EXCERPTED FROM

STEPHEN WOLFRAM
A NEW KIND OF SCIENCE

SECTION 6.5

Randomness in Class 3 Systems

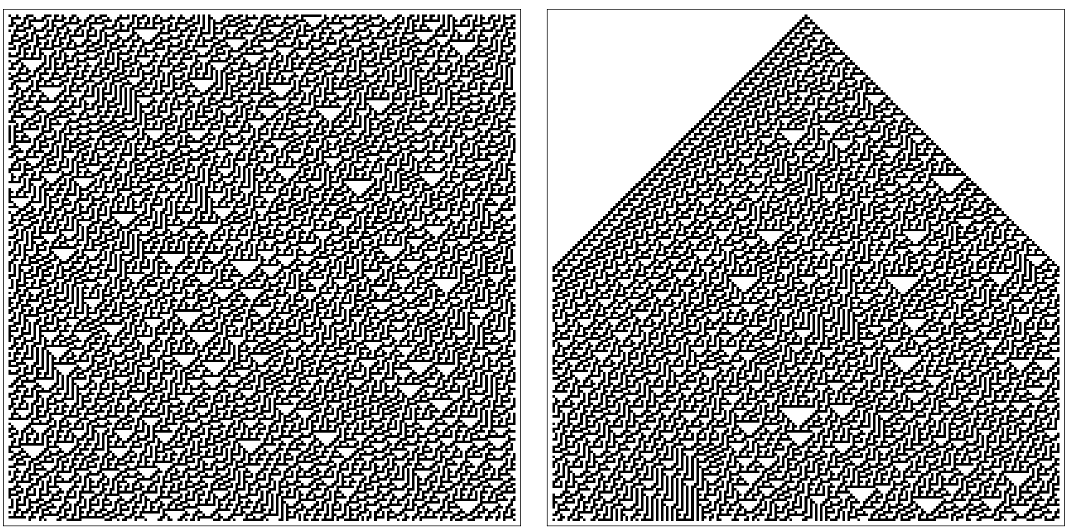
Randomness in Class 3 Systems

When one looks at class 3 systems the most obvious feature of their behavior is its apparent randomness. But where does this randomness ultimately come from? And is it perhaps all somehow just a reflection of randomness that was inserted in the initial conditions?

The presence of randomness in initial conditions—together with sensitive dependence on initial conditions—does imply at least some degree of randomness in the behavior of any class 3 system. And indeed when I first saw class 3 cellular automata I assumed that this was the basic origin of their randomness.

But the crucial point that I discovered only some time later is that random behavior can also occur even when there is no randomness in initial conditions. And indeed, in earlier chapters of this book we have already seen many examples of this fundamental phenomenon.

The pictures below now compare what happens in the rule 30 cellular automaton from page 27 if one starts from random initial conditions and from initial conditions involving just a single black cell.

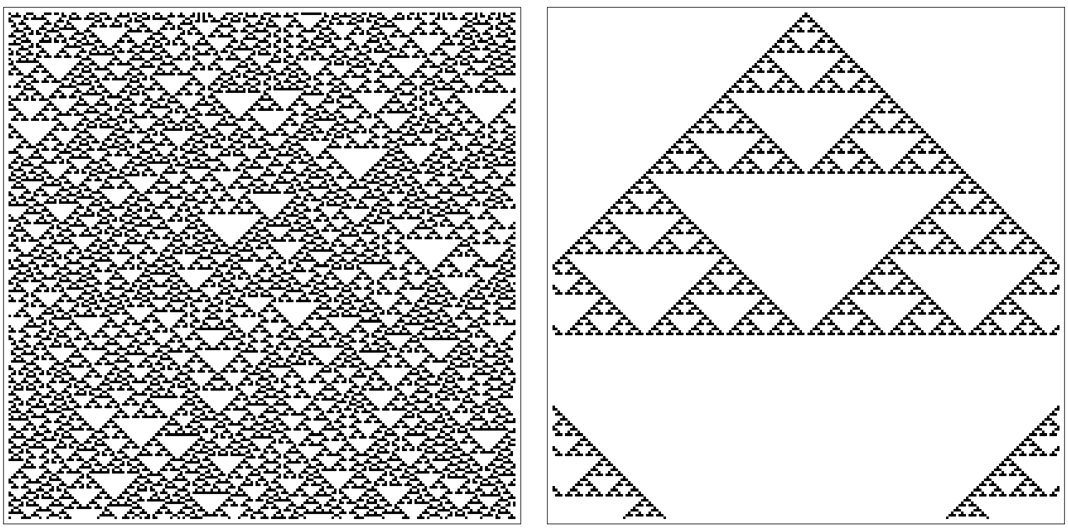


Comparison of the patterns produced by the rule 30 cellular automaton starting from random initial conditions and from simple initial conditions involving just a single black cell. Away from the edge of the second picture, the patterns look remarkably similar.

The behavior we see in the two cases rapidly becomes almost indistinguishable. In the first picture the random initial conditions certainly affect the detailed pattern that is obtained. But the crucial point is that even without any initial randomness much of what we see in the second picture still looks like typical random class 3 behavior.

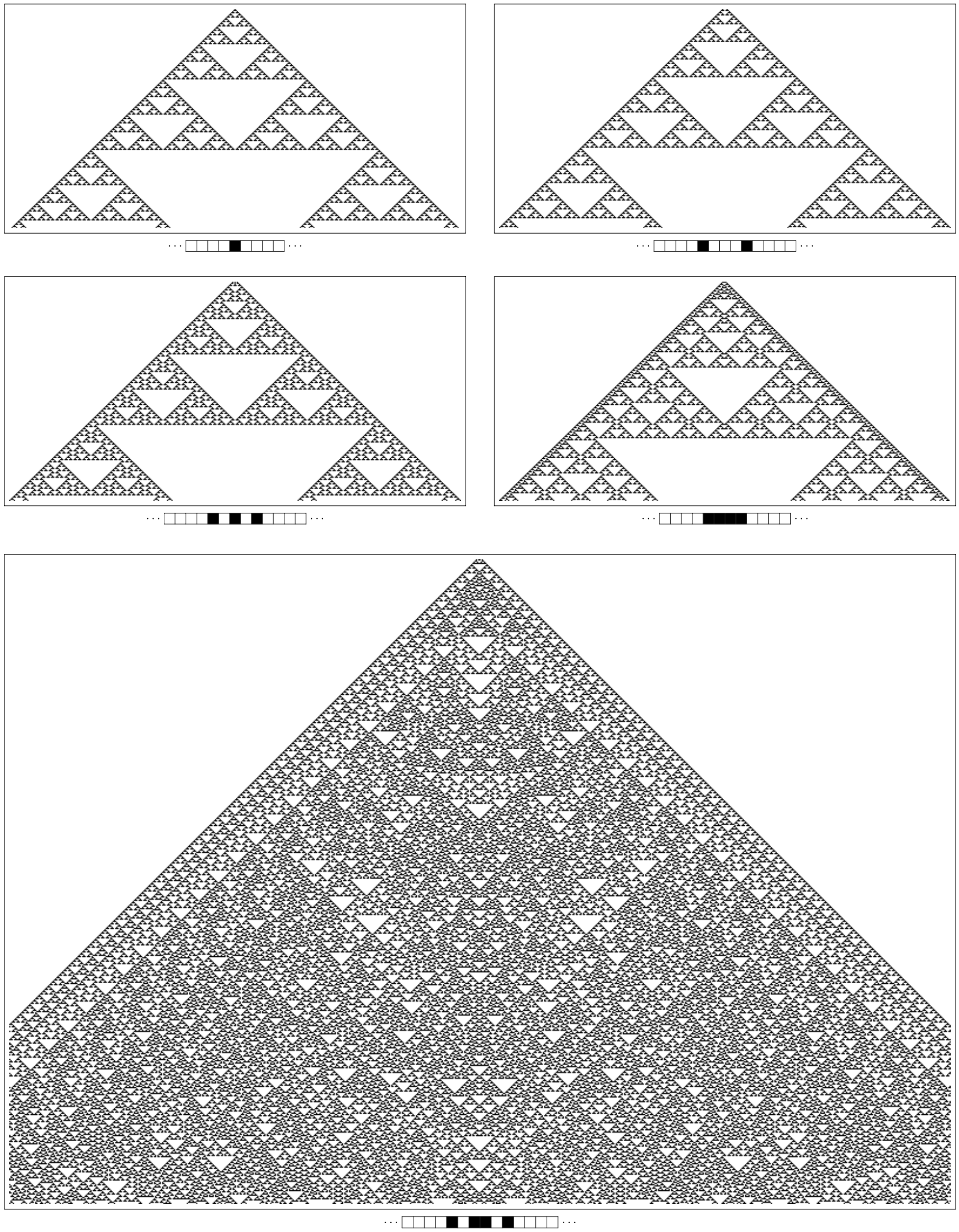
So what about other class 3 cellular automata? Do such systems always produce randomness even with simple initial conditions?

The pictures below show an example in which random class 3 behavior is obtained when the initial conditions are random, but where the pattern produced by starting with a single black cell has just a simple nested form.



Patterns produced by the rule 22 cellular automaton starting from random initial conditions and from an initial condition containing a single black cell. With random initial conditions typical class 3 behavior is seen. But with the specific initial condition shown on the right, a simple nested pattern is produced.

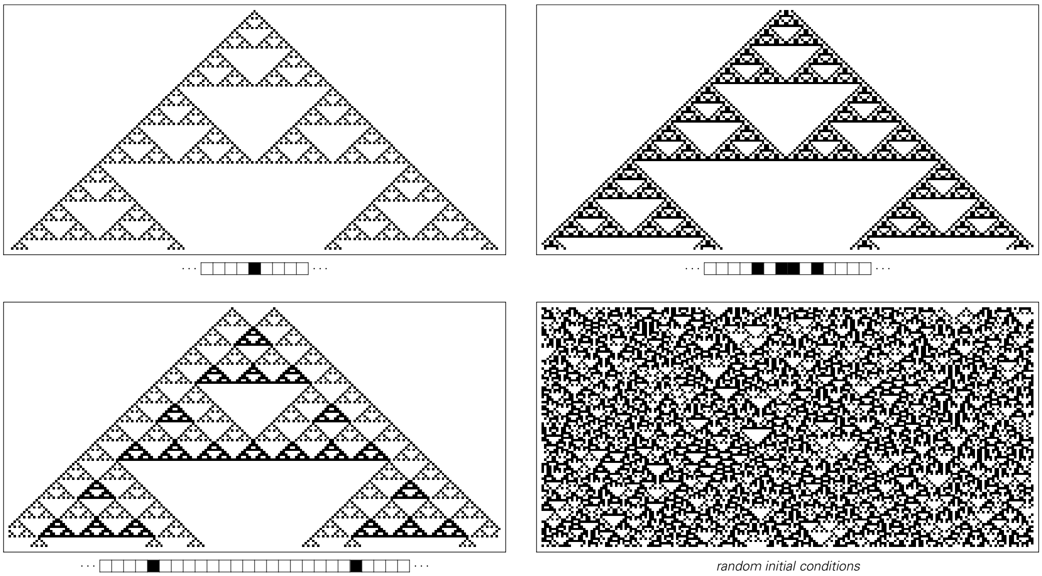
Nevertheless, the pictures on the facing page demonstrate that if one uses initial conditions that are slightly different—though still simple—then one can still see randomness in the behavior of this particular cellular automaton.



Rule 22 with various different simple initial conditions. In the top four cases, the pattern produced ultimately has a simple nested form. But in the bottom case, it is instead in many respects random, much like rule 30.

There are however a few cellular automata in which class 3 behavior is obtained with random initial conditions, but in which no significant randomness is ever produced with simple initial conditions.

The pictures below show one example. And in this case it turns out that all patterns are in effect just simple superpositions of the basic nested pattern that is obtained by starting with a single black cell.



Patterns generated by rule 90 with various initial conditions. This particular cellular automaton rule has the special property of additivity which implies that with any initial conditions the patterns that it produces can be obtained as simple superpositions of the first pattern shown above. Any initial condition that contains black cells only in a limited region will thus lead to a pattern that ultimately has a simple nested form. Unlike rule 30 or rule 22 therefore, rule 90 cannot intrinsically generate randomness starting from simple initial conditions. The randomness in the last picture shown here is thus purely a consequence of the randomness in its initial conditions. Note that the pictures above show only half as many steps of evolution as the corresponding pictures of rule 22 on the previous page.

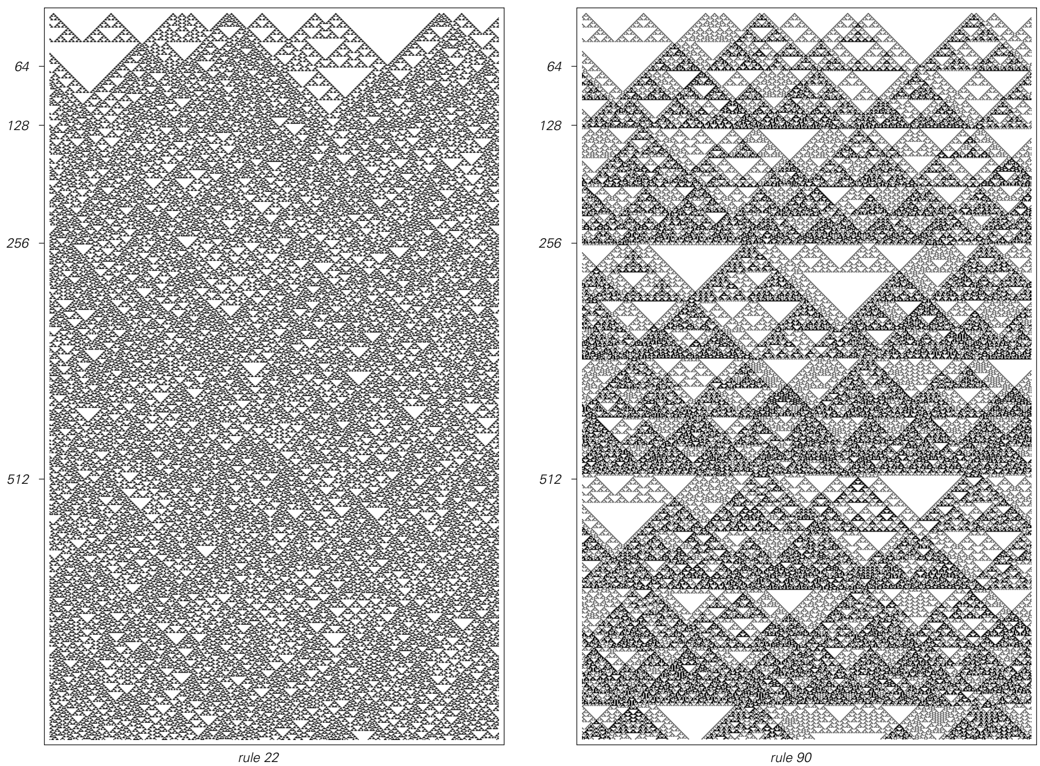
As a result, when the initial conditions involve only a limited region of black cells, the overall pattern produced always ultimately has a simple nested form. Indeed, at each of the steps where a new white triangle starts in the center, the whole pattern consists just of two copies of the region of black cells from the initial conditions.

The only way to get a random pattern therefore is to have an infinite number of randomly placed black cells in the initial conditions.

And indeed when random initial conditions are used, rule 90 does manage to produce random behavior of the kind expected in class 3.

But if there are deviations from perfect randomness in the initial conditions, then these will almost inevitably show up in the evolution of the system. And thus, for example, if the initial density of black cells is low, then correspondingly low densities will occur again at various later steps, as in the second picture below.

With rule 22, on the other hand, there is no such effect, and instead after just a few steps no visible trace remains of the low density of initial black cells.



Examples of evolution from random initial conditions with a low density of black cells. In rule 22 the low initial density has no long-term effect. But in rule 90 its effect continues forever. The reason for this difference is that in rule 22 the randomness we see is intrinsically generated by the evolution of the system, while in rule 90 it comes from randomness in the initial conditions.

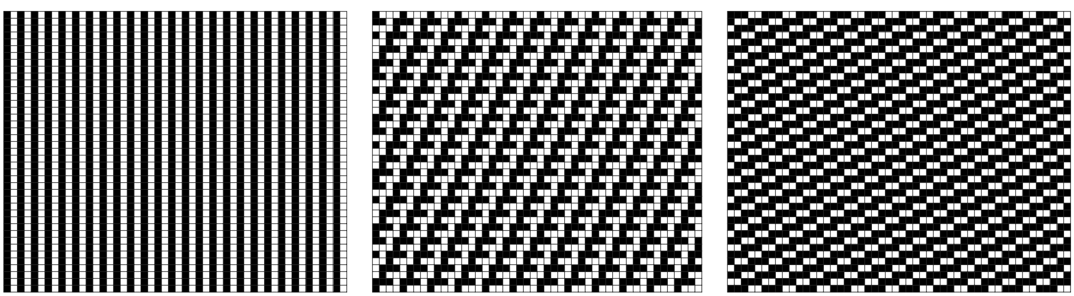
A couple of sections ago we saw that all class 3 systems have the property that the detailed patterns they produce are highly sensitive to detailed changes in initial conditions. But despite this sensitivity at the level of details, the point is that any system like rule 22 or rule 30 yields patterns whose overall properties depend very little on the form of the initial conditions that are given.

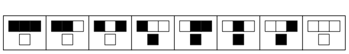
By intrinsically generating randomness such systems in a sense have a certain fundamental stability: for whatever is done to their initial conditions, they still give the same overall random behavior, with the same large-scale properties. And as we shall see in the next few chapters, there are in fact many systems in nature whose apparent stability is ultimately a consequence of just this kind of phenomenon.

Special Initial Conditions

We have seen that cellular automata such as rule 30 generate seemingly random behavior when they are started both from random initial conditions and from simple ones. So one may wonder whether there are in fact any initial conditions that make rule 30 behave in a simple way.

As a rather trivial example, one certainly knows that if its initial state is uniformly white, then rule 30 will just yield uniform white forever. But as the pictures below demonstrate, it is also possible to find less trivial initial conditions that still make rule 30 behave in a simple way.





Examples of special initial conditions that make the rule 30 cellular automaton yield simple repetitive behavior. Small patches with the same structures as shown here can be seen embedded in typical random patterns produced by rule 30. At left is a representation of rule 30. Finding initial conditions that make cellular automata yield behavior with certain repetition periods is closely related to the problem of satisfying constraints discussed on page 210.