DTC Visitor Program 2011: Exploring a cut-cell representation of orography

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Aim

To perform further testing of cut-cell representations of orography in a simple 2D flux-form atmospheric model.

Purpose

To assess whether cut-cell representations of uneven terrain would offer a useful addition to the WRF model environment.

Background

Studies have demonstrated cut-cell representations of orography can offer an alternative to traditional terrain-following approaches (e.g. [4]) for accurately simulating flow over steep mountains (e.g. [1, 3, 10, 7, 13, 6, 9]) and suggested improvement in numerical weather prediction (NWP) skill scores from using cut-cell methods in regions of significant orography ([11, 12]).

In a cut-cell model, orography is embedded within a Cartesian mesh, intersecting regularly-shaped grid-cells - see Figure 1. Away from the lower boundary all grid-cells represent cuboid-shaped volumes of atmosphere. At the lower boundary, the orography *cuts* through grid-cells such that the volume within the grid-cell that is "open" to the atmosphere may no longer be a cuboid. To handle the irregularity in the shape of open grid-volumes, the dynamical equations are solved with a finite-volume method, which takes account of the size and shape of the open faces and volumes in each grid-cell.

The work undertaken during this visit builds on earlier efforts to explore cut-cells in the context of the WRF model - work performed by Joe Klemp (NCAR) with Juergen Steppeler, Dave Dempsey and S. Lock (WRF DTC Visitor Program, 2009).

Model outline

For this work, cut-cells have been implemented in a 2D "toy" code chosen for consistency with the WRF model dynamical core, as described in [8]. In summary, the model predicts conserved quantities $U \equiv \rho u'$, $W \equiv \rho w'$, ρ and $\Theta \equiv \rho\theta'$, where ρ is the dry air density, and u', w' and θ' represent perturbations from some background state in the horizontal and vertical windspeeds and the (dry) potential temperature respectively. The governing equations are posed in mass-conserving flux-form.

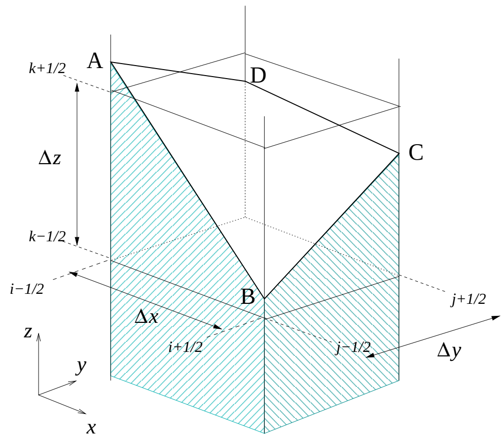

Figure 1: Example of a grid-cell centered on $(x, y, z) = [i, j, k]$ that is cut by the orographic surface $ABCD$. The shading indicates regions of the grid that lie beneath the orographic surface. Only the regions above are "open" to the atmosphere. (Figure taken from [9].)

Variables are calculated at regularly spaced grid-points with U and W staggered by a half grid-spacing in the horizontal and vertical respectively from co-located ρ and Θ (i.e. C-grid variable placement in the horizontal and Lorenz grid placement in the vertical). For consistency with other cut-cell models, the vertical coordinate-levels are specified by constant height z.

The grid-intersecting orographic surface is defined by linear sections that are continuous at grid-column boundaries as described in [10] and the model equations are discretized with a finite-volume method as in [1].

The model is hereafter referred to as "CUT_flux". Previous work has demonstrated that applying the model to a standard orographic test for stably stratified flow yields noisy solutions. During this project, the results have been further explored to diagnose the source of the problem and attempt to find a solution.

Results and discussion

The work has been based around an orographic test case well-documented in the literature (e.g. [5]). Flow is past a bell-shaped hill described by

$$h(x) = \frac{h_0}{1 + x^2/a^2},$$

where $h_0 = 400\,\mathrm{m}$ is the height of the hill, and $a = 10\,\mathrm{km}$ is the hill half-width. The atmosphere is stably stratified with a potential temperature defined by a constant Brunt-Väissälä frequency of $N = 0.01\,\mathrm{s}^{-1}$ and the background windspeed is $U_0 = 10\,\mathrm{ms}^{-1}$. Model fields should describe a vertically propagating wave centered on the hilltop.

For the basic set-up, the model has horizontal and vertical grid-spacings of $\Delta x = 2\,\mathrm{km}$ and $\Delta z = 250\,\mathrm{m}$ respectively.

Basic test

Results from CUT_flux broadly exhibit the expected features but include a distinct noisy signal - see Figure 2. The noisy signal is restricted to a region immediately upwind of the hill, and is particularly evident in the w field. Also worth noting, is a sharp grid-scale feature in the u' and θ' contours in the lee of the hill close to the lower boundary. Such features are unfamiliar from other model studies

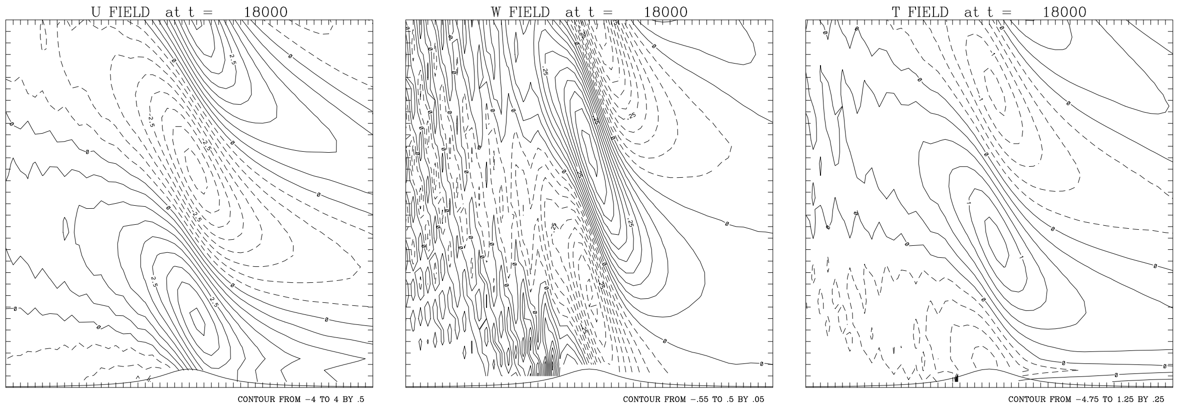

Figure 2: Contour plots of model fields u', w and θ' at model time $t = 18000\,\mathrm{s}$ from CUT_flux for flow past a 400 m high, 10 km wide hill in a stably stratified atmosphere ($N = 0.01\,\mathrm{s}^{-1}$).

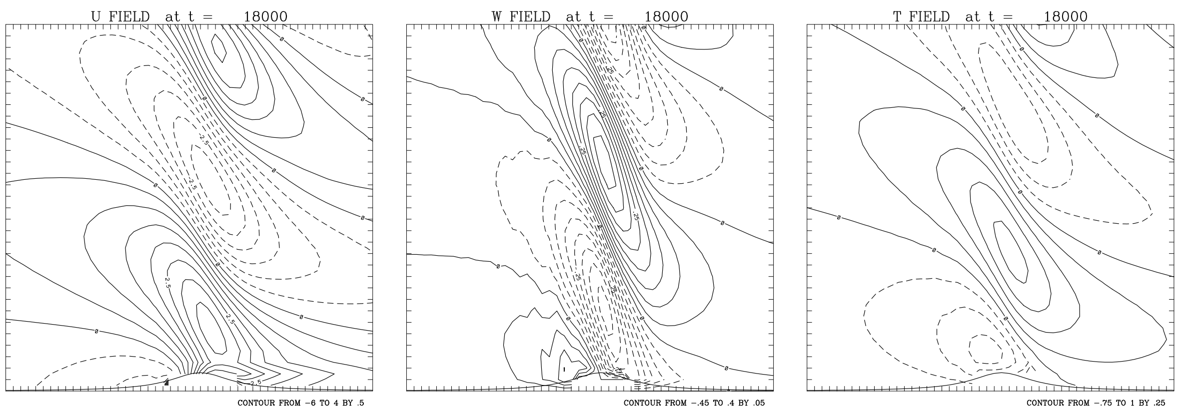

Figure 3: As in Figure 2, but for results from CUT_adv.

- neither evident in similar studies with terrain-following coordinate models, nor in the previously published studies with cut-cell models. The results prompt the question of whether there is something about the WRF model environment that is fundamentally inconsistent with the cut-cell approach.

To better understand the problem, the test was repeated using the cut-cell code demonstrated in [9], referred to hereafter as "CUT_adv" ("adv" denoting the advection-form of the model's governing equations, contrasting with the flux-form of "CUT_flux"). Model parameters associated with the numerics were defined to aim for close agreement in the two model set-ups, e.g. minimal and equivalent explicit diffusion coefficients. Results from CUT_adv are illustrated in Figure 3. There is evidence of some small oscillations in the upstream w field, but at a much reduced level than seen from CUT_flux, and no associated disturbance is evident in either the w or θ' fields. However, there is evidence of unexpected behaviour in the low-level u' field in the lee of the hill.

Examples of "good" results

Through experimentation, a number of cases were found that yield much improved results from CUT_flux:

1. repeating the basic test but varying the vertical grid-spacing to reduce the severity of the size of the cut-cells;

2. repeating the basic test but for a smaller hill (lower height);

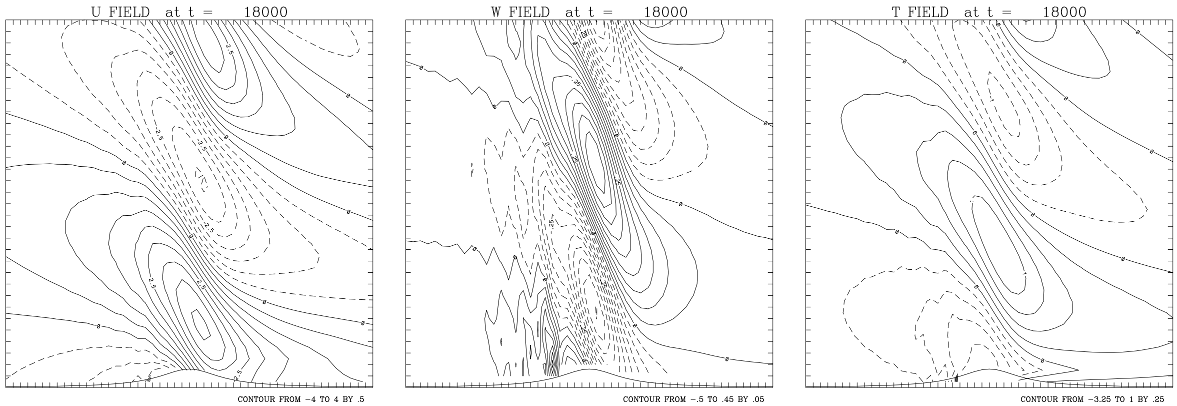

Figure 4: As for Figure 2 but reducing the time-step by an order of magnitude ($\Delta t \to 0.1\Delta t$).

3. repeating the basic test by for a neutrally stratified atmosphere ($N = 0\,\mathrm{s}^{-1}$).

Test 1: Reducing the severity of the cut-cells

Performing the basic test with $h_0 = 400\,\mathrm{m}$ but for vertical grid-spacing $\Delta z = 400\,\mathrm{m}$ and then reduced to $\Delta z = 350\,\mathrm{m}$ increases the size of the smallest grid-volume by an order of magnitude:

Case:	Δz	Minimum grid-volume
Basic	400 m	$\sim 0.01\Delta V$
Less severe cut-cells	350 m	$\sim 0.13\Delta V$

where the grid-volumes are stated relative to the volume of an un-cut grid-cell, $\Delta V \equiv \Delta x \Delta z$. The result is vastly smoother fields (plots not included here) - some noise is still evident in the w field, but much reduced; and u and θ' appear smooth. The result suggests the noise is a result of some instability associated with very small cut-cells. That being so, using a smaller time-step (Δt) for integrating cases that include very small grid-volumes should be expected to improve the solution.

Repeating the basic test, but reducing the time-step by an order of magnitude does indeed yield smoother solutions - see Figure 4 - but there is still clear evidence of a disturbance in the w field, and the θ' field still suggests some problem near the lee-side lower boundary. The length of time-step does not fully explain the problem.

Test 2: Flow over a smaller hill

The basic test is repeated but for a smaller hill-height, $h_0 = 100\,\mathrm{m}$, and reduced vertical grid-spacing $\Delta z = 50\,\mathrm{m}$. Reducing the hill-height reduces the non-linearity of the problem (see e.g. [2]). To constrain for the effect identified in Test 1, the vertical grid-spacing is chosen to yield a similarly small minimum grid-volume in both cases:

Case:	h_0	Δz	Minimum grid-volume
Basic	400 m	250 m	$\sim 0.04\Delta V$
Small hill	100 m	50 m	$\sim 0.02\Delta V$

Both cases are run with the same time-step.

The results show smooth solutions for the Small hill case - see Figure 5. There is no significant evidence of the noise exhibited by the basic test. There is still some evidence, though reduced, of strange behaviour near the lee-side lower boundary in u' and θ'. In contrast to Test 1, the very small grid-volumes in the Small hill case do not generate a noisy solution, suggesting that for this more linear case small grid-volumes are not a source of instability.

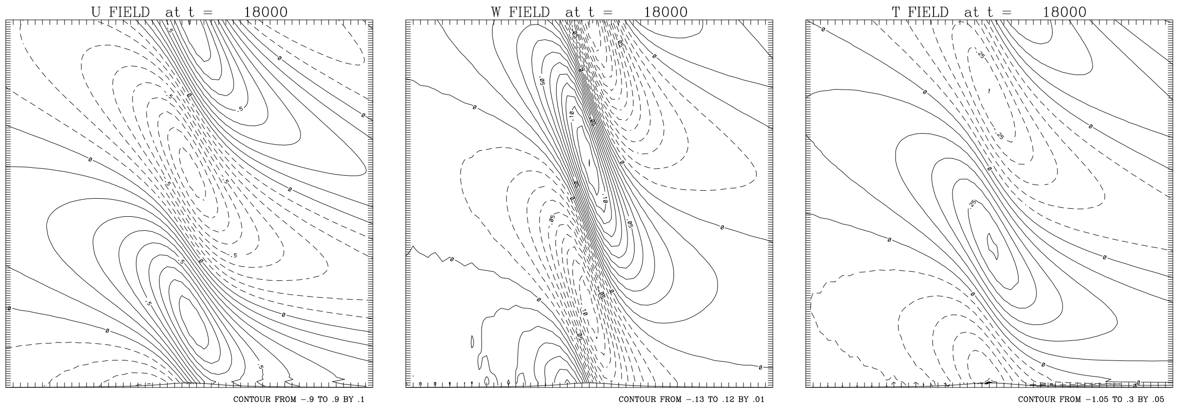

Figure 5: As for Figure 2 but for the “Small hill” case: $h_0 = 100\,\mathrm{m}$ and $\Delta z = 50\,\mathrm{m}$.

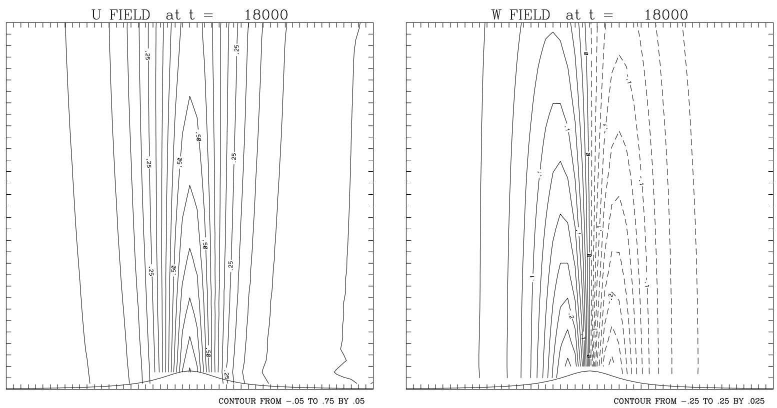

Figure 6: Plots of u' and w fields for the Neutral flow case.

Test 3: Neutral flow case

Repeating the basic set-up but for a neutrally stratified atmosphere, $N = 0\,\mathrm{s}^{-1}$, further reduces the complexity of the probem. Furthermore, the perturbed motion in the neutral flow case is focussed near the surface, meaning numerical solutions are particularly sensitive to the handling of the lower boundary and associated errors are especially evident.

Results for the neutral flow case show smooth fields with well-behaved solutions at the lower boundary (and aloft) - see Figure 6. Since the hill dimensions and grid-spacings are identical to the basic test, the associated very small minimum grid-volume is not acting as a source of instability in this instance. The result suggests that the computation of the buoyancy term, $\Delta\Theta/\Delta z$ (which is zero for the the neutral flow case), is a source of error.

Future work

Work is ongoing to:

- consider whether the buoyancy term in the cut-cells can be more accurately computed by taking better account of the mass-centre in the cut-cells;
- consider whether CUT_adv exhibits a similar sensitivity to the handling of the buoyancy term;

- explore the differences between flux-form and advection-form models in an otherwise equivalent model framework.

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