return a "diagonalMatrix", apart from cumsum() etc which return a *vector* also for **base** matrix.

- \* signature(e1 = "ddiMatrix", e2="denseMatrix"): arithmetic and other operators from the Ops group have a few dozen explicit method definitions, in order to keep the results *diagonal* in many cases, including the following:
- / signature(e1 = "ddiMatrix", e2="denseMatrix"): the result is from class ddiMatrix which is typically very desirable. Note that when e2 contains off-diagonal zeros or NAs, we implicitly use 0/x = 0, hence differing from traditional R arithmetic (where  $0/0 \mapsto \text{NaN}$ ), in order to preserve sparsity.
- **summary** (object = "diagonalMatrix"): Returns an object of S3 class "diagSummary" which is the summary of the vector object@x plus a simple heading, and an appropriate print method.

#### See Also

Diagonal() as constructor of these matrices, and isDiagonal. ddiMatrix and ldiMatrix are "actual" classes extending "diagonalMatrix".

## **Examples**

diagU2N

Transform Triangular Matrices from Unit Triangular to General Triangular and Back

#### **Description**

Transform a triangular matrix x, i.e., of class triangularMatrix, from (internally!) unit triangular ("unitriangular") to "general" triangular (diagU2N(x)) or back (diagN2U(x)). Note that the latter, diagN2U(x), also sets the diagonal to one in cases where diag(x) was not all one.

.diagU2N(x) and .diagN2U(x) assume *without* checking that x is a triangularMatrix with suitable diag slot ("U" and "N", respectively), hence they should be used with care.

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### Usage

```
diagU2N(x, cl = getClassDef(class(x)), checkDense = FALSE)
diagN2U(x, cl = getClassDef(class(x)), checkDense = FALSE)
.diagU2N(x, cl = getClassDef(class(x)), checkDense = FALSE)
.diagN2U(x, cl = getClassDef(class(x)), checkDense = FALSE)
```

## **Arguments**

x a triangularMatrix, often sparse.
 cl (optional, for speedup only:) class (definition) of x.
 checkDense logical indicating if dense (see denseMatrix) matrices should be considered at

all; i.e., when false, as per default, the result will be sparse even when x is dense.

#### **Details**

The concept of unit triangular matrices with a diag slot of "U" stems from LAPACK.

#### Value

a triangular matrix of the same class but with a different diag slot. For diagU2N (semantically) with identical entries as x, whereas in diagN2U(x), the off-diagonal entries are unchanged and the diagonal is set to all 1 even if it was not previously.

#### Note

Such internal storage details should rarely be of relevance to the user. Hence, these functions really are rather *internal* utilities.

#### See Also

```
"triangularMatrix", "dtCMatrix".
```

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dimScale

Scale the Rows and Columns of a Matrix

## **Description**

dimScale, rowScale, and colScale implement D1 %\*% x %\*% D2, D %\*% x, and x %\*% D for diagonal matrices D1, D2, and D with diagonal entries d1, d2, and d, respectively. Unlike the explicit products, these functions preserve dimnames(x) and symmetry where appropriate.

## Usage

```
dimScale(x, d1 = sqrt(1/diag(x, names = FALSE)), d2 = d1)
rowScale(x, d)
colScale(x, d)
```

## Arguments

a matrix, possibly inheriting from virtual class Matrix.
 d1, d2, d
 numeric vectors giving factors by which to scale the rows or columns of x; they are recycled as necessary.

#### **Details**

dimScale(x) (with d1 and d2 unset) is only roughly equivalent to cov2cor(x). cov2cor sets the diagonal entries of the result to 1 (exactly); dimScale does not.

### Value

The result of scaling x, currently always inheriting from virtual class dMatrix.

It inherits from triangularMatrix if and only if x does. In the special case of dimScale(x, d1, d2) with identical d1 and d2, it inherits from symmetricMatrix if and only if x does.

### Author(s)

Mikael Jagan

## See Also

cov2cor

```
n <- 6L
(x <- forceSymmetric(matrix(1, n, n)))
dimnames(x) <- rep.int(list(letters[seq_len(n)]), 2L)

d <- seq_len(n)
(D <- Diagonal(x = d))</pre>
```

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```
(scx <- dimScale(x, d)) # symmetry and 'dimnames' kept
(mmx <- D %*% x %*% D) # symmetry and 'dimnames' lost
stopifnot(identical(unname(as(scx, "generalMatrix")), mmx))

rowScale(x, d)
colScale(x, d)

dMatrix-class (Virtual) Class "dMatrix" of "double" Matrices</pre>
```

## Description

The dMatrix class is a virtual class contained by all actual classes of numeric matrices in the **Matrix** package. Similarly, all the actual classes of logical matrices inherit from the lMatrix class.

#### Slots

Common to all matrix object in the package:

Dim: Object of class "integer" - the dimensions of the matrix - must be an integer vector with exactly two non-negative values.

Dimnames: list of length two; each component containing NULL or a character vector length equal the corresponding Dim element.

#### Methods

```
There are (relatively simple) group methods (see, e.g., Arith)
Arith signature(e1 = "dMatrix", e2 = "dMatrix"): ...
Arith signature(e1 = "dMatrix", e2 = "numeric"): ...
Arith signature(e1 = "numeric", e2 = "dMatrix"): ...
Math signature(x = "dMatrix"): ...
Math2 signature(x = "dMatrix", digits = "numeric"): this group contains round() and signif().
Compare signature(e1 = "numeric", e2 = "dMatrix"): ...
Compare signature(e1 = "dMatrix", e2 = "numeric"): ...
Compare signature(e1 = "dMatrix", e2 = "dMatrix"): ...
Summary signature(x = "dMatrix"): The "Summary" group contains the seven functions max(),
    min(), range(), prod(), sum(), any(), and all().
The following methods are also defined for all double matrices:
expm signature(x = "dMatrix"): computes the "Matrix Exponential", see expm.
The following methods are defined for all logical matrices:
which signature(x = "lsparseMatrix") and many other subclasses of "lMatrix": as the base
     function which(x, arr.ind) returns the indices of the TRUE entries in x; if arr.ind is true,
    as a 2-column matrix of row and column indices. Since Matrix version 1.2-9, if useNames is
    true, as by default, with dimnames, the same as base::which.
```

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### See Also

The nonzero-pattern matrix class nMatrix, which can be used to store non-NA logical matrices even more compactly.

The numeric matrix classes dgeMatrix, dgCMatrix, and Matrix.

drop@(x, tol=1e-10) is sometimes preferable to (and more efficient than) zapsmall(x, digits=10).

## **Examples**

```
showClass("dMatrix")
set.seed(101)
round(Matrix(rnorm(28), 4,7), 2)
M <- Matrix(rlnorm(56, sd=10), 4,14)
(M. <- zapsmall(M))
table(as.logical(M. == 0))</pre>
```

dmperm

Dulmage-Mendelsohn Permutation / Decomposition

## Description

For any  $n \times m$  (typically) sparse matrix x compute the Dulmage-Mendelsohn row and columns permutations which at first splits the n rows and m columns into coarse partitions each; and then a finer one, reordering rows and columns such that the permutated matrix is "as upper triangular" as possible.

### Usage

```
dmperm(x, nAns = 6L, seed = 0L)
```

## **Arguments**

```
x a typically sparse matrix; internally coerced to either "dgCMatrix" or "dtCMatrix".

nAns an integer specifying the length of the resulting list. Must be 2, 4, or 6.

seed an integer code in -1,0,1; determining the (initial) permutation; by default, seed = 0, no (or the identity) permutation; seed = -1 uses the "reverse" permutation k:1; for seed = 1, it is a random permutation (using R's RNG, seed, etc).
```

## **Details**

See the book section by Tim Davis; page 122–127, in the References.

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#### Value

p integer vector with the permutation p, of length nrow(x).

q integer vector with the permutation q, of length ncol(x).

r integer vector of length nb+1, where block k is rows r[k] to r[k+1]-1 in A[p,q].

s integer vector of length nb+1, where block k is cols s[k] to s[k+1]-1 in A[p,q].

rr5 integer vector of length 5, defining the coarse row decomposition.

cc5 integer vector of length 5, defining the coarse column decomposition.

#### Author(s)

Martin Maechler, with a lot of "encouragement" by Mauricio Vargas.

#### References

Section 7.4 *Dulmage-Mendelsohn decomposition*, pp. 122 ff of Timothy A. Davis (2006) *Direct Methods for Sparse Linear Systems*, SIAM Series "Fundamentals of Algorithms".

#### See Also

Schur, the class of permutation matrices; "pMatrix".

a named list with (by default) 6 components,

```
set.seed(17)
(S9 <- rsparsematrix(9, 9, nnz = 10, symmetric=TRUE)) # dsCMatrix
str( dm9 <- dmperm(S9) )</pre>
(S9p \leftarrow with(dm9, S9[p, q]))
## looks good, but *not* quite upper triangular; these, too:
str( dm9.0 <- dmperm(S9, seed=-1)) # non-random too.
str( dm9_1 <- dmperm(S9, seed= 1)) # a random one
## The last two permutations differ, but have the same effect!
(S9p0 \leftarrow with(dm9.0, S9[p, q])) # .. hmm ..
stopifnot(all.equal(S9p0, S9p))# same as as default, but different from the random one
set.seed(11)
(M <- triu(rsparsematrix(9,11, 1/4)))</pre>
dM <- dmperm(M); with(dM, M[p, q])</pre>
(Mp <- M[sample.int(nrow(M)), sample.int(ncol(M))])</pre>
dMp <- dmperm(Mp); with(dMp, Mp[p, q])</pre>
set.seed(7)
(n7 \leftarrow rsparsematrix(5, 12, nnz = 10, rand.x = NULL))
str(dm.7 <- dmperm(n7))
stopifnot(exprs = {
 lengths(dm.7[1:2]) == dim(n7)
```

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dpoMatrix-class

Positive Semi-definite Dense (Packed | Non-packed) Numeric Matrices

### **Description**

- The "dpoMatrix" class is the class of positive-semidefinite symmetric matrices in nonpacked storage.
- The "dppMatrix" class is the same except in packed storage. Only the upper triangle or the lower triangle is required to be available.
- The "corMatrix" and "copMatrix" classes represent correlation matrices. They extend "dpoMatrix" and "dppMatrix", respectively, with an additional slot sd allowing restoration of the original covariance matrix.

# **Objects from the Class**

Objects can be created by calls of the form new("dpoMatrix", ...) or from crossprod applied to an "dgeMatrix" object.

#### Slots

- uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.
- x: Object of class "numeric". The numeric values that constitute the matrix, stored in column-major order.
- Dim: Object of class "integer". The dimensions of the matrix which must be a two-element vector of non-negative integers.

Dimnames: inherited from class "Matrix"

- factors: Object of class "list". A named list of factorizations that have been computed for the matrix.
- sd: (for "corMatrix" and "copMatrix") a numeric vector of length n containing the (original)  $\sqrt{var(.)}$  entries which allow reconstruction of a covariance matrix from the correlation matrix.

#### **Extends**

```
Class "dsyMatrix", directly.
Classes "dgeMatrix", "symmetricMatrix", and many more by class "dsyMatrix".
```

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#### Methods

chol signature(x = "dpoMatrix"): Returns (and stores) the Cholesky decomposition of x, see chol.

**determinant** signature(x = "dpoMatrix"): Returns the determinant of x, via chol(x), see above.

rcond signature(x = "dpoMatrix", norm = "character"): Returns (and stores) the reciprocal
 of the condition number of x. The norm can be "0" for the one-norm (the default) or "I" for
 the infinity-norm. For symmetric matrices the result does not depend on the norm.

```
solve signature(a = "dpoMatrix", b = "...."), and
```

**solve** signature(a = "dppMatrix", b = "....") work via the Cholesky composition, see also the Matrix solve-methods.

**Arith** signature(e1 = "dpoMatrix", e2 = "numeric") (and quite a few other signatures): The result of ("elementwise" defined) arithmetic operations is typically *not* positive-definite anymore. The only exceptions, currently, are multiplications, divisions or additions with *positive* length(.) == 1 numbers (or logicals).

#### Note

Currently the validity methods for these classes such as getValidity(getClass("dpoMatrix")) for efficiency reasons only check the diagonal entries of the matrix – they may not be negative. This is only necessary but not sufficient for a symmetric matrix to be positive semi-definite.

A more reliable (but often more expensive) check for positive semi-definiteness would look at the signs of diag(BunchKaufman(.)) (with some tolerance for very small negative values), and for (strict) positive definiteness at something like !inherits(tryCatch(chol(.), error=identity), "error"). Indeed, when *coercing* to these classes, a version of Cholesky() or chol() is typically used, e.g., see selectMethod("coerce", c(from="dsyMatrix", to="dpoMatrix")).

## See Also

Classes dsyMatrix and dgeMatrix; further, Matrix, rcond, chol, solve, crossprod.

drop0 67

drop0	Drop Non-Structural Zeros from a Sparse Matrix	

# Description

Deletes "non-structural" zeros (i.e., zeros stored explicitly, in memory) from a sparse matrix and returns the result.

# Usage

```
drop0(x, tol = 0, is.Csparse = NA, give.Csparse = TRUE)
```

## **Arguments**

х	a Matrix, typically inheriting from virtual class sparseMatrix. denseMatrix and traditional vectors and matrices are coerced to CsparseMatrix, with zeros dropped automatically, hence users passing such x should consider as(x, "CsparseMatrix") instead, notably in the tol = 0 case.
tol	a non-negative number. If $\boldsymbol{x}$ is numeric, then entries less than or equal to tol in absolute value are deleted.
is.Csparse	a logical used only if give.Csparse is TRUE, indicating if x already inherits from virtual class CsparseMatrix, in which case coercion is not attempted, permitting some (typically small) speed-up.
give.Csparse	a logical indicating if the result must inherit from virtual class CsparseMatrix. If FALSE and x inherits from RsparseMatrix, TsparseMatrix, or indMatrix, then the result preserves the class of x. The default value is TRUE only for backwards compatibility.

# Value

A sparseMatrix, the result of deleting non-structural zeros from x, possibly after coercion.

# Note

drop0 is sometimes called in conjunction with zapsmall, e.g., when dealing with sparse matrix products; see the example.

## See Also

Function sparseMatrix, for constructing objects inheriting from virtual class sparseMatrix; nnzero.

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### **Examples**

dsCMatrix-class

Numeric Symmetric Sparse (column compressed) Matrices

### **Description**

The dsCMatrix class is a class of symmetric, sparse numeric matrices in the compressed, columnoriented format. In this implementation the non-zero elements in the columns are sorted into increasing row order.

The dsTMatrix class is the class of symmetric, sparse numeric matrices in triplet format.

#### **Objects from the Class**

```
Objects can be created by calls of the form new("dsCMatrix", ...) or new("dsTMatrix", ...), or automatically via e.g., as(*, "symmetricMatrix"), or (for dsCMatrix) also from Matrix(.). Creation "from scratch" most efficiently happens via sparseMatrix(*, symmetric=TRUE).
```

## Slots

uplo: A character object indicating if the upper triangle ("U") or the lower triangle ("L") is stored.

- i: Object of class "integer" of length nnZ (*half* number of non-zero elements). These are the row numbers for each non-zero element in the lower triangle of the matrix.
- p: (only in class "dsCMatrix":) an integer vector for providing pointers, one for each column, see the detailed description in CsparseMatrix.
- j: (only in class "dsTMatrix":) Object of class "integer" of length nnZ (as i). These are the column numbers for each non-zero element in the lower triangle of the matrix.
- x: Object of class "numeric" of length nnZ the non-zero elements of the matrix (to be duplicated for full matrix).

factors: Object of class "list" - a list of factorizations of the matrix.

Dim: Object of class "integer" - the dimensions of the matrix - must be an integer vector with exactly two non-negative values.

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#### **Extends**

Both classes extend classes and symmetricMatrix dsparseMatrix directly; dsCMatrix further directly extends CsparseMatrix, where dsTMatrix does TsparseMatrix.

#### Methods

```
solve signature(a = "dsCMatrix", b = "...."): x < - solve(a,b) solves Ax = b for x; see solve-methods.
```

chol signature(x = "dsCMatrix", pivot = "logical"): Returns (and stores) the Cholesky decomposition of x, see chol.

Cholesky signature(A = "dsCMatrix",...): Computes more flexibly Cholesky decompositions, see Cholesky.

**determinant** signature(x = "dsCMatrix", logarithm = "missing"): Evaluate the determinant of x on the logarithm scale. This creates and stores the Cholesky factorization.

**determinant** signature(x = "dsCMatrix", logarithm = "logical"): Evaluate the determinant of x on the logarithm scale or not, according to the logarithm argument. This creates and stores the Cholesky factorization.

- t signature(x = "dsCMatrix"): Transpose. As for all symmetric matrices, a matrix for which the upper triangle is stored produces a matrix for which the lower triangle is stored and vice versa, i.e., the uplo slot is swapped, and the row and column indices are interchanged.
- t signature(x = "dsTMatrix"): Transpose. The uplo slot is swapped from "U" to "L" or vice versa, as for a "dsCMatrix", see above.

#### See Also

Classes dgCMatrix, dgTMatrix, dgeMatrix and those mentioned above.

```
mm <- Matrix(toeplitz(c(10, 0, 1, 0, 3)), sparse = TRUE)
mm # automatically dsCMatrix
str(mm)
mT <- as(as(mm, "generalMatrix"), "TsparseMatrix")

## Either
(symM <- as(mT, "symmetricMatrix")) # dsT
(symC <- as(symM, "CsparseMatrix")) # dsC
## or
sT <- Matrix(mT, sparse=TRUE, forceCheck=TRUE) # dsT

sym2 <- as(symC, "TsparseMatrix")
## --> the same as 'symM', a "dsTMatrix"
```

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dsparseMatrix-class

Virtual Class "dsparseMatrix" of Numeric Sparse Matrices

## Description

The Class "dsparseMatrix" is the virtual (super) class of all numeric sparse matrices.

## **Slots**

```
Dim: the matrix dimension, see class "Matrix".Dimnames: see the "Matrix" class.x: a numeric vector containing the (non-zero) matrix entries.
```

### **Extends**

```
Class "dMatrix" and "sparseMatrix", directly. Class "Matrix", by the above classes.
```

#### See Also

the documentation of the (non virtual) sub classes, see showClass("dsparseMatrix"); in particular, dgTMatrix, dgCMatrix, and dgRMatrix.

## **Examples**

```
showClass("dsparseMatrix")
```

dsRMatrix-class

Symmetric Sparse Compressed Row Matrices

## **Description**

The dsRMatrix class is a class of symmetric, sparse matrices in the compressed, row-oriented format. In this implementation the non-zero elements in the rows are sorted into increasing column order.

## **Objects from the Class**

```
These "..RMatrix" classes are currently still mostly unimplemented!

Objects can be created by calls of the form new("dsRMatrix", ...).
```

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### Slots

uplo: A character object indicating if the upper triangle ("U") or the lower triangle ("L") is stored. At present only the lower triangle form is allowed.

- j: Object of class "integer" of length nnzero (number of non-zero elements). These are the row numbers for each non-zero element in the matrix.
- p: Object of class "integer" of pointers, one for each row, to the initial (zero-based) index of elements in the row.

factors: Object of class "list" - a list of factorizations of the matrix.

x: Object of class "numeric" - the non-zero elements of the matrix.

Dim: Object of class "integer" - the dimensions of the matrix - must be an integer vector with exactly two non-negative values.

Dimnames: List of length two, see Matrix.

### **Extends**

Classes dsparseMatrix, symmetricMatrix, and RsparseMatrix, directly.

Class "dMatrix", by class "dsparseMatrix"; class "sparseMatrix", by classes "dsparseMatrix" and "RsparseMatrix".

#### Methods

forceSymmetric signature(x = "dsRMatrix", uplo = "missing"): a trivial method just returning x

**forceSymmetric** signature(x = "dsRMatrix", uplo = "character"): if uplo == x@uplo, this trivially returns x; otherwise t(x).

### See Also

the classes dgCMatrix, dgTMatrix, and dgeMatrix.

72 dsyMatrix-class

dsyMatrix-class

Symmetric Dense (Packed or Unpacked) Numeric Matrices

## **Description**

- The "dsyMatrix" class is the class of symmetric, dense matrices in non-packed storage and
- "dspMatrix" is the class of symmetric dense matrices in *packed* storage, see pack(). Only the upper triangle or the lower triangle is stored.

## **Objects from the Class**

Objects can be created by calls of the form new("dsyMatrix", ...) or new("dspMatrix", ...), respectively.

#### **Slots**

- uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.
- x: Object of class "numeric". The numeric values that constitute the matrix, stored in column-major order.
- Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), see the Matrix.
- factors: Object of class "list". A named list of factorizations that have been computed for the matrix.

#### **Extends**

```
"dsyMatrix" extends class "dgeMatrix", directly, whereas "dspMatrix" extends class "ddenseMatrix", directly.
```

Both extend class "symmetricMatrix", directly, and class "Matrix" and others, *in*directly, use showClass("dsyMatrix"), e.g., for details.

## Methods

solve-methods.

```
norm signature(x = "dspMatrix", type = "character"), or x = "dsyMatrix" or type = "missing":
    Computes the matrix norm of the desired type, see, norm.
```

**rcond** signature(x = "dspMatrix", type = "character"), or x = "dsyMatrix" or type = "missing": Computes the reciprocal condition number, rcond().

```
solve signature(a = "dspMatrix", b = "...."), and

solve signature(a = "dsyMatrix", b = "...."): x < - solve(a,b) solves Ax = b for x; see
```

t signature(x = "dsyMatrix"): Transpose; swaps from upper triangular to lower triangular storage, i.e., the uplo slot from "U" to "L" or vice versa, the same as for all symmetric matrices.

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#### See Also

The *positive* (*Semi-*)*definite* dense (packed or non-packed numeric matrix classes dpoMatrix, dppMatrix and corMatrix,

Classes dgeMatrix and Matrix; solve, norm, rcond, t

## **Examples**

```
## Only upper triangular part matters (when uplo == "U" as per default)
(sy2 \leftarrow new("dsyMatrix", Dim = as.integer(c(2,2)), x = c(14, NA,32,77)))
str(t(sy2)) # uplo = "L", and the lower tri. (i.e. NA is replaced).
chol(sy2) #-> "Cholesky" matrix
(sp2 <- pack(sy2)) # a "dspMatrix"</pre>
## Coercing to dpoMatrix gives invalid object:
sy3 <- new("dsyMatrix", Dim = as.integer(c(2,2)), x = c(14, -1, 2, -7))
try(as(sy3, "dpoMatrix")) # -> error: not positive definite
## 4x4 example
m <- matrix(0,4,4); m[upper.tri(m)] <- 1:6</pre>
(sym \leftarrow m+t(m)+diag(11:14, 4))
(S1 <- pack(sym))
(S2 <- t(S1))
stopifnot(all(S1 == S2)) # equal "seen as matrix", but differ internally :
str(S1)
S2@x
```

dtCMatrix-class

Triangular, (compressed) sparse column matrices

# **Description**

The "dtCMatrix" class is a class of triangular, sparse matrices in the compressed, column-oriented format. In this implementation the non-zero elements in the columns are sorted into increasing row order.

The "dtTMatrix" class is a class of triangular, sparse matrices in triplet format.

# **Objects from the Class**

Objects can be created by calls of the form new("dtCMatrix", ...) or calls of the form new("dtTMatrix", ...), but more typically automatically via Matrix() or coercions such as as(x, "triangularMatrix").

## Slots

uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.

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- diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N"; see triangularMatrix.
- p: (only present in "dtCMatrix":) an integer vector for providing pointers, one for each column, see the detailed description in CsparseMatrix.
- i: Object of class "integer" of length nnzero (number of non-zero elements). These are the row numbers for each non-zero element in the matrix.
- j: Object of class "integer" of length nnzero (number of non-zero elements). These are the column numbers for each non-zero element in the matrix. (Only present in the dtTMatrix class.)
- x: Object of class "numeric" the non-zero elements of the matrix.
- Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), inherited from the Matrix, see there.

#### **Extends**

Class "dgCMatrix", directly. Class "triangularMatrix", directly. Class "dMatrix", "sparseMatrix", and more by class "dgCMatrix" etc, see the examples.

#### Methods

```
solve signature(a = "dtCMatrix", b = "...."): sparse triangular solve (aka "backsolve" or "for-
wardsolve"), see solve-methods.

t signature(x = "dtCMatrix"): returns the transpose of x

t signature(x = "dtTMatrix"): returns the transpose of x
```

#### See Also

Classes dgCMatrix, dgTMatrix, dgeMatrix, and dtrMatrix.

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dtpMatrix-class

Packed Triangular Dense Matrices - "dtpMatrix"

## **Description**

The "dtpMatrix" class is the class of triangular, dense, numeric matrices in packed storage. The "dtrMatrix" class is the same except in nonpacked storage.

#### **Objects from the Class**

Objects can be created by calls of the form new("dtpMatrix", ...) or by coercion from other classes of matrices.

#### Slots

uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.

diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N"; see triangularMatrix.

x: Object of class "numeric". The numeric values that constitute the matrix, stored in column-major order. For a packed square matrix of dimension  $d \times d$ , length(x) is of length d(d+1)/2 (also when diag == "U"!).

Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), inherited from the Matrix, see there.

#### **Extends**

Class "ddenseMatrix", directly. Class "triangularMatrix", directly. Class "dMatrix" and more by class "ddenseMatrix" etc, see the examples.

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## Methods

```
%*% signature(x = "dtpMatrix", y = "dgeMatrix"): Matrix multiplication; ditto for several
    other signature combinations, see showMethods("%*%", class = "dtpMatrix").

determinant signature(x = "dtpMatrix", logarithm = "logical"): the determinant(x) trivially is prod(diag(x)), but computed on log scale to prevent over- and underflow.

diag signature(x = "dtpMatrix"): ...

norm signature(x = "dtpMatrix", type = "character"): ...

rcond signature(x = "dtpMatrix", norm = "character"): ...

solve signature(a = "dtpMatrix", b = "..."): efficiently using internal backsolve or forward-
    solve, see solve-methods.

t signature(x = "dtpMatrix"): t(x) remains a "dtpMatrix", lower triangular if x is upper tri-
    angular, and vice versa.
```

#### See Also

Class dtrMatrix

#### **Examples**

dtRMatrix-class

Triangular Sparse Compressed Row Matrices

# **Description**

The dtRMatrix class is a class of triangular, sparse matrices in the compressed, row-oriented format. In this implementation the non-zero elements in the rows are sorted into increasing columnd order.

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## **Objects from the Class**

This class is currently still mostly unimplemented!

Objects can be created by calls of the form new("dtRMatrix", ...).

#### **Slots**

uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular. At present only the lower triangle form is allowed.

diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N"; see triangularMatrix.

- j: Object of class "integer" of length nnzero(.) (number of non-zero elements). These are the row numbers for each non-zero element in the matrix.
- p: Object of class "integer" of pointers, one for each row, to the initial (zero-based) index of elements in the row. (Only present in the dsRMatrix class.)
- x: Object of class "numeric" the non-zero elements of the matrix.

Dim: The dimension (a length-2 "integer")

Dimnames: corresponding names (or NULL), inherited from the Matrix, see there.

## **Extends**

Class "dgRMatrix", directly. Class "dsparseMatrix", by class "dgRMatrix". Class "dMatrix", by class "dgRMatrix". Class "sparseMatrix", by class "dgRMatrix". Class "Matrix", by class "dgRMatrix".

#### Methods

No methods currently with class "dsRMatrix" in the signature.

## See Also

Classes dgCMatrix, dgTMatrix, dgeMatrix

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dtrMatrix-class

Triangular, dense, numeric matrices

# **Description**

The "dtrMatrix" class is the class of triangular, dense, numeric matrices in nonpacked storage. The "dtpMatrix" class is the same except in packed storage, see pack().

## **Objects from the Class**

Objects can be created by calls of the form new("dtrMatrix", ...).

#### Slots

- uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.
- diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N"; see triangularMatrix.
- x: Object of class "numeric". The numeric values that constitute the matrix, stored in column-major order.
- Dim: Object of class "integer". The dimensions of the matrix which must be a two-element vector of non-negative integers.

#### Extends

Class "ddenseMatrix", directly. Class "triangularMatrix", directly. Class "Matrix" and others, by class "ddenseMatrix".

## Methods

Among others (such as matrix products, e.g. ?crossprod-methods),

```
norm signature(x = "dtrMatrix", type = "character"): ..
rcond signature(x = "dtrMatrix", norm = "character"): ..
```

**solve** signature(a = "dtrMatrix", b = "...."): efficiently use a "forwardsolve" or backsolve for a lower or upper triangular matrix, respectively, see also solve-methods.

+, -, \*, ..., ==, >=, ... all the Ops group methods are available. When applied to two triangular matrices, these return a triangular matrix when easily possible.

#### See Also

Classes ddenseMatrix, dtpMatrix, triangularMatrix

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## **Examples**

```
(m <- rbind(2:3, 0:-1))
(M <- as(m, "generalMatrix"))</pre>
(T <- as(M, "triangularMatrix")) # formally upper triangular
(T2 <- as(t(M), "triangularMatrix"))</pre>
stopifnot(T@uplo == "U", T2@uplo == "L", identical(T2, t(T)))
m <- matrix(0,4,4); m[upper.tri(m)] <- 1:6</pre>
(t1 <- Matrix(m+diag(,4)))</pre>
str(t1p <- pack(t1))
(t1pu \leftarrow diagN2U(t1p))
stopifnot(exprs = {
   inherits(t1 , "dtrMatrix"); validObject(t1)
   inherits(t1p, "dtpMatrix"); validObject(t1p)
   inherits(t1pu, "dtCMatrix"); validObject(t1pu)
   t1pu@x == 1:6
   all(t1pu == t1p)
   identical((t1pu - t1)@x, numeric())# sparse all-0
})
```

expand-methods

**Expand Matrix Factorizations** 

## **Description**

expand1 and expand2 construct matrix factors from objects specifying matrix factorizations. Such objects typically do not store the factors explicitly, employing instead a compact representation to save memory.

## Usage

```
expand1(x, which, ...) expand2(x, ...) expand (x, ...)
```

#### **Arguments**

x a matrix factorization, typically inheriting from virtual class MatrixFactorization.
 which a character string indicating a matrix factor.
 further arguments passed to or from methods.

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## **Details**

Methods for expand are retained only for backwards compatibility with **Matrix** < 1.6-0. New code should use expand1 and expand2, whose methods provide more control and behave more consistently. Notably, expand2 obeys the rule that the product of the matrix factors in the returned list should reproduce (within some tolerance) the factorized matrix, *including* its dimnames.

Hence if x is a matrix and y is its factorization, then

```
all.equal(as(x, "matrix"), as(Reduce(`%*%`, expand2(y)), "matrix"))
```

should in most cases return TRUE.

#### Value

expand1 returns an object inheriting from virtual class Matrix, representing the factor indicated by which, always without row and column names.

expand2 returns a list of factors, typically with names using conventional notation, as in list(L=, U=). The first and last factors get the row and column names of the factorized matrix, which are preserved in the Dimnames slot of x.

#### Methods

The following table lists methods for expand1 together with allowed values of argument which.

Methods for expand2 and expand are described below. Factor names and classes apply also to expand1.

```
expand2 signature(x = "CHMsimpl"): expands the factorization A = P_1'L_1DL_1'P_1 = P_1'LL'P_1 as list(P1., L1, D, L1., P1) (the default) or as list(P1., L, L., P1), depending on optional logical argument LDL. P1 and P1. are pMatrix, L1, L1., L, and L. are dtCMatrix, and D is a ddiMatrix.
```

expand2 signature(x = "CHMsuper"): as CHMsimpl, but the triangular factors are stored as dgCMatrix.

```
expand2 signature(x = "p?Cholesky"): expands the factorization A = L_1DL'_1 = LL' as list(L1, D, L1.) (the default) or as list(L, L.), depending on optional logical argument LDL. L1, L1., L, and L. are dtrMatrix or dtpMatrix, and D is a ddiMatrix.
```

```
expand2 signature(x = "p?BunchKaufman"): expands the factorization A = UD_UU' = LD_LL' where U = \prod_{k=1}^{b_U} P_k U_k and L = \prod_{k=1}^{b_L} P_k L_k as list(U, DU, U.) or list(L, DL, L.), depending on x@uplo. If optional argument complete is TRUE, then an unnamed list giving the full expansion with 2b_U + 1 or 2b_L + 1 matrix factors is returned instead. P_k are represented
```

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as pMatrix,  $U_k$  and  $L_k$  are represented as dtCMatrix, and  $D_U$  and  $D_L$  are represented as dsCMatrix.

- expand2 signature(x = "Schur"): expands the factorization A = QTQ' as list(Q, T, Q.). Q and Q. are x@Q and t(x@Q) modulo Dimnames, and T is x@T.
- expand2 signature(x = "sparseLU"): expands the factorization  $A = P_1'LUP_2'$  as list(P1., L, U, P2.). P1. and P2. are pMatrix, and L and U are dtCMatrix.
- expand2 signature(x = "denseLU"): expands the factorization  $A = P'_1LU$  as list(P1., L, U). P1. is a pMatrix, and L and U are dtrMatrix if square and dgeMatrix otherwise.
- expand2 signature(x = "sparseQR"): expands the factorization  $A = P_1'QRP_2' = P_1'Q_1R_1P_2'$  as list(P1., Q, R, P2.) or list(P1., Q1, R1, P2.), depending on optional logical argument complete. P1. and P2. are pMatrix, Q and Q1 are dgeMatrix, R is a dgCMatrix, and R1 is a dtCMatrix.
- expand signature(x = "CHMfactor"): as expand2, but returning list(P, L). expand(x)[["P"]] and expand2(x)[["P1"]] represent the same permutation matrix  $P_1$  but have opposite margin slots and inverted perm slots. The components of expand(x) do not preserve x@Dimnames.
- expand signature(x = "sparseLU"): as expand2, but returning list(P, L, U, Q). expand(x)[["Q"]] and expand2(x)[["P2."]] represent the same permutation matrix  $P_2'$  but have opposite margin slots and inverted perm slots. expand(x)[["P"]] represents the permutation matrix  $P_1$  rather than its transpose  $P_1'$ ; it is expand2(x)[["P1."]] with an inverted perm slot. expand(x)[["L"]] and expand2(x)[["L"]] represent the same unit lower triangular matrix L, but with diag slot equal to "N" and "U", respectively. expand(x)[["L"]] and expand(x)[["U"]] store the permuted first and second components of x@Dimnames in their Dimnames slots.
- expand signature(x = "denseLU"): as expand2, but returning list(L, U, P). expand(x)[["P"]] and expand2(x)[["P1."]] are identical modulo Dimnames. The components of expand(x) do not preserve x@Dimnames.

# See Also

The virtual class MatrixFactorization of matrix factorizations.

Generic functions Cholesky, BunchKaufman, Schur, 1u, and qr for computing factorizations.

```
showMethods("expand1", inherited = FALSE)
showMethods("expand2", inherited = FALSE)
set.seed(0)

(A <- Matrix(rnorm(9L, 0, 10), 3L, 3L))
(lu.A <- lu(A))
(e.lu.A <- expand2(lu.A))
stopifnot(exprs = {
    is.list(e.lu.A)
    identical(names(e.lu.A), c("P1.", "L", "U"))
    all(sapply(e.lu.A, is, "Matrix"))
    all.equal(as(A, "matrix"), as(Reduce(`%*%`, e.lu.A), "matrix"))
})

## 'expand1' and 'expand2' give equivalent results modulo</pre>
```

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```
## dimnames and representation of permutation matrices;
## see also function 'alt' in example("Cholesky-methods")
(a1 <- sapply(names(e.lu.A), expand1, x = lu.A, simplify = FALSE))
all.equal(a1, e.lu.A)
## see help("denseLU-class") and others for more examples</pre>
```

expm-methods

Matrix Exponential

## **Description**

Compute the exponential of a matrix.

# Usage

expm(x)

#### **Arguments**

Χ

a matrix, typically inheriting from the dMatrix class.

#### **Details**

The exponential of a matrix is defined as the infinite Taylor series  $expm(A) = I + A + A^2/2! + A^3/3! + ...$  (although this is definitely not the way to compute it). The method for the dgeMatrix class uses Ward's diagonal Pade' approximation with three step preconditioning, a recommendation from Moler & Van Loan (1978) "Nineteen dubious ways...".

## Value

The matrix exponential of x.

# Author(s)

This is a translation of the implementation of the corresponding Octave function contributed to the Octave project by A. Scottedward Hodel <A.S. Hodel@Eng. Auburn. EDU>. A bug in there has been fixed by Martin Maechler.

## References

```
https://en.wikipedia.org/wiki/Matrix_exponential
```

Cleve Moler and Charles Van Loan (2003) Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later. *SIAM Review* **45**, 1, 3–49. doi:10.1137/S00361445024180

for historical reference mostly:

Moler, C. and Van Loan, C. (1978) Nineteen dubious ways to compute the exponential of a matrix. *SIAM Review* **20**, 4, 801–836. doi:10.1137/1020098

Eric W. Weisstein et al. (1999) *Matrix Exponential*. From MathWorld, https://mathworld.wolfram.com/MatrixExponential.html

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#### See Also

Package **expm**, which provides newer (in some cases faster, more accurate) algorithms for computing the matrix exponential via its own (non-generic) function expm(). **expm** also implements logm(), sqrtm(), etc.

Generic function Schur.

# **Examples**

```
(m1 <- Matrix(c(1,0,1,1), ncol = 2))
(e1 <- expm(m1)) ; e <- exp(1)
stopifnot(all.equal(e1@x, c(e,0,e,e), tolerance = 1e-15))
(m2 <- Matrix(c(-49, -64, 24, 31), ncol = 2))
(e2 <- expm(m2))
(m3 <- Matrix(cbind(0,rbind(6*diag(3),0))))# sparse!
(e3 <- expm(m3)) # upper triangular</pre>
```

externalFormats

Read and write external matrix formats

## **Description**

Read matrices stored in the Harwell-Boeing or MatrixMarket formats or write sparseMatrix objects to one of these formats.

# Usage

```
readHB(file)
readMM(file)
writeMM(obj, file, ...)
```

## Arguments

file

obj a real sparse matrix

for writeMM - the name of the file to be written. For readHB and readMM the name of the file to read, as a character scalar. The names of files storing matrices

in the Harwell-Boeing format usually end in ".rua" or ".rsa". Those storing

matrices in the MatrixMarket format usually end in ".mtx". Alternatively, readHB and readMM accept connection objects.

... optional additional arguments. Currently none are used in any methods.

# Value

The readHB and readMM functions return an object that inherits from the "Matrix" class. Methods for the writeMM generic functions usually return NULL and, as a side effect, the matrix obj is written to file in the MatrixMarket format (writeMM).

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#### Note

The Harwell-Boeing format is older and less flexible than the MatrixMarket format. The function writeHB was deprecated and has now been removed. Please use writeHM instead.

Note that these formats do *not* know anything about dimnames, hence these are dropped by writeMM().

A very simple way to export small sparse matrices S, is to use summary(S) which returns a data. frame with columns i, j, and possibly x, see summary in sparseMatrix-class, and an example below.

#### References

```
https://math.nist.gov/MatrixMarket/
https://sparse.tamu.edu/
```

```
str(pores <- readMM(system.file("external/pores_1.mtx", package = "Matrix")))</pre>
str(utm <- readHB(system.file("external/utm300.rua" , package = "Matrix")))</pre>
str(lundA <- readMM(system.file("external/lund_a.mtx" , package = "Matrix")))
str(lundA <- readHB(system.file("external/lund_a.rsa" , package = "Matrix")))</pre>
## https://math.nist.gov/MatrixMarket/data/Harwell-Boeing/counterx/counterx.htm
str(jgl <- readMM(system.file("external/jgl009.mtx" , package = "Matrix")))</pre>
## NOTE: The following examples take quite some time
## ---- even on a fast internet connection:
if(FALSE) {
## The URL has been corrected, but we need an untar step:
u. <- url("https://www.cise.ufl.edu/research/sparse/RB/Boeing/msc00726.tar.gz")</pre>
str(sm <- readHB(gzcon(u.)))</pre>
}
data(KNex, package = "Matrix")
## Store as MatrixMarket (".mtx") file, here inside temporary dir./folder:
(MMfile <- file.path(tempdir(), "mmMM.mtx"))</pre>
writeMM(KNex$mm, file=MMfile)
file.info(MMfile)[,c("size", "ctime")] # (some confirmation of the file's)
## very simple export - in triplet format - to text file:
data(CAex, package = "Matrix")
s.CA <- summary(CAex)</pre>
s.CA \# shows (i, j, x) [columns of a data frame]
message("writing to ", outf <- tempfile())</pre>
write.table(s.CA, file = outf, row.names=FALSE)
## and read it back -- showing off sparseMatrix():
str(dd <- read.table(outf, header=TRUE))</pre>
## has columns (i, j, x) \rightarrow we can use via do.call() as arguments to sparseMatrix():
mm <- do.call(sparseMatrix, dd)</pre>
stopifnot(all.equal(mm, CAex, tolerance=1e-15))
```

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Multiplication by Factors from Matrix Factorizations

# **Description**

Multiplies a matrix or vector on the left or right by a factor from a matrix factorization or its transpose.

#### **Usage**

```
facmul(x, factor, y, trans = FALSE, left = TRUE, ...)
```

# Arguments

Χ	a MatrixFactorization object.
factor	a character string indicating a factor in the factorization represented by $x$ , typically an element of names(expand2( $x$ ,)).
У	a matrix or vector to be multiplied on the left or right by the factor or its transpose.
trans	a logical indicating if the transpose of the factor should be used, rather than the factor itself.
left	a logical indicating if the y should be multiplied on the left by the factor, rather than on the right.
	further arguments passed to or from methods.

#### **Details**

facmul is experimental and currently no methods are exported from Matrix.

## Value

The value of op(M) % % y or y % % op(M), depending on left, where M is the factor (always without dimnames) and op(M) is M or t(M), depending on trans.

```
## Conceptually, methods for 'facmul' _would_ behave as follows ...
## Not run:
n <- 3L
x <- lu(Matrix(rnorm(n * n), n, n))
y <- rnorm(n)
L <- unname(expand2(x)[[nm <- "L"]])
stopifnot(exprs = {
    all.equal(facmul(x, nm, y, trans = FALSE, left = TRUE), L %*% y)
    all.equal(facmul(x, nm, y, trans = FALSE, left = FALSE), y %*% L)
    all.equal(facmul(x, nm, y, trans = TRUE, left = TRUE), crossprod(L, y))
    all.equal(facmul(x, nm, y, trans = TRUE, left = FALSE), tcrossprod(y, L))</pre>
```

```
})
## End(Not run)
```

fastMisc

"Low Level" Coercions and Methods

# Description

"Semi-API" functions used internally by **Matrix**, often to bypass S4 dispatch and avoid the associated overhead. These are exported to provide this capability to expert users. Typical users should continue to rely on S4 generic functions to dispatch suitable methods, by calling, e.g., as(., <class>) for coercions.

# Usage

```
.M2kind(from, kind = ".", sparse = NA)
.M2gen(from, kind = ".")
.M2sym(from, ...)
.M2tri(from, ...)
.M2diag(from)
.M2v(from)
.M2m(from)
.M2unpacked(from)
.M2packed(from)
.M2C(from)
.M2R(from)
.M2T(from)
.M2V(from)
.m2V(from, kind = ".")
.sparse2dense(from, packed = FALSE)
.diag2dense(from, kind = ".", shape = "t", packed = FALSE, uplo = "U")
.ind2dense(from, kind = "n")
.m2dense(from, class = ".ge", uplo = "U", diag = "N", trans = FALSE)
.dense2sparse(from, repr = "C")
.diag2sparse(from, kind = ".", shape = "t", repr = "C", uplo = "U")
.ind2sparse(from, kind = "n", repr = ".")
.m2sparse(from, class = ".gC", uplo = "U", diag = "N", trans = FALSE)
.tCRT(x, lazy = TRUE)
.diag.dsC(x, Chx = Cholesky(x, LDL = TRUE), res.kind = "diag")
```

```
.solve.dgC.lu (a, b, tol = .Machine$double.eps, check = TRUE)
.solve.dgC.qr (a, b, order = 3L, check = TRUE)
.solve.dgC.chol(a, b, check = TRUE)
.updateCHMfactor(object, parent, mult = 0)
```

# Arguments

from, x, a, b	a Matrix, matrix, or vector.
kind	a string (".", ",", "n", "1", or "d") specifying the "kind" of the result. "." indicates that the kind of from should be preserved. "," is equivalent to "z" if from is complex and to "d" otherwise. "n" indicates that the result should inherit from nMatrix or nsparseVector (and so on).
shape	a string (".", "g", "s", or "t") specifying the "shape" of the result. "." indicates that the shape of from should be preserved. "g" indicates that the result should inherit from generalMatrix (and so on).
repr	a string (".", "C", "R", or "T") specifying the sparse representation of the result. "." is accepted only by .ind2sparse and indicates the most efficient representation, which is "C" ("R") for margin = 2 (1). "C" indicates that the result should inherit from CsparseMatrix (and so on).
packed	a logical indicating if the result should inherit from packedMatrix rather than from unpackedMatrix. It is ignored for from inheriting from generalMatrix.
sparse	a logical indicating if the result should inherit from sparseMatrix rather than from denseMatrix. If NA, then the result will be formally sparse if and only if from is.
uplo	a string ("U" or "L") indicating whether the result (if symmetric or triangular) should store the upper or lower triangle of from. The elements of from in the opposite triangle are ignored.
diag	a string ("N" or "U") indicating whether the result (if triangular) should be formally nonunit or unit triangular. In the unit triangular case, the diagonal elements of from are ignored.
trans	a logical indicating if the result should be a 1-row matrix rather than a 1-column matrix where from is a vector but not a matrix.
class	a string whose first three characters specify the class of the result. It should match the pattern "^[.nld](ge sy tr sp tp)" for .m2dense and "^[.nld][gst][CRT]" for .m2sparse, where "." in the first position is equivalent to "1" for logical arguments and "d" for numeric arguments.
	optional arguments passed to isSymmetric or isTriangular.
lazy	a logical indicating if the transpose should be constructed with minimal allocation, but possibly <i>without</i> preserving representation.
Chx	optionally, the Cholesky( $x$ ,) factorization of $x$ . If supplied, then $x$ is unused.
res.kind	<pre>a string in c("trace", "sumLog", "prod", "min", "max", "range", "diag", "diagBack").</pre>

see lu-methods. tol see qr-methods. order check a logical indicating if the first argument should be tested for inheritance from dgCMatrix and coerced if necessary. Set to FALSE for speed only if it is known to already inherit from dgCMatrix. object a Cholesky factorization inheriting from virtual class CHMfactor, almost always the result of a call to generic function Cholesky. parent an object of class dsCMatrix or class dgCMatrix. mult a numeric vector of postive length. Only the first element is used, and that must be finite.

#### **Details**

Functions with names of the form .<A>2<B> implement coercions from virtual class A to the "nearest" non-virtual subclass of virtual class B, where the virtual classes are abbreviated as follows:

```
M Matrix
V sparseVector
m matrix
v vector
dense denseMatrix
unpacked unpackedMatrix
packed packedMatrix
sparse CsparseMatrix, RsparseMatrix, or TsparseMatrix
C CsparseMatrix
R RsparseMatrix
T TsparseMatrix
gen generalMatrix
sym symmetricMatrix
tri triangularMatrix
diag diagonalMatrix
ind indMatrix
```

Abbreviations should be seen as a guide, rather than as an exact description of behaviour. Notably, .m2dense, .m2sparse, and .m2V accept vectors that are not matrices.

.tCRT(x): If lazy = TRUE, then .tCRT constructs the transpose of x using the most efficient representation, which for 'CRT' is 'RCT'. If lazy = FALSE, then .tCRT preserves the representation of x, behaving as the corresponding methods for generic function t.

.diag.dsC(x): .diag.dsC computes (or uses if Chx is supplied) the Cholesky factorization of x as LDL' in order to calculate one of several possible statistics from the diagonal entries of D. See res.kind under 'Arguments'.

.solve.dgC.\*(a, b): .solve.dgC.lu(a, b) needs a square matrix a. .solve.dgC.qr(a, b) needs a "long" matrix a, with nrow(a) >= ncol(a). .solve.dgC.chol(a, b) needs a "wide" matrix a, with nrow(a) <= ncol(a).

All three may be used to solve sparse linear systems directly. Only . solve . dgC . qr and . solve . dgC . chol be used to solve sparse *least squares* problems.

.updateCHMfactor(object, parent, mult): .updateCHMfactor updates object with the result of Cholesky factorizing F(parent) + mult[1] \* diag(nrow(parent)), i.e., F(parent) plus mult[1] times the identity matrix, where F = identity if parent is a dsCMatrix and F = tcrossprod if parent is a dgCMatrix. The nonzero pattern of F(parent) must match that of S if object = Cholesky(S, ...).

```
D. \leftarrow diag(x = c(1, 1, 2, 3, 5, 8))
D.0 <- Diagonal(x = c(0, 0, 0, 3, 5, 8))
S. <- toeplitz(as.double(1:6))</pre>
C. <- new("dgCMatrix", Dim = c(3L, 4L),
          p = c(0L, 1L, 1L, 1L, 3L), i = c(1L, 0L, 2L), x = c(-8, 2, 3))
stopifnot(exprs = {
    identical(.M2tri (D.), as(D., "triangularMatrix"))
    identical(.M2sym (D.), as(D., "symmetricMatrix"))
    identical(.M2diag(D.), as(D., "diagonalMatrix"))
    identical(.M2kind(C., "l"),
              as(C., "lMatrix"))
    identical(.M2kind(.sparse2dense(C.), "1"),
              as(as(C., "denseMatrix"), "lMatrix"))
    identical(.diag2sparse(D.0, ".", "t", "C"),
              .dense2sparse(.diag2dense(D.0, ".", "t", TRUE), "C"))
    identical(.M2gen(.diag2dense(D.0, ".", "s", FALSE)),
               .sparse2dense(.M2gen(.diag2sparse(D.0, ".", "s", "T"))))
    identical(S.,
              .M2m(.m2sparse(S., ".sR")))
    identical(S. * lower.tri(S.) + diag(1, 6L),
              .M2m(.m2dense (S., ".tr", "L", "U")))
    identical(.M2R(C.), .M2R(.M2T(C.)))
    identical(.tCRT(C.), .M2R(t(C.)))
})
A <- tcrossprod(C.)/6 + Diagonal(3, 1/3); A[1,2] <- 3; A
stopifnot(exprs = {
    is.numeric( x. <- c(2.2, 0, -1.2) )
    all.equal(x., .solve.dgC.lu(A, c(1,0,0), check=FALSE))
    all.equal(x., .solve.dgC.qr(A, c(1,0,0), check=FALSE))
})
## Solving sparse least squares:
X <- rbind(A, Diagonal(3)) # design matrix X (for L.S.)</pre>
Xt \leftarrow t(X)
                            # *transposed* X (for L.S.)
(y \leftarrow drop(crossprod(Xt, 1:3)) + c(-1,1)/1000) # small rand.err.
```

```
str(solveCh <- .solve.dgC.chol(Xt, y, check=FALSE)) # Xt *is* dgC..
stopifnot(exprs = {
    all.equal(solveCh$coef, 1:3, tol = 1e-3)# rel.err ~ 1e-4
    all.equal(solveCh$coef, drop(solve(tcrossprod(Xt), Xt %*% y)))
    all.equal(solveCh$coef, .solve.dgC.qr(X, y, check=FALSE))
})</pre>
```

forceSymmetric-methods

Force a Matrix to 'symmetricMatrix' Without Symmetry Checks

## **Description**

Force a square matrix x to a symmetricMatrix, without a symmetry check as it would be applied for as(x, "symmetricMatrix").

## Usage

```
forceSymmetric(x, uplo)
```

# **Arguments**

x any square matrix (of numbers), either ""traditional"" (matrix) or inheriting from Matrix.

uplo optional string, "U" or "L" indicating which "triangle" half of x should determine the result. The default is "U" unless x already has a uplo slot (i.e., when it is symmetricMatrix, or triangularMatrix), where the default will be x@uplo.

#### Value

a square matrix inheriting from class symmetricMatrix.

#### See Also

symmpart for the symmetric part of a matrix, or the coercions as(x, <symmetricMatrix class>).

```
## Hilbert matrix
i <- 1:6
h6 <- 1/outer(i - 1L, i, "+")
sd <- sqrt(diag(h6))
hh <- t(h6/sd)/sd # theoretically symmetric
isSymmetric(hh, tol=0) # FALSE; hence
try( as(hh, "symmetricMatrix") ) # fails, but this works fine:
H6 <- forceSymmetric(hh)
## result can be pretty surprising:
(M <- Matrix(1:36, 6))</pre>
```

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```
forceSymmetric(M) # symmetric, hence very different in lower triangle
(tm <- tril(M))
forceSymmetric(tm)</pre>
```

formatSparseM

Formatting Sparse Numeric Matrices Utilities

# **Description**

Utilities for formatting sparse numeric matrices in a flexible way. These functions are used by the format and print methods for sparse matrices and can be applied as well to standard R matrices. Note that *all* arguments but the first are optional.

formatSparseM() is the main "workhorse" of formatSpMatrix, the format method for sparse matrices.

.formatSparseSimple() is a simple helper function, also dealing with (short/empty) column names construction.

# Usage

## **Arguments**

X	an R object inheriting from class sparseMatrix.
zero.print	character which should be used for <i>structural</i> zeroes. The default "." may occasionally be replaced by " " (blank); using "0" would look almost like print()ing of non-sparse matrices.
align	a string specifying how the {\tt zero.print} codes should be aligned, see {\tt formatSpMatrix}.
m	(optional) a (standard R) matrix version of x.
asLogical	should the matrix be formatted as a logical matrix (or rather as a numeric one); mostly for formatSparseM().
uniDiag	logical indicating if the diagonal entries of a sparse unit triangular or unit- diagonal matrix should be formatted as "I" instead of "1" (to emphasize that the 1's are "structural").
digits	significant digits to use for printing, see print.default.
сх	(optional) character matrix; a formatted version of $x$ , still with strings such as "0.00" for the zeros.
iN0	(optional) integer vector, specifying the location of the <i>non</i> -zeroes of x.

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#### Value

a character matrix like cx, where the zeros have been replaced with (padded versions of) zero.print. As this is a *dense* matrix, do not use these functions for really large (really) sparse matrices!

## Author(s)

Martin Maechler

#### See Also

formatSpMatrix which calls formatSparseM() and is the format method for sparse matrices. printSpMatrix which is used by the (typically implicitly called) show and print methods for sparse matrices.

#### **Examples**

generalMatrix-class

Class "generalMatrix" of General Matrices

# **Description**

Virtual class of "general" matrices; i.e., matrices that do not have a known property such as symmetric, triangular, or diagonal.

# **Objects from the Class**

A virtual Class: No objects may be created from it.

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# **Slots**

Dim, Dimnames inherited from virtual class Matrix.

factors a list of MatrixFactorization objects caching factorizations of the matrix. Typically, it is initialized as an empty list and updated "automagically" whenever a factorization is computed.

# **Extends**

```
Class "Matrix", directly.
```

## See Also

Virtual classes symmetricMatrix, triangularMatrix, and diagonalMatrix.

Hilbert

Generate a Hilbert matrix

# Description

Generate the n by n symmetric Hilbert matrix. Because these matrices are ill-conditioned for moderate to large n, they are often used for testing numerical linear algebra code.

## Usage

```
Hilbert(n)
```

## **Arguments**

n

a non-negative integer.

## Value

the n by n symmetric Hilbert matrix as a "dpoMatrix" object.

## See Also

the class dpoMatrix

```
Hilbert(6)
```

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image-methods

Methods for image() in Package 'Matrix'

## **Description**

Methods for function image in package **Matrix**. An image of a matrix simply color codes all matrix entries and draws the  $n \times m$  matrix using an  $n \times m$  grid of (colored) rectangles.

The **Matrix** package image methods are based on levelplot() from package **lattice**; hence these methods return an "object" of class "trellis", producing a graphic when (auto-) print()ed.

## Usage

# **Arguments**

X	a Matrix object, i.e., fulfilling is(x, "Matrix").
xlim, ylim	x- and y-axis limits; may be used to "zoom into" matrix. Note that $x, y$ "feel reversed": ylim is for the rows (= 1st index) and xlim for the columns (= 2nd index). For convenience, when the limits are integer valued, they are both extended by $\emptyset.5$ ; also, ylim is always used decreasingly.
aspect	aspect ratio specified as number $(y/x)$ or string; see levelplot.
sub, xlab, ylab	axis annotation with sensible defaults; see plot.default.
cuts	number of levels the range of matrix values would be divided into.
useRaster	logical indicating if raster graphics should be used (instead of the tradition rectangle vector drawing). If true, panel.levelplot.raster (from lattice package) is used, and the colorkey is also done via rasters, see also levelplot and possibly grid.raster.  Note that using raster graphics may often be faster, but can be slower, depending
	on the matrix dimensions and the graphics device (dimensions).
useAbs	logical indicating if abs(x) should be shown; if TRUE, the former (implicit) default, the default col.regions will be grey colors (and no colorkey drawn). The default is FALSE unless the matrix has no negative entries.
colorkey	logical indicating if a color key aka 'legend' should be produced. Default is to draw one, unless useAbs is true. You can also specify a list, see levelplot, such aslist(raster=TRUE) in the case of rastering.

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col.regions	vector of gradually varying colors; see levelplot.
lwd	(only used when useRaster is false:) non-negative number or NULL (default), specifying the line-width of the rectangles of each non-zero matrix entry (drawn by grid.rect). The default depends on the matrix dimension and the device size.
border.col	color for the border of each rectangle. NA means no border is drawn. When NULL as by default, border.col <- if(lwd < .01) NA else NULL is used. Consider using an opaque color instead of NULL which corresponds to grid::get.gpar("col").
•••	further arguments passed to methods and levelplot, notably at for specifying (possibly non equidistant) cut values for dividing the matrix values (superseding cuts above).

#### Value

as all **lattice** graphics functions, image(<Matrix>) returns a "trellis" object, effectively the result of levelplot().

## Methods

All methods currently end up calling the method for the dgTMatrix class. Use showMethods(image) to list them all.

#### See Also

levelplot, and print.trellis from package lattice.

```
showMethods(image)
## And if you want to see the method definitions:
showMethods(image, includeDefs = TRUE, inherited = FALSE)
data(CAex, package = "Matrix")
image(CAex, main = "image(CAex)") -> imgC; imgC
stopifnot(!is.null(leg <- imgC$legend), is.list(leg$right)) # failed for 2 days ..</pre>
image(CAex, useAbs=TRUE, main = "image(CAex, useAbs=TRUE)")
cCA <- Cholesky(crossprod(CAex), Imult = .01)</pre>
## See ?print.trellis --- place two image() plots side by side:
print(image(cCA, main="Cholesky(crossprod(CAex), Imult = .01)"),
      split=c(x=1,y=1,nx=2, ny=1), more=TRUE)
print(image(cCA, useAbs=TRUE),
      split=c(x=2,y=1,nx=2,ny=1))
data(USCounties, package = "Matrix")
image(USCounties)# huge
image(sign(USCounties))## just the pattern
   # how the result looks, may depend heavily on
   # the device, screen resolution, antialiasing etc
   # e.g. x11(type="Xlib") may show very differently than cairo-based
```

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```
## Drawing borders around each rectangle;
   # again, viewing depends very much on the device:
image(USCounties[1:400,1:200], lwd=.1)
## Using (xlim,ylim) has advantage : matrix dimension and (col/row) indices:
image(USCounties, c(1,200), c(1,400), lwd=.1)
image(USCounties, c(1,300), c(1,200), lwd=.5)
image(USCounties, c(1,300), c(1,200), lwd=.01)
## These 3 are all equivalent :
(I1 <- image(USCounties, c(1,100), c(1,100), useAbs=FALSE))
I2 <- image(USCounties, c(1,100), c(1,100), useAbs=FALSE,</pre>
                                                                   border.col=NA)
I3 <- image(USCounties, c(1,100), c(1,100), useAbs=FALSE, lwd=2, border.col=NA)
stopifnot(all.equal(I1, I2, check.environment=FALSE),
          all.equal(I2, I3, check.environment=FALSE))
## using an opaque border color
image(USCounties, c(1,100), c(1,100), useAbs=FALSE, lwd=3, border.col = adjustcolor("skyblue", 1/2))
if(interactive() || nzchar(Sys.getenv("R_MATRIX_CHECK_EXTRA"))) {
## Using raster graphics: For PDF this would give a 77 MB file,
## however, for such a large matrix, this is typically considerably
## *slower* (than vector graphics rectangles) in most cases :
if(doPNG <- !dev.interactive())</pre>
  png("image-USCounties-raster.png", width=3200, height=3200)
image(USCounties, useRaster = TRUE) # should not suffer from anti-aliasing
if(doPNG)
  dev.off()
  ## and now look at the *.png image in a viewer you can easily zoom in and out
}#only if(doExtras)
```

index-class

Virtual Class "index" of Index Vectors

## **Description**

Class index is a virtual class designating index vectors, or "subscripts", for (possibly named) vectors and arrays. It is typically used in signatures of methods for the subscript and subassignment operators, namely [ and [<-. It is implemented as a *union* of the atomic vector classes numeric, logical, and character.

## See Also

```
[, [-methods, and [<--methods.
```

```
showClass("index")
```

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# Description

The indMatrix class is the class of row and column *index* matrices, stored as 1-based integer index vectors. A row (column) index matrix is a matrix whose rows (columns) are standard unit vectors. Such matrices are useful when mapping observations to discrete sets of covariate values.

Multiplying a matrix on the left by a row index matrix is equivalent to indexing its rows, i.e., sampling the rows "with replacement". Analogously, multiplying a matrix on the right by a column index matrix is equivalent to indexing its columns. Indeed, such products are implemented in **Matrix** as indexing operations; see 'Details' below.

A matrix whose rows *and* columns are standard unit vectors is called a *permutation* matrix. This special case is designated by the pMatrix class, a direct subclass of indMatrix.

#### **Details**

The transpose of an index matrix is an index matrix with identical perm but opposite margin. Hence the transpose of a row index matrix is a column index matrix, and vice versa.

The cross product of a row index matrix R and itself is a diagonal matrix whose diagonal entries are the the number of entries in each column of R.

Given a row index matrix R with perm slot p, a column index matrix C with perm slot q, and a matrix M with conformable dimensions, we have

```
RM
      = R %*% M
                         = M[p,]
MC
         M %*% C
                         = M[, q]
C'M = crossprod(C, M)
                         = M[q,]
MR'
     = tcrossprod(M, R) = M[, p]
R'R
        crossprod(R)
                         = Diagonal(x=tabulate(p, ncol(R)))
      =
CC'
        tcrossprod(C)
                         = Diagonal(x=tabulate(q, nrow(C)))
```

Operations on index matrices that result in index matrices will accordingly return an indMatrix. These include products of two column index matrices and (equivalently) column-indexing of a column index matrix (when dimensions are not dropped). Most other operations on indMatrix treat them as sparse nonzero pattern matrices (i.e., inheriting from virtual class nsparseMatrix). Hence vector-valued subsets of indMatrix, such as those given by diag, are always of type "logical".

# **Objects from the Class**

Objects can be created explicitly with calls of the form new("indMatrix", ...), but they are more commonly created by coercing 1-based integer index vectors, with calls of the form as(., "indMatrix"); see 'Methods' below.

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#### **Slots**

margin an integer, either 1 or 2, specifying whether the matrix is a row (1) or column (2) index.

perm a 1-based integer index vector, i.e., a vector of length Dim[margin] with elements taken from 1:Dim[1+margin%2].

Dim, Dimnames inherited from virtual superclass Matrix.

#### **Extends**

Classes "sparseMatrix" and "generalMatrix", directly.

#### Methods

- coerce signature(from = "numeric", to = "indMatrix"): supporting typical indMatrix construction from a vector of positive integers. Row indexing is assumed.
- coerce signature(from = "list", to = "indMatrix"): supporting indMatrix construction for row *and* column indexing, including index vectors of length 0 and index vectors whose maximum is less than the number of rows or columns being indexed.
- coerce signature(from = "indMatrix", to = "matrix"): coercion to a traditional matrix of logical type, with FALSE and TRUE in place of 0 and 1.
- t signature(x = "indMatrix"): the transpose, which is an indMatrix with identical perm but opposite margin.
- rowSums,rowMeans,colSums,colMeans signature(x = "indMatrix"): row and column sums and means.
- rbind2,cbind2 signature(x = "indMatrix", y = "indMatrix"): row-wise catenation of two row index matrices with equal numbers of columns and column-wise catenation of two column index matrices with equal numbers of rows.
- **kronecker** signature(X = "indMatrix", Y = "indMatrix"): Kronecker product of two row index matrices or two column index matrices, giving the row or column index matrix corresponding to their "interaction".

#### Author(s)

Fabian Scheipl at 'uni-muenchen.de', building on the existing class pMatrix after a nice hike's conversation with Martin Maechler. Methods for crossprod(x, y) and kronecker(x, y) with both arguments inheriting from indMatrix were made considerably faster thanks to a suggestion by Boris Vaillant. Diverse tweaks by Martin Maechler and Mikael Jagan, notably the latter's implementation of margin, prior to which the indMatrix class was designated only for row index matrices.

## See Also

Subclass pMatrix of permutation matrices, a special case of index matrices; virtual class nMatrix of nonzero pattern matrices, and its subclasses.

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## **Examples**

```
p1 <- as(c(2,3,1), "pMatrix")
(sm1 \leftarrow as(rep(c(2,3,1), e=3), "indMatrix"))
stopifnot(all(sm1 == p1[rep(1:3, each=3),]))
## row-indexing of a <pMatrix> turns it into an <indMatrix>:
class(p1[rep(1:3, each=3),])
set.seed(12) # so we know '10' is in sample
## random index matrix for 30 observations and 10 unique values:
(s10 <- as(sample(10, 30, replace=TRUE), "indMatrix"))</pre>
## Sample rows of a numeric matrix :
(mm <- matrix(1:10, nrow=10, ncol=3))
s10 %*% mm
set.seed(27)
IM1 <- as(sample(1:20, 100, replace=TRUE), "indMatrix")</pre>
IM2 <- as(sample(1:18, 100, replace=TRUE), "indMatrix")</pre>
(c12 <- crossprod(IM1,IM2))</pre>
## same as cross-tabulation of the two index vectors:
stopifnot(all(c12 - unclass(table(IM1@perm, IM2@perm)) == 0))
# 3 observations, 4 implied values, first does not occur in sample:
as(2:4, "indMatrix")
# 3 observations, 5 values, first and last do not occur in sample:
as(list(2:4, 5), "indMatrix")
as(sm1, "nMatrix")
s10[1:7, 1:4] # gives an "ngTMatrix" (most economic!)
s10[1:4, ] # preserves "indMatrix"-class
I1 <- as(c(5:1,6:4,7:3), "indMatrix")
I2 <- as(7:1, "pMatrix")</pre>
(I12 <- rbind(I1, I2))
stopifnot(is(I12, "indMatrix"),
          identical(I12, rbind(I1, I2)),
 colSums(I12) == c(2L, 2:4, 4:2))
```

 ${\tt invertPerm}$ 

Utilities for Permutation Vectors

# Description

invertPerm and signPerm compute the inverse and sign of a length-n permutation vector. isPerm tests if a length-n integer vector is a valid permutation vector. asPerm coerces a length-m transposition vector to a length-n permutation vector, where  $m \le n$ .

invertPerm

## Usage

```
invertPerm(p, off = 1L, ioff = 1L)
signPerm(p, off = 1L)
isPerm(p, off = 1L)
asPerm(pivot, off = 1L, ioff = 1L, n = length(pivot))
invPerm(p, zero.p = FALSE, zero.res = FALSE)
```

## **Arguments**

р	an integer vector of length n.
pivot	an integer vector of length m.
off	an integer offset, indicating that p is a permutation of off+0:(n-1) or that pivot contains m values sampled with replacement from off+0:(n-1).
ioff	an integer offset, indicating that the result should be a permutation of $ioff+0:(n-1)$ .
n	a integer greater than or equal to $m$ , indicating the length of the result. Transpositions are applied to a permutation vector vector initialized as $seq_len(n)$ .
zero.p	a logical. Equivalent to off=0 if TRUE and off=1 if FALSE.
zero.res	a logical. Equivalent to ioff=0 if TRUE and ioff=1 if FALSE.

#### **Details**

invertPerm(p, off, ioff=1) is equivalent to order(p) or sort.list(p) for all values of off. For the default value off=1, it returns the value of p after  $p[p] \leftarrow seq_along(p)$ .

invPerm is a simple wrapper around invertPerm, retained for backwards compatibility.

#### Value

```
By default, i.e., with off=1 and ioff=1:
```

invertPerm(p) returns an integer vector of length length(p) such that p[invertPerm(p)] and invertPerm(p)[p] are both seq\_along(p), i.e., the identity permutation.

signPerm(p) returns 1 if p is an even permutation and -1 otherwise (i.e., if p is odd).

isPerm(p) returns TRUE if p is a permutation of seq\_along(p) and FALSE otherwise.

asPerm(pivot) returns the result of transposing elements i and pivot[i] of a permutation vector initialized as  $seq_len(n)$ , for i in  $seq_along(pivot)$ .

## See Also

Class pMatrix of permutation matrices.

```
p <- sample(10L) # a random permutation vector
ip <- invertPerm(p)
s <- signPerm(p)</pre>
```

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```
## 'p' and 'ip' are indeed inverses:
stopifnot(exprs = {
    isPerm(p)
    isPerm(ip)
    identical(s, 1L) || identical(s, -1L)
    identical(s, signPerm(ip))
    identical(p[ip], 1:10)
    identical(ip[p], 1:10)
    identical(invertPerm(ip), p)
})
## Product of transpositions (1 \ 2)(2 \ 1)(4 \ 3)(6 \ 8)(10 \ 1) = (3 \ 4)(6 \ 8)(1 \ 10)
pivot <- c(2L, 1L, 3L, 3L, 5L, 8L, 7L, 8L, 9L, 1L)
q <- asPerm(pivot)</pre>
stopifnot(exprs = {
    identical(q, c(10L, 2L, 4L, 3L, 5L, 8L, 7L, 6L, 9L, 1L))
    identical(q[q], seq\_len(10L)) # because the permutation is odd:
    signPerm(q) == -1L
})
invPerm # a less general version of 'invertPerm'
```

is.na-methods

is.na(), is.finite() Methods for 'Matrix' Objects

## **Description**

Methods for generic functions anyNA(), is.na(), is.nan(), is.infinite(), and is.finite(), for objects inheriting from virtual class Matrix or sparseVector.

# Usage

```
## S4 method for signature 'denseMatrix'
is.na(x)
## S4 method for signature 'sparseMatrix'
is.na(x)
## S4 method for signature 'diagonalMatrix'
is.na(x)
## S4 method for signature 'indMatrix'
is.na(x)
## S4 method for signature 'sparseVector'
is.na(x)
## S4 method for signature 'sparseVector'
is.na(x)
## ...
## and likewise for anyNA, is.nan, is.infinite, is.finite
```

# Arguments x

an R object, here a sparse or dense matrix or vector.

is.null.DN

## Value

For is.\*(), an nMatrix or nsparseVector matching the dimensions of x and specifying the positions in x of (some subset of) NA, NaN, Inf, and -Inf. For anyNA(), TRUE if x contains NA or NaN and FALSE otherwise.

#### See Also

```
NA, NaN, Inf
```

## **Examples**

is.null.DN

Are the Dimnames dn NULL-like?

# Description

Are the dimnames dn NULL-like?

is.null.DN(dn) is less strict than is.null(dn), because it is also true (TRUE) when the dimnames dn are "like" NULL, or list(NULL, NULL), as they can easily be for the traditional R matrices (matrix) which have no formal class definition, and hence much freedom in how their dimnames look like.

## Usage

```
is.null.DN(dn)
```

## **Arguments**

```
dn dimnames() of a matrix-like R object.
```

## Value

```
logical TRUE or FALSE.
```

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#### Note

This function is really to be used on "traditional" matrices rather than those inheriting from Matrix, as the latter will always have dimnames list(NULL, NULL) exactly, in such a case.

#### Author(s)

Martin Maechler

#### See Also

```
is.null, dimnames, matrix.
```

#### **Examples**

```
m1 \leftarrow m2 \leftarrow m3 \leftarrow m4 \leftarrow m \leftarrow
    matrix(round(100 * rnorm(6)), 2, 3)
dimnames(m1) <- list(NULL, NULL)</pre>
dimnames(m2) <- list(NULL, character())</pre>
dimnames(m3) <- rev(dimnames(m2))</pre>
dimnames(m4) <- rep(list(character()),2)</pre>
m4 # prints absolutely identically to m
c.o <- capture.output</pre>
cm <- c.o(m)
stopifnot(exprs = {
    m == m1; m == m2; m == m3; m == m4
identical(cm, c.o(m1)); identical(cm, c.o(m2))
identical(cm, c.o(m3)); identical(cm, c.o(m4))
})
hasNoDimnames <- function(.) is.null.DN(dimnames(.))</pre>
stopifnot(exprs = {
    hasNoDimnames(m)
    hasNoDimnames(m1); hasNoDimnames(m2)
    hasNoDimnames(m3); hasNoDimnames(m4)
    hasNoDimnames(Matrix(m) -> M)
    hasNoDimnames(as(M, "sparseMatrix"))
})
```

# Description

isSymmetric tests whether its argument is a symmetric square matrix, by default tolerating some numerical fuzz and requiring symmetric [dD]imnames in addition to symmetry in the mathematical sense. isSymmetric is a generic function in **base**, which has a method for traditional matrices of implicit class "matrix". Methods are defined here for various proper and virtual classes in **Matrix**, so that isSymmetric works for all objects inheriting from virtual class "Matrix".

## Usage

```
## S4 method for signature 'denseMatrix'
isSymmetric(object, checkDN = TRUE, ...)
## S4 method for signature 'CsparseMatrix'
isSymmetric(object, checkDN = TRUE, ...)
## S4 method for signature 'RsparseMatrix'
isSymmetric(object, checkDN = TRUE, ...)
## S4 method for signature 'TsparseMatrix'
isSymmetric(object, checkDN = TRUE, ...)
## S4 method for signature 'diagonalMatrix'
isSymmetric(object, checkDN = TRUE, ...)
## S4 method for signature 'indMatrix'
isSymmetric(object, checkDN = TRUE, ...)
## S4 method for signature 'dgeMatrix'
isSymmetric(object, checkDN = TRUE, tol = 100 * .Machine$double.eps, tol1 = 8 * tol, ...)
## S4 method for signature 'dgCMatrix'
isSymmetric(object, checkDN = TRUE, tol = 100 * .Machine$double.eps, ...)
```

## **Arguments**

object	a "Matrix".
checkDN	a logical indicating whether symmetry of the Dimnames slot of object should be checked.
tol, tol1	numerical tolerances allowing <i>approximate</i> symmetry of numeric (rather than logical) matrices. See also isSymmetric.matrix.
	further arguments passed to methods (typically methods for all.equal).

# Details

The Dimnames slot of object, say dn, is considered to be symmetric if and only if

- dn[[1]] and dn[[2]] are identical or one is NULL; and
- ndn <- names (dn) is NULL or ndn[1] and ndn[2] are identical or one is the empty string "".

Hence list(a=nms, a=nms) is considered to be *symmetric*, and so too are list(a=nms, NULL) and list(NULL, a=nms).

Note that this definition is *looser* than that employed by isSymmetric.matrix, which requires dn[1] and dn[2] to be identical, where dn is the dimnames attribute of a traditional matrix.

# Value

A logical, either TRUE or FALSE (never NA).

#### See Also

forceSymmetric; symmpart and skewpart; virtual class "symmetricMatrix" and its subclasses.

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## **Examples**

```
isSymmetric(Diagonal(4)) # TRUE of course
M <- Matrix(c(1,2,2,1), 2,2)
isSymmetric(M) # TRUE (*and* of formal class "dsyMatrix")
isSymmetric(as(M, "generalMatrix")) # still symmetric, even if not "formally"
isSymmetric(triu(M)) # FALSE

## Look at implementations:
showMethods("isSymmetric", includeDefs = TRUE) # includes S3 generic from base</pre>
```

isTriangular-methods Test whether a Matrix is Triangular or Diagonal

# **Description**

isTriangular and isDiagonal test whether their argument is a triangular or diagonal matrix, respectively. Unlike the analogous isSymmetric, these two functions are generically from **Matrix** rather than base. Hence **Matrix** defines methods for traditional matrices of implicit class "matrix" in addition to matrices inheriting from virtual class "Matrix".

By our definition, triangular and diagonal matrices are *square*, i.e., they have the same number of rows and columns.

## Usage

```
isTriangular(object, upper = NA, ...)
isDiagonal(object)
```

#### **Arguments**

object an R object, typically a matrix.

upper a logical, either TRUE or FALSE, in which case TRUE is returned only for upper

or lower triangular object; or otherwise NA (the default), in which case TRUE is

returned for any triangular object.

... further arguments passed to methods (currently unused by **Matrix**).

#### Value

```
A logical, either TRUE or FALSE (never NA).
```

If object is triangular and upper is NA, then isTriangular returns TRUE with an attribute kind, either "U" or "L", indicating that object is **u**pper or **l**ower triangular, respectively. Users should not rely on how kind is determined for diagonal matrices, which are both upper and lower triangular.

## See Also

```
isSymmetric; virtual classes "triangularMatrix" and "diagonalMatrix" and their subclasses.
```

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## **Examples**

```
isTriangular(Diagonal(4))
## is TRUE: a diagonal matrix is also (both upper and lower) triangular
(M <- Matrix(c(1,2,0,1), 2,2))
isTriangular(M) # TRUE (*and* of formal class "dtrMatrix")
isTriangular(as(M, "generalMatrix")) # still triangular, even if not "formally"
isTriangular(crossprod(M)) # FALSE

isDiagonal(matrix(c(2,0,0,1), 2,2)) # TRUE

## Look at implementations:
showMethods("isTriangular", includeDefs = TRUE)
showMethods("isDiagonal", includeDefs = TRUE)</pre>
```

KhatriRao

Khatri-Rao Matrix Product

## **Description**

Computes Khatri-Rao products for any kind of matrices.

The Khatri-Rao product is a column-wise Kronecker product. Originally introduced by Khatri and Rao (1968), it has many different applications, see Liu and Trenkler (2008) for a survey. Notably, it is used in higher-dimensional tensor decompositions, see Bader and Kolda (2008).

#### Usage

```
KhatriRao(X, Y = X, FUN = "*", sparseY = TRUE, make.dimnames = FALSE)
```

## Arguments

X, Y	matrices of with the same number of columns.
FUN	the (name of the) function to be used for the column-wise Kronecker products, see kronecker, defaulting to the usual multiplication.
sparseY	logical specifying if Y should be coerced and treated as sparseMatrix. Set this to FALSE, e.g., to distinguish structural zeros from zero entries.
make.dimnames	logical indicating if the result should inherit dimnames from X and Y in a simple

## Value

```
a "CsparseMatrix", say R, the Khatri-Rao product of X (n \times k) and Y (m \times k), is of dimension (n \cdot m) \times k, where the j-th column, R[, j] is the kronecker product kronecker(X[, j], Y[, j]).
```

## Note

The current implementation is efficient for large sparse matrices.

way.

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#### Author(s)

Original by Michael Cysouw, Univ. Marburg; minor tweaks, bug fixes etc, by Martin Maechler.

#### References

Khatri, C. G., and Rao, C. Radhakrishna (1968) Solutions to Some Functional Equations and Their Applications to Characterization of Probability Distributions. *Sankhya: Indian J. Statistics, Series A* **30**, 167–180.

Bader, Brett W, and Tamara G Kolda (2008) Efficient MATLAB Computations with Sparse and Factored Tensors. *SIAM J. Scientific Computing* **30**, 205–231.

#### See Also

kronecker.

```
## Example with very small matrices:
m \leftarrow matrix(1:12,3,4)
d \leftarrow diag(1:4)
KhatriRao(m,d)
KhatriRao(d,m)
dimnames(m) <- list(LETTERS[1:3], letters[1:4])</pre>
KhatriRao(m,d, make.dimnames=TRUE)
KhatriRao(d,m, make.dimnames=TRUE)
dimnames(d) <- list(NULL, paste0("D", 1:4))</pre>
KhatriRao(m,d, make.dimnames=TRUE)
KhatriRao(d,m, make.dimnames=TRUE)
dimnames(d) <- list(paste0("d", 10*1:4), paste0("D", 1:4))</pre>
(Kmd <- KhatriRao(m,d, make.dimnames=TRUE))</pre>
(Kdm <- KhatriRao(d,m, make.dimnames=TRUE))</pre>
nm <- as(m, "nsparseMatrix")</pre>
nd <- as(d, "nsparseMatrix")</pre>
KhatriRao(nm,nd, make.dimnames=TRUE)
KhatriRao(nd,nm, make.dimnames=TRUE)
stopifnot(dim(KhatriRao(m,d)) == c(nrow(m)*nrow(d), ncol(d)))
## border cases / checks:
zm <- nm; zm[] <- FALSE # all FALSE matrix</pre>
stopifnot(all(K1 <- KhatriRao(nd, zm) == 0), identical(dim(K1), c(12L, 4L)),</pre>
           all(K2 <- KhatriRao(zm, nd) == 0), identical(dim(K2), c(12L, 4L)))
d0 <- d; d0[] <- 0; m0 <- Matrix(d0[-1,])
stopifnot(all(K3 <- KhatriRao(d0, m) == 0), identical(dim(K3), dim(Kdm)),</pre>
  all(K4 <- KhatriRao(m, d0) == 0), identical(dim(K4), dim(Kmd)),</pre>
  all(KhatriRao(d0, d0) == 0), all(KhatriRao(m0, d0) == 0),
  all(KhatriRao(d0, m0) == 0), all(KhatriRao(m0, m0) == 0),
  identical(dimnames(KhatriRao(m, d0, make.dimnames=TRUE)), dimnames(Kmd)))
## a matrix with "structural" and non-structural zeros:
```

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```
m01 < - \text{new}("dgCMatrix", i = c(0L, 2L, 0L, 1L), p = c(0L, 0L, 0L, 2L, 4L),
           Dim = 3:4, x = c(1, 0, 1, 0))
D4 <- Diagonal(4, x=1:4) # "as" d
DU <- Diagonal(4)# unit-diagonal: uplo="U"
(K5 <- KhatriRao( d, m01))
K5d <- KhatriRao( d, m01, sparseY=FALSE)</pre>
K5Dd <- KhatriRao(D4, m01, sparseY=FALSE)</pre>
K5Ud <- KhatriRao(DU, m01, sparseY=FALSE)</pre>
(K6 <- KhatriRao(diag(3),</pre>
                                 t(m01)))
K6D <- KhatriRao(Diagonal(3), t(m01))</pre>
K6d <- KhatriRao(diag(3),</pre>
                                 t(m01), sparseY=FALSE)
K6Dd <- KhatriRao(Diagonal(3), t(m01), sparseY=FALSE)</pre>
stopifnot(exprs = {
    all(K5 == K5d)
    identical(cbind(c(7L, 10L), c(3L, 4L)),
               which(K5 != 0, arr.ind = TRUE, useNames=FALSE))
    identical(K5d, K5Dd)
    identical(K6, K6D)
    all(K6 == K6d)
    identical(cbind(3:4, 1L),
               which(K6 != 0, arr.ind = TRUE, useNames=FALSE))
    identical(K6d, K6Dd)
})
```

KNex

Koenker-Ng Example Sparse Model Matrix and Response Vector

## **Description**

A model matrix mm and corresponding response vector y used in an example by Koenker and Ng. The matrix mm is a sparse matrix with 1850 rows and 712 columns but only 8758 non-zero entries. It is a "dgCMatrix" object. The vector y is just numeric of length 1850.

#### Usage

```
data(KNex)
```

# References

Roger Koenker and Pin Ng (2003). SparseM: A sparse matrix package for R; *J. of Statistical Software*, **8** (6), doi:10.18637/jss.v008.i06

```
data(KNex, package = "Matrix")
class(KNex$mm)
dim(KNex$mm)
image(KNex$mm)
str(KNex)
```

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```
system.time( # a fraction of a second
  sparse.sol <- with(KNex, solve(crossprod(mm), crossprod(mm, y))))
head(round(sparse.sol,3))
## Compare with QR-based solution ("more accurate, but slightly slower"):
system.time(
  sp.sol2 <- with(KNex, qr.coef(qr(mm), y) ))
all.equal(sparse.sol, sp.sol2, tolerance = 1e-13) # TRUE</pre>
```

kronecker-methods

Methods for Function 'kronecker()' in Package 'Matrix'

## **Description**

Computes Kronecker products for objects inheriting from "Matrix".

In order to preserver sparseness, we treat 0 \* NA as 0, not as NA as usually in R (and as used for the **base** function kronecker).

### Methods

```
kronecker signature(X = "Matrix", Y = "ANY") .......
kronecker signature(X = "ANY", Y = "Matrix") .......
kronecker signature(X = "diagonalMatrix", Y = "ANY") .......
kronecker signature(X = "sparseMatrix", Y = "ANY") .......
kronecker signature(X = "TsparseMatrix", Y = "TsparseMatrix") .......
kronecker signature(X = "dgTMatrix", Y = "dgTMatrix") .......
kronecker signature(X = "dtTMatrix", Y = "dtTMatrix") .......
kronecker signature(X = "indMatrix", Y = "indMatrix") .......
```

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ldenseMatrix-class

Virtual Class "IdenseMatrix" of Dense Logical Matrices

## **Description**

ldenseMatrix is the virtual class of all dense logical (S4) matrices. It extends both denseMatrix and lMatrix directly.

### **Slots**

```
x: logical vector containing the entries of the matrix.
Dim, Dimnames: see Matrix.
```

## **Extends**

```
Class "lMatrix", directly. Class "denseMatrix", directly. Class "Matrix", by class "lMatrix". Class "Matrix", by class "denseMatrix".
```

## Methods

```
as.vector signature(x = "ldenseMatrix", mode = "missing"): ...
which signature(x = "ndenseMatrix"), semantically equivalent to base function which(x, arr.ind);
    for details, see the lMatrix class documentation.
```

### See Also

Class lgeMatrix and the other subclasses.

## **Examples**

```
showClass("ldenseMatrix")
as(diag(3) > 0, "ldenseMatrix")
```

ldiMatrix-class

Class "ldiMatrix" of Diagonal Logical Matrices

# **Description**

The class "ldiMatrix" of logical diagonal matrices.

# **Objects from the Class**

Objects can be created by calls of the form new("ldiMatrix", ...) but typically rather via Diagonal.

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## Slots

```
x: "logical" vector.
diag: "character" string, either "U" or "N", see ddiMatrix.
Dim,Dimnames: matrix dimension and dimnames, see the Matrix class description.
```

# **Extends**

```
Class "diagonalMatrix" and class "lMatrix", directly.
Class "sparseMatrix", by class "diagonalMatrix".
```

### See Also

Classes ddiMatrix and diagonalMatrix; function Diagonal.

# **Examples**

```
(lM <- Diagonal(x = c(TRUE,FALSE,FALSE)))
str(lM)#> gory details (slots)

crossprod(lM) # numeric
(nM <- as(lM, "nMatrix"))
crossprod(nM) # pattern sparse</pre>
```

lgeMatrix-class

Class "IgeMatrix" of General Dense Logical Matrices

# **Description**

This is the class of general dense logical matrices.

# **Slots**

x: Object of class "logical". The logical values that constitute the matrix, stored in column-major order.

Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), see the Matrix class.

factors: Object of class "list". A named list of factorizations that have been computed for the matrix.

## **Extends**

```
Class "ldenseMatrix", directly. Class "lMatrix", by class "ldenseMatrix". Class "denseMatrix", by class "ldenseMatrix". Class "Matrix", by class "ldenseMatrix". Class "Matrix", by class "ldenseMatrix".
```

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### Methods

Currently, mainly t() and coercion methods (for as(.)); use, e.g., showMethods(class="lgeMatrix") for details.

### See Also

Non-general logical dense matrix classes such as ltrMatrix, or lsyMatrix; *sparse* logical classes such as lgCMatrix.

# **Examples**

```
showClass("lgeMatrix")
str(new("lgeMatrix"))
set.seed(1)
(lM <- Matrix(matrix(rnorm(28), 4,7) > 0))# a simple random lgeMatrix
set.seed(11)
(lC <- Matrix(matrix(rnorm(28), 4,7) > 0))# a simple random lgCMatrix
as(lM, "CsparseMatrix")
```

lsparseMatrix-class Sparse logical matrices

**Description** 

The lsparseMatrix class is a virtual class of logical sparse matrices, i.e., sparse matrices with entries TRUE, FALSE, or NA.

These can be stored in the "triplet" form (class TsparseMatrix, subclasses lgTMatrix, lsTMatrix, and ltTMatrix) or in compressed column-oriented form (class CsparseMatrix, subclasses lgCMatrix, lsCMatrix, and ltCMatrix) or—rarely—in compressed row-oriented form (class RsparseMatrix, subclasses lgRMatrix, lsRMatrix, and ltRMatrix). The second letter in the name of these non-virtual classes indicates general, symmetric, or triangular.

## **Details**

Note that triplet stored (TsparseMatrix) matrices such as lgTMatrix may contain duplicated pairs of indices (i,j) as for the corresponding numeric class dgTMatrix where for such pairs, the corresponding x slot entries are added. For logical matrices, the x entries corresponding to duplicated index pairs (i,j) are "added" as well if the addition is defined as logical or, i.e., "TRUE + TRUE |-> TRUE" and "TRUE + FALSE |-> TRUE". Note the use of asUniqueT() for getting an internally unique representation without duplicated (i,j) entries.

## **Objects from the Class**

Objects can be created by calls of the form new("lgCMatrix", ...) and so on. More frequently objects are created by coercion of a numeric sparse matrix to the logical form, e.g. in an expression x != 0.

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The logical form is also used in the symbolic analysis phase of an algorithm involving sparse matrices. Such algorithms often involve two phases: a symbolic phase wherein the positions of the non-zeros in the result are determined and a numeric phase wherein the actual results are calculated. During the symbolic phase only the positions of the non-zero elements in any operands are of interest, hence any numeric sparse matrices can be treated as logical sparse matrices.

#### **Slots**

- x: Object of class "logical", i.e., either TRUE, NA, or FALSE.
- uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular. Present in the triangular and symmetric classes but not in the general class.
- diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N" for non-unit. The implicit diagonal elements are not explicitly stored when diag is "U". Present in the triangular classes only.
- p: Object of class "integer" of pointers, one for each column (row), to the initial (zero-based) index of elements in the column. Present in compressed column-oriented and compressed row-oriented forms only.
- i: Object of class "integer" of length nnzero (number of non-zero elements). These are the row numbers for each TRUE element in the matrix. All other elements are FALSE. Present in triplet and compressed column-oriented forms only.
- j: Object of class "integer" of length nnzero (number of non-zero elements). These are the column numbers for each TRUE element in the matrix. All other elements are FALSE. Present in triplet and compressed row-oriented forms only.

Dim: Object of class "integer" - the dimensions of the matrix.

### Methods

```
coerce signature(from = "dgCMatrix", to = "lgCMatrix")
t signature(x = "lgCMatrix"): returns the transpose of x
which signature(x = "lsparseMatrix"), semantically equivalent to base function which(x, arr.ind); for details, see the lMatrix class documentation.
```

## See Also

the class dgCMatrix and dgTMatrix

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```
nmG.1 <- as(mmG.1, "nMatrix") # <<< has "TRUE" also where mmG.1 had FALSE
## from logical to "double"
dmG.1 \leftarrow as(mmG.1, "dMatrix") # has '0' and back:
lmG.1 <- as(dmG.1, "lMatrix")</pre>
stopifnot(identical(nmG.1, as((KNex $ mm) != 0,"nMatrix")),
          validObject(lmG.1),
          identical(lmG.1, mmG.1))
class(xnx <- crossprod(nmG.1))# "nsC.."</pre>
class(xlx <- crossprod(mmG.1))# "dsC.." : numeric</pre>
is0 \leftarrow (xlx == 0)
mean(as.vector(is0))# 99.3% zeros: quite sparse, but
table(xlx@x == 0)# more than half of the entries are (non-structural!) 0
stopifnot(isSymmetric(xlx), isSymmetric(xnx),
          ## compare xnx and xlx : have the *same* non-structural 0s :
          sapply(slotNames(xnx),
                 function(n) identical(slot(xnx, n), slot(xlx, n))))
```

lsyMatrix-class

Symmetric Dense Logical Matrices

## **Description**

The "lsyMatrix" class is the class of symmetric, dense logical matrices in non-packed storage and "lspMatrix" is the class of of these in packed storage. In the packed form, only the upper triangle or the lower triangle is stored.

## **Objects from the Class**

Objects can be created by calls of the form new("lsyMatrix", ...).

### Slots

uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.

x: Object of class "logical". The logical values that constitute the matrix, stored in column-major order.

Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), see the Matrix class.

factors: Object of class "list". A named list of factorizations that have been computed for the matrix.

### Extends

Both extend classes "ldenseMatrix" and "symmetricMatrix", directly; further, class "Matrix" and others, *in*directly. Use showClass("lsyMatrix"), e.g., for details.

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### Methods

Currently, mainly t() and coercion methods (for as(.); use, e.g., showMethods(class="lsyMatrix") for details.

#### See Also

```
lgeMatrix, Matrix, t
```

### **Examples**

```
(M2 <- Matrix(c(TRUE, NA, FALSE, FALSE), 2, 2)) # logical dense (ltr)
str(M2)
# can
(sM <- M2 | t(M2)) # "lge"
as(sM, "symmetricMatrix")
str(sM <- as(sM, "packedMatrix")) # packed symmetric</pre>
```

ltrMatrix-class

Triangular Dense Logical Matrices

## **Description**

The "ltrMatrix" class is the class of triangular, dense, logical matrices in nonpacked storage. The "ltpMatrix" class is the same except in packed storage.

### **Slots**

x: Object of class "logical". The logical values that constitute the matrix, stored in column-major order.

uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.

diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N"; see triangularMatrix.

Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), see the Matrix class.

factors: Object of class "list". A named list of factorizations that have been computed for the matrix.

### **Extends**

Both extend classes "ldenseMatrix" and "triangularMatrix", directly; further, class "Matrix", "lMatrix" and others, *in*directly. Use showClass("ltrMatrix"), e.g., for details.

# Methods

Currently, mainly t() and coercion methods (for as(.); use, e.g., showMethods(class="ltrMatrix") for details.

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## See Also

Classes lgeMatrix, Matrix; function t

## **Examples**

```
showClass("ltrMatrix")

str(new("ltpMatrix"))
(lutr <- as(upper.tri(matrix(, 4, 4)), "ldenseMatrix"))
str(lutp <- pack(lutr)) # packed matrix: only 10 = 4*(4+1)/2 entries
!lutp # the logical negation (is *not* logical triangular !)
## but this one is:
stopifnot(all.equal(lutp, pack(!!lutp)))</pre>
```

lu-methods

Methods for LU Factorization

# Description

Computes the pivoted LU factorization of an  $m \times n$  real matrix A, which has the general form

$$P_1AP_2 = LU$$

or (equivalently)

$$A = P_1' L U P_2'$$

where  $P_1$  is an  $m \times m$  permutation matrix,  $P_2$  is an  $n \times n$  permutation matrix, L is an  $m \times \min(m, n)$  unit lower trapezoidal matrix, and U is a  $\min(m, n) \times n$  upper trapezoidal matrix.

Methods for denseMatrix are built on LAPACK routine dgetrf, which does not permute columns, so that  $P_2$  is an identity matrix.

Methods for sparseMatrix are built on CXSparse routine cs\_lu, which requires m=n, so that L and U are triangular matrices.

## Usage

```
lu(x, ...)
## S4 method for signature 'dgeMatrix'
lu(x, warnSing = TRUE, ...)
## S4 method for signature 'dgCMatrix'
lu(x, errSing = TRUE, order = NA_integer_,
    tol = 1, ...)
## S4 method for signature 'dsyMatrix'
lu(x, cache = TRUE, ...)
## S4 method for signature 'dsCMatrix'
lu(x, cache = TRUE, ...)
## S4 method for signature 'matrix'
lu(x, ...)
```

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## Arguments

X	a finite matrix or Matrix to be factorized, which must be square if sparse.
warnSing	a logical indicating if a warning should be signaled for singular x. Used only by methods for dense matrices.
errSing	a logical indicating if an error should be signaled for singular x. Used only by methods for sparse matrices.
order	an integer in 0:3 passed to CXSparse routine cs_sqr, indicating a strategy for choosing the column permutation $P_2$ . 0 means no column permutation. 1, 2, and 3 indicate a fill-reducing ordering of $A+A'$ , $\tilde{A}'\tilde{A}$ , and $A'A$ , where $\tilde{A}$ is $A$ with "dense" rows removed. NA (the default) is equivalent to 2 if tol == 1 and 1 otherwise. Do not set to 0 unless you know that the column order of $A$ is already sensible.
tol	a number. The original pivot element is used if its absolute value exceeds tol * a, where a is the maximum in absolute value of the other possible pivot elements. Set tol < 1 only if you know what you are doing.
cache	a logical indicating if the result should be cached in x@factors[["LU"]]. Note that caching is experimental and that only methods for classes extending generalMatrix or symmetricMatrix will have this argument.
	further arguments passed to or from methods.

## **Details**

What happens when x is determined to be near-singular differs by method. The method for class dgeMatrix completes the factorization, warning if warnSing = TRUE and in any case returning a valid denseLU object. Users of this method can detect singular x with a suitable warning handler; see tryCatch. In contrast, the method for class dgCMatrix abandons further computation, throwing an error if errSing = TRUE and otherwise returning NA. Users of this method can detect singular x with an error handler or by setting errSing = FALSE and testing for a formal result with is(., "sparseLU").

### Value

An object representing the factorization, inheriting from virtual class LU. The specific class is denseLU unless x inherits from virtual class sparseMatrix, in which case it is sparseLU.

### References

The LAPACK source code, including documentation; see <a href="https://netlib.org/lapack/double/dgetrf.f">https://netlib.org/lapack/double/dgetrf.f</a>.

Davis, T. A. (2006). *Direct methods for sparse linear systems*. Society for Industrial and Applied Mathematics. doi:10.1137/1.9780898718881

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

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## See Also

Classes denseLU and sparseLU and their methods.

Classes dgeMatrix and dgCMatrix.

Generic functions expand1 and expand2, for constructing matrix factors from the result.

Generic functions Cholesky, BunchKaufman, Schur, and qr, for computing other factorizations.

## **Examples**

```
showMethods("lu", inherited = FALSE)
set.seed(0)
## ---- Dense ------
(A1 <- Matrix(rnorm(9L), 3L, 3L))
(lu.A1 \leftarrow lu(A1))
(A2 <- round(10 * A1[, -3L]))
(lu.A2 \leftarrow lu(A2))
## A ~ P1' L U in floating point
str(e.lu.A2 \leftarrow expand2(lu.A2), max.level = 2L)
stopifnot(all.equal(A2, Reduce(`%*%`, e.lu.A2)))
## ---- Sparse -------
A3 <- as(readMM(system.file("external/pores_1.mtx", package = "Matrix")),
        "CsparseMatrix")
(lu.A3 \leftarrow lu(A3))
## A ~ P1' L U P2' in floating point
str(e.lu.A3 <- expand2(lu.A3), max.level = 2L)</pre>
stopifnot(all.equal(A3, Reduce(`%*%`, e.lu.A3)))
```

mat2triplet

Map Matrix to its Triplet Representation

# Description

From an R object coercible to "TsparseMatrix", typically a (sparse) matrix, produce its triplet representation which may collapse to a "Duplet" in the case of binary aka pattern, such as "nMatrix" objects.

## Usage

```
mat2triplet(x, uniqT = FALSE)
```

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## **Arguments**

Х	any R object for which as(x, "TsparseMatrix") works; typically a matrix of one of the <b>Matrix</b> package matrices.
uniqT	logical indicating if the triplet representation should be 'unique' in the sense of asUniqueT(byrow=FALSE).

### Value

A list, typically with three components,

```
    vector of row indices for all non-zero entries of x
    vector of columns indices for all non-zero entries of x
    vector of all non-zero entries of x; exists only when as(x, "TsparseMatrix") is not a "nsparseMatrix".
```

Note that the order of the entries is determined by the coercion to "TsparseMatrix" and hence typically with increasing j (and increasing i within ties of j).

### Note

The mat2triplet() utility was created to be a more efficient and more predictable substitute for summary(<sparseMatrix>). UseRs have wrongly expected the latter to return a data frame with columns i and j which however is wrong for a "diagonalMatrix".

### See Also

```
The summary() method for "sparseMatrix", summary, sparseMatrix-method.

mat2triplet() is conceptually the inverse function of spMatrix and (one case of) sparseMatrix.
```

```
mat2triplet # simple definition

i <- c(1,3:8); j <- c(2,9,6:10); x <- 7 * (1:7)
  (Ax <- sparseMatrix(i, j, x = x)) ## 8 x 10 "dgCMatrix"
  str(trA <- mat2triplet(Ax))
  stopifnot(i == sort(trA$i), sort(j) == trA$j, x == sort(trA$x))

D <- Diagonal(x=4:2)
  summary(D)
  str(mat2triplet(D))</pre>
```

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matmult-methods

Matrix (Cross) Products (of Transpose)

## **Description**

The basic matrix product, %\*% is implemented for all our Matrix and also for sparseVector classes, fully analogously to R's base matrix and vector objects.

The functions crossprod and tcrossprod are matrix products or "cross products", ideally implemented efficiently without computing t(.)'s unnecessarily. They also return symmetricMatrix classed matrices when easily detectable, e.g., in crossprod(m), the one argument case.

tcrossprod() takes the cross-product of the transpose of a matrix. tcrossprod(x) is formally equivalent to, but faster than, the call x % % t(x), and so is tcrossprod(x, y) instead of x % % t(y).

*Boolean* matrix products are computed via either %&% or boolArith = TRUE.

## Usage

## **Arguments**

X	a matrix-like object
У	a matrix-like object, or for $[t]$ crossprod() NULL (by default); the latter case is formally equivalent to $y = x$ .
boolArith	logical, i.e., NA, TRUE, or FALSE. If true the result is (coerced to) a pattern matrix, i.e., "nMatrix", unless there are NA entries and the result will be a "lMatrix". If false the result is (coerced to) numeric. When NA, currently the default, the result is a pattern matrix when x and y are "nsparseMatrix" and numeric otherwise.
	potentially more arguments passed to and from methods.

### **Details**

For some classes in the Matrix package, such as dgCMatrix, it is much faster to calculate the cross-product of the transpose directly instead of calculating the transpose first and then its cross-product. boolArith = TRUE for regular ("non cross") matrix products, %\*% cannot be specified. Instead, we provide the %&% operator for *boolean* matrix products.

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### Value

A Matrix object, in the one argument case of an appropriate *symmetric* matrix class, i.e., inheriting from symmetricMatrix.

#### Methods

```
%*% signature(x = "dgeMatrix", y = "dgeMatrix"): Matrix multiplication; ditto for several
other signature combinations, see showMethods("%*%", class = "dgeMatrix").
```

crossprod signature(x = "dgeMatrix", y = "dgeMatrix"): ditto for several other signatures,
 use showMethods("crossprod", class = "dgeMatrix"), matrix crossproduct, an efficient
 version of t(x) %\*% y.

crossprod signature(x = "CsparseMatrix", y = "missing") returns t(x) %\*% x as an dsCMatrix
 object.

crossprod signature(x = "TsparseMatrix", y = "missing") returns t(x) %\*% x as an dsCMatrix
 object.

crossprod,tcrossprod signature(x = "dtrMatrix", y = "matrix") and other signatures, see "%\*%"
above.

#### Note

boolArith = TRUE, FALSE or NA has been newly introduced for **Matrix** 1.2.0 (March 2015). Its implementation has still not been tested extensively. Notably the behaviour for sparse matrices with x slots containing extra zeros had not been documented previously, see the %8% help page.

Currently, boolArith = TRUE is implemented via CsparseMatrix coercions which may be quite inefficient for dense matrices. Contributions for efficiency improvements are welcome.

### See Also

tcrossprod in R's base, and crossprod and \*\*. Matrix package \*& for boolean matrix product methods.

```
## A random sparse "incidence" matrix :
m <- matrix(0, 400, 500)
set.seed(12)
m[runif(314, 0, length(m))] <- 1
mm <- as(m, "CsparseMatrix")
object.size(m) / object.size(mm) # smaller by a factor of > 200
## tcrossprod() is very fast:
system.time(tCmm <- tcrossprod(mm))# 0 (PIII, 933 MHz)
system.time(cm <- crossprod(t(m))) # 0.16
system.time(cm. <- tcrossprod(m)) # 0.02</pre>
```

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```
stopifnot(cm == as(tCmm, "matrix"))
## show sparse sub matrix
tCmm[1:16, 1:30]
```

Matrix

Construct a Classed Matrix

# **Description**

Construct a Matrix of a class that inherits from Matrix.

# Usage

# Arguments

data	an optional numeric data vector or matrix.
nrow	when data is not a matrix, the desired number of rows
ncol	when data is not a matrix, the desired number of columns
byrow	logical. If FALSE (the default) the matrix is filled by columns, otherwise the matrix is filled by rows.
dimnames	a dimnames attribute for the matrix: a list of two character components. They are set if not NULL (as per default).
sparse	logical or NULL, specifying if the result should be sparse or not. By default, it is made sparse when more than half of the entries are 0.
doDiag	logical indicating if a diagonalMatrix object should be returned when the resulting matrix is diagonal ( <i>mathematically</i> ). As class diagonalMatrix extends sparseMatrix, this is a natural default for all values of sparse.
	Otherwise, if doDiag is false, a dense or sparse (depending on sparse) <i>symmet-ric</i> matrix will be returned.
forceCheck	logical indicating if the checks for structure should even happen when data is already a "Matrix" object.

# **Details**

If either of nrow or ncol is not given, an attempt is made to infer it from the length of data and the other parameter. Further, Matrix() makes efforts to keep logical matrices logical, i.e., inheriting from class lMatrix, and to determine specially structured matrices such as symmetric, triangular or diagonal ones. Note that a *symmetric* matrix also needs symmetric dimnames, e.g., by specifying dimnames = list(NULL, NULL), see the examples.

Most of the time, the function works via a traditional (*full*) matrix. However, Matrix(0, nrow, ncol) directly constructs an "empty" sparseMatrix, as does Matrix(FALSE, \*).

Although it is sometime possible to mix unclassed matrices (created with matrix) with ones of class "Matrix", it is much safer to always use carefully constructed ones of class "Matrix".

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### Value

Returns matrix of a class that inherits from "Matrix". Only if data is not a matrix and does not already inherit from class Matrix are the arguments nrow, ncol and byrow made use of.

#### See Also

The classes Matrix, symmetricMatrix, triangularMatrix, and diagonalMatrix; further, matrix.

Special matrices can be constructed, e.g., via sparseMatrix (sparse), bdiag (block-diagonal), bandSparse (banded sparse), or Diagonal.

```
Matrix(0, 3, 2)
                            # 3 by 2 matrix of zeros -> sparse
Matrix(0, 3, 2, sparse=FALSE)# -> 'dense'
## 4 cases - 3 different results :
Matrix(0, 2, 2)
                             # diagonal !
Matrix(0, 2, 2, sparse=FALSE)# (ditto)
                              doDiag=FALSE)# -> sparse symm. "dsCMatrix"
Matrix(0, 2, 2,
Matrix(0, 2, 2, sparse=FALSE, doDiag=FALSE)# -> dense symm. "dsyMatrix"
                            # a 3 by 2 matrix (+ integer warning)
Matrix(1:6, 3, 2)
Matrix(1:6 + 1, nrow=3)
## logical ones:
Matrix(diag(4) > 0) # -> "ldiMatrix" with diag = "U"
Matrix(diag(4) > 0, sparse=TRUE) # (ditto)
Matrix(diag(4) >= 0) # -> "lsyMatrix" (of all 'TRUE')
## triangular
13 <- upper.tri(matrix(,3,3))</pre>
(M <- Matrix(13))  # -> "ltCMatrix"
                    # -> "ltrMatrix"
Matrix(! 13)
as(13, "CsparseMatrix")# "lgCMatrix"
Matrix(1:9, nrow=3,
       dimnames = list(c("a", "b", "c"), c("A", "B", "C")))
(I3 <- Matrix(diag(3)))# identity, i.e., unit "diagonalMatrix"
str(I3) # note 'diag = "U"' and the empty 'x' slot
(A \leftarrow cbind(a=c(2,1), b=1:2))# symmetric *apart* from dimnames
Matrix(A)
                             # hence 'dgeMatrix'
(As <- Matrix(A, dimnames = list(NULL, NULL)))# -> symmetric
forceSymmetric(A) # also symmetric, w/ symm. dimnames
stopifnot(is(As, "symmetricMatrix"),
          is(Matrix(0, 3,3), "sparseMatrix"),
          is(Matrix(FALSE, 1,1), "sparseMatrix"))
```

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Matrix-class

Virtual Class "Matrix" of Matrices

# **Description**

The Matrix class is a class contained by all actual classes in the **Matrix** package. It is a "virtual" class.

### Slots

Dim an integer vector of length 2 giving the dimensions of the matrix.

Dimnames a list of length 2. Each element must be NULL or a character vector of length equal to the corresponding element of Dim.

#### Methods

```
determinant signature(x = "Matrix", logarithm = "missing"): and
```

determinant signature(x = "Matrix", logarithm = "logical"): compute the (log) determinant of x. The method chosen depends on the actual Matrix class of x. Note that det also works for all our matrices, calling the appropriate determinant() method. The Matrix::det is an exact copy of base::det, but in the correct namespace, and hence calling the S4-aware version of determinant().).

diff signature(x = "Matrix"): As diff() for traditional matrices, i.e., applying diff() to each
column.

dim signature(x = "Matrix"): extract matrix dimensions dim.

dim<- signature(x = "Matrix", value = "ANY"): where value is integer of length 2. Allows to reshape Matrix objects, but only when prod(value) == prod(dim(x)).

dimnames signature(x = "Matrix"): extract dimnames.

**dimnames<-** signature(x = "Matrix", value = "list"): set the dimnames to a list of length 2, see dimnames<-.

**length** signature(x = "Matrix"): simply defined as prod(dim(x)) (and hence of mode "double").

show signature(object = "Matrix"): show method for printing. For printing sparse matrices,
 see printSpMatrix.

zapsmall signature(x = "Matrix"): typically used for "dMatrix": round() matrix entries such
that (relatively) very small entries become zero exactly.

image signature(object = "Matrix"): draws an image of the matrix entries, using levelplot()
from package lattice.

head signature(object = "Matrix"): return only the "head", i.e., the first few rows.

**tail** signature(object = "Matrix"): return only the "*tail*", i.e., the last few rows of the respective matrix.

as.matrix, as.array signature(x = "Matrix"): the same as as(x, "matrix"); see also the note
below.

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```
as.vector signature(x = "Matrix", mode = "missing"): as.vector(m) should be identical to
    as.vector(as(m, "matrix")), implemented more efficiently for some subclasses.
```

```
as(x, "vector"), as(x, "numeric") etc, similarly.
```

coerce signature(from = "ANY", to = "Matrix"): This relies on a correct as.matrix() method
for from.

There are many more methods that (conceptually should) work for all "Matrix" objects, e.g., colSums, rowMeans. Even base functions may work automagically (if they first call as.matrix() on their principal argument), e.g., apply, eigen, svd or kappa all do work via coercion to a "traditional" (dense) matrix.

#### Note

Loading the Matrix namespace "overloads" as.matrix and as.array in the **base** namespace by the equivalent of function(x) as(x, "matrix"). Consequently, as.matrix(m) or as.array(m) will properly work when m inherits from the "Matrix" class — *also* for functions in package **base** and other packages. E.g., apply or outer can therefore be applied to "Matrix" matrices.

## Author(s)

Douglas Bates <a href="mailto:bates@stat.wisc.edu">bates@stat.wisc.edu</a> and Martin Maechler

### See Also

the classes dgeMatrix, dgCMatrix, and function Matrix for construction (and examples). Methods, e.g., for kronecker.

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Matrix-notyet	Virtual Classes Not Yet Really Implemented and Used
---------------	---

# Description

iMatrix is the virtual class of all integer (S4) matrices. It extends the Matrix class directly. zMatrix is the virtual class of all complex (S4) matrices. It extends the Matrix class directly.

## **Examples**

```
showClass("iMatrix")
showClass("zMatrix")
```

MatrixClass

The Matrix (Super-) Class of a Class

# **Description**

Return the (maybe super-)class of class cl from package **Matrix**, returning character(0) if there is none.

## Usage

# Arguments

```
cl string, class name

cld its class definition

...Matrix logical indicating if the result must be of pattern "[dlniz]..Matrix" where the first letter "[dlniz]" denotes the content kind.

dropVirtual logical indicating if virtual classes are included or not.

... further arguments are passed to .selectSuperClasses().
```

### Value

```
a character string
```

# Author(s)

Martin Maechler, 24 Mar 2009

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### See Also

Matrix, the mother of all Matrix classes.

### **Examples**

```
mkA <- setClass("A", contains="dgCMatrix")
(A <- mkA())
stopifnot(identical(
         MatrixClass("A"),
         "dgCMatrix"))</pre>
```

MatrixFactorization-class

Virtual Class "MatrixFactorization" of Matrix Factorizations

# **Description**

MatrixFactorization is the virtual class of factorizations of  $m \times n$  matrices A, having the general form

$$P_1AP_2 = A_1 \cdots A_p$$

or (equivalently)

$$A = P_1' A_1 \cdots A_p P_2'$$

where  $P_1$  and  $P_2$  are permutation matrices. Factorizations requiring symmetric A have the constraint  $P_2 = P_1'$ , and factorizations without row or column pivoting have the constraints  $P_1 = I_m$  and  $P_2 = I_n$ , where  $I_m$  and  $I_n$  are the  $m \times m$  and  $n \times n$  identity matrices.

CholeskyFactorization, BunchKaufmanFactorization, SchurFactorization, LU, and QR are the virtual subclasses of MatrixFactorization containing all Cholesky, Bunch-Kaufman, Schur, LU, and QR factorizations, respectively.

## Slots

Dim an integer vector of length 2 giving the dimensions of the factorized matrix.

Dimnames a list of length 2 preserving the dimnames of the factorized matrix. Each element must be NULL or a character vector of length equal to the corresponding element of Dim.

### Methods

dim signature(x = "MatrixFactorization"): returns x@Dim.

dimnames signature(x = "MatrixFactorization"): returns x@Dimnames.

dimnames<- signature(x = "MatrixFactorization", value = "NULL"): returns x with x@Dimnames
 set to list(NULL, NULL).</pre>

dimnames<- signature(x = "MatrixFactorization", value = "list"): returns x with x@Dimnames
 set to value.</pre>

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### See Also

Classes extending CholeskyFactorization, namely Cholesky, pCholesky, and CHMfactor.

Classes extending BunchKaufmanFactorization, namely BunchKaufman and pBunchKaufman.

Classes extending SchurFactorization, namely Schur.

Classes extending LU, namely denseLU and sparseLU.

Classes extending QR, namely sparseQR.

Generic functions Cholesky, BunchKaufman, Schur, 1u, and qr for computing factorizations.

Generic functions expand1 and expand2 for constructing matrix factors from MatrixFactorization objects.

## **Examples**

```
showClass("MatrixFactorization")
```

ndenseMatrix-class

Dim, Dimnames: see Matrix.

Virtual Class "ndenseMatrix" of Dense Logical Matrices

# **Description**

ndenseMatrix is the virtual class of all dense logical (S4) matrices. It extends both denseMatrix and lMatrix directly.

### **Slots**

```
x: logical vector containing the entries of the matrix.
```

### **Extends**

```
Class "nMatrix", directly. Class "denseMatrix", directly. Class "Matrix", by class "nMatrix". Class "Matrix", by class "denseMatrix".
```

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### Methods

```
%*% signature(x = "nsparseMatrix", y = "ndenseMatrix"): ...
%*% signature(x = "ndenseMatrix", y = "nsparseMatrix"): ...
crossprod signature(x = "nsparseMatrix", y = "ndenseMatrix"): ...
crossprod signature(x = "ndenseMatrix", y = "nsparseMatrix"): ...
as.vector signature(x = "ndenseMatrix", mode = "missing"): ...
diag signature(x = "ndenseMatrix"): extracts the diagonal as for all matrices, see the generic diag().
which signature(x = "ndenseMatrix"), semantically equivalent to base function which(x, arr.ind); for details, see the lMatrix class documentation.
```

### See Also

Class ngeMatrix and the other subclasses.

# **Examples**

```
showClass("ndenseMatrix")
as(diag(3) > 0, "ndenseMatrix")# -> "nge"
```

nearPD

Nearest Positive Definite Matrix

## Description

Compute the nearest positive definite matrix to an approximate one, typically a correlation or variance-covariance matrix.

## Usage

# **Arguments**

x numeric  $n \times n$  approximately positive definite matrix, typically an approximation to a correlation or covariance matrix. If x is not symmetric (and ensureSymmetry is not false), symmpart(x) is used.

corr logical indicating if the matrix should be a *correlation* matrix.

nearPD

keepDiag	logical, generalizing corr: if TRUE, the resulting matrix should have the same diagonal $(diag(x))$ as the input matrix.
base.matrix	logical indicating if the resulting mat component should be a <b>base</b> matrix or (by default) a Matrix of class dpoMatrix.
do2eigen	logical indicating if a posdefify() eigen step should be applied to the result of the Higham algorithm.
doSym	logical indicating if $X \leftarrow (X + t(X))/2$ should be done, after $X \leftarrow tcrossprod(Qd, Q)$ ; some doubt if this is necessary.
doDykstra	logical indicating if Dykstra's correction should be used; true by default. If false, the algorithm is basically the direct fixpoint iteration $Y_k = P_U(P_S(Y_{k-1}))$ .
only.values	logical; if TRUE, the result is just the vector of eigenvalues of the approximating matrix.
ensureSymmetry	logical; by default, symmpart(x) is used whenever isSymmetric(x) is not true. The user can explicitly set this to TRUE or FALSE, saving the symmetry test. Beware however that setting it FALSE for an asymmetric input x, is typically nonsense!
eig.tol	defines relative positiveness of eigenvalues compared to largest one, $\lambda_1$ . Eigenvalues $\lambda_k$ are treated as if zero when $\lambda_k/\lambda_1 \leq eig.tol$ .
conv.tol	convergence tolerance for Higham algorithm.
posd.tol	tolerance for enforcing positive definiteness (in the final posdefify step when do2eigen is TRUE).
maxit	maximum number of iterations allowed.
conv.norm.type	convergence norm type (norm(*, type)) used for Higham algorithm. The default is "I" (infinity), for reasons of speed (and back compatibility); using "F" is more in line with Higham's proposal.
trace	logical or integer specifying if convergence monitoring should be traced.

## **Details**

This implements the algorithm of Higham (2002), and then (if do2eigen is true) forces positive definiteness using code from posdefify. The algorithm of Knol and ten Berge (1989) (not implemented here) is more general in that it allows constraints to (1) fix some rows (and columns) of the matrix and (2) force the smallest eigenvalue to have a certain value.

Note that setting corr = TRUE just sets diag(.) <- 1 within the algorithm.

Higham (2002) uses Dykstra's correction, but the version by Jens Oehlschlägel did not use it (accidentally), and still gave reasonable results; this simplification, now only used if doDykstra = FALSE, was active in nearPD() up to Matrix version 0.999375-40.

### Value

If only.values = TRUE, a numeric vector of eigenvalues of the approximating matrix; Otherwise, as by default, an S3 object of class "nearPD", basically a list with components

mat a matrix of class dpoMatrix, the computed positive-definite matrix.

eigenvalues numeric vector of eigenvalues of mat.

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corr logical, just the argument corr.

normF the Frobenius norm (norm(x-X, "F")) of the difference between the original

and the resulting matrix.

iterations number of iterations needed.

converged logical indicating if iterations converged.

## Author(s)

Jens Oehlschlägel donated a first version. Subsequent changes by the Matrix package authors.

### References

Cheng, Sheung Hun and Higham, Nick (1998) A Modified Cholesky Algorithm Based on a Symmetric Indefinite Factorization; *SIAM J. Matrix Anal.*\ Appl., **19**, 1097–1110.

Knol DL, ten Berge JMF (1989) Least-squares approximation of an improper correlation matrix by a proper one. *Psychometrika* **54**, 53–61.

Higham, Nick (2002) Computing the nearest correlation matrix - a problem from finance; *IMA Journal of Numerical Analysis* **22**, 329–343.

### See Also

A first version of this (with non-optional corr=TRUE) has been available as nearcor(); and more simple versions with a similar purpose posdefify(), both from package **sfsmisc**.

```
## Higham(2002), p.334f - simple example
A \leftarrow matrix(1, 3,3); A[1,3] \leftarrow A[3,1] \leftarrow 0
n.A <- nearPD(A, corr=TRUE, do2eigen=FALSE)</pre>
n.A[c("mat", "normF")]
n.A.m <- nearPD(A, corr=TRUE, do2eigen=FALSE, base.matrix=TRUE)$mat
                                                  #=----
stopifnot(exprs = {
  all.equal(n.A$mat[1,2], 0.760689917)
  all.equal(n.A$normF, 0.52779033, tolerance=1e-9)
 all.equal(n.A.m, unname(as.matrix(n.A$mat)), tolerance = 1e-15)# seen rel.d.= 1.46e-16
})
set.seed(27)
m <- matrix(round(rnorm(25),2), 5, 5)</pre>
m \leftarrow m + t(m)
diag(m) \leftarrow pmax(0, diag(m)) + 1
(m <- round(cov2cor(m), 2))</pre>
str(near.m <- nearPD(m, trace = TRUE))</pre>
round(near.m$mat, 2)
norm(m - near.m$mat) # 1.102 / 1.08
if(requireNamespace("sfsmisc")) {
   m2 <- sfsmisc::posdefify(m) # a simpler approach</pre>
   norm(m - m2) # 1.185, i.e., slightly "less near"
}
```

ngeMatrix-class

```
round(nearPD(m, only.values=TRUE), 9)
## A longer example, extended from Jens' original,
## showing the effects of some of the options:
pr <- Matrix(c(1,</pre>
                      0.477, 0.644, 0.478, 0.651, 0.826,
                            0.516, 0.233, 0.682, 0.75,
               0.477, 1,
               0.644, 0.516, 1, 0.599, 0.581, 0.742,
               0.478, 0.233, 0.599, 1,
                                           0.741, 0.8,
               0.651, 0.682, 0.581, 0.741, 1,
                                                  0.798,
               0.826, 0.75, 0.742, 0.8, 0.798, 1),
             nrow = 6, ncol = 6)
nc. <- nearPD(pr, conv.tol = 1e-7) # default</pre>
nc.$iterations # 2
nc.1 <- nearPD(pr, conv.tol = 1e-7, corr = TRUE)</pre>
nc.1$iterations # 11 / 12 (!)
      <- nearPD(pr, conv.tol = 1e-15)
str(ncr)# still 2 iterations
ncr.1 <- nearPD(pr, conv.tol = 1e-15, corr = TRUE)</pre>
ncr.1 $ iterations # 27 / 30 !
ncF <- nearPD(pr, conv.tol = 1e-15, conv.norm = "F")</pre>
stopifnot(all.equal(ncr, ncF))# norm type does not matter at all in this example
## But indeed, the 'corr = TRUE' constraint did ensure a better solution;
## cov2cor() does not just fix it up equivalently :
norm(pr - cov2cor(ncr\$mat)) # = 0.09994
norm(pr -
               ncr.1$mat) # = 0.08746 / 0.08805
### 3) a real data example from a 'systemfit' model (3 eq.):
(load(system.file("external", "symW.rda", package="Matrix"))) # "symW"
dim(symW) # 24 x 24
class(symW)# "dsCMatrix": sparse symmetric
if(dev.interactive()) image(symW)
EV <- eigen(symW, only=TRUE)$values
summary(EV) ## looking more closely {EV sorted decreasingly}:
tail(EV)# all 6 are negative
EV2 <- eigen(sWpos <- nearPD(symW)$mat, only=TRUE)$values
stopifnot(EV2 > 0)
if(requireNamespace("sfsmisc")) {
    plot(pmax(1e-3,EV), EV2, type="o", log="xy", xaxt="n", yaxt="n")
    for(side in 1:2) sfsmisc::eaxis(side)
    plot(pmax(1e-3,EV), EV2, type="o", log="xy")
abline(0, 1, col="red3", lty=2)
```

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## **Description**

This is the class of general dense nonzero-pattern matrices, see nMatrix.

## **Slots**

x: Object of class "logical". The logical values that constitute the matrix, stored in column-major order.

Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), see the Matrix class.

factors: Object of class "list". A named list of factorizations that have been computed for the matrix.

### **Extends**

Class "ndenseMatrix", directly. Class "lMatrix", by class "ndenseMatrix". Class "denseMatrix", by class "ndenseMatrix". Class "Matrix", by class "ndenseMatrix". Class "Matrix", by class "ndenseMatrix".

#### Methods

Currently, mainly t() and coercion methods (for as(.)); use, e.g., showMethods(class="ngeMatrix") for details.

### See Also

Non-general logical dense matrix classes such as ntrMatrix, or nsyMatrix; *sparse* logical classes such as ngCMatrix.

# **Examples**

```
showClass("ngeMatrix")
## "lgeMatrix" is really more relevant
```

nMatrix-class

Class "nMatrix" of Non-zero Pattern Matrices

# **Description**

The nMatrix class is the virtual "mother" class of all *non-zero pattern* (or simply *pattern*) matrices in the **Matrix** package.

### **Slots**

Common to all matrix object in the package:

Dim: Object of class "integer" - the dimensions of the matrix - must be an integer vector with exactly two non-negative values.

Dimnames: list of length two; each component containing NULL or a character vector length equal the corresponding Dim element.

nnzero-methods

## Methods

coerce signature(from = "matrix", to = "nMatrix"): Note that these coercions (must) coerce
 NAs to non-zero, hence conceptually TRUE. This is particularly important when sparseMatrix
 objects are coerced to "nMatrix" and hence to nsparseMatrix.

\_\_\_\_

Additional methods contain group methods, such as

```
Ops signature(e1 = "nMatrix", e2 = "...."),...
Arith signature(e1 = "nMatrix", e2 = "...."),...
Compare signature(e1 = "nMatrix", e2 = "...."),...
Logic signature(e1 = "nMatrix", e2 = "...."),...
Summary signature(x = "nMatrix", "..."),...
```

### See Also

The classes lMatrix, nsparseMatrix, and the mother class, Matrix.

# **Examples**

```
getClass("nMatrix")
L3 <- Matrix(upper.tri(diag(3)))
L3 # an "ltCMatrix"
as(L3, "nMatrix") # -> ntC*
## similar, not using Matrix()
as(upper.tri(diag(3)), "nMatrix")# currently "ngTMatrix"
```

nnzero-methods

The Number of Non-Zero Values of a Matrix

# Description

Returns the number of non-zero values of a numeric-like R object, and in particular an object x inheriting from class Matrix.

## Usage

```
nnzero(x, na.counted = NA)
```

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## Arguments

x an R object, typically inheriting from class Matrix or numeric.
na.counted a logical describing how NAs should be counted. There are three possible

settings for na.counted: **TRUE** NAs *are* counted as non-zero (since "they are not zero").

**NA** (default)the result will be NA if there are NA's in x (since "NA's are not known, i.e., *may be* zero").

**FALSE** NAs are *omitted* from x before the non-zero entries are counted.

For sparse matrices, you may often want to use na. counted = TRUE.

### Value

the number of non zero entries in x (typically integer).

Note that for a *symmetric* sparse matrix S (i.e., inheriting from class symmetricMatrix), nnzero(S) is typically *twice* the length(S@x).

## Methods

signature(x = "ANY") the default method for non-Matrix class objects, simply counts the number 0s in x, counting NA's depending on the na. counted argument, see above.

signature(x = "denseMatrix") conceptually the same as for traditional matrix objects, care has to be taken for "symmetricMatrix" objects.

signature(x = "diagonalMatrix"), and signature(x = "indMatrix") fast simple methods for
 these special "sparseMatrix" classes.

signature(x = "sparseMatrix") typically, the most interesting method, also carefully taking "symmetricMatrix" objects into account.

### See Also

The Matrix class also has a length method; typically, length(M) is much larger than nnzero(M) for a sparse matrix M, and the latter is a better indication of the *size* of M.

```
drop0, zapsmall.
```

```
m <- Matrix(0+1:28, nrow = 4)
m[-3,c(2,4:5,7)] <- m[ 3, 1:4] <- m[1:3, 6] <- 0
(mT <- as(m, "TsparseMatrix"))
nnzero(mT)
(S <- crossprod(mT))
nnzero(S)
str(S) # slots are smaller than nnzero()
stopifnot(nnzero(S) == sum(as.matrix(S) != 0))# failed earlier

data(KNex, package = "Matrix")
M <- KNex$mm
class(M)</pre>
```

norm-methods

```
dim(M)
length(M); stopifnot(length(M) == prod(dim(M)))
nnzero(M) # more relevant than length
## the above are also visible from
str(M)
```

norm-methods

Matrix Norms

# Description

Computes a matrix norm of x, using Lapack for dense matrices. The norm can be the one ("0", or "1") norm, the infinity ("I") norm, the Frobenius ("F") norm, the maximum modulus ("M") among elements of a matrix, or the spectral norm or 2-norm ("2"), as determined by the value of type.

# Usage

```
norm(x, type, ...)
```

# Arguments

x a real or complex matrix.

type A character indicating the type of norm desired.

"0", "o" or "1" specifies the one norm, (maximum absolute column sum);

"I" or "i" specifies the infinity norm (maximum absolute row sum);

"F" **or** "f" specifies the Frobenius norm (the Euclidean norm of x treated as if it were a vector);

"M" or "m" specifies the maximum modulus of all the elements in x; and

"2" specifies the "spectral norm" aka "2-norm", which is the largest singular value (svd) of x.

The default is "0". Only the first character of type[1] is used.

... further arguments passed to or from other methods.

#### **Details**

For dense matrices, the methods eventually call the Lapack functions dlange, dlansy, dlantr, zlange, zlansy, and zlantr.

### Value

A numeric value of class "norm", representing the quantity chosen according to type.

### References

Anderson, E., et al. (1994). LAPACK User's Guide, 2nd edition, SIAM, Philadelphia.

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### See Also

onenormest(), an *approximate* randomized estimate of the 1-norm condition number, efficient for large sparse matrices.

The norm() function from R's base package.

### **Examples**

```
x <- Hilbert(9)</pre>
norm(x)# = "0" = "1"
stopifnot(identical(norm(x), norm(x, "1")))
norm(x, "I")# the same, because 'x' is symmetric
allnorms <- function(x) {
    ## norm(NA, "2") did not work until R 4.0.0
    do2 \leftarrow getRversion() \ge "4.0.0" \mid | !anyNA(x)
    vapply(c("1", "I", "F", "M", if(do2) "2"), norm, 0, x = x)
}
allnorms(x)
allnorms(Hilbert(10))
i \leftarrow c(1,3:8); j \leftarrow c(2,9,6:10); x \leftarrow 7 * (1:7)
A <- sparseMatrix(i, j, x = x)
                                                        ## 8 x 10 "dgCMatrix"
(sA <- sparseMatrix(i, j, x = x, symmetric = TRUE)) ## 10 x 10 "dsCMatrix"
(tA <- sparseMatrix(i, j, x = x, triangular= TRUE)) ## 10 x 10 "dtCMatrix"
(allnorms(A) \rightarrow nA)
allnorms(sA)
allnorms(tA)
stopifnot(all.equal(nA, allnorms(as(A, "matrix"))),
 all.equal(nA, allnorms(tA))) # because tA == rbind(A, 0, 0)
A. <- A; A.[1,3] <- NA
stopifnot(is.na(allnorms(A.))) # gave error
```

nsparseMatrix-class Sparse "pattern" Matrices

## **Description**

The nsparseMatrix class is a virtual class of sparse "pattern" matrices, i.e., binary matrices conceptually with TRUE/FALSE entries. Only the positions of the elements that are TRUE are stored.

These can be stored in the "triplet" form (TsparseMatrix, subclasses ngTMatrix, nsTMatrix, and ntTMatrix which really contain pairs, not triplets) or in compressed column-oriented form (class CsparseMatrix, subclasses ngCMatrix, nsCMatrix, and ntCMatrix) or—rarely—in compressed row-oriented form (class RsparseMatrix, subclasses ngRMatrix, nsRMatrix, and ntRMatrix). The second letter in the name of these non-virtual classes indicates general, symmetric, or triangular.

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### **Objects from the Class**

Objects can be created by calls of the form new("ngCMatrix", ...) and so on. More frequently objects are created by coercion of a numeric sparse matrix to the pattern form for use in the symbolic analysis phase of an algorithm involving sparse matrices. Such algorithms often involve two phases: a symbolic phase wherein the positions of the non-zeros in the result are determined and a numeric phase wherein the actual results are calculated. During the symbolic phase only the positions of the non-zero elements in any operands are of interest, hence numeric sparse matrices can be treated as sparse pattern matrices.

#### Slots

- uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular. Present in the triangular and symmetric classes but not in the general class.
- diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N" for non-unit. The implicit diagonal elements are not explicitly stored when diag is "U". Present in the triangular classes only.
- p: Object of class "integer" of pointers, one for each column (row), to the initial (zero-based) index of elements in the column. Present in compressed column-oriented and compressed row-oriented forms only.
- i: Object of class "integer" of length nnzero (number of non-zero elements). These are the row numbers for each TRUE element in the matrix. All other elements are FALSE. Present in triplet and compressed column-oriented forms only.
- j: Object of class "integer" of length nnzero (number of non-zero elements). These are the column numbers for each TRUE element in the matrix. All other elements are FALSE. Present in triplet and compressed row-oriented forms only.

Dim: Object of class "integer" - the dimensions of the matrix.

# Methods

coerce signature(from = "dgCMatrix", to = "ngCMatrix"), and many similar ones; typically
you should coerce to "nsparseMatrix" (or "nMatrix"). Note that coercion to a sparse pattern
matrix records all the potential non-zero entries, i.e., explicit ("non-structural") zeroes are
coerced to TRUE, not FALSE, see the example.

```
t signature(x = "ngCMatrix"): returns the transpose of x
```

which signature(x = "lsparseMatrix"), semantically equivalent to base function which(x, arr.ind); for details, see the lMatrix class documentation.

### See Also

the class dgCMatrix

```
(m <- Matrix(c(0,0,2:0), 3,5, dimnames=list(LETTERS[1:3],NULL))) ## ``extract the nonzero-pattern of (m) into an nMatrix'': nm <- as(m, "nsparseMatrix") ## \rightarrow will be a "ngCMatrix" str(nm) # no 'x' slot
```

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nsyMatrix-class

Symmetric Dense Nonzero-Pattern Matrices

## **Description**

The "nsyMatrix" class is the class of symmetric, dense nonzero-pattern matrices in non-packed storage and "nspMatrix" is the class of of these in packed storage. Only the upper triangle or the lower triangle is stored.

### **Objects from the Class**

Objects can be created by calls of the form new("nsyMatrix", ...).

## **Slots**

uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.

x: Object of class "logical". The logical values that constitute the matrix, stored in column-major order.

Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), see the Matrix class.

factors: Object of class "list". A named list of factorizations that have been computed for the matrix.

# **Extends**

```
"nsyMatrix" extends class "ngeMatrix", directly, whereas "nspMatrix" extends class "ndenseMatrix", directly.
```

Both extend class "symmetricMatrix", directly, and class "Matrix" and others, *in*directly, use showClass("nsyMatrix"), e.g., for details.

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## Methods

Currently, mainly t() and coercion methods (for as(.); use, e.g., showMethods(class="nsyMatrix") for details.

### See Also

```
ngeMatrix, Matrix, t
```

# **Examples**

ntrMatrix-class

Triangular Dense Logical Matrices

### Description

The "ntrMatrix" class is the class of triangular, dense, logical matrices in nonpacked storage. The "ntpMatrix" class is the same except in packed storage.

### **Slots**

x: Object of class "logical". The logical values that constitute the matrix, stored in column-major order.

uplo: Object of class "character". Must be either "U", for upper triangular, and "L", for lower triangular.

diag: Object of class "character". Must be either "U", for unit triangular (diagonal is all ones), or "N"; see triangularMatrix.

Dim,Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), see the Matrix class.

factors: Object of class "list". A named list of factorizations that have been computed for the matrix.

## Extends

```
"ntrMatrix" extends class "ngeMatrix", directly, whereas "ntpMatrix" extends class "ndenseMatrix", directly.
```

Both extend Class "triangularMatrix", directly, and class "denseMatrix", "lMatrix" and others, *in*directly, use showClass("nsyMatrix"), e.g., for details.

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## Methods

Currently, mainly t() and coercion methods (for as(.); use, e.g., showMethods(class="ntrMatrix") for details.

### See Also

Classes ngeMatrix, Matrix; function t

## **Examples**

```
showClass("ntrMatrix")

str(new("ntpMatrix"))
(nutr <- as(upper.tri(matrix(, 4, 4)), "ndenseMatrix"))
str(nutp <- pack(nutr)) # packed matrix: only 10 = 4*(4+1)/2 entries
!nutp # the logical negation (is *not* logical triangular !)
## but this one is:
stopifnot(all.equal(nutp, pack(!!nutp)))</pre>
```

pack-methods

Representation of Packed and Unpacked Dense Matrices

# **Description**

pack() coerces dense symmetric and dense triangular matrices from unpacked format (storing the full matrix) to packed format (storing only one of the upper and lower triangles). unpack() performs the reverse coercion. The two formats are formalized by the virtual classes "packedMatrix" and "unpackedMatrix".

## Usage

```
pack(x, ...)
## S4 method for signature 'dgeMatrix'
pack(x, symmetric = NA, upperTri = NA, ...)
## S4 method for signature 'lgeMatrix'
pack(x, symmetric = NA, upperTri = NA, ...)
## S4 method for signature 'ngeMatrix'
pack(x, symmetric = NA, upperTri = NA, ...)
## S4 method for signature 'matrix'
pack(x, symmetric = NA, upperTri = NA, ...)
## s4 method for signature 'matrix'
pack(x, symmetric = NA, upperTri = NA, ...)
```

pack-methods

## **Arguments**

X	A dense symmetric or dense triangular matrix.
	For pack(): typically an "unpackedMatrix" or a standard "matrix", though "packedMatrix" are allowed and returned unchanged.
	<pre>For unpack(): typically a "packedMatrix", though "unpackedMatrix" are allowed and returned unchanged.</pre>
symmetric	logical (including NA) optionally indicating whether x is symmetric (or triangular).
upperTri	(for triangular $x$ only) logical (including NA) indicating whether $x$ is upper (or lower) triangular.
	further arguments passed to or from other methods.

#### **Details**

pack(x) checks matrices x *not* inheriting from one of the virtual classes "symmetricMatrix" "triangularMatrix" for symmetry (via isSymmetric()) then for upper and lower triangularity (via isTriangular()) in order to identify a suitable coercion. Setting one or both of symmetric and upperTri to TRUE or FALSE rather than NA allows skipping of irrelevant tests for large matrices known to be symmetric or (upper or lower) triangular.

Users should *not* assume that pack() and unpack() are inverse operations. Specifically, y <- unpack(pack(x)) may not reproduce an "unpackedMatrix" x in the sense of identical(). See the examples.

# Value

```
For pack(): a "packedMatrix" giving the condensed representation of x.For unpack(): an "unpackedMatrix" giving the full storage representation of x.
```

```
showMethods("pack")
(s <- crossprod(matrix(sample(15), 5,3))) # traditional symmetric matrix</pre>
(sp \leftarrow pack(s))
mt <- as.matrix(tt <- tril(s))</pre>
(pt <- pack(mt))</pre>
stopifnot(identical(pt, pack(tt)),
  dim(s) == dim(sp), all(s == sp),
  dim(mt) == dim(pt), all(mt == pt), all(mt == tt))
showMethods("unpack")
(cp4 <- chol(Hilbert(4))) # is triangular</pre>
tp4 <- pack(cp4) # [t]riangular [p]acked
str(tp4)
(unpack(tp4))
stopifnot(identical(tp4, pack(unpack(tp4))))
z1 \leftarrow new("dsyMatrix", Dim = c(2L, 2L), x = as.double(1:4), uplo = "U")
z2 <- unpack(pack(z1))</pre>
```

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```
stopifnot(!identical(z1, z2), \#\_not\_ identical\\ all(z1 == z2)) \# but mathematically equal\\ cbind(z1@x, z2@x) \# (unused!) lower triangle is "lost" in translation
```

packedMatrix-class

Virtual Class "packedMatrix" of Packed Dense Matrices

## Description

Class "packedMatrix" is the *virtual* class of dense symmetric or triangular matrices in "packed" format, storing only the choose(n+1,2) == n\*(n+1)/2 elements of the upper or lower triangle of an n-by-n matrix. It is used to define common methods for efficient subsetting, transposing, etc. of its *proper* subclasses: currently "[dln]spMatrix" (packed symmetric), "[dln]tpMatrix" (packed triangular), and subclasses of these, such as "dppMatrix".

### Slots

```
uplo: "character"; either "U", for upper triangular, and "L", for lower. Dim, Dimnames: as all Matrix objects.
```

### **Extends**

```
Class "denseMatrix", directly. Class "Matrix", by class "denseMatrix", distance 2.
```

### Methods

```
pack signature(x = "packedMatrix"): ...
unpack signature(x = "packedMatrix"): ...
isSymmetric signature(object = "packedMatrix"): ...
isTriangular signature(object = "packedMatrix"): ...
isDiagonal signature(object = "packedMatrix"): ...
t signature(x = "packedMatrix"): ...
diag signature(x = "packedMatrix"): ...
diag<- signature(x = "packedMatrix"): ...</pre>
```

## Author(s)

Mikael Jagan

### See Also

```
pack and unpack; its virtual "complement" "unpackedMatrix"; its proper subclasses "dspMatrix",
"ltpMatrix", etc.
```

```
showClass("packedMatrix")
showMethods(classes = "packedMatrix")
```

pMatrix-class

pMatrix-class

Permutation matrices

## **Description**

The pMatrix class is the class of *permutation* matrices, stored as 1-based integer permutation vectors. A permutation matrix is a square matrix whose rows *and* columns are all standard unit vectors. It follows that permutation matrices are a special case of *index* matrices (hence pMatrix is defined as a direct subclass of indMatrix).

Multiplying a matrix on the left by a permutation matrix is equivalent to permuting its rows. Analogously, multiplying a matrix on the right by a permutation matrix is equivalent to permuting its columns. Indeed, such products are implemented in **Matrix** as indexing operations; see 'Details' below.

### **Details**

By definition, a permutation matrix is both a row index matrix and a column index matrix. However, the perm slot of a pMatrix cannot be used interchangeably as a row index vector and column index vector. If margin=1, then perm is a row index vector, and the corresponding column index vector can be computed as invPerm(perm), i.e., by inverting the permutation. Analogously, if margin=2, then perm and invPerm(perm) are column and row index vectors, respectively.

Given an n-by-n row permutation matrix P with perm slot p and a matrix M with conformable dimensions, we have

```
PM
      = P %*% M
                            M[p,]
MP
     = M %*% P
                         = M[, i(p)]
P'M
                         = M[i(p),]
        crossprod(P, M)
MP'
        tcrossprod(M, P)
                         = M[, p]
P'P
      = crossprod(P)
                         = Diagonal(n)
PP'
         tcrossprod(P)
                         = Diagonal(n)
```

where i := invPerm.

# **Objects from the Class**

Objects can be created explicitly with calls of the form new("pMatrix", ...), but they are more commonly created by coercing 1-based integer index vectors, with calls of the form as(., "pMatrix"); see 'Methods' below.

### **Slots**

margin,perm inherited from superclass indMatrix. Here, perm is an integer vector of length Dim[1] and a permutation of 1:Dim[1].

Dim, Dimnames inherited from virtual superclass Matrix.

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#### **Extends**

```
Class "indMatrix", directly.
```

#### Methods

- coerce signature(from = "numeric", to = "pMatrix"): supporting typical pMatrix construction from a vector of positive integers, specifically a permutation of 1:n. Row permutation is assumed.
- t signature(x = "pMatrix"): the transpose, which is a pMatrix with identical perm but opposite margin. Coincides with the inverse, as permutation matrices are orthogonal.
- solve signature(a = "pMatrix", b = "missing"): the inverse permutation matrix, which is a
  pMatrix with identical perm but opposite margin. Coincides with the transpose, as permutation matrices are orthogonal. See showMethods("solve", classes = "pMatrix") for more
  signatures.
- **determinant** signature(x = "pMatrix", logarithm = "logical"): always returning 1 or -1, as permutation matrices are orthogonal. In fact, the result is exactly the *sign* of the permutation.

#### See Also

Superclass indMatrix of index matrices, for many inherited methods; invPerm, for computing inverse permutation vectors.

```
(pm1 <- as(as.integer(c(2,3,1)), "pMatrix"))</pre>
t(pm1) # is the same as
solve(pm1)
pm1 %*% t(pm1) # check that the transpose is the inverse
stopifnot(all(diag(3) == as(pm1 %*% t(pm1), "matrix")),
          is.logical(as(pm1, "matrix")))
set.seed(11)
## random permutation matrix :
(p10 <- as(sample(10), "pMatrix"))</pre>
## Permute rows / columns of a numeric matrix :
(mm < - round(array(rnorm(3 * 3), c(3, 3)), 2))
mm %*% pm1
pm1 %*% mm
try(as(as.integer(c(3,3,1)), "pMatrix"))# Error: not a permutation
as(pm1, "TsparseMatrix")
p10[1:7, 1:4] # gives an "ngTMatrix" (most economic!)
## row-indexing of a <pMatrix> keeps it as an <indMatrix>:
p10[1:3, ]
```

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printSpMatrix

Format and Print Sparse Matrices Flexibly

## Description

Format and print sparse matrices flexibly. These are the "workhorses" used by the format, show and print methods for sparse matrices. If x is large, printSpMatrix2(x) calls printSpMatrix() twice, namely, for the first and the last few rows, suppressing those in between, and also suppresses columns when x is too wide.

printSpMatrix() basically prints the result of formatSpMatrix().

## Usage

# Arguments

Χ	an R object inheriting from class sparseMatrix.
digits	significant digits to use for printing, see print.default, the default, NULL, corresponds to using getOption("digits").
maxp	integer, default from options(max.print), influences how many entries of large matrices are printed at all. Typically should not be smaller than around 1000; values smaller than 100 are silently "rounded up" to 100.
cld	the class definition of $x$ ; must be equivalent to $getClassDef(class(x))$ and exists mainly for possible speedup.
zero.print	character which should be printed for $structural$ zeroes. The default "." may occasionally be replaced by " " (blank); using "0" would look almost like print()ing of non-sparse matrices.

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col.names

logical or string specifying if and how column names of x should be printed, possibly abbreviated. The default is taken from options("sparse.colnames") if that is set, otherwise FALSE unless there are less than ten columns. When TRUE the full column names are printed.

When col.names is a string beginning with "abb" or "sub" and ending with an integer n (i.e., of the form "abb... <n>"), the column names are abbreviate()d or substring()ed to (target) length n, see the examples.

note.dropping.colnames

logical specifying, when col.names is FALSE if the dropping of the column names should be noted. TRUE by default

names should be noted, TRUE by default.

uniDiag logical indicating if the diagonal entries of a sparse unit triangular or unit-

diagonal matrix should be formatted as "I" instead of "1" (to emphasize that

the 1's are "structural").

col.trailer a string to be appended to the right of each column; this is typically made use of

by show(<sparseMatrix>) only, when suppressing columns.

suppRows, suppCols

logicals or NULL, for printSpMatrix2() specifying if rows or columns should be suppressed in printing. If NULL, sensible defaults are determined from dim(x) and options(c("width", "max.print")). Setting both to FALSE may be a

very bad idea.

align a string specifying how the zero print codes should be aligned, i.e., padded as

strings. The default, "fancy", takes some effort to align the typical zero.print = "." with the position of  $\emptyset$ , i.e., the first decimal (one left of decimal point) of the numbers printed, whereas align = "right" just makes use of print(\*,

right = TRUE).

width number, a positive integer, indicating the approximately desired (line) width of

the output, see also fitWidth.

fitWidth logical indicating if some effort should be made to match the desired width or

temporarily enlarge that if deemed necessary.

... unused optional arguments.

## Details

**formatSpMatrix:** If x is large, only the first rows making up the approximately first maxp entries is used, otherwise all of x. .formatSparseSimple() is applied to (a dense version of) the matrix. Then, formatSparseM is used, unless in trivial cases or for sparse matrices without x slot.

## Value

formatSpMatrix()

returns a character matrix with possibly empty column names, depending on col.names etc, see above.

printSpMatrix\*()

return x invisibly, see invisible.

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### Author(s)

Martin Maechler

#### See Also

the virtual class sparseMatrix and the classes extending it; maybe sparseMatrix or spMatrix as simple constructors of such matrices.

The underlying utilities formatSparseM and .formatSparseSimple() (on the same page).

## **Examples**

gr-methods

Methods for QR Factorization

## **Description**

Computes the pivoted QR factorization of an  $m \times n$  real matrix A, which has the general form

$$P_1AP_2 = QR$$

or (equivalently)

$$A = P_1' Q R P_2'$$

where  $P_1$  and  $P_2$  are permutation matrices,  $Q = \prod_{j=1}^n H_j$  is an  $m \times m$  orthogonal matrix equal to the product of n Householder matrices  $H_j$ , and R is an  $m \times n$  upper trapezoidal matrix.

denseMatrix use the default method implemented in **base**, namely qr.default. It is built on LINPACK routine dqrdc and LAPACK routine dgeqp3, which do not pivot rows, so that  $P_1$  is an identity matrix.

Methods for sparseMatrix are built on CXSparse routines cs\_sqr and cs\_qr, which require  $m \ge n$ .

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## Usage

```
qr(x, ...)
## S4 method for signature 'dgCMatrix'
qr(x, order = 3L, ...)
```

## **Arguments**

x a finite matrix or Matrix to be factorized, satisfying  $nrow(x) \ge ncol(x)$  if

sparse.

order an integer in 0:3 passed to CXSparse routine cs\_sqr, indicating a strategy for

choosing the column permutation  $P_2$ . 0 means no column permutation. 1, 2, and 3 indicate a fill-reducing ordering of A + A',  $\tilde{A}'\tilde{A}$ , and A'A, where  $\tilde{A}$  is A with "dense" rows removed. Do not set to 0 unless you know that the column

order of A is already sensible.

... further arguments passed to or from methods.

#### **Details**

If x is sparse and structurally rank deficient, having structural rank r < n, then x is augmented with (n-r) rows of (partly non-structural) zeros, such that the augmented matrix has structural rank n. This augmented matrix is factorized as described above:

$$P_1 A P_2 = P_1 \begin{bmatrix} A_0 \\ 0 \end{bmatrix} P_2 = QR$$

where  $A_0$  denotes the original, user-supplied  $(m - (n - r)) \times n$  matrix.

## Value

An object representing the factorization, inheriting from virtual S4 class QR or S3 class qr. The specific class is qr unless x inherits from virtual class sparseMatrix, in which case it is sparseQR.

### References

Davis, T. A. (2006). *Direct methods for sparse linear systems*. Society for Industrial and Applied Mathematics. doi:10.1137/1.9780898718881

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Class sparseQR and its methods.

Class dgCMatrix.

Generic function qr from **base**, whose default method qr.default "defines" the S3 class qr of dense QR factorizations.

Generic functions expand1 and expand2, for constructing matrix factors from the result.

Generic functions Cholesky, BunchKaufman, Schur, and 1u, for computing other factorizations.

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```
showMethods("qr", inherited = FALSE)
## Rank deficient: columns 3 {b2} and 6 {c3} are "extra"
M \leftarrow as(cbind(a1 = 1,
              b1 = rep(c(1, 0), each = 3L),
              b2 = rep(c(0, 1), each = 3L),
              c1 = rep(c(1, 0, 0), 2L),
              c2 = rep(c(0, 1, 0), 2L),
              c3 = rep(c(0, 0, 1), 2L)),
        "CsparseMatrix")
rownames(M) <- paste0("r", seq_len(nrow(M)))</pre>
b <- 1:6
eps <- .Machine$double.eps</pre>
## .... [1] full rank .....
## ===> a least squares solution of A x = b exists
        and is unique _in exact arithmetic_
(A1 <- M[, -c(3L, 6L)])
(qr.A1 \leftarrow qr(A1))
stopifnot(exprs = {
    rankMatrix(A1) == ncol(A1)
    { d1 <- abs(diag(qr.A1@R)); sum(d1 < max(d1) * eps) == 0L }
    rcond(crossprod(A1)) >= eps
    all.equal(gr.coef(gr.A1, b), drop(solve(crossprod(A1), crossprod(A1, b))))
    all.equal(qr.fitted(qr.A1, b) + qr.resid(qr.A1, b), b)
})
## .... [2] numerically rank deficient with full structural rank ......
## ===> a least squares solution of A x = b does not
##
        exist or is not unique _in exact arithmetic_
(A2 \leftarrow M)
(qr.A2 \leftarrow qr(A2))
stopifnot(exprs = {
    rankMatrix(A2) == ncol(A2) - 2L
    { d2 \leftarrow abs(diag(qr.A2@R)); sum(d2 < max(d2) * eps) == 2L }
    rcond(crossprod(A2)) < eps</pre>
    ## 'qr.coef' computes unique least squares solution of "nearby" problem
    ## Z \times = b for some full rank Z \sim A, currently without warning {FIXME} !
    tryCatch({ qr.coef(qr.A2, b); TRUE }, condition = function(x) FALSE)
    all.equal(qr.fitted(qr.A2, b) + qr.resid(qr.A2, b), b)
})
## .... [3] numerically and structurally rank deficient .....
## ===> factorization of _augmented_ matrix with
##
        full structural rank proceeds as in [2]
```

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```
NB: implementation details are subject to change; see (*) below
A3 <- M
A3[, c(3L, 6L)] <- 0
(qr.A3 <- qr(A3)) # with a warning ... "additional 2 row(s) of zeros"
stopifnot(exprs = {
    ## sparseQR object preserves the unaugmented dimensions (*)
    dim(qr.A3) == dim(A3)
    dim(qr.A3@V) == dim(A3) + c(2L, 0L)
    dim(qr.A3@R) == dim(A3) + c(2L, 0L)
    ## The augmented matrix remains numerically rank deficient
    rankMatrix(A3) == ncol(A3) - 2L
    { d3 \leftarrow abs(diag(qr.A3@R)); sum(d3 < max(d3) * eps) == 2L }
    rcond(crossprod(A3)) < eps</pre>
})
## Auxiliary functions accept and return a vector or matrix
## with dimensions corresponding to the unaugmented matrix (*),
## in all cases with a warning
qr.coef (qr.A3, b)
qr.fitted(qr.A3, b)
qr.resid (qr.A3, b)
## .... [4] yet more examples .....
## By disabling column pivoting, one gets the "vanilla" factorization
## A = Q^{-} R, where Q^{-} := P1' Q is orthogonal because P1 and Q are
(qr.A1.pp <- qr(A1, order = 0L)) # partial pivoting
ae1 <- function(a, b, ...) all.equal(as(a, "matrix"), as(b, "matrix"), ...)</pre>
ae2 <- function(a, b, ...) ae1(unname(a), unname(b), ...)</pre>
stopifnot(exprs = {
    length(qr.A1 @q) == ncol(A1)
    length(qr.A1.pp@q) == 0L # indicating no column pivoting
    ae2(A1[, qr.A1@q + 1L], qr.Q(qr.A1 ) %*% qr.R(qr.A1 ))
    ae2(A1
                          , qr.Q(qr.A1.pp) %*% qr.R(qr.A1.pp))
})
```

rankMatrix

Rank of a Matrix

#### **Description**

Compute 'the' matrix rank, a well-defined functional in theory(\*), somewhat ambiguous in practice. We provide several methods, the default corresponding to Matlab's definition.

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(\*) The rank of a  $n \times m$  matrix A, rk(A), is the maximal number of linearly independent columns (or rows); hence  $rk(A) \leq min(n, m)$ .

### Usage

## **Arguments**

x numeric matrix, of dimension  $n \times m$ , say.

tol

nonnegative number specifying a (relative, "scalefree") tolerance for testing of "practically zero" with specific meaning depending on method; by default, max(dim(x)) \* .Machine\$double.eps is according to Matlab's default (for its only method which is our method="tolNorm2").

method

a character string specifying the computational method for the rank, can be abbreviated:

"tolNorm2": the number of singular values >= tol \* max(sval);

"qrLINPACK": for a dense matrix, this is the rank of qr(x, tol, LAPACK=FALSE) (which is qr(...)\$rank);

This ("qr\*", dense) version used to be *the* recommended way to compute a matrix rank for a while in the past.

For sparse x, this is equivalent to "gr.R".

"qr.R": this is the rank of triangular matrix R, where qr() uses LAPACK or a "sparseQR" method (see qr-methods) to compute the decomposition QR. The rank of R is then defined as the number of "non-zero" diagonal entries  $d_i$  of R, and "non-zero"s fulfill  $|d_i| \geq \text{tol} \cdot \max(|d_i|)$ .

"qr": is for back compatibility; for dense x, it corresponds to "qrLINPACK", whereas for sparse x, it uses "qr.R".

For all the "qr\*" methods, singular values sval are not used, which may be crucially important for a large sparse matrix x, as in that case, when sval is not specified, the default, computing svd() currently coerces x to a dense matrix.

"useGrad": considering the "gradient" of the (decreasing) singular values, the index of the *smallest* gap.

"maybeGrad": choosing method "useGrad" only when that seems *reasonable*; otherwise using "tolNorm2".

sval

numeric vector of non-increasing singular values of x; typically unspecified and computed from x when needed, i.e., unless method = "qr".

warn.t

logical indicating if rankMatrix() should warn when it needs t(x) instead of x. Currently, for method = "qr" only, gives a warning by default because the caller often could have passed t(x) directly, more efficiently.

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warn.qr	in the $QR$ cases (i.e., if method starts with "qr"), rankMatrix() calls qr2rankMarix(, do.warn = warn.qr), see below.
qr	an R object resulting from $qr(x,)$ , i.e., typically inheriting from class "qr" or "sparseQR".
isBqr	logical indicating if qr is resulting from <b>base</b> qr(). (Otherwise, it is typically from <b>Matrix</b> package sparse qr.)
do.warn	logical; if true, warn about non-finite diagonal entries in the $R$ matrix of the $QR$ decomposition. Do not change lightly!

#### **Details**

qr2rankMatrix() is typically called from rankMatrix() for the "qr"\* methods, but can be used directly - much more efficiently in case the qr-decomposition is available anyway.

#### Value

If x is a matrix of all  $\emptyset$  (or of zero dimension), the rank is zero; otherwise, typically a positive integer in 1:min(dim(x)) with attributes detailing the method used.

There are rare cases where the sparse QR decomposition "fails" in so far as the diagonal entries of R, the  $d_i$  (see above), end with non-finite, typically NaN entries. Then, a warning is signalled (unless warn.gr/do.warn is not true) and NA (specifically, NA\_integer\_) is returned.

#### Note

For large sparse matrices x, unless you can specify sval yourself, currently method = "qr" may be the only feasible one, as the others need sval and call svd() which currently coerces x to a denseMatrix which may be very slow or impossible, depending on the matrix dimensions.

Note that in the case of sparse x, method = "qr", all non-strictly zero diagonal entries  $d_i$  where counted, up to including **Matrix** version 1.1-0, i.e., that method implicitly used tol = 0, see also the set.seed(42) example below.

### Author(s)

Martin Maechler; for the "\*Grad" methods building on suggestions by Ravi Varadhan.

## See Also

```
qr, svd.
```

```
rankMatrix(cbind(1, 0, 1:3)) # 2
(meths <- eval(formals(rankMatrix)$method))
## a "border" case:
H12 <- Hilbert(12)
rankMatrix(H12, tol = 1e-20) # 12; but 11 with default method & tol.
sapply(meths, function(.m.) rankMatrix(H12, method = .m.))</pre>
```

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```
qr.R qrLINPACK qr useGrad maybeGrad
## tolNorm2
        11
               11
                         12
                                12
                                         11
## The meaning of 'tol' for method="qrLINPACK" and *dense* x is not entirely "scale free"
rMQL <- function(ex, M) rankMatrix(M, method="qrLINPACK",tol = 10^-ex)</pre>
rMQR <- function(ex, M) rankMatrix(M, method="qr.R",</pre>
                                                         tol = 10^-ex)
sapply(5:15, rMQL, M = H12) # result is platform dependent
## 7 7 8 10 10 11 11 11 12 12 12 {x86_64}
sapply(5:15, rMQL, M = 1000 * H12) # not identical unfortunately
## 7 7 8 10 11 11 12 12 12 12 12
sapply(5:15, rMQR, M = H12)
## 5 6 7 8 8 9 9 10 10 11 11
sapply(5:15, rMQR, M = 1000 * H12) # the *same*
## "sparse" case:
M15 <- kronecker(diag(x=c(100,1,10)), Hilbert(5))
sapply(meths, function(.m.) rankMatrix(M15, method = .m.))
#--> all 15, but 'useGrad' has 14.
sapply(meths, function(.m.) rankMatrix(M15, method = .m., tol = 1e-7)) # all 14
## "large" sparse
n <- 250000; p <- 33; nnz <- 10000
L <- sparseMatrix(i = sample.int(n, nnz, replace=TRUE),</pre>
                  j = sample.int(p, nnz, replace=TRUE),
                  x = rnorm(nnz)
(st1 <- system.time(r1 <- rankMatrix(L)))</pre>
                                                          # warning+ ~1.5 sec (2013)
(st2 <- system.time(r2 <- rankMatrix(L, method = "qr"))) # considerably faster!</pre>
r1[[1]] == print(r2[[1]]) ## --> ( 33 TRUE )
## another sparse-"qr" one, which ``failed'' till 2013-11-23:
set.seed(42)
f1 <- factor(sample(50, 1000, replace=TRUE))</pre>
f2 <- factor(sample(50, 1000, replace=TRUE))</pre>
f3 <- factor(sample(50, 1000, replace=TRUE))
D <- t(do.call(rbind, lapply(list(f1,f2,f3), as, 'sparseMatrix')))
dim(D); nnzero(D) ## 1000 x 150 // 3000 non-zeros (= 2%)
                                  method='qr') == 148,
stopifnot(rankMatrix(D,
 rankMatrix(crossprod(D),method='qr') == 148)
## zero matrix has rank 0 :
stopifnot(sapply(meths, function(.m.)
                        rankMatrix(matrix(0, 2, 2), method = .m.)) == 0)
```

rcond-methods

Estimate the Reciprocal Condition Number

# **Description**

Estimate the reciprocal of the condition number of a matrix.

This is a generic function with several methods, as seen by showMethods(rcond).

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# Usage

```
rcond(x, norm, ...)
## S4 method for signature 'sparseMatrix,character'
rcond(x, norm, useInv=FALSE, ...)
```

#### **Arguments**

x an R object that inherits from the Matrix class.

norm character string indicating the type of norm to be used in the estimate. The

default is "0" for the 1-norm ("0" is equivalent to "1"). For sparse matrices, when useInv=TRUE, norm can be any of the kinds allowed for norm; otherwise,

the other possible value is "I" for the infinity norm, see also norm.

useInv logical (or "Matrix" containing solve(x)). If not false, compute the reciprocal

condition number as  $1/(\|x\| \cdot \|x^{-1}\|)$ , where  $x^{-1}$  is the inverse of x, solve(x). This may be an efficient alternative (only) in situations where solve(x) is fast

(or known), e.g., for (very) sparse or triangular matrices.

Note that the result may differ depending on useInv, as per default, when it is

false, an approximation is computed.

... further arguments passed to or from other methods.

#### Value

An estimate of the reciprocal condition number of x.

#### BACKGROUND

The condition number of a regular (square) matrix is the product of the norm of the matrix and the norm of its inverse (or pseudo-inverse).

More generally, the condition number is defined (also for non-square matrices A) as

$$\kappa(A) = \frac{\max_{\|v\|=1} \|Av\|}{\min_{\|v\|=1} \|Av\|}.$$

Whenever x is *not* a square matrix, in our method definitions, this is typically computed via rcond(qr.R(qr(X)), ...) where X is x or t(x).

The condition number takes on values between 1 and infinity, inclusive, and can be viewed as a factor by which errors in solving linear systems with this matrix as coefficient matrix could be magnified.

rcond() computes the *reciprocal* condition number  $1/\kappa$  with values in [0, 1] and can be viewed as a scaled measure of how close a matrix is to being rank deficient (aka "singular").

Condition numbers are usually estimated, since exact computation is costly in terms of floating-point operations. An (over) estimate of reciprocal condition number is given, since by doing so overflow is avoided. Matrices are well-conditioned if the reciprocal condition number is near 1 and ill-conditioned if it is near zero.

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#### References

Golub, G., and Van Loan, C. F. (1989). *Matrix Computations*, 2nd edition, Johns Hopkins, Baltimore

#### See Also

norm, kappa() from package **base** computes an *approximate* condition number of a "traditional" matrix, even non-square ones, with respect to the p = 2 (Euclidean) norm. solve.

condest, a newer *approximate* estimate of the (1-norm) condition number, particularly efficient for large sparse matrices.

```
x \leftarrow Matrix(rnorm(9), 3, 3)
rcond(x)
## typically "the same" (with more computational effort):
1 / (norm(x) * norm(solve(x)))
rcond(Hilbert(9)) # should be about 9.1e-13
## For non-square matrices:
rcond(x1 \leftarrow cbind(1,1:10)) # 0.05278
rcond(x2 \leftarrow cbind(x1, 2:11))# practically 0, since x2 does not have full rank
## sparse
(S1 <- Matrix(rbind(0:1,0, diag(3:-2))))
rcond(S1)
m1 <- as(S1, "denseMatrix")
all.equal(rcond(S1), rcond(m1))
## wide and sparse
rcond(Matrix(cbind(0, diag(2:-1))))
## Large sparse example -----
m \leftarrow Matrix(c(3,0:2), 2,2)
M <- bdiag(kronecker(Diagonal(2), m), kronecker(m,m))</pre>
36*(iM <- solve(M)) # still sparse
MM <- kronecker(Diagonal(10), kronecker(Diagonal(5),kronecker(m,M)))</pre>
dim(M3 <- kronecker(bdiag(M,M),MM)) # 12'800 ^ 2</pre>
if(interactive()) ## takes about 2 seconds if you have >= 8 GB RAM
  system.time(r <- rcond(M3))</pre>
## whereas this is *fast* even though it computes solve(M3)
system.time(r. <- rcond(M3, useInv=TRUE))</pre>
if(interactive()) ## the values are not the same
  c(r, r.) # 0.05555 0.013888
## for all 4 norms available for sparseMatrix :
cbind(rr <- sapply(c("1","I","F","M"),</pre>
             function(N) rcond(M3, norm=N, useInv=TRUE)))
```

rep2abI

rep2abI

Replicate Vectors into 'abIndex' Result

# **Description**

```
rep2abI(x, times) conceptually computes rep.int(x, times) but with an abIndex class result.
```

# Usage

```
rep2abI(x, times)
```

# Arguments

x numeric vector

times integer (valued) scalar: the number of repetitions

#### Value

```
a vector of class abIndex
```

## See Also

```
rep.int(), the base function; abIseq, abIndex.
```

# **Examples**

```
(ab <- rep2abI(2:7, 4))
stopifnot(identical(as(ab, "numeric"),
    rep(2:7, 4)))</pre>
```

rleDiff-class

Class "rleDiff" of rle(diff(.)) Stored Vectors

# **Description**

Class "rleDiff" is for compactly storing long vectors which mainly consist of *linear* stretches. For such a vector x, diff(x) consists of *constant* stretches and is hence well compressable via rle().

# **Objects from the Class**

```
Objects can be created by calls of the form new("rleDiff", \ldots).
```

Currently experimental, see below.

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# **Slots**

```
first: A single number (of class "numLike", a class union of "numeric" and "logical").
rle: Object of class "rle", basically a list with components "lengths" and "values", see
    rle(). As this is used to encode potentially huge index vectors, lengths may be of type
    double here.
```

#### Methods

There is a simple show method only.

#### Note

This is currently an *experimental* auxiliary class for the class abIndex, see there.

#### See Also

```
rle, abIndex.
```

# **Examples**

```
showClass("rleDiff")
ab <- c(abIseq(2, 100), abIseq(20, -2))
ab@rleD # is "rleDiff"</pre>
```

rsparsematrix

Random Sparse Matrix

# **Description**

Generate a random sparse matrix efficiently. The default has rounded gaussian non-zero entries, and rand. x = NULL generates random patter m matrices, i.e. inheriting from matrix.

# Usage

## **Arguments**

nrow, ncol	number of rows and columns, i.e., the matrix dimension (dim).
density	optional number in $[0,1]$ , the density is the proportion of non-zero entries among all matrix entries. If specified it determines the default for nnz, otherwise nnz needs to be specified.
nnz	number of non-zero entries, for a sparse matrix typically considerably smaller than nrow*ncol. Must be specified if density is not.

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symmetric	logical indicating if result should be a matrix of class symmetricMatrix. Note that in the symmetric case, nnz denotes the number of non zero entries of the upper (or lower) part of the matrix, including the diagonal.
rand.x	NULL or the random number generator for the x slot, a function such that rand.x(n) generates a numeric vector of length n. Typical examples are rand.x = rnorm, or rand.x = runif; the default is nice for didactical purposes.
	optionally further arguments passed to sparseMatrix(), notably repr.

## **Details**

The algorithm first samples "encoded" (i, j)s without replacement, via one dimensional indices, if not symmetric sample.int(nrow\*ncol, nnz), then—if rand.x is not NULL—gets x <- rand.x(nnz) and calls sparseMatrix(i=i, j=j, x=x, ...). When rand.x=NULL, sparseMatrix(i=i, j=j, ...) will return a pattern matrix (i.e., inheriting from nsparseMatrix).

## Value

a sparseMatrix, say M of dimension (nrow, ncol), i.e., with dim(M) == c(nrow, ncol), if symmetric is not true, with nzM <- nnzero(M) fulfilling nzM <= nnz and typically, nzM == nnz.

## Author(s)

Martin Maechler

```
set.seed(17)# to be reproducible
M <- rsparsematrix(8, 12, nnz = 30) # small example, not very sparse
M
M1 <- rsparsematrix(1000, 20, nnz = 123, rand.x = runif)
summary(M1)

## a random *symmetric* Matrix
(S9 <- rsparsematrix(9, 9, nnz = 10, symmetric=TRUE)) # dsCMatrix
nnzero(S9)# ~ 20: as 'nnz' only counts one "triangle"

## a random patter*n* aka boolean Matrix (no 'x' slot):
(n7 <- rsparsematrix(5, 12, nnz = 10, rand.x = NULL))

## a [T]riplet representation sparseMatrix:
T2 <- rsparsematrix(40, 12, nnz = 99, repr = "T")
head(T2)</pre>
```

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RsparseMatrix-class Class "RsparseMatrix" of Sparse Matrices in Row-compressed Form

## Description

The "RsparseMatrix" class is the virtual class of all sparse matrices coded in sorted compressed row-oriented form. Since it is a virtual class, no objects may be created from it. See showClass("RsparseMatrix") for its subclasses.

#### Slots

- j: Object of class "integer" of length nnzero (number of non-zero elements). These are the row numbers for each non-zero element in the matrix.
- p: Object of class "integer" of pointers, one for each row, to the initial (zero-based) index of elements in the row.

Dim, Dimnames: inherited from the superclass, see sparseMatrix.

#### **Extends**

Class "sparseMatrix", directly. Class "Matrix", by class "sparseMatrix".

### Methods

Originally, **few** methods were defined on purpose, as we rather use the CsparseMatrix in **Matrix**. Then, more methods were added but *beware* that these typically do *not* return "RsparseMatrix" results, but rather Csparse\* or Tsparse\* ones; e.g., R[i, j] <- v for an "RsparseMatrix" R works, but after the assignment, R is a (triplet) "TsparseMatrix".

```
t signature(x = "RsparseMatrix"): ...
coerce signature(from = "RsparseMatrix", to = "CsparseMatrix"): ...
coerce signature(from = "RsparseMatrix", to = "TsparseMatrix"): ...
```

## See Also

its superclass, sparseMatrix, and, e.g., class dgRMatrix for the links to other classes.

```
showClass("RsparseMatrix")
```

Schur-class 161

Schur-class

Schur Factorizations

# **Description**

Schur is the class of Schur factorizations of  $n \times n$  real matrices A, having the general form

$$A = QTQ'$$

where Q is an orthogonal matrix and T is a block upper triangular matrix with  $1 \times 1$  or  $2 \times 2$  diagonal blocks specifying the real and complex conjugate eigenvalues of A. The column vectors of Q are the Schur vectors of A, and T is the Schur form of A.

The Schur factorization generalizes the spectral decomposition of normal matrices A, whose Schur form is block diagonal, to arbitrary square matrices.

#### **Details**

The matrix A and its Schur form T are *similar* and thus have the same spectrum. The eigenvalues are computed trivially as the eigenvalues of the diagonal blocks of T.

#### Slots

Dim, Dimnames inherited from virtual class MatrixFactorization.

Q an orthogonal matrix, inheriting from virtual class Matrix.

T a block upper triangular matrix, inheriting from virtual class Matrix. The diagonal blocks have dimensions 1-by-1 or 2-by-2.

EValues a numeric or complex vector containing the eigenvalues of the diagonal blocks of T, which are the eigenvalues of T and consequently of the factorized matrix.

## Extends

Class SchurFactorization, directly. Class MatrixFactorization, by class SchurFactorization, distance 2.

### Instantiation

Objects can be generated directly by calls of the form new("Schur", ...), but they are more typically obtained as the value of Schur(x) for x inheriting from Matrix (often dgeMatrix).

## Methods

```
determinant signature(from = "Schur", logarithm = "logical"): computes the determinant
    of the factorized matrix A or its logarithm.
expand1 signature(x = "Schur"): see expand1-methods.
expand2 signature(x = "Schur"): see expand2-methods.
solve signature(a = "Schur", b = .): see solve-methods.
```

Schur-methods

## References

The LAPACK source code, including documentation; see <a href="https://netlib.org/lapack/double/dgees.f">https://netlib.org/lapack/double/dgees.f</a>.

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Class dgeMatrix.

Generic functions Schur, expand1 and expand2.

# **Examples**

```
showClass("Schur")
set.seed(0)
n <- 4L
(A <- Matrix(rnorm(n * n), n, n))</pre>
## With dimnames, to see that they are propagated :
dimnames(A) <- list(paste0("r", seq_len(n)),</pre>
                    paste0("c", seq_len(n)))
(sch.A <- Schur(A))
str(e.sch.A <- expand2(sch.A), max.level = 2L)</pre>
## A ~ Q T Q' in floating point
stopifnot(exprs = {
    identical(names(e.sch.A), c("Q", "T", "Q."))
    all.equal(A, with(e.sch.A, Q %*% T %*% Q.))
})
## Factorization handled as factorized matrix
b <- rnorm(n)
stopifnot(all.equal(det(A), det(sch.A)),
          all.equal(solve(A, b), solve(sch.A, b)))
## One of the non-general cases:
Schur(Diagonal(6L))
```

Schur-methods

Methods for Schur Factorization

# **Description**

Computes the Schur factorization of an  $n \times n$  real matrix A, which has the general form

$$A = QTQ'$$

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where Q is an orthogonal matrix and T is a block upper triangular matrix with  $1 \times 1$  and  $2 \times 2$  diagonal blocks specifying the real and complex conjugate eigenvalues of A. The column vectors of Q are the Schur vectors of A, and T is the Schur form of A.

Methods are built on LAPACK routine dgees.

#### Usage

```
Schur(x, vectors = TRUE, ...)
```

# Arguments

x a finite square matrix or Matrix to be factorized.

vectors a logical. If TRUE (the default), then Schur vectors are computed in addition to

the Schur form.

... further arguments passed to or from methods.

#### Value

An object representing the factorization, inheriting from virtual class SchurFactorization if vectors = TRUE. Currently, the specific class is always Schur in that case. An exception is if x is a traditional matrix, in which case the result is a named list containing Q, T, and EValues slots of the Schur object.

If vectors = FALSE, then the result is the same named list but without Q.

### References

The LAPACK source code, including documentation; see <a href="https://netlib.org/lapack/double/dgees.f">https://netlib.org/lapack/double/dgees.f</a>.

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press, doi:10.56021/9781421407944

# See Also

Class Schur and its methods.

Class dgeMatrix.

Generic functions expand1 and expand2, for constructing matrix factors from the result.

Generic functions Cholesky, BunchKaufman, 1u, and qr, for computing other factorizations.

```
showMethods("Schur", inherited = FALSE)
set.seed(0)

Schur(Hilbert(9L)) # real eigenvalues

(A <- Matrix(round(rnorm(25L, sd = 100)), 5L, 5L))
(sch.A <- Schur(A)) # complex eigenvalues</pre>
```

164 solve-methods

solve-methods

Methods in Package Matrix for Function solve

## **Description**

Methods for generic function solve for solving linear systems of equations, i.e., for X in AX = B, where A is a square matrix and X and B are matrices with dimensions consistent with A.

### Usage

```
solve(a, b, ...)
## S4 method for signature 'dgeMatrix, ANY'
solve(a, b, tol = .Machine$double.eps, ...)
## S4 method for signature 'dgCMatrix, missing'
solve(a, b, sparse = TRUE, ...)
## S4 method for signature 'dgCMatrix, matrix'
solve(a, b, sparse = FALSE, ...)
## S4 method for signature 'dgCMatrix,denseMatrix'
solve(a, b, sparse = FALSE, ...)
## S4 method for signature 'dgCMatrix, sparseMatrix'
solve(a, b, sparse = TRUE, ...)
## S4 method for signature 'denseLU,dgeMatrix'
solve(a, b, ...)
## S4 method for signature 'BunchKaufman,dgeMatrix'
solve(a, b, ...)
## S4 method for signature 'Cholesky,dgeMatrix'
solve(a, b, ...)
```

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```
## S4 method for signature 'sparseLU,dgCMatrix'
solve(a, b, tol = .Machine$double.eps, ...)
## S4 method for signature 'sparseQR,dgCMatrix'
solve(a, b, ...)
## S4 method for signature 'CHMfactor,dgCMatrix'
solve(a, b, system = c("A", "LDLt", "LD", "DLt", "L", "Lt", "D", "P", "Pt"), ...)
```

## **Arguments**

a	a finite square matrix or Matrix containing the coefficients of the linear system, or otherwise a MatrixFactorization, in which case methods behave (by default) as if the factorized matrix were specified.
b	a vector, sparseVector, matrix, or Matrix satisfying NROW(b) == nrow(a), giving the right-hand side(s) of the linear system. Vectors b are treated as length(b)-by-1 matrices. If b is missing, then methods take b to be an identity matrix.
tol	a non-negative number. For a inheriting from <code>denseMatrix</code> , an error is signaled if the reciprocal one-norm condition number (see <code>rcond</code> ) of a is less than tol, indicating that a is near-singular. For a of class <code>sparseLU</code> , an error is signaled if the ratio $\min(d)/\max(d)$ is less than tol, where $d = abs(diag(a@U))$ . (Interpret with care, as this ratio is a cheap heuristic and <i>not</i> in general equal to or even proportional to the reciprocal one-norm condition number.) Setting tol = $0$ disables the test.
sparse	a logical indicating if the result should be formally sparse, i.e., if the result should inherit from virtual class <code>sparseMatrix</code> . Only methods for sparse a and missing or matrix b have this argument. Methods for missing or sparse b use <code>sparse = TRUE</code> by default. Methods for dense b use <code>sparse = FALSE</code> by default.
system	a string specifying a linear system to be solved. Only methods for a inheriting from CHMfactor have this argument. See 'Details'.
• • •	further arguments passed to or from methods.

# Details

Methods for general and symmetric matrices a compute a triangular factorization (LU, Bunch-Kaufman, or Cholesky) and call the method for the corresponding factorization class. The factorization is sparse if a is. Methods for sparse, symmetric matrices a attempt a Cholesky factorization and perform an LU factorization only if that fails (typically because a is not positive definite).

Triangular, diagonal, and permutation matrices do not require factorization (they are already "factors"), hence methods for those are implemented directly. For triangular a, solutions are obtained by forward or backward substitution; for diagonal a, they are obtained by scaling the rows of b; and for permutations a, they are obtained by permuting the rows of b.

Methods for dense a are built on 14 LAPACK routines: class d..Matrix, where ..=(ge|tr|tp|sy|sp|po|pp), uses routines d..tri and d..trs for missing and non-missing b, respectively. A corollary is that these methods always give a dense result.

Methods for sparse a are built on CXSparse routines cs\_lsolve, cs\_usolve, and cs\_spsolve and CHOLMOD routines cholmod\_solve and cholmod\_spsolve. By default, these methods give a

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vector result if b is a vector, a sparse matrix result if b is missing or a sparse matrix, and a dense matrix result if b is a dense matrix. One can override this behaviour by setting the sparse argument, where available, but that should be done with care. Note that a sparse result may be sparse only in the formal sense and not at all in the mathematical sense, depending on the nonzero patterns of a and b. Furthermore, whereas dense results are fully preallocated, sparse results must be "grown" in a loop over the columns of b.

Methods for a of class sparseQR are simple wrappers around qr.coef, giving the *least squares* solution in overdetermined cases.

Methods for a inheriting from CHMfactor can solve systems other than the default one AX = B. The correspondence between its system argument the system actually solved is outlined in the table below. See CHMfactor-class for a definition of notation.

system	isLDL(a)=TRUE	isLDL(a)=FALSE
"A"	AX = B	AX = B
"LDLt"	$L_1DL_1'X = B$	LL'X = B
"LD"	$L_1DX = B$	LX = B
"DLt"	$DL_1'X = B$	L'X = B
"L"	$L_1X = B$	LX = B
"Lt"	$L_1'X = B$	L'X = B
"D"	DX = B	X = B
"P"	$X = P_1 B$	$X = P_1 B$
"Pt"	$X = P_1'B$	$X = P_1'B$

#### See Also

Virtual class MatrixFactorization and its subclasses.

Generic functions Cholesky, BunchKaufman, Schur, lu, and gr for *computing* factorizations.

Generic function solve from base.

Function qr.coef from **base** for computing least squares solutions of overdetermined linear systems.

```
## A close to symmetric example with "quite sparse" inverse:
n1 <- 7; n2 <- 3
dd <- data.frame(a = gl(n1,n2), b = gl(n2,1,n1*n2))# balanced 2-way
X <- sparse.model.matrix(~ -1+ a + b, dd)# no intercept --> even sparser
XXt <- tcrossprod(X)
diag(XXt) <- rep(c(0,0,1,0), length.out = nrow(XXt))

n <- nrow(ZZ <- kronecker(XXt, Diagonal(x=c(4,1))))
image(a <- 2*Diagonal(n) + ZZ %*% Diagonal(x=c(10, rep(1, n-1))))
isSymmetric(a) # FALSE
image(drop0(skewpart(a)))
image(ia0 <- solve(a, tol = 0)) # checker board, dense [but really, a is singular!]
try(solve(a, sparse=TRUE))##-> error [ TODO: assertError ]
ia. <- solve(a, sparse=TRUE, tol = 1e-19)##-> *no* error
if(R.version$arch == "x86_64")
    ## Fails on 32-bit [Fedora 19, R 3.0.2] from Matrix 1.1-0 on [FIXME ??] only
```

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sparse.model.matrix Construct Sparse Design / Model Matrices

## **Description**

Construct a sparse model or "design" matrix, from a formula and data frame (sparse.model.matrix) or a single factor (fac2sparse).

The fac2[Ss]parse() functions are utilities, also used internally in the principal user level function sparse.model.matrix().

### Usage

```
sparse.model.matrix(object, data = environment(object),
    contrasts.arg = NULL, xlev = NULL, transpose = FALSE,
    drop.unused.levels = FALSE, row.names = TRUE,
    sep = "", verbose = FALSE, ...)

fac2sparse(from, to = c("d", "l", "n"),
    drop.unused.levels = TRUE, repr = c("C", "R", "T"), giveCsparse)
fac2Sparse(from, to = c("d", "l", "n"),
    drop.unused.levels = TRUE, repr = c("C", "R", "T"), giveCsparse,
    factorPatt12, contrasts.arg = NULL)
```

## Arguments

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xlev to be used as argument of model. frame if data has no "terms" attribute.

transpose logical indicating if the *transpose* should be returned; if the transposed is used

anyway, setting transpose = TRUE is more efficient.

drop.unused.levels

should factors have unused levels dropped? The default for sparse.model.matrix has been changed to FALSE, 2010-07, for compatibility with R's standard (dense)

model.matrix().

row.names logical indicating if row names should be used.

sep character string passed to paste() when constructing column names from the

variable name and its levels.

verbose logical or integer indicating if (and how much) progress output should be printed.

... further arguments passed to or from other methods.

from (for fac2sparse():) a factor.

to a character indicating the "kind" of sparse matrix to be returned. The default,

"d" is for double.

giveCsparse **deprecated**, replaced with repr; logical indicating if the result must be a CsparseMatrix.

repr character string, one of "C", "T", or "R", specifying the sparse *repr*esentation

to be used for the result, i.e., one from the super classes CsparseMatrix, TsparseMatrix,

or RsparseMatrix.

factorPatt12 logical vector, say fp, of length two; when fp[1] is true, return "contrasted"

t(X); when fp[2] is true, the original ("dummy") t(X), i.e, the result of fac2sparse().

#### Value

a sparse matrix, extending CsparseMatrix (for fac2sparse() if repr = "C" as per default; a TsparseMatrix or RsparseMatrix, otherwise).

For fac2Sparse(), a list of length two, both components with the corresponding transposed model matrix, where the corresponding factorPatt12 is true.

fac2sparse(), the basic workhorse of sparse.model.matrix(), returns the transpose (t) of the model matrix.

## Note

model.Matrix(sparse = TRUE) from package **MatrixModels** may be nowadays be preferable to sparse.model.matrix, as model.Matrix returns an object of class modelMatrix with additional slots assign and contrasts relating to the model variables.

### Author(s)

Doug Bates and Martin Maechler, with initial suggestions from Tim Hesterberg.

### See Also

```
model.matrix in package stats, part of base R.
```

model. Matrix in package MatrixModels; see 'Note'.

as(f, "sparseMatrix") (see coerce(from = "factor", ...) in the class doc sparseMatrix) produces the *transposed* sparse model matrix for a single factor f (and *no* contrasts).

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## **Examples**

```
dd \leftarrow data.frame(a = gl(3,4), b = gl(4,1,12)) \# balanced 2-way
options("contrasts") # the default: "contr.treatment"
sparse.model.matrix(~ a + b, dd)
sparse.model.matrix(~ -1+ a + b, dd)# no intercept --> even sparser
sparse.model.matrix(~ a + b, dd, contrasts = list(a="contr.sum"))
sparse.model.matrix(~ a + b, dd, contrasts = list(b="contr.SAS"))
## Sparse method is equivalent to the traditional one :
stopifnot(all(sparse.model.matrix(~
                                      a + b, dd) ==
          Matrix(model.matrix(~ a + b, dd), sparse=TRUE)),
      all(sparse.model.matrix(\sim 0 + a + b, dd) ==
          Matrix(model.matrix(~0 + a + b, dd), sparse=TRUE)))
(ff <- gl(3,4,, c("X","Y", "Z")))
fac2sparse(ff) # 3 x 12 sparse Matrix of class "dgCMatrix"
## X 1 1 1 1 . . . . . . . .
## Y . . . . 1 1 1 1 . . . .
## Z . . . . . . . 1 1 1 1
## can also be computed via sparse.model.matrix():
f30 <- gl(3,0)
               )
f12 \leftarrow g1(3,0, 12)
stopifnot(
 all.equal(t( fac2sparse(ff) ),
   sparse.model.matrix(~ 0+ff),
    tolerance = 0, check.attributes=FALSE),
 is(M <- fac2sparse(f30, drop= TRUE), "CsparseMatrix"), dim(M) == c(0, 0),
 is(M <- fac2sparse(f30, drop=FALSE), "CsparseMatrix"), dim(M) == c(3, 0),</pre>
 is(M \leftarrow fac2sparse(f12, drop= TRUE), "CsparseMatrix"), dim(M) == c(0,12),
 is(M <- fac2sparse(f12, drop=FALSE), "CsparseMatrix"), dim(M) == c(3,12)</pre>
```

sparseLU-class

Sparse LU Factorizations

## **Description**

sparseLU is the class of sparse, row- and column-pivoted LU factorizations of  $n \times n$  real matrices A, having the general form

$$P_1AP_2 = LU$$

or (equivalently)

$$A = P_1' L U P_2'$$

where  $P_1$  and  $P_2$  are permutation matrices, L is a unit lower triangular matrix, and U is an upper triangular matrix.

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#### Slots

Dim, Dimnames inherited from virtual class MatrixFactorization.

L an object of class dtCMatrix, the unit lower triangular L factor.

U an object of class dtCMatrix, the upper triangular U factor.

p, q 0-based integer vectors of length Dim[1], specifying the permutations applied to the rows and columns of the factorized matrix. q of length 0 is valid and equivalent to the identity permutation, implying no column pivoting. Using R syntax, the matrix  $P_1AP_2$  is precisely A[p+1, q+1] (A[p+1, ] when q has length 0).

#### **Extends**

Class LU, directly. Class MatrixFactorization, by class LU, distance 2.

#### Instantiation

Objects can be generated directly by calls of the form new("sparseLU", ...), but they are more typically obtained as the value of lu(x) for x inheriting from sparseMatrix (often dgCMatrix).

#### Methods

```
determinant signature(from = "sparseLU", logarithm = "logical"): computes the determi-
nant of the factorized matrix A or its logarithm.

expand signature(x = "sparseLU"): see expand-methods.

expand1 signature(x = "sparseLU"): see expand1-methods.

expand2 signature(x = "sparseLU"): see expand2-methods.

solve signature(a = "sparseLU", b = .): see solve-methods.
```

## References

Davis, T. A. (2006). *Direct methods for sparse linear systems*. Society for Industrial and Applied Mathematics. doi:10.1137/1.9780898718881

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

### See Also

Class denseLU for dense LU factorizations.

Class dgCMatrix.

Generic functions 1u, expand1 and expand2.

## **Examples**

```
showClass("sparseLU")
set.seed(2)
A <- as(readMM(system.file("external", "pores_1.mtx", package = "Matrix")),
        "CsparseMatrix")
(n \leftarrow A@Dim[1L])
## With dimnames, to see that they are propagated :
dimnames(A) <- dn <- list(paste0("r", seq_len(n)),</pre>
                           paste0("c", seq_len(n)))
(lu.A \leftarrow lu(A))
str(e.lu.A \leftarrow expand2(lu.A), max.level = 2L)
ae1 <- function(a, b, ...) all.equal(as(a, "matrix"), as(b, "matrix"), ...)</pre>
ae2 <- function(a, b, ...) ae1(unname(a), unname(b), ...)</pre>
## A ~ P1' L U P2' in floating point
stopifnot(exprs = {
    identical(names(e.lu.A), c("P1.", "L", "U", "P2."))
    identical(e.lu.A[["P1."]],
              new("pMatrix", Dim = c(n, n), Dimnames = c(dn[1L], list(NULL)),
                  margin = 1L, perm = invertPerm(lu.A@p, 0L, 1L)))
    identical(e.lu.A[["P2."]],
              new("pMatrix", Dim = c(n, n), Dimnames = c(list(NULL), dn[2L]),
                  margin = 2L, perm = invertPerm(lu.A@q, 0L, 1L)))
    identical(e.lu.A[["L"]], lu.A@L)
    identical(e.lu.A[["U"]], lu.A@U)
    ae1(A, with(e.lu.A, P1. %*% L %*% U %*% P2.))
    ae2(A[lu.A@p + 1L, lu.A@q + 1L], with(e.lu.A, L \%*% U))
})
## Factorization handled as factorized matrix
b <- rnorm(n)
stopifnot(identical(det(A), det(lu.A)),
          identical(solve(A, b), solve(lu.A, b)))
```

sparseMatrix

General Sparse Matrix Construction from Nonzero Entries

# **Description**

User-friendly construction of sparse matrices (inheriting from virtual class CsparseMatrix, RsparseMatrix, or TsparseMatrix) from the positions and values of their nonzero entries.

This interface is recommended over direct construction via calls such as new("..[CRT]Matrix", ...).

# Usage

# Arguments

r	guments	
	i, j	integer vectors of equal length specifying the positions (row and column indices) of the nonzero (or non-TRUE) entries of the matrix. Note that, when x is non-missing, the $x_k$ corresponding to $repeated$ pairs $(i_k, j_k)$ are $added$ , for consistency with the definition of class $TsparseMatrix$ , unless use.last.ij is TRUE, in which case only the $last$ such $x_k$ is used.
	p	integer vector of pointers, one for each column (or row), to the initial (zero-based) index of elements in the column (or row). Exactly one of i, j, and p must be missing.
	x	optional, typically nonzero values for the matrix entries. If specified, then the length must equal that of i (or j) or equal 1, in which case x is recycled as necessary. If missing, then the result is a <b>n</b> onzero pattern matrix, i.e., inheriting from class nsparseMatrix.
	dims	optional length-2 integer vector of matrix dimensions. If missing, then $!index1+c(max(i), max(j))$ is used.
	dimnames	optional list of dimnames; if missing, then NULL ones are used.
	symmetric	logical indicating if the resulting matrix should be symmetric. In that case, $(i, j, p)$ should specify only one triangle (upper or lower).
	triangular	logical indicating if the resulting matrix should be triangular. In that case, $(i, j, p)$ should specify only one triangle (upper or lower).
	index1	logical. If TRUE (the default), then i and j are interpreted as 1-based indices, following the R convention. That is, counting of rows and columns starts at 1. If FALSE, then they are interpreted as 0-based indices.
	repr	character string, one of "C", "R", and "T", specifying the <b>repr</b> esentation of the sparse matrix result, i.e., specifying one of the virtual classes CsparseMatrix, RsparseMatrix, and TsparseMatrix.
	giveCsparse	( <b>deprecated</b> , replaced by repr) logical indicating if the result should inherit from CsparseMatrix or TsparseMatrix. Note that operations involving CsparseMatrix are very often (but not always) more efficient.
	check	logical indicating whether to check that the result is formally valid before returning. Do not set to FALSE unless you know what you are doing!
	use.last.ij	logical indicating if, in the case of repeated (duplicated) pairs $(i_k, j_k)$ , only the last pair should be used. FALSE (the default) is consistent with the definiton of class TsparseMatrix.

# **Details**

Exactly one of the arguments i, j and p must be missing.

In typical usage, p is missing, i and j are vectors of positive integers and x is a numeric vector. These three vectors, which must have the same length, form the triplet representation of the sparse matrix.

If i or j is missing then p must be a non-decreasing integer vector whose first element is zero. It provides the compressed, or "pointer" representation of the row or column indices, whichever is missing. The expanded form of p,  $rep(seq\_along(dp), dp)$  where dp <- diff(p), is used as the (1-based) row or column indices.

You cannot set both singular and triangular to true; rather use Diagonal() (or its alternatives, see there).

The values of i, j, p and index1 are used to create 1-based index vectors i and j from which a TsparseMatrix is constructed, with numerical values given by x, if non-missing. Note that in that case, when some pairs  $(i_k, j_k)$  are repeated (aka "duplicated"), the corresponding  $x_k$  are added, in consistency with the definition of the TsparseMatrix class, unless use.last.ij is set to true.

By default, when repr = "C", the CsparseMatrix derived from this triplet form is returned, where repr = "R" now allows to directly get an RsparseMatrix and repr = "T" leaves the result as TsparseMatrix.

The reason for returning a CsparseMatrix object instead of the triplet format by default is that the compressed column form is easier to work with when performing matrix operations. In particular, if there are no zeros in x then a CsparseMatrix is a unique representation of the sparse matrix.

## Value

A sparse matrix, by default in compressed sparse column format and (formally) without symmetric or triangular structure, i.e., by default inheriting from both CsparseMatrix and generalMatrix.

### Note

You *do* need to use index1 = FALSE (or add + 1 to i and j) if you want use the 0-based i (and j) slots from existing sparse matrices.

#### See Also

Matrix(\*, sparse=TRUE) for the constructor of such matrices from a *dense* matrix. That is easier in small sample, but much less efficient (or impossible) for large matrices, where something like sparseMatrix() is needed. Further bdiag and Diagonal for (block-)diagonal and bandSparse for banded sparse matrix constructors.

Random sparse matrices via rsparsematrix().

The standard  $R \times (*, sparse=TRUE)$ , for sparse tables and sparse.model.matrix() for building sparse model matrices.

Consider CsparseMatrix and similar class definition help files.

```
## simple example
i <- c(1,3:8); j <- c(2,9,6:10); x <- 7 * (1:7)
(A <- sparseMatrix(i, j, x = x))  ## 8 x 10 "dgCMatrix"
summary(A)</pre>
```

```
str(A) # note that *internally* 0-based row indices are used
(sA <- sparseMatrix(i, j, x = x, symmetric = TRUE)) ## 10 x 10 "dsCMatrix"
(tA <- sparseMatrix(i, j, x = x, triangular= TRUE)) ## 10 x 10 "dtCMatrix"</pre>
stopifnot( all(sA == tA + t(tA)) ,
           identical(sA, as(tA + t(tA), "symmetricMatrix")))
## dims can be larger than the maximum row or column indices
(AA \leftarrow sparseMatrix(c(1,3:8), c(2,9,6:10), x = 7 * (1:7), dims = c(10,20)))
summary(AA)
## i, j and x can be in an arbitrary order, as long as they are consistent
set.seed(1); (perm <- sample(1:7))</pre>
(A1 <- sparseMatrix(i[perm], j[perm], x = x[perm]))
stopifnot(identical(A, A1))
## The slots are 0-index based, so
try( sparseMatrix(i=A@i, p=A@p, x= seq_along(A@x)) )
## fails and you should say so: 1-indexing is FALSE:
     sparseMatrix(i=A@i, p=A@p, x= seq_along(A@x), index1 = FALSE)
## the (i,j) pairs can be repeated, in which case the x's are summed
(args \leftarrow data.frame(i = c(i, 1), j = c(j, 2), x = c(x, 2)))
(Aa <- do.call(sparseMatrix, args))</pre>
## explicitly ask for elimination of such duplicates, so
## that the last one is used:
(A. <- do.call(sparseMatrix, c(args, list(use.last.ij = TRUE))))</pre>
stopifnot(Aa[1,2] == 9, # 2+7 == 9
          A.[1,2] == 2) # 2 was *after* 7
## for a pattern matrix, of course there is no "summing":
(nA <- do.call(sparseMatrix, args[c("i","j")]))</pre>
dn <- list(LETTERS[1:3], letters[1:5])</pre>
## pointer vectors can be used, and the (i,x) slots are sorted if necessary:
m \leftarrow sparseMatrix(i = c(3,1, 3:2, 2:1), p = c(0:2, 4,4,6), x = 1:6, dimnames = dn)
str(m)
stopifnot(identical(dimnames(m), dn))
sparseMatrix(x = 2.72, i=1:3, j=2:4) # recycling x
sparseMatrix(x = TRUE, i=1:3, j=2:4) # recycling x, |--> "lgCMatrix"
## no 'x' --> patter*n* matrix:
(n <- sparseMatrix(i=1:6, j=rev(2:7)))# -> ngCMatrix
## an empty sparse matrix:
(e <- sparseMatrix(dims = c(4,6), i={}, j={}))
## a symmetric one:
(sy \leftarrow sparseMatrix(i=c(2,4,3:5), j=c(4,7:5,5), x=1:5,
                    dims = c(7,7), symmetric=TRUE))
stopifnot(isSymmetric(sy),
```

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```
identical(sy, ## switch i <-> j {and transpose }
    t( sparseMatrix(j= c(2,4,3:5), i= c(4,7:5,5), x = 1:5,
                     dims = c(7,7), symmetric=TRUE))))
## rsparsematrix() calls sparseMatrix() :
M1 <- rsparsematrix(1000, 20, nnz = 200)
summary(M1)
## pointers example in converting from other sparse matrix representations.
if(requireNamespace("SparseM") &&
   packageVersion("SparseM") >= "0.87" &&
   nzchar(dfil <- system.file("extdata", "rua_32_ax.rua", package = "SparseM"))) {</pre>
 X <- SparseM::model.matrix(SparseM::read.matrix.hb(dfil))</pre>
 XX \leftarrow sparseMatrix(j = X@ja, p = X@ia - 1L, x = X@ra, dims = X@dimension)
 validObject(XX)
 ## Alternatively, and even more user friendly :
 X. \leftarrow as(X, "Matrix") # or also
 X2 <- as(X, "sparseMatrix")</pre>
 stopifnot(identical(XX, X.), identical(X., X2))
}
```

sparseMatrix-class

Virtual Class "sparseMatrix" — Mother of Sparse Matrices

#### **Description**

Virtual Mother Class of All Sparse Matrices

# **Slots**

Dim: Object of class "integer" - the dimensions of the matrix - must be an integer vector with exactly two non-negative values.

Dimnames: a list of length two - inherited from class Matrix, see Matrix.

### **Extends**

Class "Matrix", directly.

### Methods

```
show (object = "sparseMatrix"): The show method for sparse matrices prints "structural" ze-
roes as "." using printSpMatrix() which allows further customization.
```

```
print signature(x = "sparseMatrix"), ....
```

The print method for sparse matrices by default is the same as show() but can be called with extra optional arguments, see printSpMatrix().

```
format signature(x = "sparseMatrix"), ....
```

The format method for sparse matrices, see formatSpMatrix() for details such as the extra optional arguments.

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summary (object = "sparseMatrix", uniqT=FALSE): Returns an object of S3 class "sparseSummary" which is basically a data.frame with columns (i,j,x) (or just (i,j) for nsparseMatrix class objects) with the stored (typically non-zero) entries. The print method resembles Matlab's way of printing sparse matrices, and also the MatrixMarket format, see writeMM.

cbind2 (x = \*, y = \*): several methods for binding matrices together, column-wise, see the basic cbind and rbind functions.

Note that the result will typically be sparse, even when one argument is dense and larger than the sparse one.

**rbind2** (x = \*, y = \*): binding matrices together row-wise, see cbind2 above.

determinant (x = "sparseMatrix", logarithm=TRUE): determinant() methods for sparse matrices typically work via Cholesky or lu decompositions.

**diag** (x = "sparseMatrix"): extracts the diagonal of a sparse matrix.

dim<- signature(x = "sparseMatrix", value = "ANY"): allows to reshape a sparse matrix to a
 sparse matrix with the same entries but different dimensions. value must be of length two
 and fulfill prod(value) == prod(dim(x)).</pre>

coerce signature(from = "factor", to = "sparseMatrix"): Coercion of a factor to "sparseMatrix" produces the matrix of indicator rows stored as an object of class "dgCMatrix". To obtain columns representing the interaction of the factor and a numeric covariate, replace the "x" slot of the result by the numeric covariate then take the transpose. Missing values (NA) from the factor are translated to columns of all 0s.

See also colSums, norm, ... for methods with separate help pages.

### Note

In method selection for multiplication operations (i.e. %\*% and the two-argument form of crossprod) the sparseMatrix class takes precedence in the sense that if one operand is a sparse matrix and the other is any type of dense matrix then the dense matrix is coerced to a dgeMatrix and the appropriate sparse matrix method is used.

#### See Also

sparseMatrix, and its references, such as xtabs(\*, sparse=TRUE), or sparse.model.matrix(),
for constructing sparse matrices.

T2graph for conversion of "graph" objects (package graph) to and from sparse matrices.

```
showClass("sparseMatrix") ## and look at the help() of its subclasses
M <- Matrix(0, 10000, 100)
M[1,1] <- M[2,3] <- 3.14
M ## show(.) method suppresses printing of the majority of rows

data(CAex, package = "Matrix")
dim(CAex) # 72 x 72 matrix
determinant(CAex) # works via sparse lu(.)

## factor -> t( <sparse design matrix> ) :
(fact <- gl(5, 3, 30, labels = LETTERS[1:5]))</pre>
```

```
(Xt <- as(fact, "sparseMatrix")) # indicator rows

## missing values --> all-0 columns:
f.mis <- fact
i.mis <- c(3:5, 17)
is.na(f.mis) <- i.mis

Xt != (X. <- as(f.mis, "sparseMatrix")) # differ only in columns 3:5,17
stopifnot(all(X.[,i.mis] == 0), all(Xt[,-i.mis] == X.[,-i.mis]))</pre>
```

sparseQR-class

Sparse QR Factorizations

### **Description**

sparseQR is the class of sparse, row- and column-pivoted QR factorizations of  $m \times n$  ( $m \ge n$ ) real matrices, having the general form

$$P_1AP_2 = QR = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} R_1 \\ 0 \end{bmatrix} = Q_1R_1$$

or (equivalently)

$$A = P_1'QRP_2' = P_1' \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} R_1 \\ 0 \end{bmatrix} P_2' = P_1'Q_1R_1P_2'$$

where  $P_1$  and  $P_2$  are permutation matrices,  $Q = \prod_{j=1}^n H_j$  is an  $m \times m$  orthogonal matrix ( $Q_1$  contains the first n column vectors) equal to the product of n Householder matrices  $H_j$ , and R is an  $m \times n$  upper trapezoidal matrix ( $R_1$  contains the first n row vectors and is upper triangular).

#### **Usage**

```
qrR(qr, complete = FALSE, backPermute = TRUE, row.names = TRUE)
```

#### **Arguments**

qr an object of class sparseQR, almost always the result of a call to generic function

qr with sparse x.

complete a logical indicating if R should be returned instead of  $R_1$ .

backPermute a logical indicating if R or  $R_1$  should be multiplied on the right by  $P_2'$ .

row.names a logical indicating if dimnames(qr)[1] should be propagated unpermuted to

the result. If complete = FALSE, then only the first n names are kept.

#### **Details**

The method for qr.Q does not return Q but rather the (also orthogonal) product  $P_1'Q$ . This behaviour is algebraically consistent with the **base** implementation (see qr), which can be seen by noting that qr.default in **base** does not pivot rows, constraining  $P_1$  to be an identity matrix. It follows that qr.Q(qr.default(x)) also returns  $P_1'Q$ .

Similarly, the methods for qr.qy and qr.qty multiply on the left by  $P'_1Q$  and  $Q'P_1$  rather than Q and Q'.

It is wrong to expect the values of qr.Q (or qr.R, qr.qy, qr.qty) computed from "equivalent" sparse and dense factorizations (say, qr(x) and qr(as(x, "matrix")) for x of class dgCMatrix) to compare equal. The underlying factorization algorithms are quite different, notably as they employ different pivoting strategies, and in general the factorization is not unique even for fixed  $P_1$  and  $P_2$ .

On the other hand, the values of qr.X, qr.coef, qr.fitted, and qr.resid are well-defined, and in those cases the sparse and dense computations *should* compare equal (within some tolerance).

The method for qr.R is a simple wrapper around qrR, but not back-permuting by default and never giving row names. It did not support backPermute = TRUE until **Matrix** 1.6-0, hence code needing the back-permuted result should call qrR if **Matrix** >= 1.6-0 is not known.

#### **Slots**

Dim, Dimnames inherited from virtual class MatrixFactorization.

beta a numeric vector of length Dim[2], used to construct Householder matrices; see V below.

- V an object of class dgCMatrix with Dim[2] columns. The number of rows nrow(V) is at least Dim[1] and at most Dim[1]+Dim[2]. V is lower trapezoidal, and its column vectors generate the Householder matrices  $H_j$  that compose the orthogonal Q factor. Specifically,  $H_j$  is constructed as diag(Dim[1]) beta[j] \* tcrossprod(V[, j]).
- R an object of class dgCMatrix with nrow(V) rows and Dim[2] columns. R is the upper trapezoidal R factor.
- p, q 0-based integer vectors of length nrow(V) and Dim[2], respectively, specifying the permutations applied to the rows and columns of the factorized matrix. q of length 0 is valid and equivalent to the identity permutation, implying no column pivoting. Using R syntax, the matrix  $P_1AP_2$  is precisely A[p+1, q+1] (A[p+1, ] when q has length 0).

### **Extends**

Class OR, directly. Class MatrixFactorization, by class OR, distance 2.

#### Instantiation

Objects can be generated directly by calls of the form new("sparseQR", ...), but they are more typically obtained as the value of qr(x) for x inheriting from sparseMatrix (often dgCMatrix).

## Methods

```
determinant signature(from = "sparseQR", logarithm = "logical"): computes the determinant of the factorized matrix A or its logarithm.
```

```
expand1 signature(x = "sparseQR"): see expand1-methods.
```

```
expand2 signature(x = "sparseQR"): see expand2-methods.
```

qr.Q signature(qr = "sparseQR"): returns as a dgeMatrix either  $P_1'Q$  or  $P_1'Q_1$ , depending on optional argument complete. The default is FALSE, indicating  $P_1'Q_1$ .

- qr.R signature(qr = "sparseQR"): qrR returns R,  $R_1$ , RP2', or  $R_1P2'$ , depending on optional arguments complete and backPermute. The default in both cases is FALSE, indicating  $R_1$ , for compatibility with **base**. The class of the result in that case is dtCMatrix. In the other three cases, it is dgCMatrix.
- qr.X signature(qr = "sparseQR"): returns A as a dgeMatrix, by default. If m > n and optional argument ncol is greater than n, then the result is augmented with  $P_1'QJ$ , where J is composed of columns (n+1) through ncol of the  $m \times m$  identity matrix.
- qr.coef signature(qr = "sparseQR", y = .): returns as a dgeMatrix or vector the result of multiplying y on the left by  $P_2R_1^{-1}Q_1'P_1$ .
- qr.fitted signature(qr = "sparseQR", y = .): returns as a dgeMatrix or vector the result of multiplying y on the left by  $P_1'Q_1Q_1'P_1$ .
- qr.resid signature(qr = "sparseQR", y = .): returns as a dgeMatrix or vector the result of multiplying y on the left by  $P_1'Q_2Q_2'P_1$ .
- qr.qty signature(qr = "sparseQR", y = .): returns as a dgeMatrix or vector the result of multiplying y on the left by  $Q'P_1$ .
- qr.qy signature(qr = "sparseQR", y = .): returns as a dgeMatrix or vector the result of multiplying y on the left by  $P'_1Q$ .

```
solve signature(a = "sparseQR", b = .): see solve-methods.
```

## References

Davis, T. A. (2006). *Direct methods for sparse linear systems*. Society for Industrial and Applied Mathematics. doi:10.1137/1.9780898718881

Golub, G. H., & Van Loan, C. F. (2013). *Matrix computations* (4th ed.). Johns Hopkins University Press. doi:10.56021/9781421407944

#### See Also

Class dgCMatrix.

Generic function qr from **base**, whose default method qr.default "defines" the S3 class qr of dense QR factorizations.

qr-methods for methods defined in **Matrix**.

Generic functions expand1 and expand2.

The many auxiliary functions for QR factorizations: qr.Q, qr.R, qr.X, qr.coef, qr.fitted, qr.resid, qr.qty, qr.qy, and qr.solve.

```
showClass("sparseQR")
set.seed(2)
m <- 300L</pre>
```

```
n <- 60L
A <- rsparsematrix(m, n, 0.05)
## With dimnames, to see that they are propagated :
dimnames(A) <- dn <- list(paste0("r", seq_len(m)),</pre>
                           paste0("c", seq_len(n)))
(qr.A \leftarrow qr(A))
str(e.qr.A <- expand2(qr.A, complete = FALSE), max.level = 2L)</pre>
str(E.qr.A <- expand2(qr.A, complete = TRUE), max.level = 2L)</pre>
t(sapply(e.qr.A, dim))
t(sapply(E.qr.A, dim))
## Horribly inefficient, but instructive :
slowQ <- function(V, beta) {</pre>
    d \leftarrow dim(V)
    Q \leftarrow diag(d[1L])
    if(d[2L] > 0L) {
        for(j in d[2L]:1L) {
            cat(j, "\n", sep = "")
            Q \leftarrow Q - (beta[j] * tcrossprod(V[, j])) %*% Q
        }
    }
    Q
}
ae1 <- function(a, b, ...) all.equal(as(a, "matrix"), as(b, "matrix"), ...)</pre>
ae2 <- function(a, b, ...) ae1(unname(a), unname(b), ...)
## A ~ P1' Q R P2' ~ P1' Q1 R1 P2' in floating point
stopifnot(exprs = {
    identical(names(e.qr.A), c("P1.", "Q1", "R1", "P2."))
    identical(names(E.qr.A), c("P1.", "Q" , "R" , "P2."))
    identical(e.qr.A[["P1."]],
              new("pMatrix", Dim = c(m, m), Dimnames = c(dn[1L], list(NULL)),
                   margin = 1L, perm = invertPerm(qr.A@p, 0L, 1L)))
    identical(e.qr.A[["P2."]],
              new("pMatrix", Dim = c(n, n), Dimnames = c(list(NULL), dn[2L]),
                   margin = 2L, perm = invertPerm(qr.A@q, 0L, 1L)))
    identical(e.qr.A[["R1"]], triu(E.qr.A[["R"]][seq_len(n), ]))
    identical(e.qr.A[["Q1"]],
                                    E.qr.A[["Q"]][, seq_len(n)] )
    identical(E.qr.A[["R"]], qr.A@R)
 ## ae1(E.qr.A[["Q"]], slowQ(qr.A@V, qr.A@beta))
    ae1(crossprod(E.qr.A[["Q"]]), diag(m))
    ae1(A, with(e.qr.A, P1. %*% Q1 %*% R1 %*% P2.))
    ae1(A, with(E.gr.A, P1. %*% Q %*% R %*% P2.))
    ae2(A.perm <- A[qr.A@p + 1L, qr.A@q + 1L], with(e.qr.A, Q1 %*% R1))
                                               , with(E.qr.A, Q %*% R ))
    ae2(A.perm
})
## More identities
b <- rnorm(m)
```

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```
stopifnot(exprs = {
    ae1(qrX <- qr.X
                        (qr.A ), A)
   ae2(qrQ \leftarrow qr.Q
                        (qr.A ), with(e.qr.A, P1. %*% Q1))
   ae2(
                        (qr.A ), with(e.qr.A, R1))
              qr.R
   ae2(qrc <- qr.coef (qr.A, b), with(e.qr.A, solve(R1 %*% P2., t(qrQ)) %*% b))
   ae2(qrf <- qr.fitted(qr.A, b), with(e.qr.A, tcrossprod(qrQ) %*% b))</pre>
   ae2(qrr <- qr.resid (qr.A, b), b - qrf)</pre>
   ae2(qrq <- qr.qy
                        (qr.A, b), with(E.qr.A, P1. %*% Q %*% b))
   ae2(qr.qty(qr.A, qrq), b)
})
## Sparse and dense computations should agree here
qr.Am <- qr(as(A, "matrix")) # <=> qr.default(A)
stopifnot(exprs = {
   ae2(qrX, qr.X
                      (qr.Am ))
   ae2(qrc, qr.coef (qr.Am, b))
   ae2(qrf, qr.fitted(qr.Am, b))
   ae2(qrr, qr.resid (qr.Am, b))
})
```

sparseVector

Sparse Vector Construction from Nonzero Entries

## **Description**

User friendly construction of sparse vectors, i.e., objects inheriting from class sparseVector, from indices and values of its non-zero entries.

#### Usage

```
sparseVector(x, i, length)
```

# Arguments

	• • •
will be returned.	

i integer vector (of the same length as x) specifying the indices of the non-zero (or non-TRUE) entries of the sparse vector.

length of the sparse vector.

#### **Details**

length

zero entries in x are dropped automatically, analogously as drop0() acts on sparse matrices.

## Value

```
a sparse vector, i.e., inheriting from class sparseVector.
```

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#### Author(s)

Martin Maechler

## See Also

sparseMatrix() constructor for sparse matrices; the class sparseVector.

#### **Examples**

sparseVector-class

Sparse Vector Classes

## Description

Sparse Vector Classes: The virtual mother class "sparseVector" has the five actual daughter classes "dsparseVector", "isparseVector", "lsparseVector", "nsparseVector", and "zsparseVector", where we've mainly implemented methods for the d\*, 1\* and n\* ones.

## Slots

length: class "numeric" - the length of the sparse vector. Note that "numeric" can be considerably larger than the maximal "integer", .Machine\$integer.max, on purpose.

i: class "numeric" - the (1-based) indices of the non-zero entries. Must *not* be NA and strictly sorted increasingly.

Note that "integer" is "part of" "numeric", and can (and often will) be used for non-huge sparse Vectors.

x: (for all but "nsparseVector"): the non-zero entries. This is of class "numeric" for class "dsparseVector", "logical" for class "lsparseVector", etc.

#### Methods

**length** signature(x = "sparseVector"): simply extracts the length slot.

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**show** signature(object = "sparseVector"): The show method for sparse vectors prints "structural" zeroes as "." using the non-exported prSpVector function which allows further customization such as replacing "." by " " (blank). Note that options (max.print) will influence how many entries of large sparse vectors are printed at all. as.vector signature(x = "sparseVector", mode = "character") coerces sparse vectors to "regular", i.e., atomic vectors. This is the same as as(x, "vector"). **as** ...: see coerce below coerce signature(from = "sparseVector", to = "sparseMatrix"), and coerce signature(from = "sparseMatrix", to = "sparseVector"), etc: coercions to and from sparse matrices (sparseMatrix) are provided and work analogously as in standard R, i.e., a vector is coerced to a 1-column matrix. dim<- signature(x = "sparseVector", value = "integer") coerces a sparse vector to a sparse Matrix, i.e., an object inheriting from sparseMatrix, of the appropriate dimension. **head** signature(x = "sparseVector"): as with R's (package **util**) head, head(x,n) (for  $n \ge$ 1) is equivalent to x[1:n], but here can be much more efficient, see the example. tail signature(x = "sparseVector"): analogous to head, see above. toeplitz signature(x = "sparseVector"): as toeplitz(x), produce the  $n \times n$  Toeplitz matrix from x, where n = length(x). rep signature(x = "sparseVector") repeat x, with the same argument list (x, times, length.out, each,  $\dots$ ) as the default method for rep(). which signature(x = "nsparseVector") and which signature(x = "1sparseVector") return the indices of the non-zero entries (which is trivial for sparse vectors). Ops signature(e1 = "sparseVector", e2 = "\*"): define arithmetic, compare and logic operations, (see Ops). **Summary** signature(x = "sparseVector"): define all the Summary methods. is.na, is.finite, is.infinite (x = "sparseVector"), and is.na, is.finite, is.infinite (x = "nsparseVector"): return logical or "nsparseVector" of the

**is.na, is.finite, is.infinite** (x = "nsparseVector"): return logical or "nsparseVector" of the same length as x, indicating if/where x is NA (or NaN), finite or infinite, entirely analogously to the corresponding base R functions.

**zapsmall** signature(x = "sparseVectors"): typically used for numeric sparse vector: round() entries such that (relatively) very small entries become zero exactly.

c.sparseVector() is an S3 method for all "sparseVector"s, but automatic dispatch only happens for the first argument, so it is useful also as regular R function, see the examples.

#### See Also

sparseVector() for friendly construction of sparse vectors (apart from as (\*, "sparseVector")).

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## **Examples**

```
getClass("sparseVector")
getClass("dsparseVector")
sx \leftarrow c(0,0,3, 3.2, 0,0,0,-3:1,0,0,2,0,0,5,0,0)
(ss <- as(sx, "sparseVector"))</pre>
ix <- as.integer(round(sx))</pre>
(is <- as(ix, "sparseVector")) ## an "isparseVector" (!)
(ns <- sparseVector(i= c(7, 3, 2), length = 10)) # "nsparseVector"</pre>
## rep() works too:
(ri <- rep(is, length.out= 25))</pre>
## Using `dim<-` as in base R :
dim(r) \leftarrow c(4,5) \# becomes a sparse Matrix:
## or coercion (as as.matrix() in base R):
as(ss, "Matrix")
stopifnot(all(ss == print(as(ss, "CsparseMatrix"))))
## currently has "non-structural" FALSE -- printing as ":"
(lis <- is & FALSE)
(nn <- is[is == 0]) # all "structural" FALSE</pre>
## NA-case
sN \leftarrow sx; sN[4] \leftarrow NA
(svN <- as(sN, "sparseVector"))</pre>
v \leftarrow as(c(0,0,3, 3.2, rep(0,9),-3,0,-1, rep(0,20),5,0),
         "sparseVector")
v \leftarrow rep(rep(v, 50), 5000)
set.seed(1); v[sample(v@i, 1e6)] <- 0
system.time(for(i in 1:4) hv <- head(v, 1e6))</pre>
## user system elapsed
## 0.033 0.000 0.032
system.time(for(i in 1:4) h2 \leftarrow v[1:1e6])
## user system elapsed
## 1.317 0.000 1.319
stopifnot(identical(hv, h2),
          identical(is | FALSE, is != 0),
           validObject(svN), validObject(lis), as.logical(is.na(svN[4])),
           identical(is^2 > 0, is \& TRUE),
           all(!lis), !any(lis), length(nn@i) == 0, !any(nn), all(!nn),
           sum(lis) == 0, !prod(lis), range(lis) == c(0,0))
## create and use the t(.) method:
t(x20 \le sparseVector(c(9,3:1), i=c(1:2,4,7), length=20))
(T20 \leftarrow toeplitz(x20))
stopifnot(is(T20, "symmetricMatrix"), is(T20, "sparseMatrix"),
```

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```
identical(unname(as.matrix(T20)),
                    toeplitz(as.vector(x20))))
## c() method for "sparseVector" - also available as regular function
(c1 <- c(x20, 0,0,0, -10*x20))
(c2 \leftarrow c(ns, is, FALSE))
(c3 <- c(ns, !ns, TRUE, NA, FALSE))
(c4 <- c(ns, rev(ns)))
## here, c() would produce a list {not dispatching to c.sparseVector()}
(c5 <- c.sparseVector(0,0, x20))
## checking (consistency)
.v <- as.vector
.s <- function(v) as(v, "sparseVector")</pre>
stopifnot(exprs = {
    all.equal(c1, .s(c(.v(x20), 0,0,0, -10*.v(x20))),
                                                            tol = 0)
   all.equal(c2, .s(c(.v(ns), .v(is), FALSE)),
                                                            tol = 0)
   all.equal(c3, .s(c(.v(ns), !.v(ns), TRUE, NA, FALSE)), tol = 0)
   all.equal(c4, .s(c(.v(ns), rev(.v(ns)))),
                                                            tol = 0,
              check.class = FALSE)
   all.equal(c5, .s(c(0,0, .v(x20))),
                                                            tol = 0)
})
```

spMatrix

Sparse Matrix Constructor From Triplet

#### **Description**

User friendly construction of a sparse matrix (inheriting from class TsparseMatrix) from the triplet representation.

This is much less flexible than sparseMatrix() and hence somewhat deprecated.

## Usage

```
spMatrix(nrow, ncol, i = integer(0L), j = integer(0L), x = double(0L))
```

## **Arguments**

nrow, ncol	integers specifying the desired number of rows and columns.
i, j	integer vectors of the same length specifying the locations of the non-zero (or non-TRUE) entries of the matrix.
Х	atomic vector of the same length as i and j, specifying the values of the non-zero entries.

#### Value

```
A sparse matrix in triplet form, as an R object inheriting from both TsparseMatrix and generalMatrix. The matrix M will have M[i[k], j[k]] == x[k], for k = 1, 2, ..., n, where n = length(i) and M[i', j'] == 0 for all other pairs (i', j').
```

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## See Also

Matrix(\*, sparse=TRUE) for the more usual constructor of such matrices. Then, sparseMatrix is more general and flexible than spMatrix() and by default returns a CsparseMatrix which is often slightly more desirable. Further, bdiag and Diagonal for (block-)diagonal matrix constructors.

Consider TsparseMatrix and similar class definition help files.

## **Examples**

```
## simple example
A \leftarrow spMatrix(10,20, i = c(1,3:8),
                    j = c(2,9,6:10),
                    x = 7 * (1:7)
A # a "dgTMatrix"
summary(A)
str(A) # note that *internally* 0-based indices (i,j) are used
L \leftarrow spMatrix(9, 30, i = rep(1:9, 3), 1:27,
             (1:27) %% 4 != 1)
L # an "lgTMatrix"
## A simplified predecessor of Matrix' rsparsematrix() function :
rSpMatrix <- function(nrow, ncol, nnz,</pre>
                      rand.x = function(n) round(rnorm(nnz), 2))
{
    ## Purpose: random sparse matrix
    ## -----
    ## Arguments: (nrow,ncol): dimension
               nnz : number of non-zero entries
              rand.x: random number generator for 'x' slot
     ## Author: Martin Maechler, Date: 14.-16. May 2007
     stopifnot((nnz <- as.integer(nnz)) >= 0,
              nrow \ge 0, ncol \ge 0, nnz \le nrow * ncol)
    spMatrix(nrow, ncol,
             i = sample(nrow, nnz, replace = TRUE),
             j = sample(ncol, nnz, replace = TRUE),
             x = rand.x(nnz)
}
M1 < - rSpMatrix(100000, 20, nnz = 200)
summary(M1)
```

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# **Description**

Methods for "[<-", i.e., extraction or subsetting mostly of matrices, in package Matrix.

**Note**: Contrary to standard matrix assignment in base R, in x[..] <-val it is typically an **error** (see stop) when the type or class of val would require the class of x to be changed, e.g., when x is logical, say "lsparseMatrix", and val is numeric. In other cases, e.g., when x is a "nsparseMatrix" and val is not TRUE or FALSE, a warning is signalled, and val is "interpreted" as logical, and (logical) NA is interpreted as TRUE.

#### Methods

There are *many many* more than these:

```
x = "Matrix", i = "missing", j = "missing", value= "ANY" is currently a simple fallback method implementation which ensures "readable" error messages.
x = "Matrix", i = "ANY", j = "ANY", value= "ANY" currently gives an error
x = "denseMatrix", i = "index", j = "missing", value= "numeric" ...
x = "denseMatrix", i = "index", j = "index", value= "numeric" ...
x = "denseMatrix", i = "missing", j = "index", value= "numeric" ...
```

#### See Also

[-methods for subsetting "Matrix" objects; the index class; Extract about the standard subset assignment (and extraction).

# **Examples**

```
set.seed(101)
(a \leftarrow m \leftarrow Matrix(round(rnorm(7*4), 2), nrow = 7))
a[] <- 2.2 # <<- replaces **every** entry
## as do these:
a[,] <- 3 ; a[TRUE,] <- 4
m[2, 3] <- 3.14 # simple number
m[3, 3:4] \leftarrow 3:4 # simple numeric of length 2
## sub matrix assignment:
m[-(4:7), 3:4] \leftarrow cbind(1,2:4) \#-> upper right corner of 'm'
m[3:5, 2:3] < -0
m[6:7, 1:2] <- Diagonal(2)
## rows or columns only:
m[1,] <- 10
m[,2] <- 1:7
m[-(1:6), ] \leftarrow 3:0 \# not the first 6 rows, i.e. only the 7th
as(m, "sparseMatrix")
```

subscript-methods

Methods for "[": Extraction or Subsetting in Package 'Matrix'

## **Description**

Methods for "[", i.e., extraction or subsetting mostly of matrices, in package **Matrix**.

#### Methods

There are more than these:

```
x = "Matrix", i = "missing", j = "missing", drop= "ANY" ...
x = "Matrix", i = "numeric", j = "missing", drop= "missing" ...
x = "Matrix", i = "missing", j = "numeric", drop= "missing" ...
x = "dsparseMatrix", i = "missing", j = "numeric", drop= "logical" ...
x = "dsparseMatrix", i = "numeric", j = "missing", drop= "logical" ...
x = "dsparseMatrix", i = "numeric", j = "numeric", drop= "logical" ...
```

## See Also

[<--methods for subassignment to "Matrix" objects. Extract about the standard extraction.

## **Examples**

```
str(m <- Matrix(round(rnorm(7*4),2), nrow = 7))
stopifnot(identical(m, m[]))
m[2, 3]  # simple number
m[2, 3:4] # simple numeric of length 2
m[2, 3:4, drop=FALSE] # sub matrix of class 'dgeMatrix'
## rows or columns only:
m[1,]  # first row, as simple numeric vector
m[,1:2] # sub matrix of first two columns
showMethods("[", inherited = FALSE)</pre>
```

symmetricMatrix-class Virtual Class of Symmetric Matrices in Package Matrix

## **Description**

The virtual class of symmetric matrices, "symmetricMatrix", from the package **Matrix** contains numeric and logical, dense and sparse matrices, e.g., see the examples with the "actual" subclasses.

The main use is in methods (and C functions) that can deal with all symmetric matrices, and in as(\*, "symmetricMatrix").

## **Slots**

Dim, Dimnames inherited from virtual class Matrix. See comments below about symmetry of Dimnames.

factors a list of MatrixFactorization objects caching factorizations of the matrix. Typically, it is initialized as an empty list and updated "automagically" whenever a factorization is computed.

uplo a character string, either "U" or "L" indicating that only entries in the upper or lower triangle are referenced.

#### Extends

Class "Matrix", directly.

#### Methods

**dimnames** signature(object = "symmetricMatrix"): returns *symmetric* dimnames, even when the Dimnames slot only has row or column names. This allows to save storage for large (typically sparse) symmetric matrices.

isSymmetric signature(object = "symmetricMatrix"): returns TRUE trivially.

There's a C function symmetricMatrix\_validate() called by the internal validity checking functions, and also from getValidity(getClass("symmetricMatrix")).

## Validity and dimnames

The validity checks do not require a symmetric Dimnames slot, so it can be list(NULL, <character>), e.g., for efficiency. However, dimnames() and other functions and methods should behave as if the dimnames were symmetric, i.e., with both list components identical.

#### See Also

isSymmetric which has efficient methods (isSymmetric-methods) for the **Matrix** classes. Classes triangularMatrix, and, e.g., dsyMatrix for numeric *dense* matrices, or lsCMatrix for a logical *sparse* matrix class.

# **Examples**

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```
## Methods -- applicable to all subclasses above:
showMethods(classes = "symmetricMatrix")
```

symmpart-methods

Symmetric Part and Skew(symmetric) Part of a Matrix

# Description

symmpart(x) computes the symmetric part (x + t(x))/2 and skewpart(x) the skew symmetric part (x - t(x))/2 of a square matrix x, more efficiently for specific Matrix classes.

Note that x == symmpart(x) + skewpart(x) for all square matrices – apart from extraneous NA values in the RHS.

#### Usage

```
symmpart(x)
skewpart(x)
```

## **Arguments**

Χ

a *square* matrix; either "traditional" of class "matrix", or typically, inheriting from the Matrix class.

# Details

These are generic functions with several methods for different matrix classes, use e.g., showMethods(symmpart) to see them.

If the row and column names differ, the result will use the column names unless they are (partly) NULL where the row names are non-NULL (see also the examples).

## Value

symmpart(x) returns a symmetric matrix, inheriting from symmetricMatrix or diagonalMatrix if x inherits from Matrix.

skewpart(x) returns a skew-symmetric matrix, inheriting from generalMatrix, symmetricMatrix or diagonalMatrix if x inherits from Matrix.

#### See Also

isSymmetric.

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## **Examples**

```
m <- Matrix(1:4, 2,2)
symmpart(m)
skewpart(m)

stopifnot(all(m == symmpart(m) + skewpart(m)))

dn <- dimnames(m) <- list(row = c("r1", "r2"), col = c("var.1", "var.2"))
stopifnot(all(m == symmpart(m) + skewpart(m)))
colnames(m) <- NULL
stopifnot(all(m == symmpart(m) + skewpart(m)))
dimnames(m) <- unname(dn)
stopifnot(all(m == symmpart(m) + skewpart(m)))

## investigate the current methods:
showMethods(skewpart, include = TRUE)</pre>
```

triangularMatrix-class

Virtual Class of Triangular Matrices in Package Matrix

# **Description**

The virtual class of triangular matrices, "triangularMatrix", the package **Matrix** contains *square* (nrow == ncol) numeric and logical, dense and sparse matrices, e.g., see the examples. A main use of the virtual class is in methods (and C functions) that can deal with all triangular matrices.

# Slots

uplo: String (of class "character"). Must be either "U", for upper triangular, and "L", for lower triangular.

diag: String (of class "character"). Must be either "U", for unit triangular (diagonal is all ones), or "N" for non-unit. The diagonal elements are not accessed internally when diag is "U". For denseMatrix classes, they need to be allocated though, such that the length of the x slot does not depend on diag.

Dim, Dimnames: The dimension (a length-2 "integer") and corresponding names (or NULL), inherited from the Matrix, see there.

# Extends

Class "Matrix", directly.

#### Methods

There's a C function triangularMatrix\_validity() called by the internal validity checking functions.

Currently, Schur, isSymmetric and as() (i.e. coerce) have methods with triangularMatrix in their signature.

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## See Also

isTriangular() for testing any matrix for triangularity; classes symmetricMatrix, and, e.g., dtrMatrix for numeric *dense* matrices, or ltCMatrix for a logical *sparse* matrix subclass of "triangularMatrix".

# **Examples**

```
showClass("triangularMatrix")
## The names of direct subclasses:
scl <- getClass("triangularMatrix")@subclasses
directly <- sapply(lapply(scl, slot, "by"), length) == 0
names(scl)[directly]

(m <- matrix(c(5,1,0,3), 2))
as(m, "triangularMatrix")</pre>
```

TsparseMatrix-class

Class "TsparseMatrix" of Sparse Matrices in Triplet Form

## **Description**

The "TsparseMatrix" class is the virtual class of all sparse matrices coded in triplet form. Since it is a virtual class, no objects may be created from it. See showClass("TsparseMatrix") for its subclasses.

## **Slots**

Dim, Dimnames: from the "Matrix" class,

- i: Object of class "integer" the row indices of non-zero entries in 0-base, i.e., must be in 0:(nrow(.)-1).
- j: Object of class "integer" the column indices of non-zero entries. Must be the same length as slot i and *0-based* as well, i.e., in 0:(ncol(.)-1). For numeric Tsparse matrices, (i,j) pairs can occur more than once, see dgTMatrix.

#### **Extends**

```
Class "sparseMatrix", directly. Class "Matrix", by class "sparseMatrix".
```

## Methods

Extraction ("[") methods, see [-methods.

unpackedMatrix-class 193

#### Note

Most operations with sparse matrices are performed using the compressed, column-oriented or CsparseMatrix representation. The triplet representation is convenient for creating a sparse matrix or for reading and writing such matrices. Once it is created, however, the matrix is generally coerced to a CsparseMatrix for further operations.

Note that all new(.), spMatrix and sparseMatrix(\*, repr="T") constructors for "TsparseMatrix" classes implicitly add (i.e., "sum up")  $x_k$ 's that belong to identical  $(i_k, j_k)$  pairs, see, the example below, or also "dgTMatrix".

For convenience, methods for some operations such as %\*% and crossprod are defined for TsparseMatrix objects. These methods simply coerce the TsparseMatrix object to a CsparseMatrix object then perform the operation.

#### See Also

its superclass, sparseMatrix, and the dgTMatrix class, for the links to other classes.

#### **Examples**

```
showClass("TsparseMatrix")
## or just the subclasses' names
names(getClass("TsparseMatrix")@subclasses)

T3 <- spMatrix(3,4, i=c(1,3:1), j=c(2,4:2), x=1:4)
T3 # only 3 non-zero entries, 5 = 1+4 !</pre>
```

unpackedMatrix-class Virtual Class "unpackedMatrix" of Unpacked Dense Matrices

# **Description**

Class "unpackedMatrix" is the *virtual* class of dense matrices in "unpacked" format, storing all m\*n elements of an m-by-n matrix. It is used to define common methods for efficient subsetting, transposing, etc. of its *proper* subclasses: currently "[dln]geMatrix" (unpacked general), "[dln]syMatrix" (unpacked symmetric), "[dln]trMatrix" (unpacked triangular), and subclasses of these, such as "dpoMatrix".

#### Slots

```
Dim, Dimnames: as all Matrix objects.
```

#### **Extends**

```
Class "denseMatrix", directly. Class "Matrix", by class "denseMatrix", distance 2.
```

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## Methods

```
pack signature(x = "unpackedMatrix"): ...
unpack signature(x = "unpackedMatrix"): ...
isSymmetric signature(object = "unpackedMatrix"): ...
isTriangular signature(object = "unpackedMatrix"): ...
isDiagonal signature(object = "unpackedMatrix"): ...
t signature(x = "unpackedMatrix"): ...
diag signature(x = "unpackedMatrix"): ...
diag<- signature(x = "unpackedMatrix"): ...</pre>
```

## Author(s)

Mikael Jagan

#### See Also

pack and unpack; its virtual "complement" "packedMatrix"; its proper subclasses "dsyMatrix",
"ltrMatrix", etc.

## **Examples**

```
showClass("unpackedMatrix")
showMethods(classes = "unpackedMatrix")
```

updown-methods

Updating and Downdating Sparse Cholesky Factorizations

# Description

Computes a rank-k update or downdate of a sparse Cholesky factorization

$$P_1AP_1' = L_1DL_1' = LL'$$

which for some k-column matrix C is the factorization

$$P_1(A + sCC')P_1' = \tilde{L}_1\tilde{D}\tilde{L}_1' = \tilde{L}\tilde{L}'$$

Here, s = 1 for an update and s = -1 for a downdate.

# Usage

```
updown(update, C, L)
```

USCounties 195

# **Arguments**

update	a logical (TRUE or FALSE) or character ("+" or "-") indicating if L should be updated (or otherwise downdated).
С	a finite matrix or Matrix such that tcrossprod(C) has the dimensions of L.
L	an object of class dCHMsimpl or dCHMsuper specifying a sparse Cholesky factorization.

## Value

A sparse Cholesky factorization with dimensions matching L, typically of class dCHMsimpl.

## Author(s)

Initial implementation by Nicholas Nagle, University of Tennessee.

# References

Davis, T. A., Hager, W. W. (2001). Multiple-rank modifications of a sparse Cholesky factorization. *SIAM Journal on Matrix Analysis and Applications*, 22(4), 997-1013. doi:10.1137/S0895479899357346

## See Also

Classes dCHMsimpl and dCHMsuper and their methods, notably for generic function update, which is *not* equivalent to updown(update = TRUE).

Generic function Cholesky.

# **Examples**

USCounties

Contiguity Matrix of U.S. Counties

# Description

This matrix gives the contiguities of 3111 U.S. counties, using the queen criterion of at least one shared vertex or edge.

## Usage

```
data(USCounties)
```

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#### **Format**

A 3111 × 3111 sparse, symmetric matrix of class dsCMatrix, with 9101 nonzero entries.

#### Source

GAL lattice file 'usc\_q.GAL' (retrieved in 2008 from 'http://sal.uiuc.edu/weights/zips/usc.zip' with permission from Luc Anselin for use and distribution) was read into R using function read.gal from package spdep.

Neighbour lists were augmented with row-standardized (and then symmetrized) spatial weights, using functions nb2listw and similar.listw from packages **spdep** and **spatialreg**. The resulting listw object was coerced to class dsTMatrix using as\_dsTMatrix\_listw from **spatialreg**, and subsequently to class dsCMatrix.

#### References

Ord, J. K. (1975). Estimation methods for models of spatial interaction. *Journal of the American Statistical Association*, 70(349), 120-126. doi:10.2307/2285387

## **Examples**

```
data(USCounties, package = "Matrix")
(n <- ncol(USCounties))</pre>
I <- .symDiagonal(n)</pre>
set.seed(1)
r < -50L
rho <- 1 / runif(r, 0, 0.5)
system.time(MJ0 <- sapply(rho, function(mult)</pre>
    determinant(USCounties + mult * I, logarithm = TRUE)$modulus))
## Can be done faster by updating the Cholesky factor:
C1 <- Cholesky(USCounties, Imult = 2)</pre>
system.time(MJ1 <- sapply(rho, function(mult)</pre>
    determinant(update(C1, USCounties, mult), sqrt = FALSE)$modulus))
stopifnot(all.equal(MJ0, MJ1))
C2 <- Cholesky(USCounties, super = TRUE, Imult = 2)
system.time(MJ2 <- sapply(rho, function(mult)</pre>
    determinant(update(C2, USCounties, mult), sqrt = FALSE)$modulus))
stopifnot(all.equal(MJ0, MJ2))
```

wrld\_1deg

## **Description**

This matrix gives the contiguities of 15260 one-degree grid cells of world land areas, using a criterion based on the great-circle distance between centers.

#### Usage

```
data(wrld_1deg)
```

#### **Format**

A  $15260 \times 15260$  sparse, symmetric matrix of class dsCMatrix, with 55973 nonzero entries.

#### Source

Shoreline data were read into R from the GSHHS database using function Rgshhs from package **maptools**. Antarctica was excluded. An approximately one-degree grid was generated using function Sobj\_SpatialGrid, also from **maptools**. Grid cells with centers on land were identified using the over method for classes SpatialPolygons and SpatialGrid, defined in package **sp**. Neighbours of these were identified by passing the resulting SpatialPixels object to function dnearneigh from package **spdep**, using as a cut-off a great-circle distance of sqrt(2) kilometers between centers.

Neighbour lists were augmented with row-standardized (and then symmetrized) spatial weights, using functions nb2listw and similar.listw from packages **spdep** and **spatialreg**. The resulting listw object was coerced to class dsTMatrix using as\_dsTMatrix\_listw from **spatialreg**, and subsequently to class dsCMatrix.

#### References

Ord, J. K. (1975). Estimation methods for models of spatial interaction. *Journal of the American Statistical Association*, 70(349), 120-126. doi:10.2307/2285387

# Examples

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```
stopifnot(all.equal(MJ0, MJ1))

C2 <- Cholesky(wrld_1deg, super = TRUE, Imult = 2)
system.time(MJ2 <- sapply(rho, function(mult)
    determinant(update(C2, wrld_1deg, mult), sqrt = FALSE)$modulus))
stopifnot(all.equal(MJ0, MJ2))</pre>
```

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